CHAPTER 4: RESULTS OF THE EMPIRICAL ANALYSIS

4.1 OVERVIEW OF THE CHAPTER

Different combinations of profit-, supply- and demand equations, as well as estimation methods were employed, within the framework of profit maximising normalised quadratic and translog functions. In general, FIML estimation yielded very few significant coefficients. This casts doubt on the overall specification of the equations and systems. Consequently, OLS is used to evaluate the goodness of fit of each equation that would be included in the system. Although the system methods generally yield estimates that are more efficient, the results depend on the correct specification of the equations in the system. Thus, OLS results are used as a proxy to determine which equations are possibly causing contamination of the system.

This chapter reports tabulated[11] and graphical results of single equation OLS estimation (Section 4.2.1), of profit system results (Section 4.2.2) and a discussion of the structural tests and elasticity results. To avoid unnecessary repetition, the term "not rejected" is used without specification of the level of acceptance when the tested hypothesis cannot be rejected at the 1%-, 5%-, 10%- and 15%-level of acceptance. In all the other circumstances, the level of acceptance will be specified.

4.2 THE NORMALISED QUADRATIC

From Chapter 3: Methodology, Equation 20 and the derived input demand and output supply equations (Equation 21) were estimated and the results are reported in the following sections.

[11] Values larger than absolute ten are rounded up to the nearest integer; values less than absolute ten are rounded up to the nearest two decimals. Significance of coefficients is regarded up to a 15%-level of acceptance.

4.2.1 Single Equation OLS Results

4.2.1.1 Normalised Quadratic Profit Function

The Normalised Quadratic (NQ) profit function, Equation 20, was estimated with OLS. White's Heteroskedastic Consistent Variance Covariance Matrix Estimator (HETCOV) [White, 1980] was used to account for possible heteroskedasticity in the residuals (in the absence of heteroskedasticity, this estimator reduces to the usual covariance matrix estimator). The results are reported in Table 6.

Table 6: NQ Profit function estimated with OLS and HETCOV

Line	Variable	Parameter	Coefficient	Std. Error	T-statistic	Prob.
1	Constant	α_0	-280172	149941	-1.87	0.07
2	P^*_{MLK}	α_1	-1600882	2336949	-0.69	0.50
3	P^*_{FB}	α_2	2259143	2483672	0.91	0.37
4	P^*_{FS}	α_3	-424434	293130	-1.45	0.16
5	$(P^*_{MLK})^2$	β_{11}	-1480022	626291	-2.36	0.03
6	$(P^*_{FB})^2$	β_{22}	-372012	3009809	-0.12	0.90
7	$(P^*_{FS})^2$	β_{33}	1546	56448	0.03	0.98
8	$(P^*_{MLK})(P^*_{FB})$	β_{12}	790009	1528839	0.52	0.61
9	$(P^*_{MLK})(P^*_{FS})$	β_{13}	728827	234538	3.11	0.00
10	$(P^*_{FB})(P^*_{FS})$	β_{23}	-772165	323170	-2.39	0.02
11	$(P^*_{MLK})(Z_{MPRX})$	β_{1M}	1515763	963514	1.57	0.13
12	$(P^*_{MLK})(Z_{LCAP})$	β_{1C}	0.08	0.64	0.13	0.90
13	$(P^*_{MLK})(Z_{LABR})$	β_{1L}	0.10	9.72	0.01	0.99
14	$(P^*_{FB})(Z_{MPRX})$	β_{2M}	-1652753	1087730	-1.52	0.14
15	$(P^*_{FB})(Z_{LCAP})$	β_{2C}	0.52	0.73	0.72	0.48
16	$(P^*_{FB})(Z_{LABR})$	β_{2L}	0.10	11.70	0.01	0.99
17	$(P^*_{FS})(Z_{MPRX})$	β_{3M}	279987	167750	1.67	0.11
18	$(P^*_{FS})(Z_{LCAP})$	β_{3C}	-0.34	0.11	-3.10	0.00
19	$(P^*_{FS})(Z_{LABR})$	β_{3L}	2.03	1.14	1.79	0.08
20	Dependent Variable	π^*		R^2		0.93
21	Mean	379339		R^2-adjusted		0.89
22	Standard Deviation	411305		S.E. of regression		134947
23	Sample size	48		Akaike info criterion		26.75
24	Error Sum of Squares	5.28E+11		Schwarz criterion		27.50

The results in Table 6 are not consistent with the *a priori* expectations of coefficients' signs and magnitudes. For example: it was expected that the β_{ii}-coefficients would be positive for outputs and negative for inputs, since a well-behaved profit function is concave in output prices and convex in input prices. Apart from the fact that only β_{11}, β_{13} and β_{23} are statistically significant (at the 5%-level), the β_{11} and β_{33} coefficients have unexpected signs. The substantial negative value (and statistical significance) of the intercept (α_0) as well as the large negative value of α_1 cast doubt on the appropriateness of this equation (see sections 4.2.4 and 4.2.5 for a further discussion). The estimated equation apparently explains 89% (R^2-adjusted) of the variation in profits across firms. The residual (error) sum of squares, however, is large.

In Figure 1 observations are ranked according to ascending normalised actual profit. It is evident that the specification (fitted values) follows the same trend as the actual values, although the specified equation introduces more variation than that of the actual values. The H_0 of normality in the residuals was not rejected at the 15%-level (Jarque-Bera statistic = 4.38, prob = 0.11).

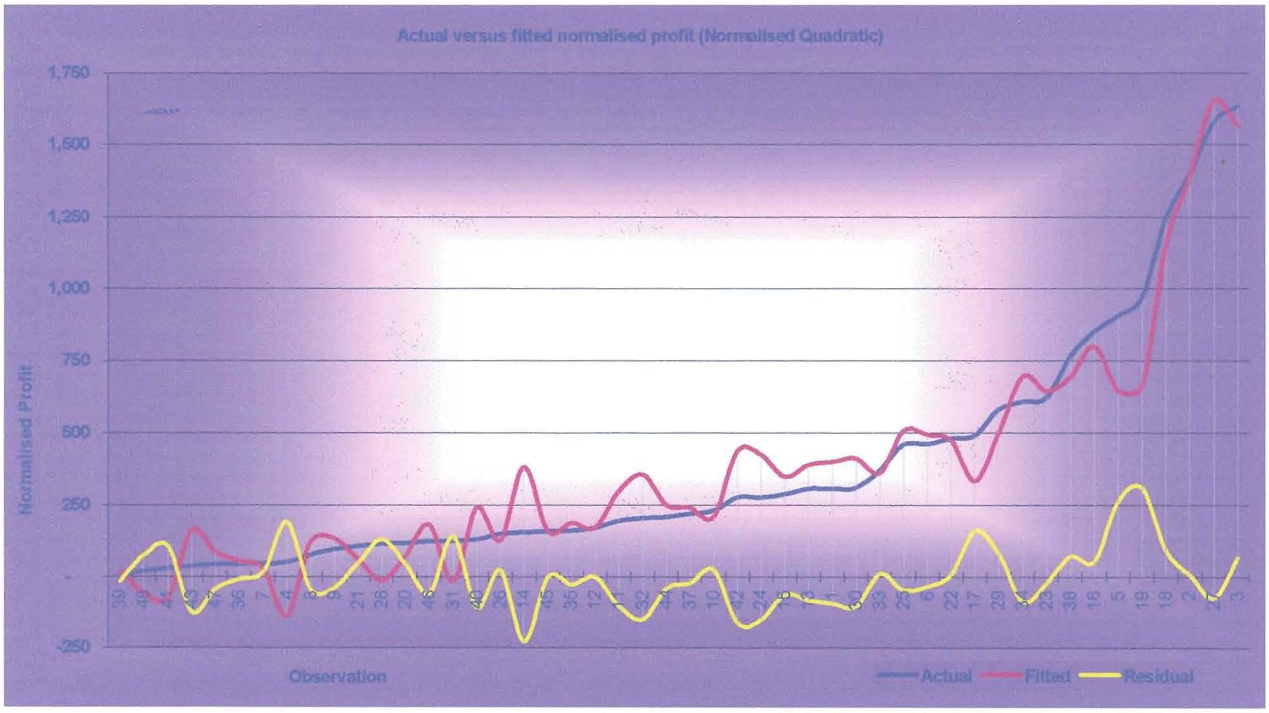

Figure 1: Normalised Quadratic Profit - result of OLS

4.2.1.2 NORMALISED QUADRATIC MILK SUPPLY

Although the specification of milk supply (Q_{MLK}) in Equation 21 provides a good fit (Adjusted $R^2 = 0.85$), it does not yield expected results. None of the price variables is significant (Table 7) –

only the livestock capital and labour quasi-fixed variables are statistically significant. The management proxy variable, however, indicates that improved management is associated with higher levels of milk production. The *a priori* expectation was that β_{11} would be significant and positive – the sign meets the expectations. In addition, it was expected that β_{12} (impact of purchased feed) and β_{13} (impact of self produced feed) would be negative – both are insignificant and only β_{13} is negative.

Table 7: NQ Milk Supply function estimated with OLS and HETCOV

Line	Variable	Parameter	Coefficient	Std. Error	T-statistic	Prob.
1	Constant	α_1	-187245	174961	-1.07	0.29
2	P^*_{MLK}	β_{11}	8370	97501	0.09	0.93
3	P^*_{FB}	β_{12}	-35192	126687	-0.28	0.78
4	P^*_{FS}	β_{13}	16150	26948	0.60	0.55
5	Z_{MPRX}	β_{1M}	87342	114255	0.76	0.45
6	Z_{LCAP}	β_{1L}	0.87	0.11	7.64	0.00
7	Z_{LABR}	β_{1C}	1.92	0.86	2.24	0.03
8	Dependent Variable	Q_{MLK}		R^2		0.87
9	Mean	627687		R^2-adjusted		0.85
10	Standard Deviation	514760		S.E. of regression		197643
11	Sample size	48		Akaike info criterion		27.36
12	Error Sum of Squares	1.60E+12		Schwarz criterion		27.63

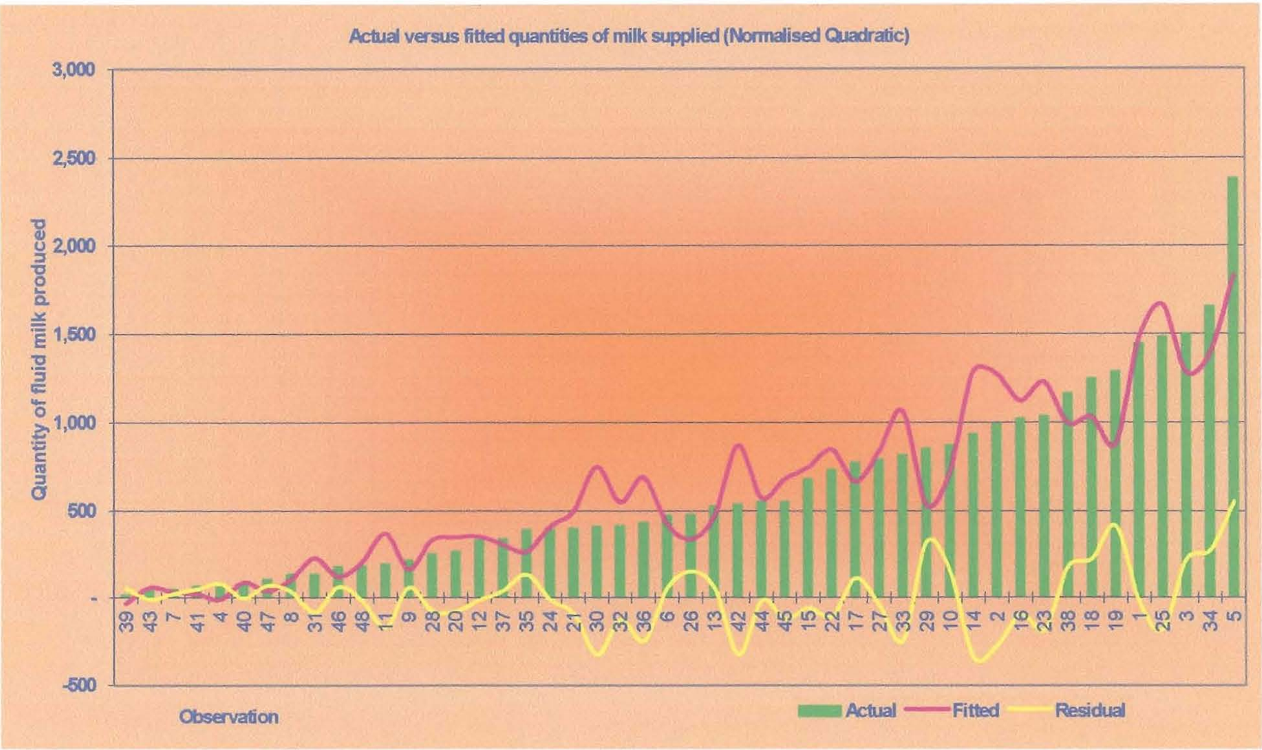

Figure 2: Normalised Quadratic - quantity of milk supplied

Figure 2 shows that although the fitted values follow the pattern of the actual quantities, there are much variation and a more erratic movement than the smooth increase in actual values. Due to the Jarque-Bera[12] statistic of 3.05 and probability of 0.22, the normality of residuals hypothesis is not rejected.

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[12] Jarque-Bera is a test statistic for testing whether the series is normally distributed or not. The test statistic measures the difference of the skewness and kurtosis of the series with those from the normal distribution. Under the null hypothesis of a normal distribution, the Jarque-Bera statistic is distributed as χ^2 with 2 degrees of freedom. The reported Probability is the probability that a Jarque-Bera statistic exceeds (in absolute value) the observed value under the null—a small probability value leads to the rejection of the null hypothesis of a normal distribution.

4.2.1.3 *NORMALISED QUADRATIC PURCHASED FEED DEMAND*

An improvement is seen in the results of the estimated quantity of purchased feed equation (Q_{FB}).

Table 8: NQ Purchased Feed Demand function estimated with OLS and HETCOV

Line	Variable	Parameter	Coefficient	Std. Error	T-statistic	Prob.
1	Constant	α_2	95952	130103	0.74	0.47
2	P^*_{MLK}	β_{12}	159209	58180	2.74	0.01
3	P^*_{FB}	β_{22}	-205936	79078	-2.60	0.01
4	P^*_{FS}	β_{13}	-4359	19262	-0.23	0.82
5	Z_{MPRX}	β_{2M}	-132465	85987	-1.54	0.13
6	Z_{LCAP}	β_{2L}	0.66	0.07	8.90	0.00
7	Z_{LABR}	β_{2C}	0.50	0.35	1.41	0.17
8	Dependent Variable	Q_{FB}		R^2		0.86
9	Mean	357580		R^2-adjusted		0.84
10	Standard Deviation	353350		S.E. of regression		139363
11	Sample size	48		Akaike info criterion		26.66
12	Error Sum of Squares	7.96E+11		Schwarz criterion		26.93

From Table 8 it is clear that the price of milk affects the demand for purchased feed positively and the price of purchased feed has a decreasing effect on the demand for the input. The coefficients are also significant at a 1% probability level. The coefficient of the price of self-produced feed does not have the expected sign (for substitutes), but is statistically insignificant. The management proxy and livestock capital both show significant influences on the demand for purchased feed. In addition to this, the adjusted-R^2 indicates that 84% of the variation in purchased feed demand is explained by the specification.

From Figure 3, it is evident that the estimated derived demand equation moves with the actual values, but models more peaks and troughs than what occurred in the observed quantities of purchased feed for the sample.

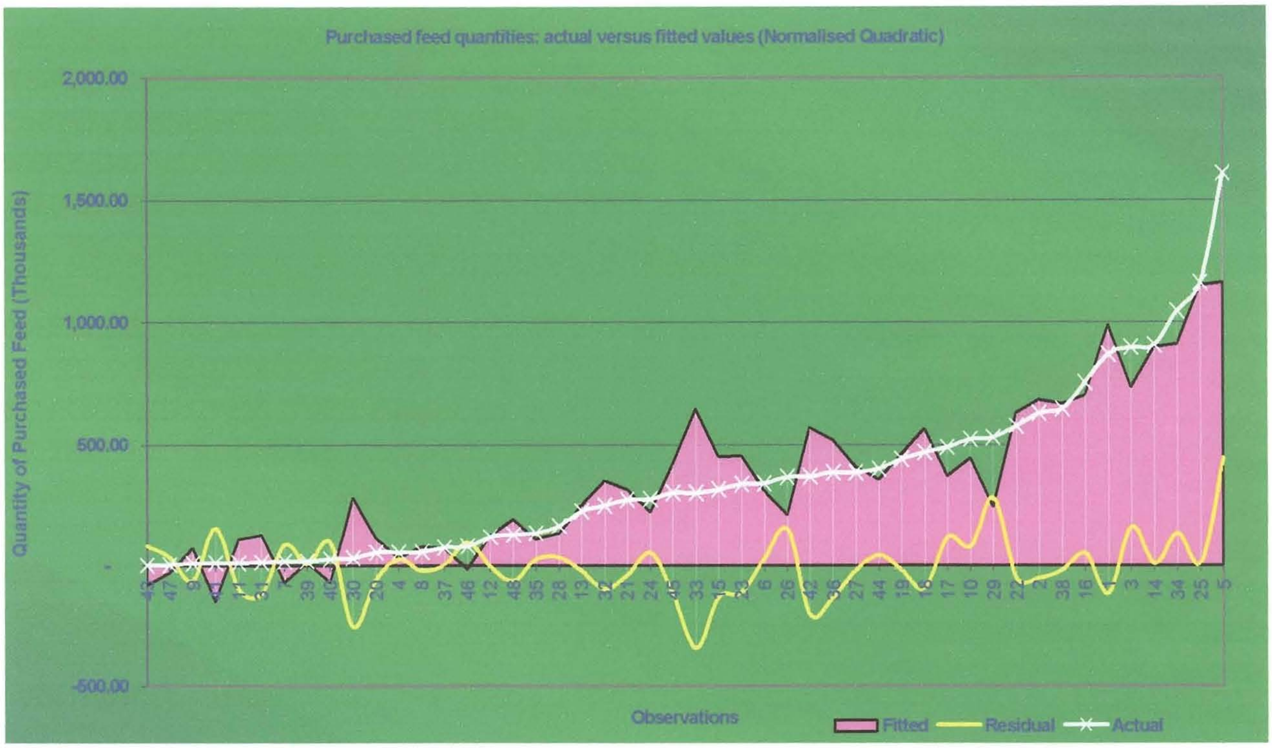

Figure 3: Normalised Quadratic - quantity of purchased feed utilised

4.2.1.4 NORMALISED QUADRATIC SELF-PRODUCED FEED DEMAND

In the case of the self produced feed demand (Table 9), only 58% of the between-farm variation is explained by the normalised quadratic specification of the demand function.

Table 9: NQ Self Produced Feed Demand function estimated with OLS and HETCOV

Line	Variable	Parameter	Coefficient	Std. Error	T-statistic	Prob.
1	Constant	α_3	59262	43680	1.36	0.18
2	P^*_{MLK}	β_{13}	-45828	49918	-0.92	0.36
3	P^*_{FB}	β_{23}	99760	64789	1.54	0.13
4	P^*_{FS}	β_{33}	-29972	7684	-3.90	0.00
5	Z_{MPRX}	β_{3M}	-25732	22768	-1.13	0.27
6	Z_{LCAP}	β_{3L}	0.18	0.04	4.06	0.00
7	Z_{LABR}	β_{3C}	-0.32	0.31	-1.05	0.30
8	Dependent Variable	Q_{FS}		R^2		0.58
9	Mean	121070		R^2-adjusted		0.52
10	Standard Deviation	96245		S.E. of regression		66779
11	Sample size	48		Akaike info criterion		25.19
12	Error Sum of Squares	1.83E+11		Schwarz criterion		25.46

Although the price of milk is not significant in this equation, it does have the expected sign, based on the results from Table 8, where purchased feed responded positively to an increase in milk prices. An *a priori* hypothesis was that purchased and self-produced feeds are substitutes. The latter results confirm this. Similarly, the significant β_{23} (at 15%-level) and β_{33} (at 1%-level) coefficients are consistent with the *a priori* expectations. Self produced feed demand responds positively to an increase in the price of purchased feed, and negatively to its own price.

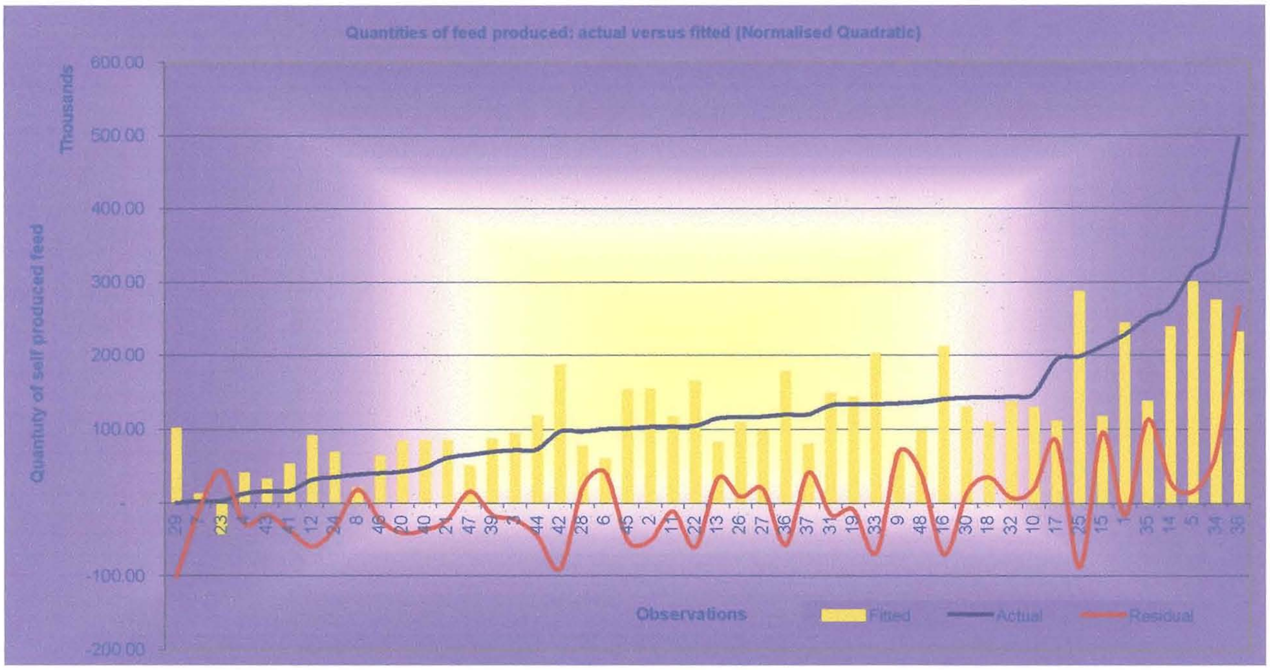

Figure 4: Observed versus fitted quantities of self-produced feed (Normalised Quadratic)

Figure 4 shows the poor explanatory power of the normalised quadratic derived demand equation in terms of self-produced feed. The fitted values follow an erratic pattern as opposed to the smooth pattern of the observed values. Consequently, substantial variation occurs in the residual values.

In Figure 5 and Figure 6 below, the respective actual and fitted values of the profit and derived demand and supply equations are graphed. In both figures (plotted in ascending magnitude of normalised profit), the same observations seem to cause deviation from the fitted pattern. There is no obvious trend in combinations of milk volume and feed use that indicates increasing profits in either the actual or the fitted series. Similarly, graphical inspection of the movement between normalised profit and the quasi-fixed variables yields no clear causational patterns.

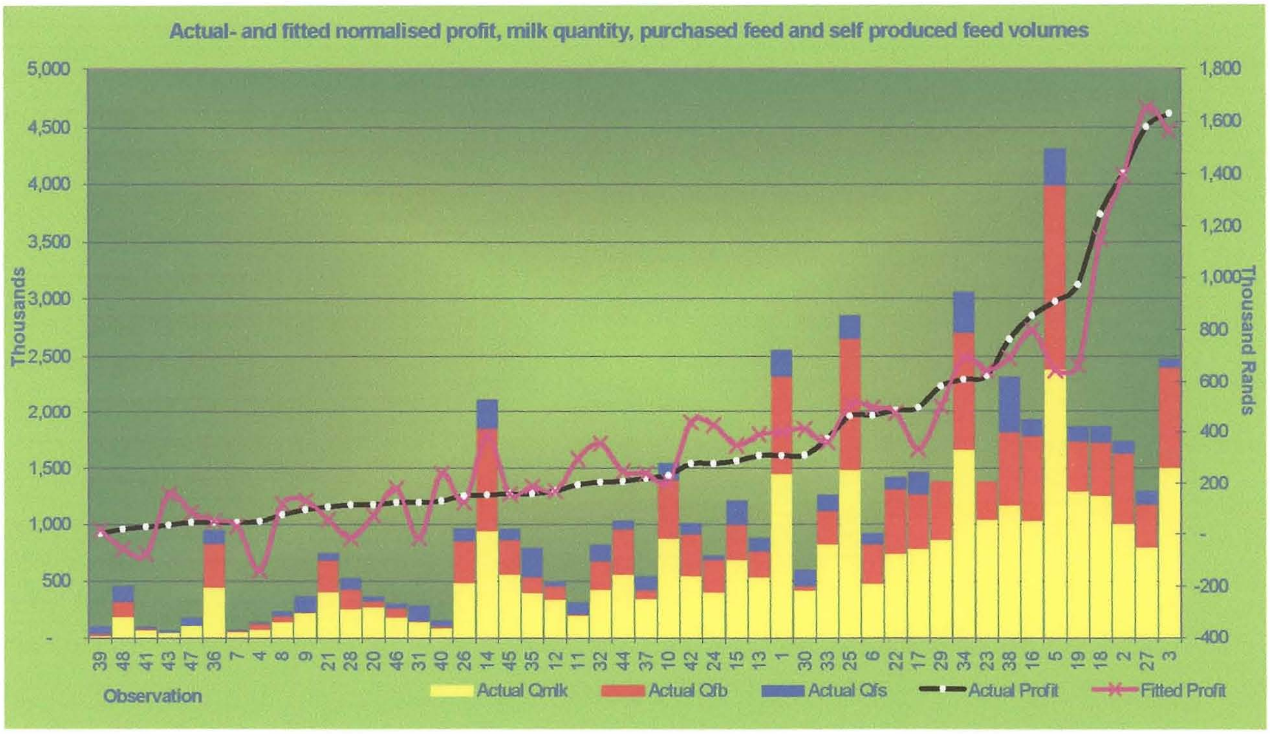

Figure 5: Actual and fitted normalised profit, and actual milk, purchased and produced feed quantities

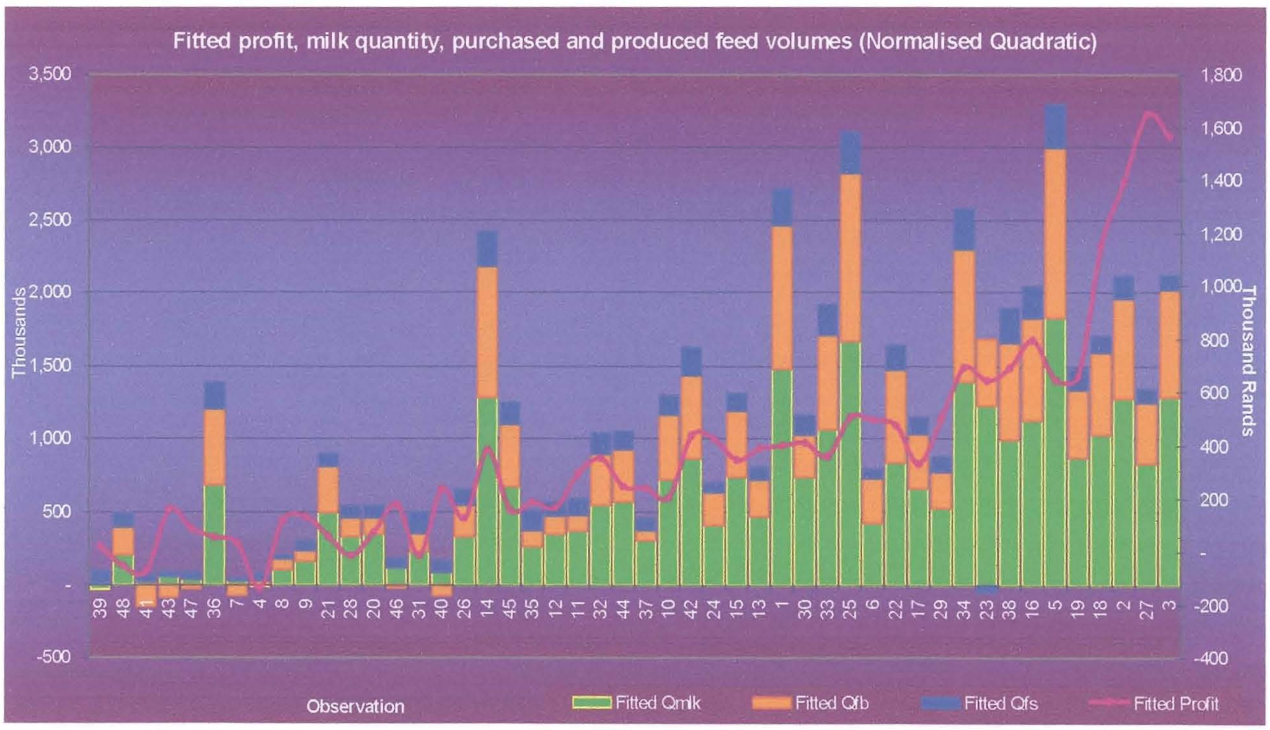

Figure 6: Fitted and actual normalised profit, and fitted milk, purchased and produced feed quantities.

The results from the single equation specification of the profit function are compared to those of the estimated demand and supply equations (Table 7 to Table 9). Wald 's coefficient test was used to establish whether the estimates differ significantly between the two OLS methods. In all cases, the null-hypothesis that the demand and supply equations' set of coefficients do not differ significantly from that of the profit function is rejected at the 5%-level.

4.2.2 *NORMALISED QUADRATIC PROFIT SYSTEM ESTIMATION RESULTS*

Application of Zellner' s Iterative Seemingly Unrelated Regression method (ISUR) (Section 3.5.2) yielded more coefficients (Table 10) that are more significant than coefficients from the OLS estimations (Table 6 to Table 9).

The β_{ii}-coefficients are all significant. However, the β_{11}-coefficient is negative instead of positive. In addition, neither the β_{12} nor the β_{23} coefficients are statistically significant – only β_{13} is significant. The substantial negative value (and statistical significance) of the intercept (α_0) as well as the large negative value of α_1 is an indication that problems of misspecification, measurement errors, exclusion of important variables, data errors, or any combination of these problems influence the results. Similar to the OLS estimation results, specification of milk supply (Q_{MLK}) does not yield expected results. The milk price and self-produced feed price variables, as well as all the quasi-fixed variables are significant (Table 10). The management proxy variable indicates that improved management is associated with higher levels of milk production.

From the results of the estimated quantity of purchased feed equation (Q_{FB}), it follows that the price of milk affects the demand for purchased feed positively and the price of purchased feed has a decreasing effect on the demand for the input. The coefficient of the price of self-produced feed has the expected sign (for the hypothesis of substitution), but is statistically insignificant. The management proxy shows a significant influence on the demand for purchased feed: improved management is associated with more intense use of purchased feed.

Table 10: NQ Profit System estimated trough Iterative Seemingly Unrelated Regression method

Line	Variable	Parameter	Coefficient	Std. Error	T-statistic	Prob.
1	Constant	α_0	-201065	59155	-3.40	0.00
2	P^*_{MLK}	α_1	-431870	84166	-5.13	0.00
3	P^*_{FB}	α_2	13725	64889	0.21	0.83
4	P^*_{FS}	α_3	-29039	45216	-0.64	0.52
5	$(P^*_{MLK})^2$	β_{11}	-95679	38672	-2.47	0.01
6	$(P^*_{FB})^2$	β_{22}	-149284	57011	-2.62	0.01
7	$(P^*_{FS})^2$	β_{33}	-24287	6592	-3.68	0.00
8	$(P^*_{MLK})(P^*_{FB})$	β_{12}	61729	46178	1.34	0.18
9	$(P^*_{MLK})(P^*_{FS})$	β_{13}	30217	9249	3.27	0.00
10	$(P^*_{FB})(P^*_{FS})$	β_{23}	12619	10039	1.26	0.21
11	$(P^*_{MLK})(Z_{MPRX})$	β_{1M}	395716	52907	7.48	0.00
12	$(P_{MLK})(Z_{LCAP})$	β_{1C}	0.07	0.04	1.83	0.07
13	$(P^*_{MLK})(Z_{LABR})$	β_{1L}	1.37	0.26	5.28	0.00
14	$(P^*_{FB})(Z_{MPRX})$	β_{2M}	59928	32276	1.86	0.07
15	$(P^*_{FB})(Z_{LCAP})$	β_{2C}	0.02	0.03	0.78	0.44
16	$(P^*_{FB})(Z_{LABR})$	β_{2L}	0.27	0.22	1.22	0.22
17	$(P^*_{FS})(Z_{MPRX})$	β_{3M}	52454	24483	2.14	0.03
18	$(P^*_{FS})(Z_{LCAP})$	β_{3C}	0.03	0.02	1.29	0.20
19	$(P^*_{FS})(Z_{LABR})$	β_{3L}	-0.40	0.20	-2	0.05
20	Dependent Variables	π^*, Q_{MLK}, Q_{FB}, Q_{FS}	Sample size	48		

In the case of the self-produced feed demand, the price of milk is significant and it has the expected sign - purchased feed responds positively to an increase in milk prices. An *a priori* hypothesis was that purchased and self-produced feeds are substitutes. This is confirmed by these results: despite the statistical insignificance of the β_{23}-coefficient, the sign is positive. Self produced feed demand responds positively to an increase in the price of purchased feed, and negatively to its own price. Higher levels of management is in this case also associated with higher levels of self-produced feed use – this is contrary to the substitutability findings, because purchased feed responds similarly to higher levels of management. According to these results, self-produced feed demand decreases with increased labour expenditure. Purchased feed, however, responds positively to increased labour expenditure – it seems that higher quality labour is associated with the use of purchased (expensive) feed.

Figure 7 shows the levels of milk production and input use associated with the estimated normalised profit. Higher profits are not necessarily associated with higher levels of milk production, or with specific ratios of input use. Based on these results, it seems that great variation exists in the production decisions between fluid milk producing units.

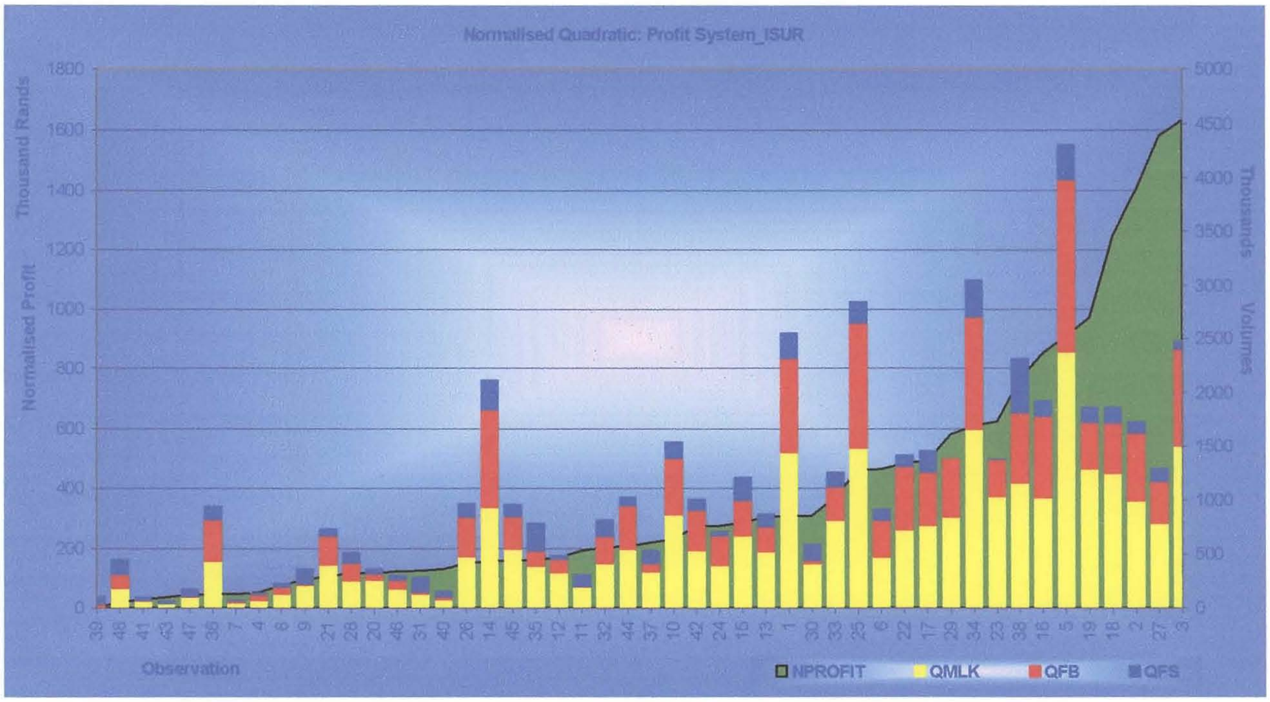

Figure 7: Estimated levels of milk production, input use and associated estimated normalised profit.

The results presented in Table 6 to Table 10 indicated that the quasi-fixed variables had a very significant relation with restricted normalised profit. Since this could be the cause for the unexpected signs of the price variables, it was decided to drop the livestock capital and labour variables, but keep the management proxy. The modified supply system was estimated with the SUR estimator [Zellner, 1962] and the results are presented in Table 11. More degrees of freedom were available due to the reduced number of parameters to be estimated.

Table 11: Modified NQ supply system

Line	Variable	Symbol	Coefficient	Std. Error	T-Statistic	Prob.
1	Constant	α_0	90851	32735	2.78	0.01
2	P^*_{MLK}	α_1	833220	305396	2.73	0.01
3	P^*_{FB}	α_2	-794149	219757	-3.61	0.00
4	P^*_{FS}	α_3	-212879	63690	-3.34	0.00
5	$(P^*_{MLK})^2$	β_{11}	216432	209180	1.03	0.30
6	$(P^*_{FB})^2$	β_{22}	451294	205623	2.19	0.03
7	$(P^*_{FS})^2$	β_{33}	33524	7927	4.23	0.00
8	$(P^*_{MLK})(P^*_{FB})$	β_{12}	-313058	188012	-1.67	0.10
9	$(P^*_{MLK})(P^*_{FS})$	β_{13}	-2474	31894	-0.08	0.94
10	$(P^*_{FB})(P^*_{FS})$	β_{23}	7635	25711	0.30	0.77
11	$(P^*_{MLK})(Z_{MPRX})$	β_{1M}	-104093	190728	-0.55	0.59
12	$(P^*_{FB})(Z_{MPRX})$	β_{2M}	258376	136037	1.90	0.06
13	$(P^*_{FS})(Z_{MPRX})$	β_{3M}	35653	38865	0.92	0.36
14	Dependent Variables		π^*, Q_{MLK}, Q_{FB}, Q_{FS}		Sample size	48

The results indicate a substantial improvement: the t-ratios improved and the coefficients of the milk price variables have the expected sign. Milk supply responds positively towards its price and negatively to increased feed prices.

4.2.3 DISCUSSION OF THE RESULTS

The results from sections 4.2.1 and 4.2.2 indicate that the normalised quadratic system results (Table 10) are generally better than the single equation results. It is likely that the unexpected results are caused by the specification of self-produced feed demand and the inherent problems associated with aggregation. Compared to the single equation results, more of the supply system variables showed significant influences on the normalised profit and on the supply and demand equations. The modified system (yielding more realistic results) was subsequently subjected to structural property tests to determine whether it conforms to the underlying economic theory.

4.2.4 TESTING THE STRUCTURAL PROPERTIES

4.2.4.1 NON-NEGATIVITY

In eight of the cases either negative profits or negative supply or demand quantities were estimated. None of these cases reported simultaneous negative profits or quantities. These results are not sufficient to classify the particular farms as non-profit maximising – small sample size bias, contamination due to aggregation and due to incorrect specification of supply or demand equations all contribute to reduced confidence in the estimation outputs.

4.2.4.2 MONOTONICITY

Evaluation of the first derivatives of the normalised profit function with respect to normalised input and output prices (at the point of approximation, Methodology, section 3.9.2) revealed that profit is monotonically increasing in milk prices ($\alpha_1 > 0$) and monotonically decreasing in purchased feed prices (α_2, $\alpha_3 < 0$).

4.2.4.3 CONVEXITY AND CONCAVITY

For convexity in all prices, it is required that the determinants of the principal minors (of the Hessian matrix of normalised profit to prices - H_{PP}) are non-negative, i.e. positive semi-definiteness of the Hessian matrix. The elements of the Hessian matrix are the β_{ij}-coefficients from Table 10:

$$H = \begin{bmatrix} \frac{\partial^2 \pi *}{(\partial P^*_{MLK})^2} & \frac{\partial^2 \pi *}{\partial P^*_{MLK} \partial P^*_{FB}} & \frac{\partial^2 \pi *}{\partial P^*_{MLK} \partial P^*_{FS}} \\ \frac{\partial^2 \pi *}{\partial P^*_{FB} \partial P^*_{MLK}} & \frac{\partial^2 \pi *}{(\partial P^*_{FB})^2} & \frac{\partial^2 \pi *}{\partial P^*_{FB} \partial P^*_{FS}} \\ \frac{\partial^2 \pi *}{\partial P^*_{FS} \partial P^*_{MLK}} & \frac{\partial^2 \pi *}{\partial P^*_{FS} \partial P^*_{FB}} & \frac{\partial^2 \pi *}{(\partial P^*_{FS})^2} \end{bmatrix} = \begin{bmatrix} 373857 & -434481 & -37738 \\ -434481 & 533709 & 38105 \\ -37738 & 38105 & 40783 \end{bmatrix}$$

$|H_1| = 373857 > 0$, $|H_2| = 10E+09 > 0$ and $|H_3| = 4E+14 > 0$, implying that H_{PP} is positive semi-definite (convexity in prices). The latter result is in accordance with the requirements for well-behaving profit functions.

4.2.4.4 Homogeneity

Homogeneity in all prices is imposed through the functional form. The function is homogenous of degree zero in prices, but not in quasi-fixed factors.

4.2.4.5 Symmetry

Symmetry was imposed during estimation, due to the small sample constraints and symmetry can be seen from the Hessian matrix: $\beta_{ij} = \beta_{ji}$.

4.2.5 Elasticity calculations

Table 12 reports the Marshallian elasticities (Equation 20) calculated from the different normalised quadratic estimations.

Table 12: Marshallian elasticities calculated from the different estimation results

Line	$E(q_i / p^*_j)$	P^*_{mlk}	P^*_{fb}	P^*_{fs}	Source
1	Q_{mlk}	-3.14	1.35	1.67	Table 6: OLS Normalised profit
2	Q_{fb}	2.94	-1.12	-3.11	Table 6: OLS Normalised profit
3	Q_{fs}	8.02	-6.85	0.02	Table 6: OLS Normalised profit
4	Q_{mlk}	0.19	0.04	0.19	Table 7: OLS Milk supply
5	Q_{fb}	0.59	-0.62	-0.02	Table 8: OLS Purchased feed demand
6	Q_{fs}	-0.50	0.89	-0.36	Table 9: OLS Self-produced feed demand
7	Q_{mlk}	-0.20	0.11	0.07	Table 10: ISUR Normalised profit system
8	Q_{fb}	0.23	-0.45	0.05	Table 10: ISUR Normalised profit system
9	Q_{fs}	0.33	0.11	-0.29	Table 10: ISUR Normalised profit system
10	Q_{mlk}	0.79	-0.74	-0.09	Table 11: ISUR Modified supply system
11	Q_{fb}	1.62	-1.60	-0.15	Table 11: ISUR Modified supply system
12	Q_{fs}	0.42	-0.34	-0.48	Table 11: ISUR Modified supply system

Line 2 indicates a plausible result: higher milk prices induce higher demand for purchased feed; own-price response is negative and cross-price response indicates that purchased and self-produced feed inputs are complements (confirmed in Line 5, Table 12). Line 6 also yields plausible results: self-produced feed demand decreases when milk prices increase (indicating a possible switch to purchased feeds) and it increases when purchased feed components become more expensive. Own-price response is negative.

From the modified system' results, milk supply elasticities are consistent with *a priori* expectations. The purchased and self-produced feed demand responses indicate complimentarity between the two inputs (contrary to the original profit system results), similar to the single equation results (Lines 2 and 3). Milk supply is consistently more intensive in purchased feed use.

Using the result from Table 12, with regard to the modified Normalised Quadratic Profit system (lines 10 to 12) and the Hicksian elasticity formulae from Equation 18 (Methodology chapter), the Hicksian input demand elasticities with respect to input prices are calculated as follows.

$$\{\eta_{lk}^{S}\} = \{\eta_{lk}\} - \{\eta_{lt}\} \times \{\eta_{th}\}^{-1} \times \{\eta_{tl}\}$$

$$= \begin{bmatrix} -1.65 & -0.15 \\ -0.34 & -0.48 \end{bmatrix} - \begin{bmatrix} 1.62 \\ 0.42 \end{bmatrix} \times [0.79]^{-1} \times [-0.74 \quad -0.09]$$

$$= \begin{bmatrix} -0.09 & 0.02 \\ 0.05 & -0.44 \end{bmatrix} = \begin{bmatrix} \eta_{FB,FB}^{S} & \eta_{FB,FS}^{S} \\ \eta_{FS,FB}^{S} & \eta_{FS,FS}^{S} \end{bmatrix}$$

The Hicksian responses confirm that both inputs are normal goods (demand decreases when prices increase) with highly inelastic compensated elasticities as opposed to the uncompensated (long run) elasticities. The inputs are gross complements, but net substitutes in the production process, with self-produced feed demand being more sensitive to purchased feed price changes than visa versa. This is in line with expectations since the price of purchased feed is determined in the open market, where the influence of self-produced feed prices play a comparatively small part. The short run (compensated) elasticities are less elastic than the long run elasticities, probably due to higher flexibility to change feeding and grazing patterns in the long-run.

The difference between uncompensated and compensated elasticities indicates the effect of the expansion process (movement to new production possibility frontiers) due to price changes and subsequent production shifts. The long-term (uncompensated) input demand responses are mainly a result of long-term adjustments.

Similarly, the Hicksian output supply elasticity with respect to output prices are as follows.

$$\{\eta_{th}^{S}\} = \{\eta_{th}\} - \{\eta_{tl}\} \times \{\eta_{lk}\}^{-1} \times \{\eta_{lt}\}$$

$$= [0.79] - [-0.74 \quad -0.09] \times \begin{bmatrix} -1.60 & -0.15 \\ -0.34 & -0.48 \end{bmatrix}^{-1} \times \begin{bmatrix} 1.62 \\ 0.42 \end{bmatrix}$$

$$= [0.04] \quad = \quad [\eta_{MLK,MLK}^{S}]$$

The short-run elasticity of milk supply with respect to its own price is positive, yet inelastic. The long-run response (0.79) is mainly due to contraction in supply (-0.83).

4.3 The Translog

4.3.1 *Single Equation OLS Results*

4.3.1.1 *Translog Profit*

This single equation (Equation 22) specification of the translog profit function succeeds in explaining 95% of the variation in the observed profits. Table 13 contains the estimation results. The results are evaluated as they pertain to the derived demand and supply equations.

Milk's share of profit is positively related to its own price (β_{11}), and to purchased feed price (β_{12}), but negatively to self produced feed prices (β_{13}) and the prices of trade animals (β_{14}). It is also negatively related to improved management (γ_{1M}), livestock capital (γ_{1C}) and labour expenditure (γ_{1L}). These results are contrary to the expectations, but they are statistically insignificant.

Coefficient β_{22} – the own price of purchased feed – is statistically significant and displays the expected sign. The share of purchased feed is positively related to milk prices and to self-produced feed prices, as expected, but also positively related to the price of traded animals – however, none of these are statistically significant. Decreased levels of profit share are associated with higher levels of management and labour expenditure.

Table 13: Translog Profit function estimated through OLS with HETCOV

Line	Variable	Parameter	Coefficient	Std. Error	T-statistic	Prob.
1	Constant	α_0	0.95	23	0.04	0.97
2	$Ln(P_{MLK})$	α_1	45	24	1.91	0.08
3	$Ln(P_{FB})$	α_2	-23	30	-0.77	0.46
4	$Ln(P_{FS})$	α_3	2.89	2.97	0.97	0.35
5	$Ln(P_{TRD})$	α_4	3.79	7.42	0.51	0.62
6	$Ln(P_{MLK})^2$	β_{11}	38	27	1.37	0.2
7	$Ln(P_{FB})^2$	β_{22}	-12	7.76	-1.53	0.15
8	$Ln(P_{FS})^2$	β_{33}	1.07	0.57	1.88	0.09
9	$Ln(P_{TRD})^2$	β_{44}	-0.03	0.77	-0.04	0.97
10	$Ln(P_{MLK})Ln(P_{FB})$	β_{12}	6.92	28	0.25	0.81
11	$Ln(P_{MLK})Ln(P_{FS})$	β_{13}	-3.04	3.05	-1.00	0.34
12	$Ln(P_{MLK})Ln(P_{TRD})$	β_{14}	-0.85	4.60	-0.18	0.86
13	$Ln(Z_{MPRX})$	β_M	-1.50	7.31	-0.21	0.84
14	$Ln(Z_{LCAP})$	β_C	1.92	3.71	0.52	0.61
15	$Ln(Z_{LABR})$	β_L	-2.32	1.29	-1.80	0.10
16	$Ln(Z_{MPRX})^2$	γ_{MM}	-7.62	5.90	-1.29	0.22
17	$Ln(Z_{LCAP})^2$	γ_{CC}	0.27	0.46	0.59	0.57
18	$Ln(Z_{LABR})^2$	γ_{LL}	0.66	0.43	1.54	0.15
19	$Ln(Z_{MPRX})Ln(Z_{LCAP})$	γ_{MC}	-1.16	0.97	-1.20	0.26
20	$Ln(Z_{MPRX})Ln(Z_{LABR})$	γ_{ML}	1.98	1.38	1.44	0.18
21	$Ln(Z_{LCAP})Ln(Z_{LABR})$	γ_{CL}	-0.36	0.39	-0.94	0.37
22	$Ln(P_{MLK})Ln(Z_{MPRX})$	γ_{1M}	-2.33	5.35	-0.44	0.67
23	$Ln(P_{MLK})Ln(Z_{LCAP})$	γ_{1C}	-3.78	2.74	-1.38	0.19
24	$Ln(P_{MLK})Ln(Z_{LABR})$	γ_{1L}	-0.09	1.60	-0.05	0.96
25	$Ln(P_{FB})Ln(Z_{MPRX})$	γ_{2M}	-7.89	8.21	-0.96	0.36
26	$Ln(P_{FB})Ln(Z_{LCAP})$	γ_{2C}	3.77	4.52	0.84	0.42
27	$Ln(P_{FB})Ln(Z_{LABR})$	γ_{2L}	-2.49	3.36	-0.74	0.47
28	$Ln(P_{FS})Ln(Z_{MPRX})$	γ_{3M}	-2.17	1.36	-1.6	0.14
29	$Ln(P_{FS})Ln(Z_{LCAP})$	γ_{3C}	-0.07	0.51	-0.13	0.9
30	$Ln(P_{FS})Ln(Z_{LABR})$	γ_{3L}	-0.07	0.76	-0.1	0.92
31	$Ln(P_{FB})Ln(P_{FS})$	β_{23}	1.85	2.82	0.66	0.52
32	$Ln(P_{FB})Ln(P_{TRD})$	β_{24}	2.13	4.75	0.45	0.66
33	$Ln(P_{FS})Ln(P_{TRD})$	β_{34}	0.42	0.72	0.59	0.57
34	$Ln(P_{TRD})Ln(Z_{MPRX})$	γ_{4M}	-0.17	1.60	-0.10	0.92
35	$Ln(P_{TRD})Ln(Z_{LCAP})$	γ_{4C}	-0.01	0.66	-0.02	0.99
36	$Ln(P_{TRD})Ln(Z_{LABR})$	γ_{4L}	-0.28	0.55	-0.51	0.62
37	Dependent Variable	$Ln(\pi)$		R^2		0.99
38	Mean	12.46		R^2-adjusted		0.95
39	Standard Deviation	1.17		S.E. of regression		0.26
40	Sample size	48		Akaike info criterion		0.21
41	Error Sum of Squares	0.73		Schwarz criterion		1.62

Self-produced feed' share in profit responds (significantly) negatively to its own price (β_{33}) and to the price of milk (β_{13}). Similarly, increased purchased feed prices (β_{23}) and increased prices for traded animals (β_{34}) would result in an increase in self-produced feed' s share of profit.

Trade income (S_{TRD}) decreases as the aggregate price increases (β_{44}) and when milk prices increase (β_{14}). When purchased and self-produced feed prices rise, the share of trade income in profit would increase (β_{24} and β_{34}). The response to increased levels of the specified quasi-fixed variables is negative in all cases (γ_{4M}, γ_{4C} and γ_{4L}).

Figure 8 shows the accuracy of the fitted equation. The fitted line follows the actual data very closely, but does introduce more peaks and troughs than what is actually observed. The errors (on a secondary scale) vary within a narrow range (0.6 and 1.3). The Jarque-Bera statistic, calculated for the H_0 of normality in the residuals, equals 3.41 at a 0.18-probability level. Thus, it is reasonable to assume normally distributed errors.

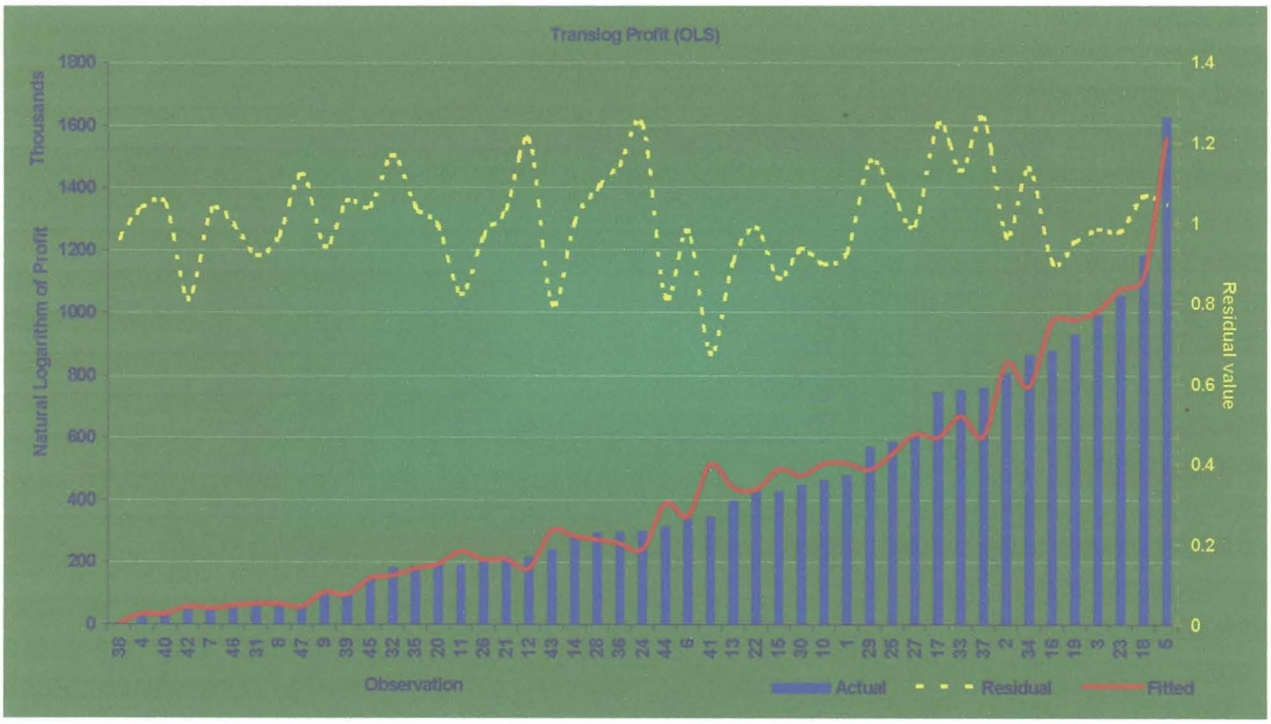

Figure 8: Actual and fitted values of profit from the OLS estimation of the translog profit function.

4.3.1.2 TRANSLOG MILK SHARE

The milk share equation, with an explanatory power of 78%, does not yield expected results. Firstly, the price of milk is negatively related to the supply of milk and increases in purchased and produced feed prices would cause increases in milk supply (although the coefficients are insignificant). Improved management and increased labour expenditure would decrease milk supply, while increases in livestock capital would increase milk supply. These results are summarised in Table 14 and Figure 9, below.

Table 14: Translog Milk Share function estimated through OLS with HETCOV

Line	Variable	Parameter	Coefficient	Std. Error	T-statistic	Prob.
1	Constant	α_1	0.89	0.96	0.93	0.36
2	$Ln(P_{MLK})$	β_{11}	-1.22	0.55	-2.21	0.03
3	$Ln(P_{FB})$	β_{12}	0.27	0.58	0.46	0.65
4	$Ln(P_{FS})$	β_{13}	0.10	0.13	0.80	0.43
5	$Ln(P_{TRD})$	β_{14}	0.11	0.16	0.69	0.50
6	$Ln(Z_{MPRX})$	γ_{1M}	-3.74	0.36	-10.33	0.00
7	$Ln(Z_{LCAP})$	γ_{1C}	0.54	0.13	4.14	0.00
8	$Ln(Z_{LABR})$	γ_{1L}	-0.41	0.11	-3.59	0.00
9	Dependent Variable	S_{MLK}		R^2		0.81
10	Mean	2.13		R^2-adjusted		0.78
11	Standard Deviation	0.86		S.E. of regression		0.41
12	Sample size	48		Akaike info criterion		1.19
13	Error Sum of Squares	6.44		Schwarz criterion		1.51

The fitted values follow the trend in the actual observations, but in most cases over or under estimate the actual values. The calculated Jarque-Bera statistic under the H_0 of normal residuals is 1.55 – which is significantly different from zero only at a 0.46- probability level. The null-hypothesis of normal residuals is not rejected.

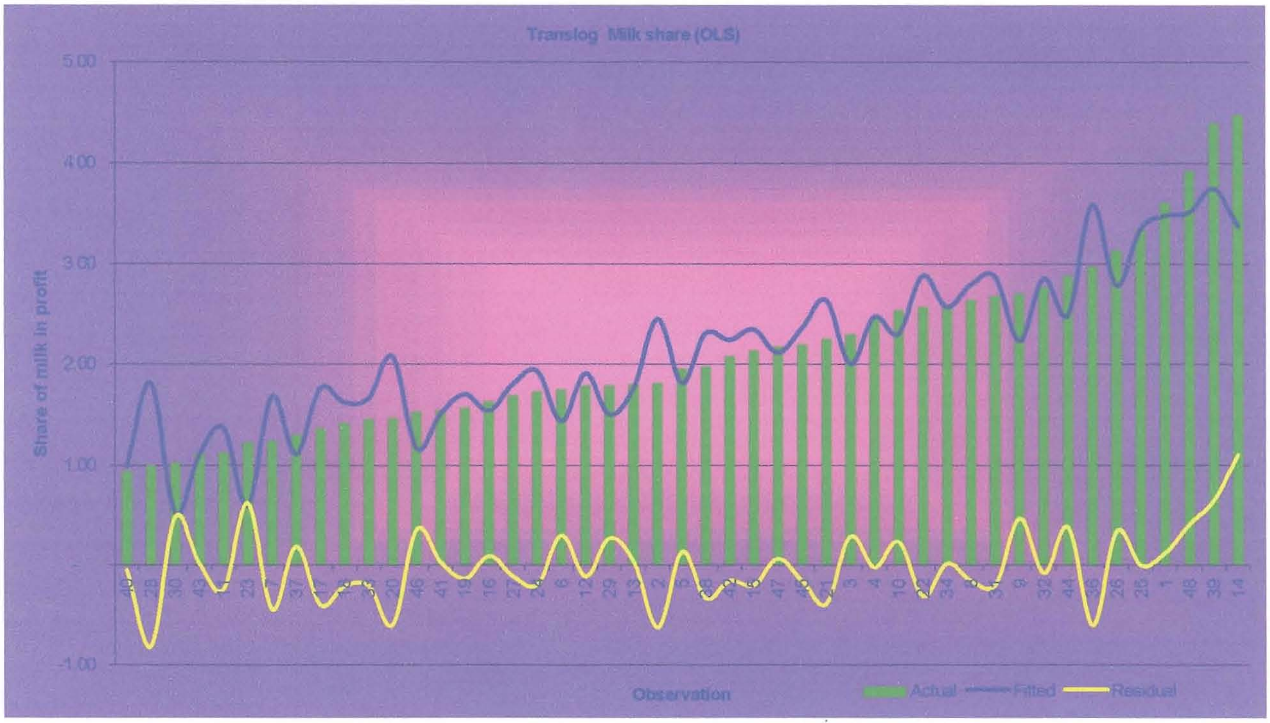

Figure 9: The actual and fitted shares of milk in restricted profit – OLS results

4.3.1.3 TRANSLOG PURCHASED FEED SHARE

In Table 15, the results of the purchased feed share equation is presented.

Table 15: Translog Purchased Feed Share function estimated through OLS with HETCOV

Line	Variable	Parameter	Coefficient	Std. Error	T-statistic	Prob.
1	$Ln(P_{FB})$	α_2	0.76	1.25	0.61	0.55
2	$Ln(P_{FB})$	β_{22}	-0.41	0.73	-0.56	0.58
3	$Ln(P_{MLK})$	β_{12}	0.29	0.79	0.37	0.71
4	$Ln(Z_{MPRX})$	γ_{2M}	2.82	0.48	5.91	0.00
5	$Ln(Z_{LCAP})$	γ_{2C}	-0.44	0.12	-3.64	0.00
67	$Ln(Z_{LABR})$	γ_{2L}	0.29	0.13	2.17	0.04
8	$Ln(P_{FS})$	β_{23}	0.02	0.14	0.17	0.87
9	$Ln(P_{TRD})$	β_{24}	-0.34	0.20	-1.72	0.09
10	Dependent Variable	S_{FB}		R^2		0.66
11	Mean	-0.94		R^2-adjusted		0.60
12	Standard Deviation	0.77		S.E. of regression		0.49
13	Sample size	48		Akaike info criterion		1.56
14	Error Sum of Squares	9.29		Schwarz criterion		1.87

Coefficient-β_{22} indicates the own-price response of the purchased feed share equation is negative, as would be expected. Milk price and self-produced price increases would increase the purchased feed share (β_{12} and β_{23}). Improved management and labour practices would increase the use of purchased feed, whilst increased expenditure on livestock capital would reduce the share of purchased feed. Figure 10[13] displays the actual versus fitted values from the estimated equation (Table 15). It is clear that observation to observation matching of fitted and actual values are quite poor, as is confirmed by the low adjusted-R^2 of 0.6.

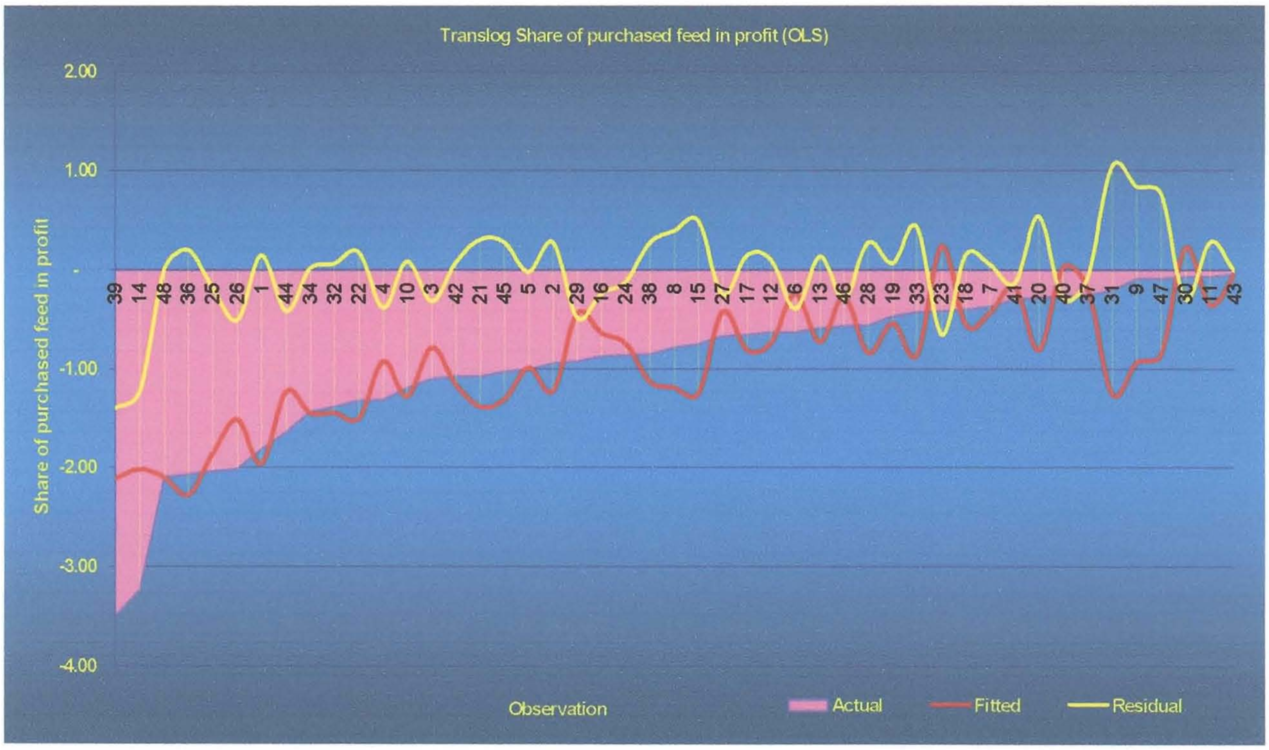

Figure 10: Actual and fitted purchased feed shares derived from the OLS single equation estimations.

[13] For theoretical accuracy, the input profit shares (negative shares of profit) are plotted as negative values - "increases in shares" refer to increases in absolute values.

4.3.1.4 TRANSLOG SELF-PRODUCED FEED SHARE

Self-produced feed' s share equation results are summarised in Table 16. Increased milk prices induce increased expenditure on self-produced feed and the latter increases when its own price rises. Increased purchased feed prices stimulate a decrease in expenditure on self-produced feeds. These results are not consistent with *a priori* expectations or with the results from the previous estimations. Management and labour is positively and significantly related to this input, while none of the price variables are significant. In addition, the adjusted-R^2 indicates that only 44% of the variation is explained by the estimated equation.

Table 16: Translog Self Produced Feed Share function estimated through OLS with HETCOV

Line	Variable	Parameter	Coefficient	Std. Error	T-statistic	Prob.
1	Constant	α_3	-2.81	1.44	-1.96	0.06
2	$Ln(P_{FS})$	β_{33}	0.09	0.15	0.62	0.54
3	$Ln(P_{MLK})$	β_{13}	1.30	1.02	1.28	0.21
4	$Ln(Z_{MPRX})$	γ_{3M}	1.83	0.55	3.35	0.00
5	$Ln(Z_{LCAP})$	γ_{3C}	-0.15	0.12	-1.2	0.24
6	$Ln(Z_{LABR})$	γ_{3L}	0.32	0.13	2.36	0.02
7	$Ln(P_{FB})$	β_{23}	-0.80	0.74	-1.09	0.28
8	$Ln(P_{TRD})$	β_{34}	0.001	0.12	0.01	0.99
9	Dependent Variable	S_{FS}		R^2		0.53
10	Mean	-0.51		R^2-adjusted		0.44
11	Standard Deviation	0.63		S.E. of regression		0.47
12	Sample size	48		Akaike info criterion		1.47
13	Error Sum of Squares	8.48		Schwarz criterion		1.78

Figure 11 portrays the goodness of fit. Clearly, the fitted values do not match the trend in the data over even a subset of the data range.

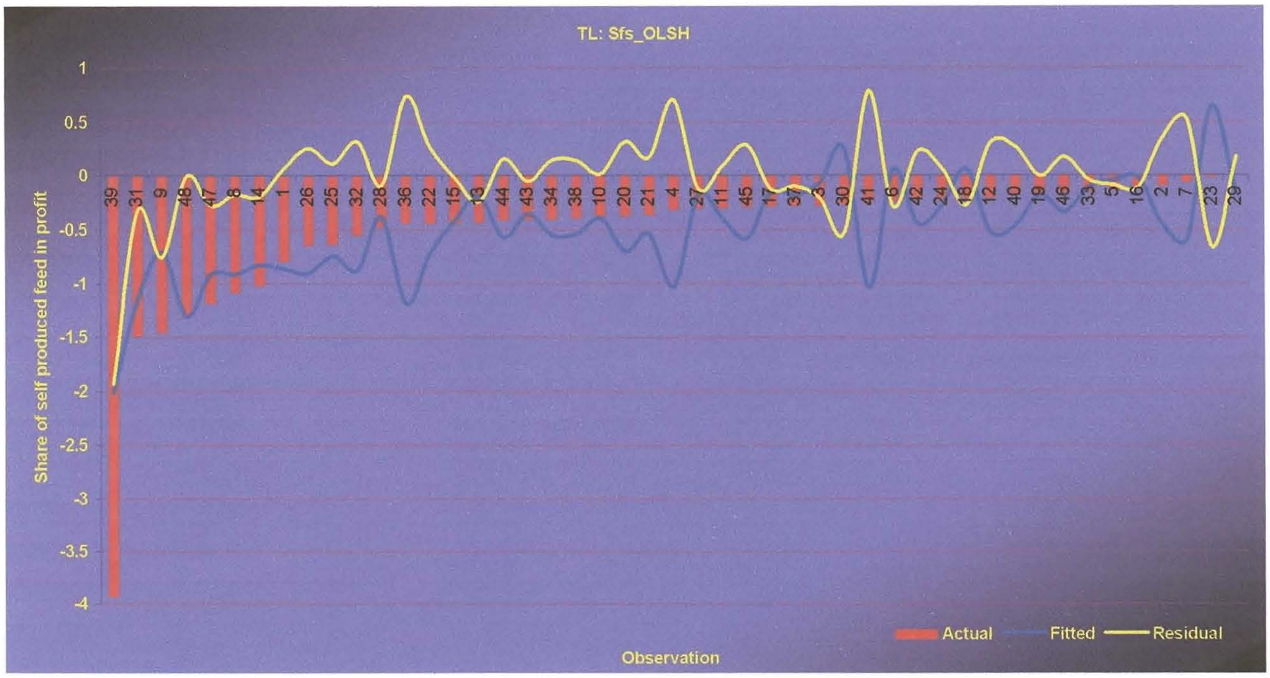

Figure 11: Actual and fitted self-produced feed shares resulting from OLS single equation estimation.

4.3.1.5 TRANSLOG TRADE INCOME SHARE

From the results in Table 17, it is clear that the trade income share equation does not provide an adequate explanation of the across-firm variation in trade income.

Table 17: Translog Trade Income Share function estimated through OLS with HETCOV

Line	Variable	Parameter	Coefficient	Std. Error	T-statistic	Prob.
1	Constant	α_4	13.58	9.70	1.40	0.17
2	Ln(P_{TRD})	β_{44}	0.54	0.83	0.66	0.52
3	Ln(P_{MLK})	β_{14}	-7.57	5.52	-1.37	0.18
4	Ln(P_{FB})	β_{24}	5.90	5.78	1.02	0.31
5	Ln(P_{FS})	β_{34}	-1.12	1.13	-0.99	0.33
6	Ln(Z_{MPRX})	γ_{4M}	-5.55	4.44	-1.25	0.22
7	Ln(Z_{LCAP})	γ_{4C}	0.24	0.51	0.47	0.64
8	Ln(Z_{LABR})	γ_{4L}	-1.17	0.96	-1.22	0.23
9	Dependent Variable	S_{TRD}		R^2		0.31
10	Mean	0.81		R^2-adjusted		0.19
11	Standard Deviation	3.59		S.E. of regression		3.22
12	Sample size	48		Akaike info criterion		5.33
13	Error Sum of Squares	405.34		Schwarz criterion		5.65

None of the variables is statistically significant. The price variables show acceptable signs: own-price response is positive; response towards increased milk prices is negative and positive toward increased purchased feed prices. Improved management and increased expenditure on labour would reduce trade income, while increased investment in livestock would increase trade income.

Figure 12 confirms the poor fit. The equation severely over or under estimates the actual data. In addition, there is not much variation in the actual values, thus reducing the estimation results.

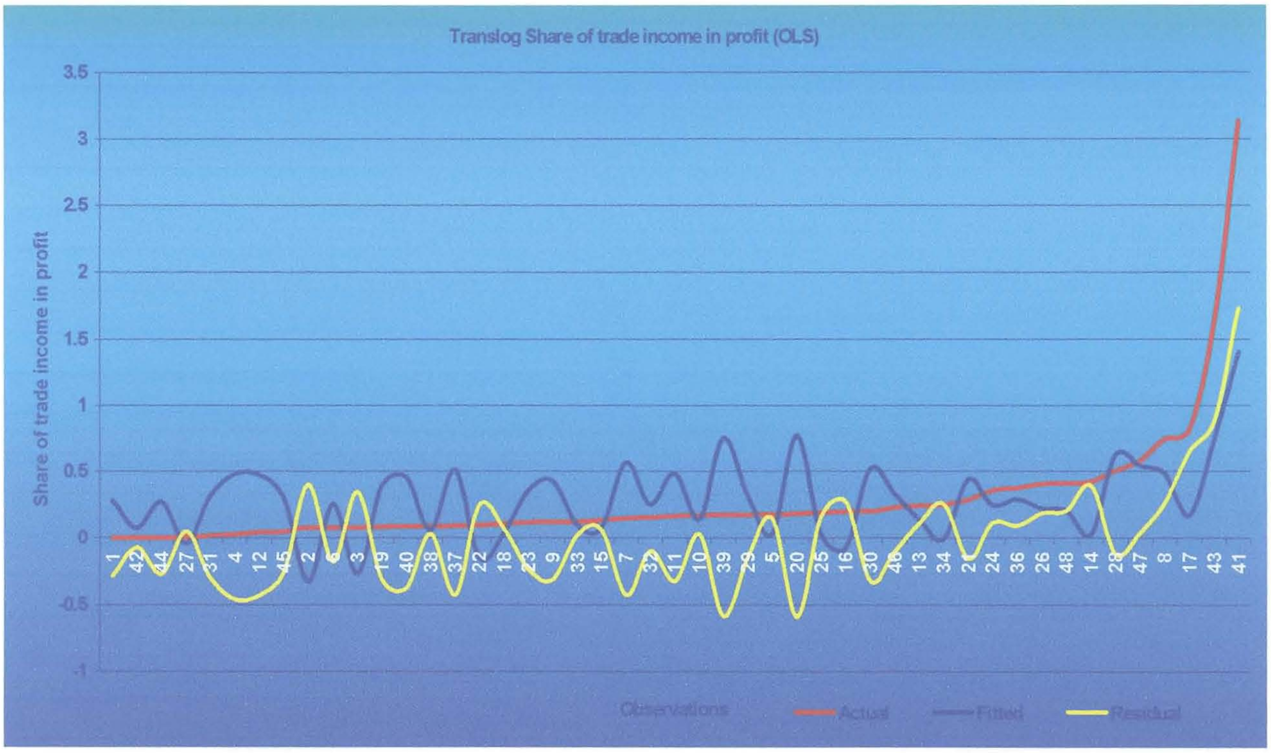

Figure 12: Actual and fitted values of trade income shares in restricted profit (OLS).

4.3.2 Profit System Estimation Results

Various combinations of the derived demand and supply equations, with or without inclusion of the profit function were estimated. The most meaningful results were obtained from the system containing the profit function, the shares of milk, purchased and self-produced feed equations – estimated using Zellner' s iterative seemingly unrelated regression method (ISUR). The trade income share equation was omitted. The results are summarised in Table 18, and discussed based on the share equation implications.

Table 18: Translog Profit System estimated with the Iterative Seemingly Unrelated Regression method

Line	Variable	Parameter	Coefficient	Std. Error	T-statistic	Prob.
1	Constant	α_0	4.59	8.09	0.57	0.57
2	$Ln(P_{MLK})$	α_1	1.31	0.88	1.49	0.14
3	$Ln(P_{FB})$	α_2	0.47	1.00	0.47	0.64
4	$Ln(P_{FS})$	α_3	-3.31	0.93	-3.55	0.00
5	$Ln(P_{TRD})$	α_4	0.91	2.34	0.39	0.70
6	$Ln(P_{MLK})^2$	β_{11}	-0.76	0.57	-1.35	0.18
7	$Ln(P_{FB})^2$	β_{22}	-0.07	0.58	-0.12	0.90
8	$Ln(P_{FS})^2$	β_{33}	-0.21	0.11	-1.82	0.07
9	$Ln(P_{TRD})^2$	β_{44}	-0.57	0.35	-1.65	0.10
10	$Ln(P_{MLK})Ln(P_{FB})$	β_{12}	-0.02	0.42	-0.04	0.97
11	$Ln(P_{MLK})Ln(P_{FS})$	β_{13}	0.27	0.10	2.77	0.01
12	$Ln(P_{MLK})Ln(P_{TRD})$	β_{14}	0.14	0.14	0.99	0.33
13	$Ln(Z_{MPRX})$	β_M	2.67	3.43	0.77	0.44
14	$Ln(Z_{LCAP})$	β_C	-0.69	1.51	-0.46	0.65
15	$Ln(Z_{LABR})$	β_L	0.86	1.04	0.82	0.41
16	$Ln(Z_{MPRX})^2$	γ_{MM}	-6.63	1.59	-4.16	0.00
17	$Ln(Z_{LCAP})^2$	γ_{CC}	0.16	0.15	1.07	0.29
18	$Ln(Z_{LABR})^2$	γ_{LL}	0.10	0.17	0.57	0.57
19	$Ln(Z_{MPRX})Ln(Z_{LCAP})$	γ_{MC}	0.36	0.60	0.60	0.55
20	$Ln(Z_{MPRX})Ln(Z_{LABR})$	γ_{ML}	-0.26	0.54	-0.47	0.64
21	$Ln(Z_{LCAP})Ln(Z_{LABR})$	γ_{CL}	-0.09	0.13	-0.71	0.48
22	$Ln(P_{MLK})Ln(Z_{MPRX})$	γ_{1M}	-3.36	0.27	-12.57	0.00
23	$Ln(P_{MLK})Ln(Z_{LCAP})$	γ_{1C}	0.48	0.11	4.42	0.00
24	$Ln(P_{MLK})Ln(Z_{LABR})$	γ_{1L}	-0.41	0.10	-4.03	0.00
25	$Ln(P_{FB})Ln(Z_{MPRX})$	γ_{2M}	2.56	0.32	8.06	0.00
26	$Ln(P_{FB})Ln(Z_{LCAP})$	γ_{2C}	-0.40	0.13	-3.11	0.00
27	$Ln(P_{FB})Ln(Z_{LABR})$	γ_{2L}	0.29	0.12	2.40	0.02
28	$Ln(P_{FS})Ln(Z_{MPRX})$	γ_{3M}	0.95	0.27	3.56	0.00
29	$Ln(P_{FS})Ln(Z_{LCAP})$	γ_{3C}	-0.03	0.12	-0.29	0.77
30	$Ln(P_{FS})Ln(Z_{LABR})$	γ_{3L}	0.29	0.12	2.47	0.01
31	$Ln(P_{FB})Ln(P_{FS})$	β_{23}	-0.14	0.12	-1.24	0.22
32	$Ln(P_{FB})Ln(P_{TRD})$	β_{24}	-0.34	0.17	-2.05	0.04
33	$Ln(P_{FS})Ln(P_{TRD})$	β_{34}	-0.09	0.15	-0.59	0.56
34	$Ln(P_{TRD})Ln(Z_{MPRX})$	γ_{4M}	0.51	0.51	1.00	0.32
35	$Ln(P_{TRD})Ln(Z_{LCAP})$	γ_{4C}	0.07	0.26	0.25	0.80
36	$Ln(P_{TRD})Ln(Z_{LABR})$	γ_{4L}	-0.23	0.20	-1.15	0.25
37	Dependent Variable	$Ln(\pi)$, S_{MLK}, S_{FB}, S_{FS}		Sample size	48	

Mixed results were obtained. While α_1 enters as a significant variable with the expected sign, the sign of β_{11} is contrary to expectations and the coefficient is insignificant. Both feed variables have significant β_{ii}-coefficients and their signs correspond to *a priori* expectations of own-price responses.

In the derived milk share equation, purchased feed prices induce negative supply responses, while self-produced feed price increases would lead to increased milk supply. Higher livestock trade prices would surprisingly cause increased milk supply. Management improvement and higher labour investments would reduce milk supply, while supply responds positively to increased livestock investment.

Evaluating the purchased feed share equation reveals that increased milk prices would reduce the demand for purchased feed inputs. Increases in self-produced feed prices and the trade prices of livestock would also reduce the demand for purchased feed. Conversely, improved management practices and higher labour expenses are associated with higher demand for purchased feed inputs. Increased livestock outlays would have a negative effect on the demand for purchased feed.

Self-produced feed shares would increase with increases in milk prices and with improved management practices and higher labour outlays. Increased livestock capital would reduce the demand for self-produced feed. Higher purchased feed prices would reduce the demand for self-produced feed – implying complementarities, not substitution between the two input groups. Higher livestock trade prices would force self-produced feed demand downwards.

Similar to the Normalised Quadratic, the highly significant quasi-fixed variables' coefficients together with unexpected signs for the price variables, prompted alternative specification of the system. Livestock capital and labour was dropped and homogeneity was imposed through normalisation of the profit function with the price of traded animals. The results of this process are presented in Table 19.

Table 19: Modified Normalised Translog profit system

Line	Variable	Parameter	Coefficient	Std. Error	T-statistic	Prob.
1	Constant	α_0	11	0.25	44	0.00
2	$Ln(P_{MLK}/P_{TRD})$	α_1	2.02	0.20	10	0.00
3	$Ln(P_{FB}/P_{TRD})$	α_2	-0.68	0.18	-3.72	0.00
4	$Ln(P_{FS}/P_{TRD})$	α_3	-0.15	0.10	-1.51	0.13
5	$Ln(P_{MLK}/P_{TRD})^2$	β_{11}	0.96	0.52	1.85	0.07
6	$Ln(P_{FB}/P_{TRD})^2$	β_{22}	1.16	0.57	2.05	0.04
7	$Ln(P_{FS}/P_{TRD})^2$	β_{33}	0.28	0.11	2.46	0.02
8	$Ln(P_{MLK}/P_{TRD})Ln(P_{FB}/P_{TRD})$	β_{12}	-1.02	0.51	-1.98	0.05
9	$Ln(P_{MLK}/P_{TRD})Ln(P_{FS}/P_{TRD})$	β_{13}	-0.45	0.16	-2.74	0.01
10	$Ln(Z_{MPRX})$	β_M	3.85	0.76	5.03	0.00
11	$Ln(Z_{MPRX})^2$	γ_{MM}	-3.32	0.58	-5.73	0.00
12	$Ln(P_{MLK}/P_{TRD})Ln(Z_{MPRX})$	γ_{1M}	-0.28	0.30	-0.94	0.35
13	$Ln(P_{FB}/P_{TRD})Ln(Z_{MPRX})$	γ_{2M}	-0.10	0.27	-0.38	0.70
14	$Ln(P_{FS}/P_{TRD})Ln(Z_{MPRX})$	γ_{3M}	-0.69	0.18	-3.82	0.00
15	$Ln(P_{FB}/P_{TRD})Ln(P_{FS}/P_{TRD})$	β_{23}	0.37	0.14	2.75	0.01
37	Dependent Variable	$Ln(\pi/P_{TRD})$, S_{MLK}, S_{FB}, S_{FS}			Sample size	48

The α_1-coefficient enters as a significant variable with the expected sign; β_{11} is statistically significant, whilst also corresponding to *a priori* expectations. Both feed variables have significant β_{ii}-coefficients. In the derived milk share equation, purchased and self-produced feed prices induce negative supply responses. The management proxy is negatively related to milk supply.

In the derived purchased feed share equation reveals that increased milk prices would reduce the demand for purchased feed inputs. Increases in self-produced feed prices would increase the demand for purchased feed. Conversely, improved management practices and higher labour expenses are associated with decreased demand for purchased feed inputs. Self-produced feed shares would decrease with increases in milk prices and with improved management. Higher purchased feed prices would increase the demand for self-produced feed – implying substitution between the two input groups.

4.3.3 Discussion of the results

Despite the high adjusted-R^2 (0.95) of the OLS estimation of the profit function (Table 13), the single equation OLS results for the share equations yielded coefficients with improved t-ratios. This improvement was downplayed by emergence of unexpected signs from the OLS estimation of the individual share equations. System estimation of the profit function and share equations produced overall improvements in variables' significance levels. However, it must be noted that the poor fit on the share of self-produced feed and on the share of trade income, casts doubt on the reliability of the system results. Yet, since the quantity of milk supplied and the level of feed administered are determined simultaneously with profit, it is believed that the system results should represent a more realistic scenario. The results from system estimation will be used to evaluate the structural properties of the profit system.

4.3.4 Testing the structural properties

4.3.4.1 Non-negativity

In four (out of forty-eight) cases, negative input quantities were estimated. None of these cases reported simultaneous negative profits or quantities and these results are not sufficient to classify the particular farms as non-profit maximising – small sample size bias, contamination due to aggregation and due to incorrect specification of supply or demand equations all contribute to reduced confidence in the estimation outputs.

4.3.4.2 Monotonicity

From the original translog supply system (Table 18), the first derivatives of the profit function with respect to input and output prices (at the point of approximation, section 3.9.2) were evaluated. Profit is monotonically increasing in milk and livestock trade prices (α_1, α_4 > 0). Profit strictly decreases in self-produced feed prices, but not in purchased feed prices. The latter is thus a violation of profit maximisation requirements.

The normalised translog supply system yielded more theoretically accurate results. Evaluation of the first derivatives of the normalised translog profit function revealed that profit is monotonically increasing in milk prices ($\alpha_1 > 0$) and monotonically decreasing in purchased and self-produced feed prices (α_2, $\alpha_3 < 0$).

4.3.4.3 CONVEXITY AND CONCAVITY

For convexity of the non-normalised profit function in all prices, the modified[14] Hessian matrix of second order derivatives of normalised profit with respect to prices (H^*_{PP}) should have non-negative determinants for the principal minors. The elements of the modified Hessian matrix are ($\gamma_{ii}+\alpha_i^2-\alpha_i$) for the ith-diagonal element, and ($\gamma_{ij}+\alpha_i\alpha_j$) for the off-diagonal elements [Capalbo, *et al.*, 1988]. The determinants $|H_1|$, $|H_2|$ and $|H_4|$ are negative, while $|H_3|$ is positive. The profit function is thus neither globally concave nor globally convex in prices. The latter result is contrary to the requirements for well-behaving profit functions. It is ascribed to the overall problems found in the data, such as small sample properties, aggregation bias and the cross-sectional nature of the data.

Stated algebraically:

$$H = \begin{bmatrix} \dfrac{\partial^2\pi}{(\partial P_{MLK})^2} & \dfrac{\partial^2\pi}{\partial P_{MLK}\partial P_{FB}} & \dfrac{\partial^2\pi}{\partial P_{MLK}\partial P_{FS}} & \dfrac{\partial^2\pi}{\partial P_{MLK}\partial P_{TRD}} \\ \dfrac{\partial^2\pi}{\partial P_{FB}\partial P_{MLK}} & \dfrac{\partial^2\pi}{(\partial P_{FB})^2} & \dfrac{\partial^2\pi}{\partial P_{FB}\partial P_{FS}} & \dfrac{\partial^2\pi}{\partial P_{FB}\partial P_{TRD}} \\ \dfrac{\partial^2\pi}{\partial P_{FS}\partial P_{MLK}} & \dfrac{\partial^2\pi}{\partial P_{FS}\partial P_{FB}} & \dfrac{\partial^2\pi}{(\partial P_{FS})^2} & \dfrac{\partial^2\pi}{\partial P_{FS}\partial P_{TRD}} \\ \dfrac{\partial^2\pi}{\partial P_{TRD}\partial P_{MLK}} & \dfrac{\partial^2\pi}{\partial P_{TRD}\partial P_{FB}} & \dfrac{\partial^2\pi}{\partial P_{TRD}\partial P_{FS}} & \dfrac{\partial^2\pi}{(\partial P_{TRD})^2} \end{bmatrix}$$

[14] The Hessian matrix is modified by dividing it through the vector of (π/p_ip_j)

$$\therefore H^*_{PP} = \begin{bmatrix} \beta_{11} + \alpha_1^2 - \alpha_1 & \beta_{12} + \alpha_1\alpha_2 & \beta_{13} + \alpha_1\alpha_3 & \beta_{14} + \alpha_1\alpha_4 \\ \beta_{21} + \alpha_2\alpha_1 & \beta_{22} + \alpha_2^2 - \alpha_2 & \beta_{23} + \alpha_2\alpha_3 & \beta_{24} + \alpha_2\alpha_4 \\ \beta_{31} + \alpha_3\alpha_1 & \beta_{32} + \alpha_3\alpha_2 & \beta_{33} + \alpha_3^2 - \alpha_3 & \beta_{34} + \alpha_3\alpha_4 \\ \beta_{41} + \alpha_4\alpha_1 & \beta_{42} + \alpha_4\alpha_2 & \beta_{43} + \alpha_4\alpha_3 & \beta_{44} + \alpha_4^2 - \alpha_4 \end{bmatrix}$$

$$= \begin{bmatrix} -0.35 & 0.60 & -4.07 & 1.33 \\ 0.60 & -0.32 & -1.70 & 0.09 \\ -4.07 & -1.70 & 14.06 & -3.10 \\ 1.33 & 0.09 & -3.10 & -0.65 \end{bmatrix}$$

The Hessian matrix of the normalised translog profit function is as follows:

$$H = \begin{bmatrix} \dfrac{\partial^2 \pi}{(\partial P^*_{MLK})^2} & \dfrac{\partial^2 \pi}{\partial P^*_{MLK} \partial P^*_{FB}} & \dfrac{\partial^2 \pi}{\partial P^*_{MLK} \partial P^*_{FS}} \\ \dfrac{\partial^2 \pi}{\partial P^*_{FB} \partial P^*_{MLK}} & \dfrac{\partial^2 \pi}{(\partial P^*_{FB})^2} & \dfrac{\partial^2 \pi}{\partial P^*_{FB} \partial P^*_{FS}} \\ \dfrac{\partial^2 \pi}{\partial P^*_{FS} \partial P^*_{MLK}} & \dfrac{\partial^2 \pi}{\partial P^*_{FS} \partial P^*_{FB}} & \dfrac{\partial^2 \pi}{(\partial P^*_{FS})^2} \end{bmatrix}$$

$$\therefore H^*_{PP} = \begin{bmatrix} \beta_{11} + \alpha_1^2 - \alpha_1 & \beta_{12} + \alpha_1\alpha_2 & \beta_{13} + \alpha_1\alpha_3 \\ \beta_{21} + \alpha_2\alpha_1 & \beta_{22} + \alpha_2^2 - \alpha_2 & \beta_{23} + \alpha_2\alpha_3 \\ \beta_{31} + \alpha_3\alpha_1 & \beta_{32} + \alpha_3\alpha_2 & \beta_{33} + \alpha_3^2 - \alpha_3 \end{bmatrix}$$

$$\begin{bmatrix} 3.27 & -2.56 & -0.69 \\ -2.56 & 2.46 & 0.46 \\ -0.69 & 046 & 0.48 \end{bmatrix}$$

These results show that the determinants $|H_1|$, $|H_2|$ and $|H_3|$ are positive. The profit function is thus globally convex in prices (i.e. positive semi-definite). The latter result conforms to the requirements for a well-behaving profit function.

4.3.4.4 HOMOGENEITY

In the non-normalised translog supply system, homogeneity in all prices was not imposed, *a priori*, but tested for afterwards. The test results are reported in Table 20, below. The results are contradictory to such an extent that neither homogeneity in all prices, nor homogeneity in quasi-fixed factors could be established.

Table 20: Results of Wald Coefficient tests for homogeneity in the translog profit system

Hypothesis	Specification	χ^2-stat	p-value	Result
H_0: $\Sigma_i\ \alpha_i = 0$	c(2)+c(3)+c(4)+c(5)=0	0.47	0.49	Fail to reject H_0
H_0: $\Sigma_i\ \beta_{ij} = 0$	C(6)+C(10)+C(11)+C(12)=0 C(7)+C(10)+C(31)+C(32)=0 C(8)+C(11)+C(31)+C(33)=0 C(9)+C(12)+C(32)+C(33)=0	26.75	0.00	Reject H_0 at all levels
H_0: $\Sigma_m\ \gamma_m = 1$	C(13)+C(14)+C(15)=1	0.75	0.39	Fail to reject H_0
H_0: $\Sigma_i\ \gamma_{im} = 0$	C(22)+C(25)+C(28)+C(34)=0 C(23)+C(26)+C(29)+C(35)=0 C(24)+C(27)+C(30)+C(36)=0	1.85	0.61	Fail to reject H_0
H_0: $\Sigma_m\ \gamma_{im} = 0$	C(22)+C(23)+C(24)=0 C(25)+C(26)+C(27)=0 C(28)+C(29)+C(30)=0 C(34)+C(35)+C(36)=0	130.40	0.00	Reject H_0 at all levels
H_0: $\Sigma_m\ \gamma_{mn} = 0$	C(16)+C(19)+C(20)=0 C(17)+C(19)+C(21)=0 C(18)+C(20)+C(21)=0	66.93	0.00	Reject H_0 at all levels

The problem of homogeneity in prices was solved through the normalisation procedure performed on the translog profit function. Homogeneity was thus imposed in the normalised translog supply system.

4.3.4.5 SYMMETRY

Symmetry was imposed during estimation, due to sample size constraints. The symmetry can be seen from the Hessian matrix: $\beta_{ij} = \beta_{ji}$.

4.3.5 ELASTICITY CALCULATIONS

Table 21 reports the Marshallian elasticities (Equation 12) calculated from the various translog profit, share and system estimations. Single equation OLS estimation of the profit function produces system responses to price changes that are as follows (Table 21, lines 1 – 4). When milk prices rise, milk supply increases; purchased feed demand decreases and self-produced feed demand increases; the volume of traded livestock also increases. An upward shift in purchased feed prices increases milk supply, livestock supply for trade and purchased feed demand, but lowers self-produced feed demand.

Table 21: Elasticities calculated from the various estimation results for the Translog specification.

Line	E (q_i / p_j)	P_{mlk}	P_{fb}	P_{fs}	P_{trd}	Source
1	Q_{mlk}	18.85	2.28	-1.94	0.41	Table 13: OLS Profit
2	Q_{fb}	-5.18	10.75	-0.45	-1.45	Table 13: OLS Profit
3	Q_{fs}	7.92	-4.46	-3.56	0.01	Table 13: OLS Profit
4	Q_{trd}	1.09	1.71	-0.00	-0.23	Table 13:OLS Profit
5	Q_{mlk}	0.58	-0.82	-0.48	0.85	Table 14: OLS Milk share
6	Q_{fb}	1.84	-2.75	-0.09	1.16	Table 15: OLS Purchased feed share
7	Q_{fs}	-0.32	0.57	-1.70	0.80	Table 16: OLS Self-produced feed share
8	Q_{trd}	-7.28	6.40	-1.92	0.48	Table 17: OLS Trade income share
9	Q_{mlk}	0.79	-0.95	-0.40	0.87	Table 18: ISUR Profit system
10	Q_{fb}	2.17	-1.87	-0.38	1.16	Table 18: ISUR Profit system
11	Q_{fs}	1.63	-0.68	-1.13	0.97	Table 18: ISUR Profit system
12	Q_{trd}	2.32	-1.37	-0.64	-0.91	Table 18: ISUR Profit system
13	Q_{mlk}	1.70	-1.53	-0.73	N/a	Table 19: ISUR Normalised profit system
14	Q_{fb}	3.46	-3.41	-0.93	N/a	Table 19: ISUR Normalised profit system
15	Q_{fs}	2.98	-1.66	-2.16	N/a	Table 19: ISUR Normalised profit system

Milk supply responds negatively towards increased self-produced feed prices and both purchased feed and self-produced feed demand declines when the price increases. Higher livestock prices increase milk supply and self-produced feed demand, but decrease purchased feed demand and volume of livestock traded. The results in line 3 seem plausible. In the other cases (except for milk's own-price response, which is highly elastic), the own- and cross-price responses are contrary to economic theory. This response system indicates complementarities between purchased and self-produced feed inputs. Furthermore, increases in milk supply (stimulated by milk price increases) favour the use of self-produced feed inputs above purchased feed inputs.

When the results from the OLS regression on the single profit share equations are evaluated as a system (Table 21, lines 5 – 8), quite different conclusions are drawn. Milk still features as a normal good (positive own-price elasticity), but is much more inelastic in this system. Milk supply expansion, however, favours the use of purchased feeds in this case. Purchased feed inputs treat self-produced feed as a complement, but self-produced feed inputs treat purchased feed inputs as substitutes in the production process.

The system estimation results (Table 18) indicate that milk (line 9) is an inelastic normal good, which is intensive in the use of purchased and self-produced feed inputs in the production process. Both inputs have negative price elasticities of demand. The demand for purchased feed inputs is more elastic with respect to milk price changes than the demand for self-produced feeds (lines 10 and 11)– probably due to the commitment of land and other factors of production into the production process of the latter input. Milk supply response is inelastic towards input price changes, more so with respect to self-produced feed (line 9). While purchased feed' s response (line 10) is elastic towards its own price, it is inelastic with respect to self-produced feed prices – the same holds for self-produced feed response (line 11). The volume of traded animals' increase when milk prices rise (gross complements)– the response is highly elastic. Higher feed prices induce contractions in the supply of livestock – these results are consistent with the complimentarity between milk production and livestock trade. However, the negative price elasticity of livestock supply is contrary to expectations.

From lines 13 to 15, the normalised supply system poses milk as a normal good with an elastic long-run own-price response. Milk supply responds negatively to input price increases, especially towards purchased feed prices. Milk production is more intensive in the use of purchased feed: higher milk prices would induce larger demand increases for purchased feed than for self-produced feed inputs. All the own-price responses adhere to theoretical requirements for normal goods. The feed inputs are gross complements (long-run).

To compute the Hicksian input demand elasticities with respect to input prices, Table 21' s results (of the profit system, lines 9 to 12) are used in the formulae from Equation 18 (Chapter 3: Methodology).

$$\{\eta_{lk}^S\} = \{\eta_{lk}\} - \{\eta_{lt}\} \times \{\eta_{th}\}^{-1} \times \{\eta_{tl}\}$$
$$= \begin{bmatrix} -1.87 & -0.38 \\ -0.68 & -1.13 \end{bmatrix} - \begin{bmatrix} 2.17 & 1.16 \\ 1.63 & 0.97 \end{bmatrix} \times \begin{bmatrix} 0.79 & 0.87 \\ 2.32 & -0.91 \end{bmatrix}^{-1} \times \begin{bmatrix} -0.95 & -0.40 \\ -1.37 & -0.64 \end{bmatrix}$$
$$= \begin{bmatrix} 0.24 & 0.53 \\ 0.95 & -0.43 \end{bmatrix} = \begin{bmatrix} \eta_{FB,FB}^S & \eta_{FB,FS}^S \\ \eta_{FS,FB}^S & \eta_{FS,FS}^S \end{bmatrix}$$

The highly elastic long-run price elasticity of purchased feed demand (-1.87) is caused by a substantial expansion effect (2.11) - the change in input use due to a movement to a new production frontier – as opposed to the inelastic short-run response (0.24). Self-produced feed displays a similar pattern (expansion effect of 0.7). While the two inputs are gross complements, they are net substitutes. This is probably due to their simultaneous importance in the milk production process. Higher purchased feed prices induce a short-run switch to self-produced feeds, but this in counteracted by the long-run expansion of purchased feed use due to milk supply increases. In the same way, increased self-produced feed prices would cause purchased feed to replace self-produced feed inputs in the short–run, but this is then balanced by the expansion effect, albeit smaller than the expansion effect of purchased feed. Milk production is clearly more intensive in the use of purchased feeds. None of the inputs are regressive in the sense that the demands for them decrease as output prices increase.

The normalised translog supply system yields the following Hicksian input demand elasticities;

$$\{\eta_{lk}^S\} = \{\eta_{lk}\} - \{\eta_{lt}\} \times \{\eta_{th}\}^{-1} \times \{\eta_{tl}\}$$
$$= \begin{bmatrix} -3.41 & -0.93 \\ -1.66 & -2.16 \end{bmatrix} - \begin{bmatrix} 3.46 \\ 2.98 \end{bmatrix} \times [1.70]^{-1} \times [-1.53 \quad -0.73]$$
$$= \begin{bmatrix} -0.31 & 0.56 \\ 1.00 & -0.89 \end{bmatrix} = \begin{bmatrix} \eta_{FB,FB}^S & \eta_{FB,FS}^S \\ \eta_{FS,FB}^S & \eta_{FS,FS}^S \end{bmatrix}$$

Long-run responses are dominated by expansion effects. The feed inputs are net substitutes in the production process. According to these results, the substitution effect is stronger when increases in purchased feed prices occur. In the long-run expansion in both inputs occur and the demand for the two components moves together.

Similarly, Hicksian output supply elasticities for the non-normalised profit function with respect to output prices are calculated as follows.

$$\{\eta_{th}^{S}\} = \{\eta_{th}\} - \{\eta_{tl}\} \times \{\eta_{lk}\}^{-1} \times \{\eta_{lt}\}$$

$$= \begin{bmatrix} 0.79 & 0.87 \\ 2.32 & -0.91 \end{bmatrix} - \begin{bmatrix} -0.95 & -0.40 \\ -1.37 & -0.64 \end{bmatrix} \times \begin{bmatrix} -1.87 & -0.38 \\ -0.68 & -1.13 \end{bmatrix}^{-1} \times \begin{bmatrix} 2.17 & 1.16 \\ 1.63 & 0.97 \end{bmatrix}$$

$$= \begin{bmatrix} -0.49 & 0.16 \\ 0.43 & -1.96 \end{bmatrix} = \begin{bmatrix} \eta_{MLK,MLK}^{S} & \eta_{MLK,TRD}^{S} \\ \eta_{TRD,MLK}^{S} & \eta_{TRD,TRD}^{S} \end{bmatrix}$$

Milk price elasticity of supply (0.79) indicates that price changes induce short-run contraction in milk supply (-0.49), followed by contraction effect (-1.28). Livestock trade is a complimentary process that exhibits short-run substitution for milk production (0.43), but this is countered by a substantial contraction effect (-1.89). In the same way, milk production substitutes for trade in the short run (due to trade price increases), but its long-run contraction effect overshadows the substitution effect.

The normalised translog supply system produces a Hicksian output supply elasticity of –0.08 (calculated as follows).

$$\{\eta_{th}^{S}\} = \{\eta_{th}\} - \{\eta_{tl}\} \times \{\eta_{lk}\}^{-1} \times \{\eta_{lt}\}$$

$$= [1.70] - [-1.53 \quad -0.73] \times \begin{bmatrix} -3.41 & -0.93 \\ -1.66 & -2.16 \end{bmatrix}^{-1} \times \begin{bmatrix} 3.46 \\ 2.98 \end{bmatrix}$$

$$= [-0.08] = [\eta_{MLK,MLK}^{S}]$$

This implies that short-run response to milk price increases is supply reducing, albeit a very in-elastic response. The highly elastic long-run response (1.70) is a result of the substantial contraction in supply (-1.78) following the short-run response.

4.4 CHOICE OF MOST APPROPRIATE FUNCTIONAL FORM

Before introduction of the modified supply systems, the results of sections 4.2.4 and 4.3.4 favoured the translog profit system (as estimated using Zellner' s Iterative Seemingly Unrelated method). However, the results of both systems led to rejection of the essential monotonicity and convexity properties and both systems suffer from aggregation, small sample and missing data problems.

On the contrary, introduction of the two alternative specifications for the Normalised Quadratic and Translog supply systems (also estimated with the ISUR method), produced results that are consistent with profit maximising producer behaviour. The long-run responses in the modified Normalised Quadratic system are more in-elastic than the Marshallian responses from the modified Normalised Translog system. In contrast, the Normalised Translog Hicksian responses are more elastic than the Normalised Quadratic's short-run responses.

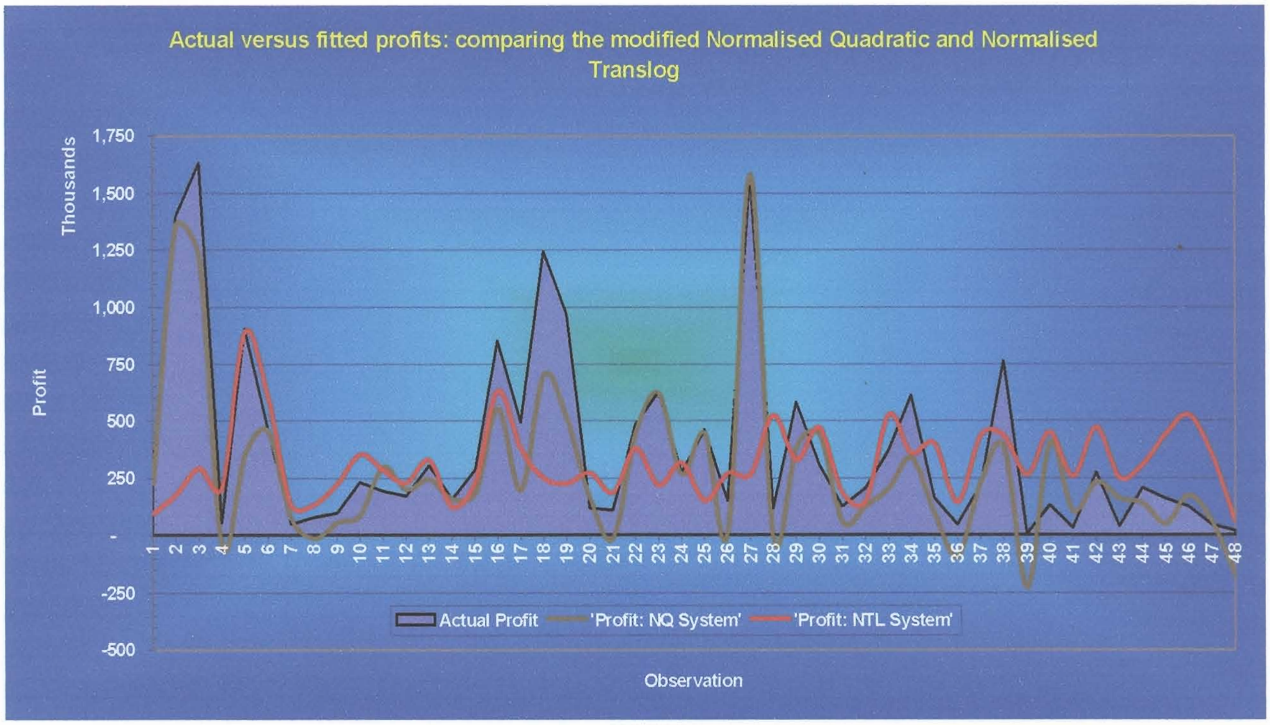

Figure 13: Comparing the Normalised Quadratic and Normalised Translog profit with actual profit levels

The Normalised Quadratic offers a closer correspondence with observed values.