CHAPTER 3: METHODOLOGY

3.1 DEFINITION OF "MILK PRODUCTION"

A milk-producing unit is defined as a whole farm or an activity of a diversified farming enterprise, in which milk products are produced. Milk production involves the dynamics of the dairy herd, which requires replacement, culling, nourishment and improvement. In addition to milk, trade in livestock forms an important part of the production process, albeit a secondary activity. The animal products are fluid milk and butterfat. The processing of milk and butterfat into other consumables (butter, milk powder, low fat or full cream milk, etc.) is not regarded as dairy production, but classifies as dairy processing. Furthermore, for the purpose of this study, only milk from bovine origin is taken into account. This is a result of the availability of information on dairy cattle as well as the overall dominance of bovine milk in the total dairy market, as opposed to milk from goats and sheep [Dairy Development Initiative, 1999].

3.2 CHOOSING BETWEEN A PRIMAL OR DUAL PARAMETRIC APPROACH

Chambers [1988] begins his description of applied production analysis with an overview of the production function (estimating the structure of production in the primal approach). However, he argued that restrictive assumptions are generally imposed on the production function and he pointed to the fact that analysts' main interest does not pertain to the production function as such, but to reliable representation and prediction of economic behaviour. He goes on to say that a restricted production function implies restrictions on the dual function.

This is where the dual approach attempts to isolate those circumstances under which purely economic phenomena can be used to reconstruct the underlying technology, given that technological restrictions are disclosed in the economic behaviour of agents. Duality implies that well-behaved cost- and profit functions are equivalent to well-behaved production functions.

Similarly, the existence of well-behaved cost- and profit functions implies the existence of well-behaved input requirement sets. A similar relationship exists between the cost and profit function: specification of a profit function implies the existence of a cost function [Lopez, 1984; Chambers, 1988]. Higgins [1986] states that the basic behavioural assumptions required when modelling production possibilities with a profit function approach are that farmers are profit maximisers and that markets are competitive. Both are realistic assumptions in the South African context.

Cost and revenue data is usually more readily available than data on physical input quantities. Therefore, parameters of the production functions can be estimated indirectly through application of duality (via a cost or profit function approach) [Leontief, 1969; Shephard, 1970; Diewert, 1971; Lau and Yotopoulos, 1972; Binswanger, 1974; Varian, 1978; Colman, 1983; Debertin, 1986].
Availability of data on primal variables is often a problem, as is the case with the present study. Only farm level budget information (cost, profit, prices) was available for this study (as opposed to regional or national aggregated data [Higgins, 1986]). Accordingly, data available for this study could only support the use of a dual approach to supply analysis.

3.2.1 PROFIT MAXIMISATION OR COST MINIMISATION

Dairy production in South Africa is not subject to quotas or any other form of centrally determined output level specification. Therefore, farmers are assumed to be maximising profits, subject to technological and environmental constraints. It could be argued that farmers minimise cost for an "expected" level of output. Since the data does not contain a time dimension, the "expected" output cannot be calculated for each farm. The underlying assumption is that the results from the two approaches will not differ significantly. The focus is, therefore, on the profit function approach.

3.3 THE THEORETICAL MODEL

Under the profit function approach, the production function is specified as $h(q,x,z)=0$, implying that $q = f(x,z)$ where h is the technology function with q as the vector of output quantities and x and z the vectors of variable and fixed factors, respectively. Denoting output (milk) price by p and the price of inputs by the vector w, the restricted profit function (in which only variable costs are deducted from gross revenues) is written as $\pi_r = p'q - w'x$ (with p' and w' the respective transposed vectors of p and w). A producer thus chooses levels of inputs and output that will maximise restricted profit (π_r), subject to the technological constraints. Algebraically, the profit maximisation problem can be specified as:

Equation 1:

$$\begin{aligned} &\underset{x,q}{Max} \quad \pi = (p'q - w'x), \\ &s.t. \quad h(q,x,z) = 0 \end{aligned}$$

The solution of the latter optimisation problem is a set of input demand and output supply equations:

Equation 2:

$$\begin{aligned} &x = x(p,w,z), \rightarrow \text{Input demand} \quad \text{and} \\ &q = q(p,w,z) \rightarrow \text{Output supply.} \\ &\text{Therefore:} \\ &\pi = p'q(p,w,z) - w'x(p,w,z) \end{aligned}$$

Ideally, the profit function should satisfy the regularity conditions that would make it a "well-behaved" profit function. These properties are: Non-negativity; continuity; twice differentiability; monotonicity - increasing (decreasing) in output (input) prices; non-decreasing in fixed inputs; convexity in prices; concavity in fixed inputs; and homogeneity of degree zero in all prices and homogeneity of degree one in all fixed factors (if the production functions exhibits constant returns to scale, CRS) [Chambers, 1991; Higgins, 1986]. When these properties are satisfied, or imposed, the profit function will be the dual function of the transformation function. Then the parameters of the profit function contain adequate information from which to infer the properties of the underlying technology (e.g. elasticities of substitution, homogeneity, etc) [Higgins, 1986].

When dealing with a single output, the normalised variable profit function is estimated. It represents the ratio of profit to the output price, as a function of relative prices of variable inputs (and outputs) and of quasi-fixed factors. In the case of a multi-output normalised profit function, the numéraire is the output price of the n^{th} commodity. Normalisation has the purpose of removing any money illusion – in other words, firms respond to relative price changes. Normalisation also reduces the demand on degrees of freedom, by effectively reducing the number of equations and parameters to estimate. Algebraically:

Equation 3: $$\pi^* = \pi^*(p^*, w^*, z), \quad with \quad w_i^* = \frac{w_i}{p_n} \quad and \quad p_i^* = \frac{p_i}{p_n}$$

where w_i denotes the price of input-i (x_i) and where p_n is the price of the single output, or the price of the n^{th} commodity in the case of multi-output. From the profit function a system of output supply and input demand equations are derived by using Hotelling' s Lemma and the first order conditions (F.O.C) for profit maximisation:

Equation 4: $$\frac{\partial \pi^*}{\partial p_i^*} = q_i, \quad and \quad \frac{\partial \pi^*}{\partial w_i^*} = -x_i$$

The symmetry in outputs and inputs is further exploited by treating inputs as negative outputs, thus simplifying notation:

Equation 5: $$q = \begin{bmatrix} q \\ -x \end{bmatrix}, \quad p = \begin{bmatrix} p^* \\ w^* \end{bmatrix} \Rightarrow q_i = \frac{\partial \pi^*(p,z)}{\partial p_i}$$

The derived supply and demand functions satisfy the symmetry (of the second order derivatives of the profit function) property, implying that:

Equation 6: $$\frac{\partial q_i}{\partial p_j} = \frac{\partial q_j}{\partial p_i}$$

Or, stated in terms of elasticities:

Equation 7: $$\frac{\partial q_i}{\partial p_j} \times \frac{p_j}{q_i} = \frac{\partial q_j}{\partial p_i} \times \frac{p_i}{q_j}$$

Similarly in natural logarithm form:

Equation 8: $$\frac{\partial \ln q_i}{\partial \ln p_j} = \frac{\partial \ln q_j}{\partial \ln p_i} \quad \therefore E_{q_i}^{p_j} = E_{q_j}^{p_i}$$

The cross price elasticities are inversely proportional to the corresponding profit shares:

Equation 9: $$\frac{E(q_i / p_j)}{E(q_j / p_i)} = \frac{s_j}{s_i}, \quad where \quad s_i = \frac{p_i q_i}{\pi}$$

In addition to these partial elasticities, the total effect of price changes comprises of a substitution and expansion effect [Higgins, 1986]. Substitution in the input case implies a movement along the isoquant, and a compromise between outputs on the transformation frontier. Expansion implies a movement from one isoquant to another, as output expands along the expansion path to the new transformation frontier [Higgins, 1986]. The corresponding types of elasticities are the Marshallian (uncompensated, long-run) and Hicksian (compensated, short-run) elasticities of substitution. Based on these elasticities, outputs and inputs can be classified into categories of gross or net substitutes or complements in the production process [Higgins, 1986; Deaton and Muellbauer, 1980].

3.4 ECONOMETRIC SPECIFICATION

3.4.1 CHOICE OF FUNCTIONAL FORM

In this study, the translog function [Christensen, *et al.*, 1973] is hypothesised as an appropriate approximation of the true profit function, as well as a good representation of the underlying production function, due to its flexibility and (consequent) wide use. This study also adopts the normalised quadratic (NQ) profit functions for comparative analysis. Both forms are estimated and the results compared to establish which form is most appropriate, given the data. The normalised quadratic form has the advantage of linearity in the parameters and simple expressions for the elasticities (evaluated at any level of prices and quantities) [Bouchet, *et al.*, 1989].

3.4.1.1 THE NORMALISED QUADRATIC PROFIT FUNCTION

In the multi-output case, profit and prices are normalised by the price of the n^{th} commodity. Algebraically it is stated as follows:

Equation 10: $\pi^* = \frac{\pi}{p_n} = \alpha_0 + \sum_i \alpha_i p_i + \frac{1}{2}\sum_{ij}\beta_{ij} p_i p_j + \sum_{im}\beta_{im} p_i z_m \quad \begin{bmatrix} i,j = 1,\cdots,n-1 \\ \beta_{ij} = \beta_{ji} \end{bmatrix}$

where $p = \begin{bmatrix} p^* \\ w^* \end{bmatrix}$ is the vector of normalised output and input prices. This type of profit function is homogenous in prices, but not in the fixed factors (z). The derived system of output supply and input demand equations is:

$$q_i = \alpha_i + \sum_j \beta_{ij} p_j + \sum_m \beta_{im} z_m \quad \text{and}$$

Equation 11: $$q_n = \pi^* - \sum_i p_i q_i = \alpha_0 - \frac{1}{2}\sum_{ij}\beta_{ij} p_i p_j$$

for commodity n, whose price was the numéraire

Normalisation allows for evaluation of supply and input responses to relative price changes. In addition, it imposes homogeneity in prices, thereby reducing pressure on degrees of freedom during estimation. Moreover, normalisation allows one equation to be dropped from the system, thus further alleviating the degrees of freedom problem. Price elasticities are computable at any value of prices and quantities, such that:

Equation 12: $E_{ij} = \frac{\beta_{ij} p_j}{q_i}, \quad i,j \neq n, \quad E_{nj} = \frac{1}{s_n}\sum_i s_i E_{ij}, \quad \text{and} \quad E_{nn} = -\sum_i E_{ni}.$

This function has the distinct advantages of linearity in the parameters and simple equations for the elasticities [Bouchet *et al.*, 1989; Thijssen, 1992; Sadoulet and De Janvry, 1995].

3.4.1.2 THE TRANSLOG PROFIT FUNCTION

The translog specification is a second-degree flexible function in prices and fixed inputs, with variable elasticities of substitution and is considered as a second order approximation of any functional form. Algebraically, the translog is specified as follows [Christensen, *et al.*, 1973; Capalbo *et al.*, 1988]:

Equation 13:

$$\ln\pi = \alpha_0 + \sum_i \alpha_i \ln p_i + \sum_m \beta_m \ln z_m + \frac{1}{2}\sum_{ij}\beta_{ij}\ln p_i \ln p_j + \frac{1}{2}\sum_{mn}\gamma_{mn}\ln z_m \ln z_n + \sum_{im}\gamma_{im}\ln p_i \ln z_m$$

Certain restrictions are required when the properties of homogeneity with respect to prices and fixed factors are imposed. The necessary restrictions are symmetry, additivity and homogeneity, respectively:

$$\beta_{ij}=\beta_{ji};\quad \gamma_{mn}=\gamma_{nm};\quad \sum_i \alpha_i = 0;\quad \sum_m \beta_m = 1;\quad \sum_i \beta_{ij} = \sum_m \gamma_{mn} = \sum_i \gamma_{im} = \sum_m \gamma_{im} = 0.$$

The system of derived factor demand and output supply is:

Equation 14: $$q_i = \frac{\pi}{p_i}\left[\alpha_i + \sum_j \beta_{ij}\ln p_j + \sum_m \gamma_{im}\ln z_m\right]$$

Elasticities are calculated as:

Equation 15: $$E_{ij} = s_j + \frac{\beta_{ij}}{s_i};\quad and \quad E_{ii} = -1 + s_i + \frac{\beta_{ii}}{s_i}$$

The translog function has an additional beneficial property. Differentiation of the profit function with respect to input or output price (Hotelling' s Lemma) yields the profit-share equation for that specific input or output [Christensen, *et al.*, 1973]. Higgins [1986] clearly shows that:

Equation 16: $$\frac{\partial \ln\pi}{\partial \ln p_i} = \frac{\partial\pi}{\partial p_i}\times\frac{p_i}{\pi} = S_i = \alpha_i + \sum_j \beta_{ij}\ln p_j + \sum_m \gamma_{im}\ln z_m \quad i = 1,\ldots,n$$

The profit shares are the basis from which to compute price elasticities of inputs and outputs (Equation 15) [Christensen, *et al.*, 1973; Thijssen, 1992; Debertin, 1986; Sadoulet and De Janvry, 1995; Binswanger, 1974]. For the translog model, Higgins [1986] defines the Marshallian and Hicksian elasticities of substitution as follows:

Equation 17:

$$Marshallian:\quad \eta_{ii} = \frac{\beta_{ii}}{S_i} + S_i - 1$$

$$\eta_{ij} = \frac{\beta_{ij}}{S_i} + S_j \quad i, j = 1,\ldots,n$$

where η_{ii} and η_{ij} represent the own-price elasticity of supply (and demand), and the cross-price elasticity of supply, respectively. The system of compensated elasticities is represented in matrix form as follows (following the notation used by Higgins, [1986]) and this applies to the Normalised Quadratic specification' s compensated elasticities as well:

Equation 18:

$$Hicksian: \quad \{\eta_{lk}^{S}\} = \{\eta_{lk}\} - \{\eta_{lt}\} \times \{\eta_{th}\}^{-1} \times \{\eta_{tl}\}$$
$$\{\eta_{th}^{S}\} = \{\eta_{th}\} - \{\eta_{tl}\} \times \{\eta_{lk}\}^{-1} \times \{\eta_{lt}\}$$

The matrices and symbols are defined as:

- $\{\eta_{lk}^{S}\}$: Hicksian input demand elasticities with respect to input prices.
- $\{\eta_{lk}\}$: Marshallian input demand elasticities with respect to input prices.
- $\{\eta_{lt}\}$: Marshallian input demand elasticities with respect to output prices.
- $\{\eta_{th}\}$: Marshallian output supply elasticities with respect to output prices.
- $\{\eta_{tl}\}$: Marshallian output supply elasticities with respect to input prices.
- $\{\eta_{th}^{S}\}$: Hicksian output supply elasticities with respect to output prices.

3.5 METHODS OF ECONOMETRIC ESTIMATION

3.5.1 SINGLE EQUATION, SYSTEM- OR FULL INFORMATION ESTIMATION PROCEDURES

Supply response from a profit function approach can be estimated through two procedures. Firstly, by estimating the profit function itself from data on farm profits, exogenous explanatory variables, prices and fixed factors. Secondly, since observations on inputs and outputs are readily available, direct estimation of the output supply and input demand equations can be done without imposing assumptions of cost minimising or profit maximising behaviour.

When following the second approach, all equations (assuming a profit-share approach) are jointly estimated except for the profit function itself. The reasoning behind this lies in the linear dependency of the profit function on the coefficients of the share equations (identification requirement). In the translog case, the dependent variables are the profit shares, which add to one. Therefore, one share equation is dropped from the system to avoid singularity of the variance-covariance matrix. The parameters of the dropped equation are derived from the estimated parameters. An additional advantage of the system approach is the avoidance of multicollinearity problems. In the profit function, square and cross product terms introduce multicollinearity, but these terms are not present in the share- or input demand and output supply equations of the system.

In terms of obtaining full information, joint estimation of the profit function and the derived functions would yield estimates that are more efficient. This places severe restrictions on the degrees of freedom and is thus not ideal in this study where the sample size is limited to forty-eight observations.

For the system of input demand and output supply functions to be compatible with profit maximisation, monotonicity and convexity of the underlying profit function, as well as homogeneity and symmetry must hold. These constraints could be imposed on the parameters during estimation of the system, or the unrestricted system could be estimated and then the theoretical constraints could be formally tested, both locally and globally [Capalbo *et al.*, 1988]. The latter effectively provides a test for profit maximising behaviour [Lopez, 1980]. On the other hand, imposing of the theoretical restrictions reduce the number of parameters to be estimated, thus alleviating the pressure on the degrees of freedom problem due to joint system estimation of these flexible forms.

3.5.2 *SINGLE EQUATION AND SYSTEM ESTIMATION TECHNIQUES*

The econometric estimation of a system of equations can be done with various techniques[10] [Johnston, 1984; Zellner, 1987; Pindyck and Rubinfield, 1991; Johnston and DiNardo, 1997; Greene, 1997]. For this study, Seemingly Unrelated Regression (SUR) and Ordinary Least Squares (OLS)

[10] Seemingly Unrelated Regressions (SUR); Full Information Maximum Likelihood (FIML); Generalised Method of Moments (GMM); Two-stage Least Squares (2SLS); Three-stage Least Squares (3SLS), Weighted Least Squares (WLS), Ordinary Least Squares (OLS), etc. See Annexure B.

estimation techniques are considered and compared to determine the appropriate estimation technique, given the data set.

While the systems approach allows for cross-equation restrictions and takes account of cross-equation error correlation, it does come at a cost. Misspecification of an equation within the system may contaminate estimates of the other parameters. When employing single equation estimation, only the parameters of the misspecified equation are affected. Therefore, both the system and single equation approach were followed and the differences are reported in the results chapter.

SUR estimation (also known as the multivariate regression or Zellner' s method) [Zellner, 1962] accounts for both heteroskedasticity and contemporaneous cross-equation error correlation. This technique is appropriate when all the right hand side variables are assumed exogenous, and when some common factors, which are not explicitly modelled, influence the disturbances across equations [Zellner, 1962; Johnston and DiNardo, 1997]. Using the Iterative-SUR makes the system indifferent to the choice of the dropped share equation. In addition, the cross-equation symmetry restrictions and possible contemporaneous correlation between the errors of the various share equations, justifies the choice of this method [Higgins, 1986; Pindyck and Rubinfeldt, 1991; Kotsoyannis, 1981; Johnston and DiNardo, 1997].

OLS system estimation minimises the sum of squared residuals for each equation, whilst considering cross-equation restrictions [EViews User Manual, 1999]. When no restrictions are imposed, the method is identical to single equation OLS. This method also provides an intuitive test for the correct specification of the different equations in the system approach. If the system estimation yields unsatisfactory results, the single equation OLS results may indicate which equation(s) causes the problems.

Each specified equation for both methods contains an additive error term that captures the unexplained difference between the profit maximising levels of input and output versus the realised levels [Higgins, 1986]. The error term will inexorably capture the effect of all variables that are not explicitly specified as well as some quality differences in inputs and outputs. No quality distinctions are reported and could thus not be incorporated. In addition, the cross-sectional nature of the data leads to the use of White's Heteroskedasticity Consistent Variance Co-variance Estimator [White, 1980] to account for possible heteroskedasticity of unknown form.

3.6 A NOTE ON PRICE VARIABILITY IN CROSS-SECTIONAL STUDIES

For estimation of a profit function or a system, sufficient price variation is necessary. While this is seldom a problem in time series data, the nature of price variation in cross-sectional studies draws attention [Higgins, 1986; Quiggin and Bui-Lan, 1984]. Difference in market prices can be the result of transaction cost such as transport or marketing cost due to differences in farms' proximity to markets. Monopoly power of processors, input suppliers or co-operatives and discounts for bulk sales or purchases may also cause variation in effective prices paid and received. The use of a profit function approach is not invalidated by these causes of price variation. However, if price variation can be ascribed to quality differences in inputs and outputs, the source of variation should be modelled, thereby removing the required variability in prices.

When dealing with aggregated inputs and outputs the price differences could be a result of the difference in the composition of the aggregates, causing the price index to be correlated with the error term (resulting in biased estimates). Similarly, difference in managerial efficiency (in interpreting market signals, timing production actions, etc.), which is not accounted for explicitly or correctly, will bias the parameter estimates [Higgins, 1986].

The assumption is made that the observed price variability is due to proximity to markets and to bulk discounts, as well as preferential contracts with processors. Furthermore, the proportion of price variability that is due to managerial differences is assumed to be captured by the management proxy.

3.7 HYPOTHESISED RELEVANT VARIABLES

3.7.1 SELECTION OF VARIABLES FOR THE EMPIRICAL ANALYSIS

The data set used for this study contains detailed production cost for fifty dairy farmers. On average, total feed cost constitutes sixty percent (60%) of the total annual cost outlay. A breakdown of self-produced (grazing and administered feed) and purchased feed types, quantities and cost is given in the data. However, data is lacking for different variables on different farms, in other words, the gaps in the data are not consistent within or between farms. The composition of labour remuneration (in terms of cash, rations, farm produce and other benefits) is detailed, but the number of hours allocated to milk production and livestock, respectively, is not recorded. Capital constitutes investment in livestock, as one group, and investment in land, fixed improvements and equipment, as the second group. The herd structure is also reported.

The lack of a time dimension precluded evaluation of technological change. In addition, the small sample size (48 farms with positive profits) places serious limitations on the size of the supply system to be estimated. Therefore, the initial choice of explanatory variables fell on two variable inputs (self-produced feed, and purchased feed) and on three quasi-fixed variables (a proxy for management, livestock capital and labour input). This choice was supported by the records of unit prices for the variable inputs for the majority of the farms. The distinction between self-produced and purchased feed is drawn, based on the assumption that the ability to produce feed lowers the input cost and thus increases profit. Similarly, the ability to purchase feed effectively expands the farm size in the short term [Burton, 1984]. Furthermore, most of the input substitution in dairy production occurs between these two input groups. It is assumed that management determines the efficient allocation of resources. A proxy for managerial ability is used, namely, restricted profit per unit of fixed cost. This measure gives an indication of a farmer' s ability to generate short-run profit sufficiently to cover medium and long-term costs.

Restricted profit was calculated as gross product value (quantity of milk multiplied by the milk price plus the value of animal trade income) minus the variable inputs (self-produced and purchased feed). The shares of the variable inputs are calculated as the total value of the input (price times quantity) divided by the restricted profit. Only those farms with positive restricted profit are considered (hence only 48 farmers out of the available fifty observations). The aggregated price of traded animals is used as the numéraire for profit normalisation.

3.7.2 *TRANSFORMATION OF VARIABLES*

When estimating the translog profit system the independent variables are expressed in their natural logarithmic form. This poses problems when some observational units report zero values for a specific input. Given the already small sample size, it was decided not to exclude these observations from the system. As a solution, the relevant input is scaled by a constant, making all observations on that input larger than zero.

The income from animal trade (as the second output activity) comprises of the sales of calves, heifers, dry cows, bulls and oxen at different free market prices. Similarly, purchased feed constitutes a mix of bought feed components and self-produced feed comprises of diverse crop mixtures. Due to the small sample size, the dynamics of each of these three processes could not be modelled. The alternative is to construct three price indices for an aggregated unit of a traded animal, an aggregate unit of purchased feed and an aggregate unit of self-produced feed per farm [Higgins, 1986]. Following the methodology of Higgins [1986] and Caves *et al.* [1982], a cross-section type Divisia index is constructed for each of the three cases. The form of the index is:

Equation 19: $$\ln P_j^k = \frac{1}{2}\sum_i^g (r_{ij}^k + \bar{r}_{ij})(\ln P_{ij}^k - \overline{\ln p_{ij}})$$

The variables in Equation 19 are defined as:

- $\ln P_j^k$: the price index for the aggregate-j (e.g. purchased feed) on the farm-k.

- r_{ij}^{k} : the share of good-i (i.e. licks in the purchased feed aggregate) in the aggregate-j on the farm-k.
- $\bar{r}_{ij}$: the average share of good-i in aggregate-j over all farms.
- $\ln P_{ij}^{k}$: the natural logarithm of the unit price for good-i in the aggregate-j on farm-k.
- $\overline{\ln p_{ij}}$: the average of the natural logarithm of the price of good-i in aggregate-j over all farms.

The index has a base of zero (the average of the sample) and for the farms that report zero values for an input or output, the average sample price was used in calculating the index.

The set of dependent and independent variables included in the supply system is given in Table 5.

Table 5: Base model system variables

Variable name	Description	Unit of measurement
Q_{mlk}	Total quantity of fluid milk produced	Litres
Q_{trd}	Total aggregate units of animals traded	Aggregated unit
Q_{fb}	Total aggregated units of purchased feed demanded	Aggregated unit
Q_{fs}	Total aggregated units of self-produced feed demanded	Aggregated unit
P_{mlk}	Unit price of a litre of milk	Rand / litre
P_{trd}	Price index for an aggregated animal unit traded	Rand / unit
P_{fb}	Price index for an aggregated unit of feed purchased	Rand / unit
P_{fs}	Price index for an aggregated unit of self-produced feed	Rand / unit
Smlk	Share of milk revenue in the restricted profit	
S_{trd}	Share of trade revenue in restricted profit	
S_{fb}	Share of purchased feed in restricted profit	
S_{fs}	Share of self produced feed in restricted profit	
MPRX	Index for management efficiency (base value is one)	Profit/ Fixed cost
LCAP	Livestock capital (average of beginning and ending stock)	Rand
LABR	Labour cost (cash, rations, payment in kind and grants)	Rand
Profit (π)	Restricted profit (Gross value of production – variable input cost)	Rand

3.8 SPECIFICATION OF THE EMPIRICAL MODEL

3.8.1 INPUT DEMAND AND OUTPUT SUPPLY EQUATIONS FOR THE NORMALISED QUADRATIC MODEL

Equation 20 shows the specification for the empirical normalised profit function. The derived supply and demand equations are represented Equation 21.

Equation 20:

$$
\begin{aligned}
\pi^* = \frac{\pi}{p_{trd}} = \alpha_0 &+ \alpha_1 p_{mlk} + \alpha_2 p_{fb} + \alpha_3 p_{fs} \\
&+ \tfrac{1}{2}(\beta_{11} p_{mlk}^2 + \beta_{22} p_{fb}^2 + \beta_{33} p_{fs}^2) \\
&+ (\beta_{12} p_{mlk} p_{fb} + \beta_{13} p_{mlk} p_{fs} + \beta_{23} p_{fb} p_{fs}) \\
&+ (\beta_{1M} p_{mlk} z_{Mprx} + \beta_{2M} p_{fb} z_{Mprx} + \beta_{3M} p_{fs} z_{Mprx}) \\
&+ (\beta_{1C} p_{mlk} z_{Lcap} + \beta_{2C} p_{fb} z_{Lcap} + \beta_{3C} p_{fs} z_{Lcap}) \\
&+ (\beta_{1L} p_{mlk} z_{Labr} + \beta_{2L} p_{fb} z_{Labr} + \beta_{3L} p_{fs} z_{Labr})
\end{aligned}
$$

Equation 21:

$$
\begin{aligned}
Q_{mlk} &= \alpha_1 + \beta_{11} p_{mlk} + \beta_{12} p_{fb} + \beta_{13} p_{fs} + \beta_{1M} z_{Mprx} + \beta_{1C} z_{Lcap} + \beta_{1L} z_{Labr} \\
Q_{fb} &= \alpha_2 + \beta_{22} p_{fb} + \beta_{12} p_{mlk} + \beta_{23} p_{fs} + \beta_{2M} z_{Mprx} + \beta_{2C} z_{Lcap} + \beta_{2L} z_{Labr} \\
Q_{fs} &= \alpha_3 + \beta_{33} p_{fs} + \beta_{13} p_{mlk} + \beta_{23} p_{fb} + \beta_{3M} z_{Mprx} + \beta_{3C} z_{Lcap} + \beta_{3L} z_{Labr}
\end{aligned}
$$

Different combinations of the four equations were estimated. Each of the four was estimated by OLS; the group of three derived equations (Equation 21) were estimated separately from and jointly with the profit function.

3.8.2 SHARE EQUATION SPECIFICATION FOR THE TRANSLOG MODEL

The translog profit function for the two-output, two variable input and three quasi-fixed input case is represented in Equation 22. The derived share equations are given in Equation 23.

Equation 22:

$$\begin{aligned}\ln\pi = {} & \alpha_0 + \alpha_1 \ln p_{mlk} + \alpha_2 \ln p_{fb} + \alpha_3 \ln p_{fs} + \alpha_4 \ln p_{trd} \\ & + \beta_M \ln z_{Mprx} + \beta_C \ln z_{Lcap} + \beta_L \ln z_{Labr} \\ & + \tfrac{1}{2}(\beta_{11} \ln p_{mlk}^2 + \beta_{22} \ln p_{fb}^2 + \beta_{33} \ln p_{fs}^2 + \beta_{44} \ln p_{trd}^2) \\ & + \beta_{12} \ln p_{mlk} \ln p_{fb} + \beta_{13} \ln p_{mlk} \ln p_{fs} + \beta_{14} \ln p_{mlk} \ln p_{trd} \\ & + \beta_{23} \ln p_{fb} \ln p_{fs} + \beta_{24} \ln p_{fb} \ln p_{trd} + \beta_{34} \ln p_{fs} \ln p_{trd} \\ & + \tfrac{1}{2}(\gamma_{MM} \ln z_{Mprx}^2 + \gamma_{CC} \ln z_{Lcap}^2 + \gamma_{LL} \ln z_{Labr}^2) \\ & + \gamma_{MC} \ln z_{Mprx} \ln z_{Lcap} + \gamma_{ML} \ln z_{Mprx} \ln z_{Labr} + \gamma_{CL} \ln z_{Lcap} \ln z_{Labr} \\ & + \gamma_{1M} \ln p_{mlk} \ln z_{Mprx} + \gamma_{1C} \ln p_{mlk} \ln z_{Lcap} + \gamma_{1L} \ln p_{mlk} \ln z_{Labr} \\ & + \gamma_{2M} \ln p_{fb} \ln z_{Mprx} + \gamma_{2C} \ln p_{fb} \ln z_{Lcap} + \gamma_{2L} \ln p_{fb} \ln z_{Labr} \\ & + \gamma_{3M} \ln p_{fs} \ln z_{Mprx} + \gamma_{3C} \ln p_{fs} \ln z_{Lcap} + \gamma_{3L} \ln p_{fs} \ln z_{Labr} \\ & + \gamma_{4M} \ln p_{trd} \ln z_{Mprx} + \gamma_{4C} \ln p_{trd} \ln z_{Lcap} + \gamma_{4L} \ln p_{trd} \ln z_{Labr}\end{aligned}$$

Equation 23:

$$\begin{aligned}S_{mlk} &= \alpha_1 + \beta_{11} \ln p_{mlk} + \beta_{12} \ln p_{fb} + \beta_{13} \ln p_{fs} + \beta_{14} \ln p_{trd} \\ &\quad + \gamma_{1M} \ln z_{Mprx} + \gamma_{1C} \ln z_{Lcap} + \gamma_{1L} \ln z_{Labr} \\ S_{fb} &= \alpha_2 + \beta_{22} \ln p_{fb} + \beta_{12} \ln p_{mlk} + \beta_{23} \ln p_{fs} + \beta_{24} \ln p_{trd} \\ &\quad + \gamma_{2M} \ln z_{Mprx} + \gamma_{2C} \ln z_{Lcap} + \gamma_{2L} \ln z_{Labr} \\ S_{fs} &= \alpha_3 + \beta_{33} \ln p_{fs} + \beta_{13} \ln p_{mlk} + \beta_{23} \ln p_{fb} + \beta_{34} \ln p_{trd} \\ &\quad + \gamma_{3M} \ln z_{Mprx} + \gamma_{3C} \ln z_{Lcap} + \gamma_{3L} \ln z_{Labr} \\ S_{trd} &= \alpha_4 + \beta_{44} \ln p_{trd} + \beta_{14} \ln p_{mlk} + \beta_{24} \ln p_{fb} + \beta_{34} \ln p_{fs} \\ &\quad + \gamma_{4M} \ln z_{Mprx} + \gamma_{4C} \ln z_{Lcap} + \gamma_{4L} \ln z_{Labr}\end{aligned}$$

Similar to the Normalised Quadratic case, different combinations of the equations were estimated. The trade-income share equation was dropped from the supply system estimations to avoid singularity of the covariance matrix.

3.9 TESTING THE PROPERTIES OF THE PROFIT FUNCTION

Under the assumptions of profit maximising behaviour with a continuous and a twice-differentiable profit function, the parameters of the estimated equations must satisfy symmetry, convexity, monotonicity and homogeneity conditions.

3.9.1 Non-negativity

The estimated input demand and output supplies and profit should be zero or positive. This was evaluated at the level of each farm. By definition, quantities are non-negative values. Similarly, a profit maximising farmer will rather produce zero output than to incur negative profits.

3.9.2 Monotonicity

This property requires that the profit function strictly increases in output prices and strictly decreases in input prices [Chambers, 1988; Capalbo *et al.*, 1988; Higgins, 1986]. This property is tested through evaluation of the first derivatives of the profit function with respect to input and output prices. In the translog case, this implies evaluation of the profit shares. For inputs, the first derivatives of the profit function with respect to the input price should be non-positive. The first derivatives of the profit function with respect to the output prices should be non-negative. Since the functions approximate the true profit function and the first derivatives are expressions in the levels of the variables, the evaluation is done at the point of approximation [Capalbo *et al.*, 1988]. In the normalised quadratic case, this implies setting the values of the variables to zero and for the translog function the values are set to one.

3.9.3 Convexity

The necessary condition for convexity is that the Hessian matrix of second order derivatives of the profit function with respect to all prices be positive semi-definite. This implies that all the principal minors must have non-negative determinants {Capalbo *et al.*, 1988]. This follows from the fact that $\partial^2\pi/\partial p_i\partial p_i = \partial Q_i/\partial p_i > 0$ and $\partial^2\pi/\partial w_i\partial w_i = -\partial X_i/\partial w_i > 0$, making the profit function convex in input and output prices (i.e. output supply is upward sloping and input demand is downward sloping). Algebraically, the Hessian matrix is represented as follows:

Equation 24:

$$H = \begin{vmatrix} \frac{\partial^2 \pi}{\partial p_i \partial p_i} & \cdots & \frac{\partial^2 \pi}{\partial p_i \partial p_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 \pi}{\partial p_n \partial p_i} & \cdots & \frac{\partial^2 \pi}{\partial p_n \partial p_n} \end{vmatrix}$$

3.9.4 HOMOGENEITY

The Wald-test was used to test for the homogeneity restrictions. The Normalised Quadratic profit function is homogeneous in prices, but not in fixed factors. For the translog profit function to be homogeneous, the symmetry condition ($\beta_{ij}=\beta_{ji}$ and $\gamma_{mn}=\gamma_{nm,}$)' the additivity restriction ($\Sigma_i\ \alpha_i=0$ and $\Sigma_m\beta_m=1$), as well as the condition that the sum of the coefficients of the squared and interaction terms are zero ($\Sigma_i\beta_{ij}=\Sigma_m\gamma_{mn}=\Sigma_i\gamma_{im}=\Sigma_m\gamma_{im}=0$) must hold. However, homogeneity in prices can also be imposed by normalising the translog profit function.

3.9.5 SYMMETRY

Symmetry is imposed due to the restricted sample size. Without the symmetry condition, there are not sufficient degrees of freedom in order to estimate all the parameters of the specified equations.

The methodology described is used to estimate different combinations of single equation and system specifications of which the results are reported in the following chapter (Chapter 4: Results).