CHAPTER 6

A STOCHASTIC MODEL OF A RELIABILITY SYSTEM WITH A HUMAN OPERATOR

6.1 INTRODUCTION

With the advancement of methods and developments in artificial intelligence, computer systems and electronic systems, we find a high degree of automation all around. Nevertheless, it cannot be overlooked that human beings are inseparable parts of systems. Radars, motor vehicles, aircrafts and ships are examples of systems which are continuously monitored by human operators. Errors that are caused by human operators are referred to as 'human-errors' (Dhillon, 1980, 1984; Yadavalli & Bekker, 2005).

Dhillon has analysed several models incorporating the concepts of human reliability (1981, 1984). Yadavalli & Bekker (2005) studied a stochastic model of a two unit system with human error and common-cause failures. Their main focus was on the estimation of the steady-state availability (both classical and Bayesian) with the assumption of human error and common-cause failures. One common feature of all the models is that they are Markovian in nature. Kumar et al. (1986) analysed systems operating in fluctuating weather conditions and subject to critical human error, the models are being Markovian. Dhillon and Rayapati (1985) studied five models for a system which needs a human operator. In their models, a human operator is assumed to be working in one of the three states: normal, moderate stress and extreme stress. In all three states, the system is assumed to be subject to two failure modes, i.e. from each state, the system may fail because of self-corrected-human error and non-self-corrected human error. The system can recover from a self-corrected-human failure state, whereas it remains in a failed state when this occurs because of non-self-corrected human error. All the underlying failure distributions are assumed to be exponential. The organization of this chapter is as follows: Section 6.2 presents the system description and relevant notation; in Section 6.3, we represent transition probability functions and sojourn times, which are used in the subsequent analyses; Section 6.4 is a study of reliability analysis and the mean time to system failure; the availability analysis is presented in Section 6.5; the study of expected number of visits to a state and the profit analysis are presented in Section 6.6.

6.2 SYSTEM DESCIPTION AND NOTATION

1. The system is operated by a human operator. The system may fail because of its built-in nature or because of the human operator.
2. The human operator working on the system can be in one of three states: normal, moderate stress or extreme stress. The human operator is more prone to commit errors while in extreme stress state than in the other two states.
3. The system is subject to two failure modes irrespective of the state of the human operator: (a) failure because of self-corrected human-error, (b) failure because of non-self-corrected human error.
4. The rates of change of human operator condition from normal to moderate stress, to extreme stress and vice versa are all exponential. Further, human errors also occur at an exponential rate.
5. Repair time distributions for the system failed from the three human operator conditions are arbitrary and different from one another.
6. Failures are statistically independent.
7. The repaired system is as good as new.

NOTATION

λ_{ij} Constant rate of occurrence of human error, where i denotes the state of human operator and j denotes the system states

α_{lm} Constant rate of change of the state of human operator from l^{th} state to m^{th} state; $l = 1, 2, 3$ and $m = 1, 2, 3$ ($l \neq m$)

$G_{ij}(t)$ Repair time distribution for the system, where i denotes the state of human operator from which the system failed and j denotes the system state; $i = 1, 2, 3$ and $j = 1, 2$.

P_{ij} One-step transition probability from state S_i to S_j

P Transition probability matrix

μ_i Mean unconditional sojourn time of the system in S_i

$\mu^{(1)}$ The diag (μ_0, μ_3, μ_6)

ξ $(1, \quad 1, \quad 1)'$, column vector

N_{ij} Total time spent in a transit state S_j before the system failure, given that the system starts in S_i

$N^{(1)}$ $\left[N_{ij}\right]$, matrix

t_i' Total time spent in up-states given that the system starts to S_i

v_i $E(t_i')$

$V^{(1)}$ $(v_0, \quad v_3, \quad v_6)'$, column vector

b_{ij} P[System is absorbed in S_j $\big|$ system started in S_i]

B $\left[b_{ij}\right]$

E	$\{0, 1, 2, \ldots, 8\}$
E_1	$\{0, 3, 6\}$, up-states
E_2	$\{1, 2, 4, 5, 7, 8\}$, down-states
Π_i	Limiting probability that the Markov-Chain in S_i, $(i \in E)$
d_i	Determinant of the minor of D
ψ_i	Limiting probability of the system being in S_i, $(i \in E)$
Y_i	Earning rate of the system per unit time in S_j
g	Expected profit per unit time in steady state
r_{ij}	Fixed transition reward for a transition from S_i to S_j

6.3 TRANSITION PROBABILITIES AND SOJOURN TIMES

At any instant, the system can be in one of the following states: (See figure 6.1)

(i) Up-states:

$S_0\,(1, 0)$, $S_3\,(2, 0)$, $S_6\,(3, 0)$,

(ii) Down-states:

$S_1\,(1, 1)$, $S_2\,(1, 2)$, $S_4\,(2, 1)$,

$S_5\,(2, 2)$, $S_7\,(3, 1)$, $S_8\,(3, 2)$.

The first symbol in parenthesis denotes the human operator state and the second symbol denotes the state of the system.

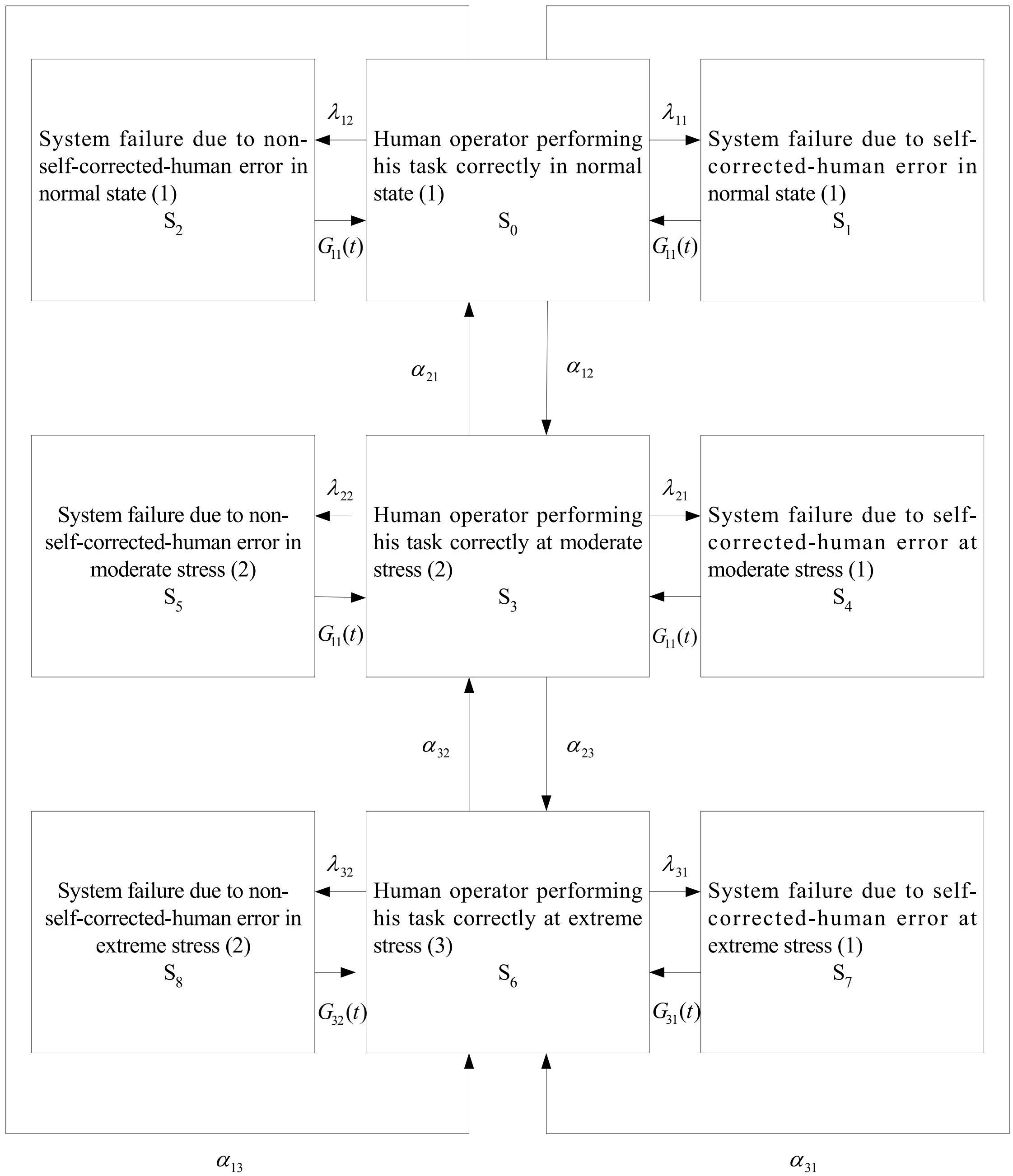

Figure 6.1: Transition Diagram

As indicated earlier, the states of human operator are (1) normal (2) moderate stress and (3) extreme stress. Similarly, the system states are 0: operative, 1: failed because of self-corrected-human error and 2: failed because of non-self-corrected-human error.

The system behavior can be described by a stochastic process $\{Z(t), t \geq 0\}$ with state space $E = \{0,1,\ldots,8\}$, where Z(*t*) denotes the state of the system at time *t*. It may be noted that the process Z(*t*), is a semi-Markovian and as such the well-known properties of Semi-Markovian Process (SMP) (see Cinlar (1975)) are applied to study the system behavior in detail.

The transition probabilities are given by:

$$P_{01} = \int_0^\infty \lambda_{11} e^{-(\lambda_{11}+\lambda_{12}+\alpha_{12}+\alpha_{13})t} dt$$

$$= \frac{\lambda_{11}}{\lambda_{11}+\lambda_{12}+\alpha_{12}+\alpha_{13}} \tag{6.1}$$

$$P_{02} = \int_0^\infty \lambda_{12} e^{-(\lambda_{12}+\lambda_{11}+\alpha_{12}+\alpha_{13})t} dt$$

$$= \frac{\lambda_{12}}{\lambda_{11}+\lambda_{12}+\alpha_{12}+\alpha_{13}} \tag{6.2}$$

$$P_{03} = \int_0^\infty \alpha_{12} e^{-(\alpha_{12}+\lambda_{11}+\lambda_{12}+\alpha_{13})t} dt$$

$$= \frac{\alpha_{12}}{\lambda_{11}+\lambda_{12}+\alpha_{12}+\alpha_{13}} \tag{6.3}$$

$$P_{06} = \int_0^\infty \alpha_{13} e^{-(\alpha_{13}+\alpha_{12}+\lambda_{11}+\lambda_{12})t} dt$$

$$= \frac{\alpha_{13}}{\alpha_{12} + \alpha_{13} + \lambda_{11} + \lambda_{12}} \tag{6.4}$$

$$P_{10} = P_{20} = 1 \tag{6.5}$$

$$P_{30} = \int_0^\infty \alpha_{21} e^{-(\alpha_{21}+\lambda_{12}+\lambda_{22}+\lambda_{23})t} dt$$

$$= \frac{\alpha_{21}}{\lambda_{21} + \lambda_{22} + \alpha_{21} + \alpha_{23}} \tag{6.6}$$

$$P_{34} = \int_0^\infty \lambda_{21} e^{-(\lambda_{21}+\lambda_{22}+\alpha_{21}+\alpha_{23})t} dt$$

$$= \frac{\lambda_{21}}{\lambda_{21} + \lambda_{22} + \alpha_{21} + \alpha_{23}} \tag{6.7}$$

$$P_{35} = \int_0^\infty \lambda_{22} e^{-(\lambda_{22}+\lambda_{21}+\alpha_{21}+\alpha_{23})t} dt$$

$$= \frac{\lambda_{22}}{\lambda_{21} + \lambda_{22} + \alpha_{21} + \alpha_{23}} \tag{6.8}$$

$$P_{36} = \int_0^\infty \alpha_{23} e^{-(\alpha_{23}+\lambda_{22}+\lambda_{21}+\alpha_{21})t} dt$$

$$= \frac{\alpha_{23}}{\lambda_{21} + \lambda_{22} + \alpha_{21} + \alpha_{23}} \tag{6.9}$$

$$P_{43} = P_{53} = 1 \tag{6.10}$$

$$P_{67} = \int_0^\infty \lambda_{31} e^{-(\lambda_{31}+\lambda_{32}+\alpha_{32}+\alpha_{31})t} dt$$

$$= \frac{\lambda_{31}}{\lambda_{31}+\lambda_{32}+\alpha_{31}+\alpha_{32}} \tag{6.11}$$

$$P_{68} = \int_0^\infty \lambda_{32} e^{-(\lambda_{32}+\lambda_{31}+\alpha_{32}+\alpha_{31})t} dt$$

$$= \frac{\lambda_{32}}{\lambda_{31}+\lambda_{32}+\alpha_{31}+\alpha_{32}} \tag{6.12}$$

$$P_{60} = \int_0^\infty \alpha_{31} e^{-(\alpha_{31}+\lambda_{31}+\lambda_{32}+\alpha_{32})t} dt$$

$$= \frac{\alpha_{31}}{\alpha_{31}+\lambda_{31}+\lambda_{32}+\alpha_{32}} \tag{6.13}$$

$$P_{63} = \int_0^\infty \alpha_{32} e^{-(\alpha_{32}+\alpha_{31}+\lambda_{31}+\lambda_{32})t} dt$$

$$= \frac{\alpha_{32}}{\lambda_{31}+\lambda_{32}+\alpha_{31}+\alpha_{32}} \tag{6.14}$$

and $\quad P_{76} = P_{86} = 1.$ (6.15)

The unconditional mean sojourn time of the system in state S_i are given below:

$$\mu_0 = \int_0^\infty e^{-(\lambda_{11}+\lambda_{12}+\alpha_{12}+\alpha_{13})t} dt$$

$$= \frac{1}{\lambda_{11}+\lambda_{12}+\alpha_{12}+\alpha_{13}} \tag{6.16}$$

$$\mu_1 = \int_0^\infty \bar{G}_{11}(t)dt = m_1 \text{ (say)} \tag{6.17}$$

$$\mu_2 = \int_0^\infty \bar{G}_{12}(t)dt = m_2 \text{ (say)} \tag{6.18}$$

$$\mu_3 = \int_0^\infty e^{-(\lambda_{21}+\lambda_{22}+\alpha_{21}+\alpha_{23})t}dt$$

$$= \frac{1}{\lambda_{21}+\lambda_{22}+\alpha_{21}+\alpha_{23}} \tag{6.19}$$

$$\mu_4 = \int_0^\infty \bar{G}_{22}(t)dt = m_4 \text{ (say)} \tag{6.20}$$

$$\mu_5 = \int_0^\infty \bar{G}_{22}(t)dt = m_5 \text{ (say)} \tag{6.21}$$

$$\mu_6 = \int_0^\infty e^{-(\lambda_{31}+\lambda_{32}+\alpha_{32}+\alpha_{31})t}dt$$

$$= \frac{1}{\lambda_{31}+\lambda_{32}+\alpha_{32}+\alpha_{31}} \tag{6.22}$$

$$\mu_7 = \int_0^\infty \bar{G}_{31}(t)dt = m_7 \text{ (say)} \tag{6.23}$$

and $\mu_8 = \int_0^\infty \bar{F}_{32}(t)dt = m_8$ (say). (6.24)

6.4 RELIABILITY ANALYSIS

We need to find the MTSF using reliability analysis. To obtain MTSF, we convert the down-states of the system into absorbing states, so that the transition probability matrix

(TPM) is steadily available in a canonical form and to obtain MTSF by the SMP approach (Agarwal et al. (1986)).

$$\boldsymbol{P} = \begin{array}{c} \\ 1 \\ 2 \\ 4 \\ 5 \\ 7 \\ 8 \\ 0 \\ 3 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccccc|ccc} 1 & 2 & 4 & 5 & 6 & 7 & 8 & & 0 & 3 & 6 \end{array} \\ \left[\begin{array}{c|c} I & O \\ \hline R & Q \end{array}\right] \end{array}$$

$$\boldsymbol{R} = \begin{array}{c} \\ 0 \\ 3 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 4 & 5 & 7 & 8 \end{array} \\ \begin{bmatrix} P_{01} & P_{02} & 0 & 0 & 0 & 0 \\ 0 & P_{34} & P_{35} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & P_{67} & P_{68} \end{bmatrix} \end{array}$$

$$\boldsymbol{Q} = \begin{array}{c} \\ 0 \\ 3 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 3 & 6 \end{array} \\ \begin{bmatrix} 0 & P_{03} & P_{06} \\ P_{30} & 0 & P_{36} \\ P_{60} & P_{63} & 0 \end{bmatrix} \end{array}$$

Then we know that (See Agarwal et al. (1986))

$$\boldsymbol{N}^{(1)} = (\boldsymbol{I} - \boldsymbol{Q})^{-1} \boldsymbol{M}^{(1)}$$

$$\boldsymbol{V}^{(1)} = \boldsymbol{N}^{(1)}$$

The elements of $\boldsymbol{V}^{(1)}$ produces the MTSF, given that the system starts in a transient state.

Specifically,

$$\boldsymbol{N}^{(1)} = \frac{1}{D}\begin{bmatrix} (1-P_{36}P_{63})\mu_0 & (P_{03}+P_{06}P_{63})\mu_3 & (P_{06}+P_{03}P_{36})\mu_6 \\ (P_{30}+P_{60}P_{36})\mu_0 & (1-P_{06}P_{60})\mu_3 & (P_{36}+P_{06}P_{30})\mu_6 \\ (P_{60}+P_{30}P_{63})\mu_0 & (P_{63}+P_{03}P_{60})\mu_3 & (1-P_{03}P_{30})\mu_6 \end{bmatrix} = V^{(1)} \tag{6.25}$$

$$\boldsymbol{V}^{(1)} = \frac{1}{D}\begin{bmatrix} (1-P_{36}P_{63})\mu_0 + (P_{03}+P_{06}P_{63})\mu_3 + (P_{06}+P_{03}P_{36})\mu_6 \\ (P_{30}+P_{60}P_{36})\mu_0 + (1-P_{06}P_{60})\mu_3 + (P_{36}+P_{06}P_{30})\mu_6 \\ (P_{60}+P_{30}P_{63})\mu_0 + (P_{63}+P_{03}P_{60})\mu_3 + (1-P_{03}P_{30})\mu_6 \end{bmatrix} \tag{6.26}$$

where

$$D = \left[(1-P_{36}P_{63}) - P_{30}(P_{03}+P_{06}P_{63}) + P_{60}(P_{06}+P_{03}P_{36})\right].$$

After substituting the required values into (6.25) and (6.26), we obtain

$$V_0 = \frac{B_0}{B} \tag{6.27}$$

$$V_3 = \frac{B_3}{B} \tag{6.28}$$

$$V_6 = \frac{B_6}{B} \tag{6.29}$$

where

$$B_0 = M_1 + (\alpha_{21}+\alpha_{12}+\alpha_{23})M_1M_2 + (\alpha_{32}+\alpha_{31}+\alpha_{13})M_1M_3 + PM_1M_2M_3$$

$$B_3 = M_2 + (\alpha_{31}+\alpha_{32}+\alpha_{23})M_2M_3 + (\alpha_{21}+\alpha_{12}+\alpha_{13})M_2M_3 + PM_1M_2M_3$$

$$B_6 = M_3 + (\alpha_{31}+\alpha_{12}+\alpha_{13})M_1M_3 + (\alpha_{32}+\alpha_{23}+\alpha_{21})M_2M_3 + PM_1M_2M_3$$

$$B = 1 + (\alpha_{12}+\alpha_{13})M_1 + (\alpha_{23}+\alpha_{31})M_2$$

$$+ (\alpha_{31}+\alpha_{32})M_3 + K_1M_2M_3 + K_2M_1M_3 + K_3M_1M_2$$

$$
\begin{aligned}
K_1 &= \alpha_{31}(\alpha_{23}+\alpha_{21})+\alpha_{21}\alpha_{32} \\
K_2 &= \alpha_{12}(\alpha_{32}+\alpha_{31})+\alpha_{32}\alpha_{13} \\
K_3 &= \alpha_{23}(\alpha_{12}+\alpha_{13})+\alpha_{21}\alpha_{32}
\end{aligned}
\tag{6.30}
$$

$$
\begin{aligned}
P = {} & (\alpha_{12}\alpha_{23}+\alpha_{23}\alpha_{31}+\alpha_{31}\alpha_{12})+(\alpha_{23}\alpha_{13}+\alpha_{32}\alpha_{21}+\alpha_{21}\alpha_{13}) \\
& +(\alpha_{12}\alpha_{32}+\alpha_{31}\alpha_{21}+\alpha_{23}\alpha_{13})
\end{aligned}
\tag{6.31}
$$

$$
M_1 = \frac{1}{\lambda_{11}+\lambda_{12}}; \quad M_2 = \frac{1}{\lambda_{21}+\lambda_{22}} \quad ; \ M_3 = \frac{1}{\lambda_{31}+\lambda_{32}}.
$$

In fact, M_1, M_2 and M_3 are MTSF's of respective decomposed subsystems, corresponding to normal, moderate and extreme stress states. It can be noted that (6.27) is in agreement with Dhillon and Rayapati (1985).

According to Agarwal et al. (1986), the absorption probabilities, i.e. the probabilities that the process starting from S_i (i = 0, 3, 6) enters the absorbing states S_j (j = 1, 2, 4, 5, 7, 8) are given by the matrix B = [b_{ij}], namely

$$
B = [I-Q]^{-1} R
$$

$$
= \frac{1}{D}
\begin{array}{c}
\\ 0 \\ 3 \\ 6
\end{array}
\begin{array}{c}
\begin{array}{cccccc} \quad 1 \quad & \quad 2 \quad & \quad 4 \quad & \quad 5 \quad & \quad 7 \quad & \quad 8 \quad \end{array} \\
\begin{bmatrix}
P_{01}C_{01} & P_{02}C_{01} & P_{34}C_{02} & P_{35}C_{02} & P_{67}C_{03} & P_{68}C_{03} \\
P_{01}C_{31} & P_{02}C_{31} & P_{34}C_{34} & P_{35}C_{32} & P_{67}C_{03} & P_{68}C_{33} \\
P_{01}C_{61} & P_{02}C_{61} & P_{34}C_{62} & P_{35}C_{62} & P_{67}C_{63} & P_{68}C_{63}
\end{bmatrix}
\end{array}
$$

where

$$
C_{01} = (1-P_{36}P_{63}); \qquad C_{02} = (P_{03}+P_{06}P_{63});
$$

$$
C_{03} = (P_{06}+P_{03}P_{36}); \ C_{04} = (P_{30}+P_{60}P_{36});
$$

$$
C_{32} = (1-P_{06}P_{60}); \qquad C_{33} = (P_{36}+P_{06}P_{30});
$$

$$
C_{61} = (P_{60}+P_{30}P_{63}); \ C_{62} = (P_{63}+P_{03}P_{60});
$$

$$
C_{63} = (1-P_{03}P_{30}). \tag{6.32}
$$

6.5 AVAILABILITY ANALYSIS

To obtain the steady-state availability A_∞, the limiting probabilities α_i's are required

$$\psi_i = \frac{\mu_i \pi_i}{\sum_{i \in E} \mu_i \pi_i}$$

$$= \frac{\mu_i d_i}{\sum_{i \in E} \mu_i d_i} \quad ; \quad i \in E_1 \tag{6.33}$$

$$A_\infty = \psi_0 + \psi_3 + \psi_6 . \tag{6.34}$$

Since d_i's are the determinants of the minors of D,

$$d_0 = (1 - P_{34} - P_{35})(1 - P_{67} - P_{68}) - P_{63}P_{36} \tag{6.35}$$

$$\begin{aligned} d_1 = (1 - P_{02})[(1 - P_{34} - P_{35})(1 - P_{67} - P_{68}) - P_{63}P_{36}] \\ - P_{30}[P_{03}(1 - P_{67} - P_{68}) + P_{63}P_{36}] \\ - P_{60}[P_{03}P_{36} + P_{06}(1 - P_{34} - P_{35})] \end{aligned} \tag{6.36}$$

$$\begin{aligned} d_2 = (1 - P_{01})[(1 - P_{34} - P_{35})(1 - P_{67} - P_{68}) - P_{63}P_{36}] \\ - P_{30}[P_{03}(1 - P_{67} - P_{68}) + P_{63}P_{36}] \\ - P_{60}[P_{03}P_{36} + P_{06}(1 - P_{34} - P_{35})] \end{aligned} \tag{6.37}$$

$$d_3 = (1 - P_{01} - P_{02})(1 - P_{67} - P_{68}) - P_{60}P_{06} \tag{6.38}$$

$$\begin{aligned} d_4 = (1 - P_{01} - P_{02})[(1 - P_{35})(1 - P_{67} - P_{68}) - P_{63}P_{36}] \\ - P_{30}[P_{03}(1 - P_{67} - P_{68}) + P_{63}P_{06}] \\ - P_{60}P_{06}(1 - P_{34}) - P_{60}P_{03}P_{36} \end{aligned} \tag{6.39}$$

$$\begin{aligned} d_5 = (1 - P_{01} - P_{02})[(1 - P_{34})(1 - P_{67} - P_{68}) - P_{63}P_{36}] \\ - P_{30}[P_{03}(1 - P_{67} - P_{68}) + P_{63}P_{06}] \end{aligned}$$

$$-P_{60}P_{06}(1-P_{34})-P_{60}P_{03}P_{36} \tag{6.40}$$

$$d_6=(1-P_{01}-P_{02})(1-P_{34}-P_{35})-P_{30}P_{04} \tag{6.41}$$

$$\begin{aligned} d_7=&(1-P_{01}-P_{02})[(1-P_{34}-P_{35})(1-P_{68})-P_{63}P_{36}] \\ &-P_{30}[P_{03}(1-P_{68})+P_{06}P_{63}] \\ &-P_{60}[P_{03}P_{36}+P_{06}(1-P_{34}-P_{35})] \end{aligned} \tag{6.42}$$

$$\begin{aligned} d_8=&(1-P_{01}-P_{02})[(1-P_{34}-P_{35})(1-P_{67})-P_{63}P_{36}] \\ &-P_{30}[P_{03}(1-P_{67})+P_{06}P_{63}] \\ &-P_{60}[P_{03}P_{36}+P_{06}(1-P_{34}-P_{35})]. \end{aligned} \tag{6.43}$$

Using the d_i's, the steady-state availability A_∞, can be obtained as

$$A_\infty=\frac{K_1+K_2+K_3}{\frac{K_1}{A_1}+\frac{K_2}{A_2}+\frac{K_3}{A_3}} \tag{6.44}$$

where $$A_1=\frac{1}{1+a_{11}+a_{12}}$$

$$A_2=\frac{1}{1+a_{21}+a_{22}}$$

$$A_3=\frac{1}{1+a_{31}+a_{32}}$$

$$a_{11}=\lambda_{11}M_1\ ;\quad a_{12}=\lambda_{12}M_2\ ;$$

$$a_{21}=\lambda_{21}M_4\ ;\quad a_{22}=\lambda_{22}M_5\ ;$$

$$a_{31}=\lambda_{31}M_7\ ;\quad a_{32}=\lambda_{32}M_8\,.$$

In fact, A_1, A_2 and A_3 are the steady-state availabilities of the decomposed sub-systems corresponding to normal, moderate and extreme stress states respectively. It may be noted that A_∞ is the harmonic mean of A_1, A_2 and A_3 with the weights K_1, K_2 and K_3 which are functions of α_i's. Also

(i) $\underset{1\le i\le 3}{Min}\, A_i \le A_\infty \le \underset{1\le i\le 3}{Max}\, A_i$

(ii) The above range of A_∞ is independent of α_i's.

6.6 EXPECTED NUMBER OF VISITS TO A STATE AND EXPECTED PROFIT

$$\delta_i = \frac{\mu_i}{\sum_{i\in E}\mu_i d_i}$$

Hence, from equations 6.16 – 6.24 and 6.35 – 6.43,

$$\delta_0 = \frac{K}{(\lambda_{11}+\lambda_{12}+\alpha_{12}+\alpha_{13})T}$$

$$\delta_1 = \frac{M_1 K}{T}$$

$$\delta_2 = \frac{M_2 K}{T}$$

$$\delta_3 = \frac{K}{(\lambda_{21}+\lambda_{22}+\alpha_{21}+\alpha_{23})T}$$

$$\delta_4 = \frac{m_4 K}{T}$$

$$\delta_5 = \frac{m_5 K}{T}$$

$$\delta_6 = \frac{K}{(\lambda_{31}+\lambda_{32}+\alpha_{32}+\alpha_{31})T}$$

$$\delta_7 = \frac{M_7 K}{T}$$

and $$\delta_8 = \frac{M_8 K}{T}$$

where

$$K = (\lambda_{11} + \lambda_{12} + \alpha_{12} + \alpha_{13})(\lambda_{21} + \lambda_{22} + \alpha_{21} + \alpha_{23})(\lambda_{31} + \lambda_{32} + \alpha_{31} + \alpha_{32})$$

$$T = \frac{K_1}{A_1} + \frac{K_2}{A_2} + \frac{K_3}{A_3}.$$

We follow the same approach as in Agarwal (1988), to find the expected profit

$$g = \frac{\sum_i \pi_i \mu_i \mathrm{V}_i}{\sum_i \pi_i \mathrm{V}_i}$$

$$\mu_i \mathrm{V}_i = \sum_i P_{ij} r_{ij} + y_i \mu_i \ .$$

Hence g can be calculated as

$$g = \frac{Z}{T}$$

where

$$g = \frac{K_1}{(\lambda_{11} + \lambda_{12} + \alpha_{12} + \alpha_{13})}[\lambda_{11} r_{01} + \lambda_{12} r_{02} + \alpha_{12} r_{03} + \alpha_{13} r_{06} + y_0]$$

$$+ \frac{\lambda_{11} K_1}{M_1}[M_1 r_{10} + y_1] + \frac{\lambda_{12} K_1}{M_2}[M_2 r_{20} + y_2]$$

$$+ \frac{K_2}{(\lambda_{21} + \lambda_{22} + \alpha_{23} + \alpha_{21})}[\lambda_{21} r_{34} + \lambda_{22} r_{35} + \alpha_{23} r_{36} + \alpha_{21} r_{30} + y_3]$$

$$+ \frac{\lambda_{21} K_2}{M_4}[M_4 r_{43} + y_4] + \frac{\lambda_{22} K_2}{M_5}[M_5 r_{53} + y_5]$$

$$+ \frac{K_3}{(\lambda_{31} + \lambda_{32} + \alpha_{32} + \alpha_{31})}[\lambda_{31} r_{67} + \lambda_{32} r_{68} + \alpha_{32} r_{63} + \alpha_{32} r_{60} + y_6]$$

$$+\frac{\lambda_{31}K_3}{M_7}[M_7 r_{76}+y_7]+\frac{\lambda_{32}K_3}{M_8}[M_8 r_{86}+y_8].$$

6.7 NUMERICAL ILLUSTRATION

As a numerical illustration, the behaviour of q_0, the mean time to system filure, has been studied with respect to changes in M_1, M_2 and M_3, the mean times to failure of decomposed sub-systems. It may be noted that MTSF increases as M_1 increases but there is always an upper bound on the MTSF when other factors M_2 and M_3 are kept constant. This is convincing, as, after a certain change, any improvement in M_1 is not likely to improve the MTSF value. In fact, for fixed values of M_2 and M_3, q_0 takes the form

$$q_0=\frac{M_1}{a+bM_2}\leq\frac{1}{b}$$

where a and b are functions of M_2 and M_3 and thus may be treated as constants as long as M_2 and M_3 are fixed. The above inequality gives the least upper bound for q_0.

For example, for $M_2 = 100$, least upper bounds for q_0 are given as follows for varying M_3:

M_3	25	40	60	100	...	∞
q_0	97.03	126.48	155.65	194.09	...	324.80

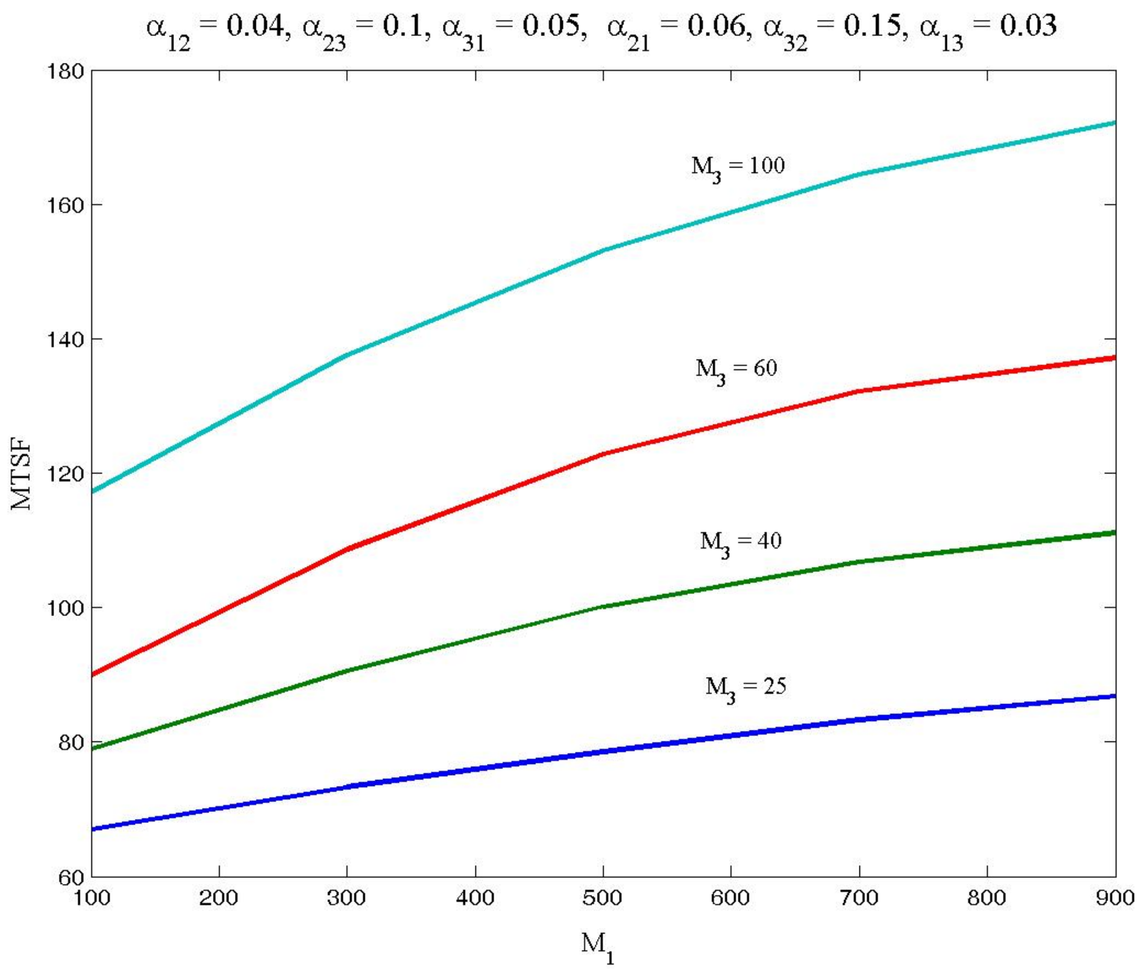

Figure 6.1

As the Mean time to failure of decomposed subsystem 1 (M_1) increases Mean Time to System Failure (MTSF) is an increasing function of M_1 (for different values of M_3 and fixed values of α_{12}, α_{23}, α_{31}, α_{21}, α_{32} and α_{13}).

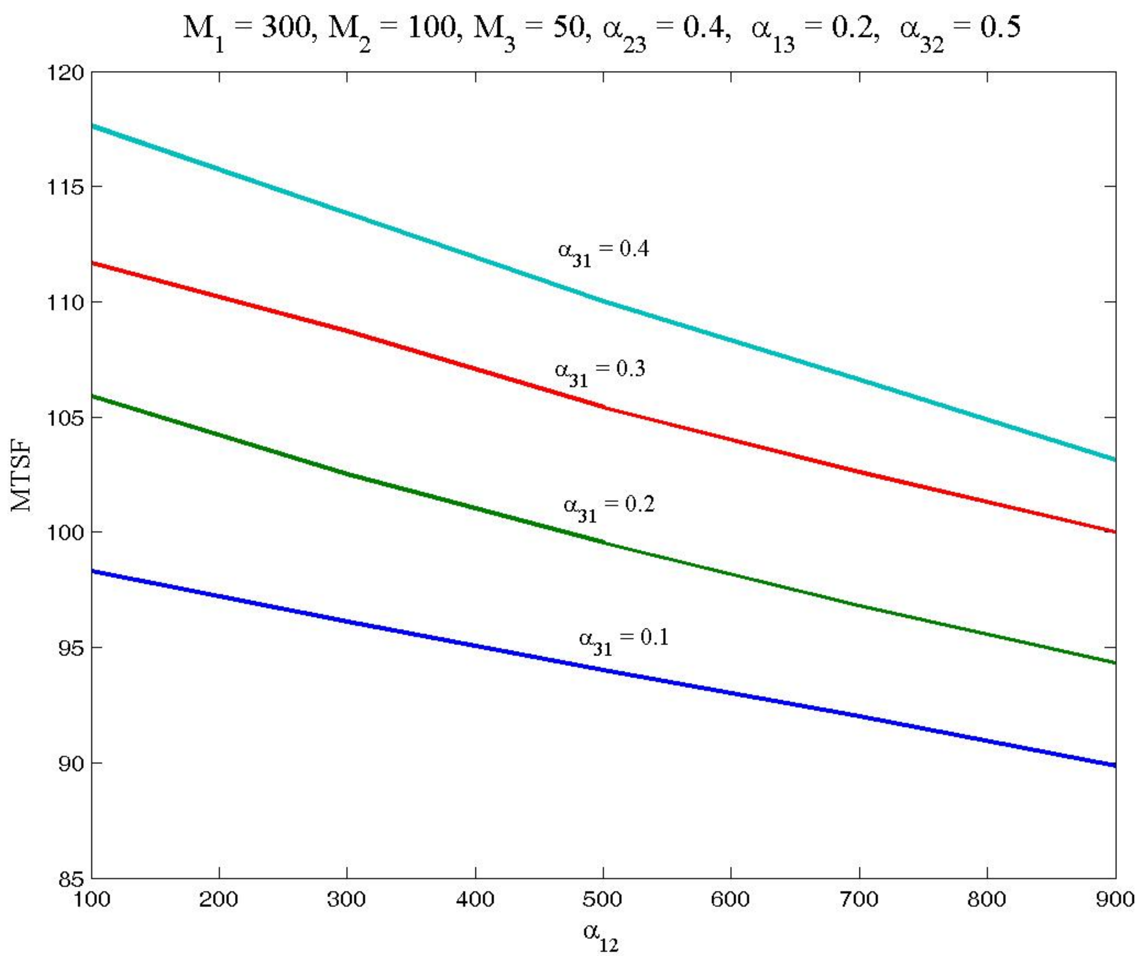

Figure 6.2

As the constant rate of change from first to second state, α_{12}, increases Mean Time to System Failure (MTSF) is a decreasing function of α_{12} (for different values of α_{31} and fixed values of M_1, M_2, M_3, α_{23}, α_{13}, α_{21}, and α_{32}).

6.8 CONCLUSION

A repairable system with a human operator is considered. The operator could be in one of the three states – normal, moderate stress and extreme stress. The system can fail due to self-corrected and non-self-corrected errors. With the help of a semi-Markovian process and a regeneration point technique various characteristics, like availability and MTSF are obtained (results in Figures 6.1 – 6.2). A cost-benefit analysis is also obtained for such a system.