CHAPTER 5

COST ANALYSIS OF A THREE-UNIT STANDBY REDUNDANT SYSTEM

5.1 INTRODUCTION

In the study of standby redundant systems, two unit systems have been examined extensively in the past. However, the study of *n*-unit redundant systems has received much less attention because of the built-in difficulties in the analysis. Kistner and Subramanian (1974) considered an *n*-unit warm standby system with a single repair facility. In this case, the pdf of the life time of the online unit was taken to be arbitrary while all the other distributions were exponential; these results were later extended to cover the case of several repair facilities by Subramanian, Venkatakrishnan and Kistner (1976). In the dual problem, viz., the *n*-unit system in which the pdf's of the repair time is arbitrary has been studied by Gopalan (1975). Gupta, Bajaj and Singh (1986) have studied the cost-benefit analysis of a single three unit redundant system with inspection, delayed replacement and two types of repair. Kalpakam et al. (1987) have considered a multi-component system in which *n* identical units connected in series; one needed for the system function, the units being supported by *m* spares and a single repair facility. Subramanian et al. (1987) studied a *n*-unit system in which the pdf of the life time is arbitrary and with the varying repair rate. Gupta and Bansal (1991) have analysed a cost function for a three unit standby system subject to random shocks and linear failure rates. It can be seen that in almost all the articles on standby systems in which the number of units is greater than two, at least one of the associated distributions is taken to be exponential.

The study of *n*-unit systems, even in the case of cold standbys, appears to be rather complicated when the pdf of both the life time of the online unit and that of the repair time are arbitrary. However, in this chapter, we study the case when n=3 and obtain elegantly many interesting performance measures.

The organisation of the chapter is as follows: In section 5.2, assumptions and notation are given and in section 5.3, the various system measures are obtained. In section 5.4 special cases are considered and in section 5.5 a comprehensive cost function is constructed. In section 5.6 numerical results are given to illustrate some of the results obtained.

5.2 ASSUMPTIONS AND NOTATION

5.2.1 ASSUMPTIONS

1. The system consists of three identical units with a single repair facility. Each individual unit performs the system function satisfactorily.
2. Initially at t=0, one unit is switched online and the other two units are installed as cold standbys. The initial condition is denoted by E_0.
3. After the completion of repair, a unit is installed back into the system as cold standby if at the epoch another unit functions online; else it is installed as the online unit.

The following table 5.1 describes all possible events:

Event	N(t-0)	N(t)	The system is
E_1	0	1	operable
E_2	1	2	operable
E_3	2	3	just not operable
E_4	2	1	operable
E_5	3	2	just operable
E_6	1	0	operable

Table 5.1

5.2.2 NOTATION

The following functions are defined only for regenerative events E_i.

$N_i(t)$ = Number of events of the type E_i in (0, t]

$U_i(t)$ = P[System unavailable at time t | E_i at t= 0]

$$M_i(t) = \lim_{\Delta \to 0} \frac{P[\text{a repair commencement in } (t,\ t + \Delta) | E_i \ at\ t = 0]}{\Delta}$$

$$\varphi(t) = \lim_{\Delta \to 0} \frac{P[\text{first system failure in } (t,\ t + \Delta) | E_i \ at\ t = 0]}{\Delta}$$

$$P_{ij}(t) = \lim_{\Delta \to 0} \frac{P[N_j(t+\Delta) = 1, N_k(t) = 0, k = 1,5 | E_i \ at\ t = 0]}{\Delta} \quad , \quad \text{for } i, j = 1, 5$$

$$P_{i3}(t) = \lim_{\Delta \to 0} \frac{P[N_3(t+\Delta) = 1, N_k(t) = 0, k = 1,3,5 | E_i \ at\ t = 0]}{\Delta}$$

$\Pi_{i3}(t)$ = P[System in down state at time t, no E_1 or E_5 events in (0, t]$|$ E_i at t= 0]

$f(\cdot)$ = pdf of the life time of a unit, while operating online

$g(\cdot)$ = pdf of the repair time of a failed unit.

5.3 ANALYSIS

It is noted that the events E_1 and E_5 listed in table 5.1 are regenerative, while the rest are not.

5.3.1 AUXILLIARY FUNCTIONS

The following expressions for the probabilities $P_{ij}(t)$'s and Π_{ij} (t)'s can be obtained:

By definition, $P_{ij}(t)$ (i, j = 1, 5) denote the probability that an E_j event occurring in (t, t + dt) given that an event E_i had occurred at time t = 0 and that no E_1 or E_5 event occurs in (0, t]. Similarly $P_{i3}(t)dt$ refers to the probability of a system breakdown on E_i event had occurred at time t = 0 and that no E_1 or E_5 event occurs in (0, t]. Hence,

$$P_{11}(t) = f(t)G(t) + \sum_{n=1}^{\infty} \int\int \ldots \int f(u_1)g(v_1)f(u_2-u_1)g(v_2-v_1)\ldots g(v_n-v_{n-1}) f(t-u_n)G(t-v_n)du_1dv_1\ldots du_ndv_n, \quad 0 \le u_1 \le v_1 \le \ldots \le u_n \le v_n \le t. \quad (5.1)$$

This equation is obtained by considering the following two mutually exclusive and exhaustive cases:

(a) The repair of a unit commenced at t = 0 is completed before the online unit fails

(b) The online unit fails before this repair completion.

In this case, since by definition of $P_{11}(t)$, no E_3 event (since no E_5 event can occur) can occur in (0, t], the third unit cannot fail before the repair completion. This way a sequence of E_2 and E_4 can occur any number of times before the repair completion. This way a sequence of E_2 and E_4 can occur any number of times before the ultimate occurrence of an E_1 event.

By similar arguments, we get

$$P_{15}(t) = \int_0^t f(u)F(t-u)g(t)du + \sum_{n=1}^{\infty} \int \cdots \int f(u_1)g(v_1)f(u_2-u_1)g(v_2-v_1)\ldots f(u_{n+1}-u_n)$$
$$F(t-u_{n+1})g(t-v_n)du_1dv_1\ldots du_ndv_ndu_{n+1} \quad (5.2)$$

$$P_{13}(t) = \int_0^t f(u)f(t-u)G(t)du + \sum_{n=1}^{\infty} \int \cdots \int f(u_1)g(v_1)f(u_2-u_1)g(v_2-v_1)\ldots f(u_{n+1}-u_n)$$
$$f(t-u_{n+1})G(t-v_n)du_1dv_1\ldots du_ndv_ndu_{n+1}. \quad (5.3)$$

Also, by its definition, $\Pi_{i3}(t)$ refer to the state probabilities of the system being down given that at time t = 0, E_i had occurred and that no E_1 or E_5 event occurred in (0, t]. We have

$$\Pi_{i3}(t) = \int_0^t f(u)F(t-u)G(t)du + \sum_{n=1}^{\infty} \int \cdots \int f(u_1)g(v_1)f(u_2-u_1)g(v_2-v_1)\ldots f(u_{n+1}-u_n)$$
$$F(t-u_{n+1})G(t-v_n)du_1dv_1\ldots du_ndv_ndu_{n+1} \quad (5.4)$$

$$P_{51}(t) = \int_0^t g(u)G(t-u)f(t)du + \sum_{n=1}^{\infty} \int \cdots \int g(u_1)f(v_1)g(u_2-u_1)f(v_2-v_1)\ldots g(u_{n+1}-u_n)$$
$$f(t-v_n)G(t-v_{n+1})du_1dv_1\ldots du_ndv_ndu_{n+1}. \quad (5.5)$$

For all these expressions, the integrations have to be performed for $0 \le u_1 \le v_1 \le \ldots \le u_n \le v_n \le u_{n+1} < t$ while for the following expressions, it has to be performed for $0 \le u_1 \le v_1 \le \ldots \le u_n \le v_n < t$.

$$P_{55}(t) = g(t)F(t) + \sum_{n=1}^{\infty} \int \cdots \int g(u_1)f(v_n)g(u_2-u_1)f(v_2-v_1)\ldots g(u_n-u_{n-1})$$
$$f(v_n-v_{n-1})F(t-v_n)g(t-u_n)du_1dv_1\ldots du_ndv_n \quad (5.6)$$

$$P_{53}(t) = f(t)G(t) + \sum_{n=1}^{\infty} \int \cdots \int g(u_1)f(v_1)g(u_2-u_1)f(v_2-v_1)\ldots g(u_n-u_{n-1})$$

$$f(v_n - v_{n-1}) f(t - v_n) G(t - u_n) du_1 dv_1 \ldots du_n dv_n \qquad (5.7)$$

$$\Pi_{53}(\mathrm{t}) = F(t)G(t) + \sum_{n=1}^{\infty} \int\int \ldots \int g(u_1) f(v_1) g(u_2 - u_1) f(v_2 - v_1) \ldots g(u_n - u_{n-1})$$
$$f(v_n - v_{n-1}) F(t - v_n) G(t - u_n) du_1 dv_1 \ldots du_n dv_n \;. \qquad (5.8)$$

5.3.2 RELIABILITY ANALYSIS

We have

$$\varphi_0(t) = f(t) © \varphi_1(t)$$

$$\varphi_1(t) = P_{11}(t) © \varphi_1(t) + P_{13}(t) \,.$$
(5.9)

The equation $\varphi(t)$ is derived by observing the fact that the online unit has to fail before t, if there is to be a system failure in (t, t + Δ). The equation for $\varphi_1(t)$ is obtained by considering the following mutually exclusive and exhaustive cases:

(a) E_1 event occurs in (u, u + du), u < t

(b) no E_1 event occurs before t and the system fails in (t, t + Δ).

Hence the reliability of the system is given by

$$R_0(t) = \int_t^{\infty} \varphi_0(u) du \,. \qquad (5.10)$$

5.3.3 AVAILABILITY ANALYSIS

It is easier to write the equations governing the unavailability of the system. We have, by arguments similar to those in reliability analysis:

$$U_0(t) = f(t) © U_1(t)$$

$$U_1(t) = P_{11}(t) © U_1(t) + \ P_{15}(t) © U_1(t) + \ \pi_{13}(t)$$

$$U_5(t) = P_{51}(t) © U_1(t) + \ P_{55}(t) © U_5(t) + \ \pi_{53}(t) .$$
(5.11)

Solving the equations in (5.11), by using the Laplace transform technique, we get

$$U_0^*(s) = f^*(s)\frac{P_{15}^*(s)\Pi_{53}^*(s) + \Pi_{13}^*(s)[1 - P_{55}^*(s)]}{[1 - P_{11}^*(s)][1 - P_{55}^*(s)] - P_{15}^*(s)P_{51}^*(s)} .$$

By inverting $U_0^*(s)$, wet get $U_0(t)$.

The steady state availability is given by $A_0 = 1 – U_0$ where U_0 is the steady state value of U_0 (t) obtained by using the relation

$$\lim_{s \to 0} sU_0^*(s) = U_0 .$$

5.3.4 MEASURES OF SYSTEM PERFORMANCE

5.3.4.1 EXPECTED NUMBER OF TRANSITIONS FROM STATE 0 TO STATE 1 in (0, t]

The expected number of visits by the repairman in (0, t] is given by $\int_0^t V_0(u)du$.

The equations governing $V_i(t)$ are:

$$V_0(t) = f(t) © V_1(t) + f(t)$$

$$V_1(t) = P_{11}(t) + \ P_{11}(t) © V_1(t) + \ P_{15}(t) © V_5(t)$$

and $$V_5(t) = P_{51}(t) © V_1(t) + \ P_{55}(t) © V_5(t) .$$
(5.12)

$V_0(t)$ can be obtained using the Laplace transform technique.

5.3.4.2 EXPECTED NUMBER OF REPAIRS COMMENCED IN (0, t]

The expected number of repairs commenced in (0, t] is given by $\int_0^t M_0(u)du$. The governing equations for $M_0(t)$ are:

$$M_0(t) = f(t) + f(t) © M_1(t)$$

$$M_1(t) = P_{11}(t) + P_{11}(t) © M_1(t) + P_{15}(t) + P_{15}(t) © M_5(t) + \xi_{121}(t)$$

$$M_5(t) = P_{51}(t) © M_1(t) + P_{55}(t) + P_{55}(t) © M_5(t) + \xi_{21}(t) \quad (5.13)$$

where

$$\xi_{121}(t) = \int_0^t f(u)F(t-u)g(t)du + \sum_{n=2}^{\infty} \int \dots \int f(u_1)g(v_1)f(u_2-u_1)g(v_2-v_1)\dots f(u_n-u_{n-1}) F(t-u_n)g(t-v_{n-1})du_1dv_1\dots dv_{n-1}dv_n$$

$$\xi_{21}(t) = g(t)F(t) + \sum_{n=1}^{\infty} \int \dots \int g(u_1)f(v_1)g(u_2-u_1)f(v_2-v_1)\dots g(u_n-u_{n-1}) f(v_n-v_{n-1})F(t-v_n)g(t-v_n)du_1\dots du_n dv_n .$$

These integrals are to be evaluated for $0 \le u_1 \le v_1 \le \dots \le u_n \le v_n \le u_{n+1} \le t$.

5.3.4.3 EXPECTED NUMBER OF REPAIRS COMPLETED IN (0, t]

The expected number of repairs completed in (0, t] is given by $\int_0^t M_0(u)du$. We have

$$M_0(t) = f(t) © M_1(t)$$

$$M_1(t) = P_{11}(t) © M_1(t) + P_{15}(t) © M_5(t) + \eta_1(t)$$

$$M_2(t) = P_{51}(t) \copyright M_1(t) + \quad P_{55}(t) + P_{55}(t) \copyright M_5(t) + \eta_2(t)$$

(5.14)

where

$$\eta_1(t) = g(t)F(t) + \sum_{n=1}^{\infty} \int \cdots \int f(u_1)g(v_1)f(u_2 - u_1)f(v_2 - v_1)\ldots f(u_n - u_{n-1})$$

$$\{g(t - v_{n-1}) + g(v_n - v_{n-1})g(t - v_n)F(t - u_n)du_1dv_1\ldots du_ndv_n \,;$$

$$0 \le u_1 \le v_1 \le \ldots \le u_n \le v_n \le t$$

$$\eta_2(t) = g(t)F(t) + \int_0^t g(u)g(t-u)F(t)du + \sum_{n=1}^{\infty} \int \cdots \int g(u_1)f(v_1)g(u_2 - u_1)$$

$$f(v_2 - v_1)\ldots f(v_n - v_{n-1})\{g(t - u_n) + g(u_{n+1} - u_n)g(t - v_{n+1})$$

$$F(t - v_n)du_1dv_1\ldots du_ndv_ndv_{n+1}.$$

$M_0(t)$ can be solved from this set of equations.

5.3.4.4 EXPECTED NUMBER OF SYSTEM BREAKDOWNS IN (0, t]

The expected number of system breakdowns is given by $\int_0^t D_0(u)du$. We have

$$D_0(t) = f(t) \copyright D_1(t)$$

$$D_1(t) = P_{11}(t) \copyright D_1(t) + \quad P_{15}(t) \copyright D_5(t) + P_{13}(t)$$

$$D_2(t) = P_{51}(t) \copyright D_1(t) + \quad P_{55}(t) \copyright D_5(t) + P_{53}(t)$$

(5.15)

By Laplace transforms technique, we get $D_0(t)$.

5.3.4.5 EXPECTED NUMBER OF SYSTEM RECOVERIES IN (0, t]

The expected number of system recoveries is given by $\int_0^t S_0(u)du$. In this case, the govening equations for $S_0(t)$ are:

$$S_0(t) = f(t) © S_1(t)$$

$$S_1(t) = P_{11}(t) © S_1(t) + P_{15}(t) © S_5(t) + P_{15}(t)$$

and $S_2(t) = P_{51}(t) © S_1(t) + P_{55}(t) © S_5(t) + P_{55}(t)$.
(5.16)

Equations (5.16) can be solved for $S_0(t)$.

REMARK:

It is noted that the steady state value of these expected numbers also represent the respective expected numbers per unit time in the steady state. Further, in the steady state, the expected number per unit time of the system breakdowns and recoveries are equal while that of the repair commencements is equal to repair completions.

5.4 SPECIAL CASES:

In this section, we consider two important special cases of the general model studied.

5.4.1 MODEL 1

All the results obtained in section 5.3 are deduced for the special case where the life time distribution of the online unit is general and the repair time distribution is exponential.

By setting g(t) $= \mu e^{-\mu t}$, the various P_{ij}(t) and Π_{i3}(t)'s reduce to simpler form. For illustration purposes, we consider P_{11}(t):

Substituting g(t) $= \mu e^{-\mu t}$ in equation (5.1), we have

$$P_{11}(t) = f(t)[1-e^{-\mu t}] + \sum_{n=1}^{\infty} \int_0^t \int_0^{v_1} \ldots \int_0^{v_1} f(u_1) f(u_2-u_1) f(u_n-u_{n-1}) f(t-u_n) \mu^{n-1}$$

$$[\mu e^{-\mu v_n} - \mu e^{-\mu t}] du_1 dv_1 \ldots du_n dv_n \, .$$

Changing the order of integration, we have

$$P_{11}(t) = \quad f(t)[1-e^{-\mu t}] + \sum_{n=1}^{\infty} \int_0^t \int_0^{u_n} \ldots \int_0^{u_2} f(u_1) \mu(u_2-u_1) f(u_2-u_1) \ldots f(u_n-u_{n-1})$$

$$f(u_n-u_{n-1}) f(t-u_n) e^{-u\mu_n}[1-\mu e^{-\mu(t-u_n)} - f(t-u_n) e^{-\mu(t-\mu_n)}] du_1 dv_1 \ldots du_n$$

which gives

$$P_{11}(t) = f(t)[1-e^{-\mu t}] + f(t)e^{-\mu t} \copyright \sum_{n=1}^{\infty} \{f(t)\mu t e^{-\mu t}\}^{(n-1)} \copyright f(t)[1-e^{-\mu t} - \mu t e^{-\mu t}] \, .$$

The simplified expressions for the other $P_{ij}(t)$'s and $\Pi_{i3}(t)$'s are obtained by similar arguments. By substituting these expressions in the corresponding integral equations and solving them, we get the results for the various system characteristics as:

$$\text{MTSF} = \frac{-f^{*'}(0)[1+f^*(\mu)+\mu f^{*'}(\mu)+\{f^*(\mu)\}^2]}{\{f^*(\mu)\}^2}$$

$$A_0 = \frac{-\mu f^{*'}(0)[1+\mu f^{*'}(\mu)]}{\{f^*(\mu)\}^2 - \mu^2 f^{*'}(0) f^{*'}(\mu) - \mu f^{*'}(0)}$$

$$V_0 = \frac{\mu^2[1+\mu f^{*'}(\mu) - f^*(\mu)]}{\{f^*(\mu)\}^2 - \mu^2 f^{*'}(0) f^{*'}(\mu) - \mu f^{*'}(0)}$$

$$m_0 = M_0 = \frac{\mu[1+\mu f^{*'}(\mu)]}{\{f^*(\mu)\}^2 - \mu^2 f^{*'}(0) f^{*'}(\mu) - \mu f^{*'}(0)}$$

$$D_0 = S_0 = \frac{\mu\{f^{*'}(\mu)\}^2}{\{f^*(\mu)\}^2 - \mu^2 f^{*'}(0) f^{*'}(\mu) - \mu f^{*'}(0)} \, .$$

As is to be expected, m_0 and D_0 are equal to M_0 and S_0 respectively.

5.4.2 MODEL 2

In this section, the various system characteristics are deduced for the special case when the life time distributions of the online unit is exponential with parameter λ and the repair time distribution is general. By following the same procedure as in Model 1, all $P_{ij}(t)$'s and $\Pi_{i3}(t)$'s reduce to simpler form and the system measures become

$$\text{MTSF} = \frac{3 + 2\lambda g^{*}(\lambda) - 2g^{*}(\lambda)}{\lambda[1 - g^{*}(\lambda) + \lambda g^{*'}(\lambda)]}$$

$$A_0 = \frac{[1 + \lambda g^{*}(\lambda)]}{\{g^{*}(\lambda)\}^2 - \lambda^2 g^{*'}(0) f^{*'}(\lambda) - \lambda g^{*'}(0)}$$

$$V_0 = \frac{\lambda \{g^{*'}(\lambda)\}^2}{\{g^{*}(\lambda)\}^2 - \lambda^2 g^{*'}(0) g^{*'}(\lambda) - \lambda g^{*'}(0)}$$

$$m_0 = M_0 = \frac{\lambda[1 + g^{*'}(\lambda)]}{\{g^{*}(\lambda)\}^2 - \lambda^2 g^{*'}(0) g^{*'}(\lambda) - \lambda g^{*'}(0)}$$

$$D_0 = S_0 = \frac{\lambda[1 - g^{*}(\lambda) + \lambda g^{*'}(\lambda)]}{\{g^{*}(\lambda)\}^2 - \lambda^2 g^{*'}(0) g^{*'}(\lambda) - \lambda g^{*'}(0)}.$$

5.5 COST ANALYSIS

In this section, we construct a comprehensive cost function per unit time in the steady state.

1. The costs due to the visits by the repairman to the repair facility per unit time is βV_0, where β is the cost per visit.

2. The cost associated with the repair rate is $r\ (\frac{1}{MRT})$, where r (> 0) is the constant of proportionality associated with the mean repair rate.

$$CF = \alpha U_0 + \beta V_0 + V(\frac{1}{MRT}) + \eta D_0 \qquad (5.17)$$

This cost function is to be optimised with respect to the control parameter MRT within some known bounds.

5.6 NUMERICAL RESULTS

In this section, some of the results obtained for models 1 and 2 are illustrated with numerical examples. We consider the following special cases for this purpose

MODEL 1

We assume that

$$f(t) = \frac{a_1 a_2}{a_2 - a_1}(e^{-a_1 t} - e^{-a_2 t}); \qquad a_1 > 0, a_2 > 0$$

In figures 5.1 to 5.6, three cases are considered for each characteristic corresponding to three different mean failure times to three different failure times of the online unit; (a_1, a_2) were chosen randomly in increasing order, namely

$(a_1, a_2):(0.058,0.2),(0.067,0.2),(0.076,0.2)$.

MODEL 2

In this model, we assume that

$$g(t) = \mu^2 t e^{-\mu t}.$$

Then MRT is $\frac{2}{\mu}$. For this model also, three cases are considered for each characteristic corresponding to the three values of mean failure times, viz., 0.058, 0.067 and 0.076. Figures 5.7 to 5.12 gives the variation of the various characteristics when MRT is varied. The results demonstrate the following results, viz., as the MRT of a failed unit increase, for the assumed parametric structure thereby giving a unique optimal.

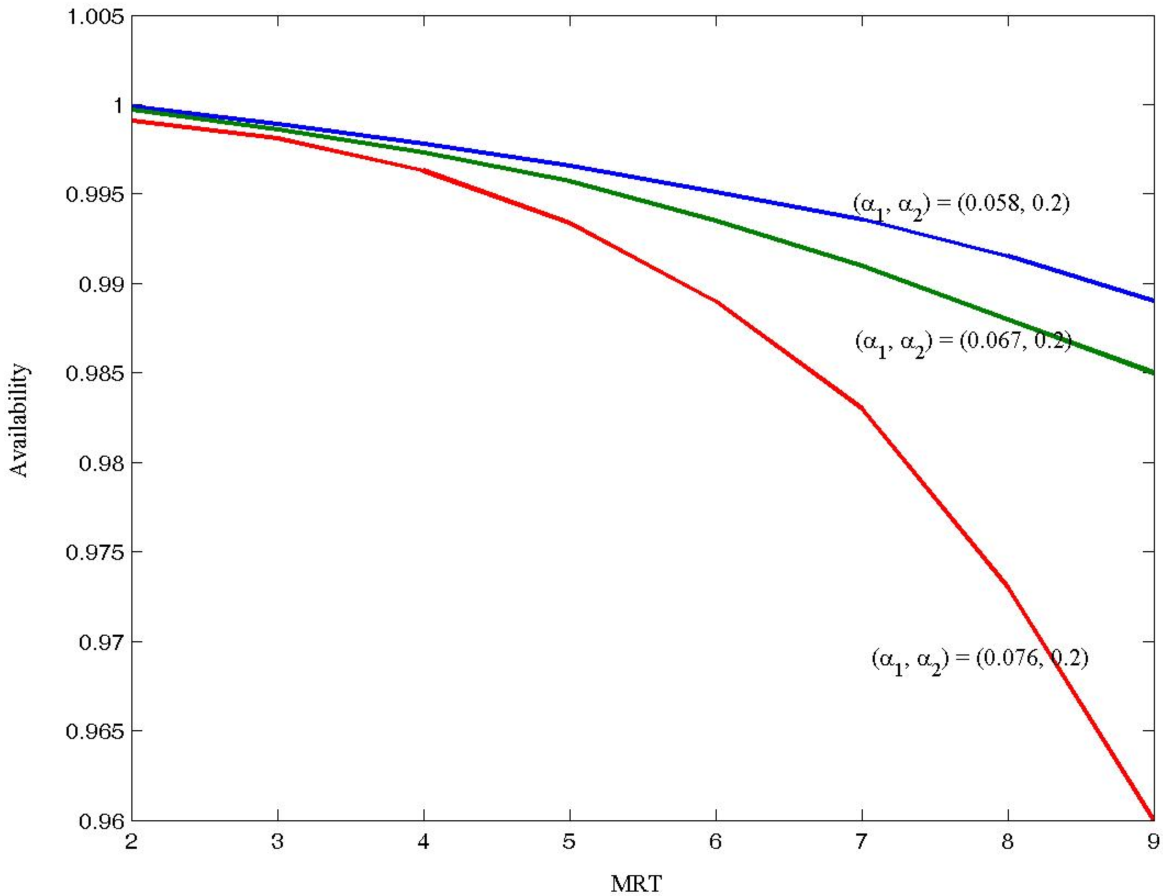

Figure 5.1 (Model 1)

As the Mean Repair Time (MRT) increases the steady-state availability is a decreasing function of MRT (for the different values of α_1 and α_2).

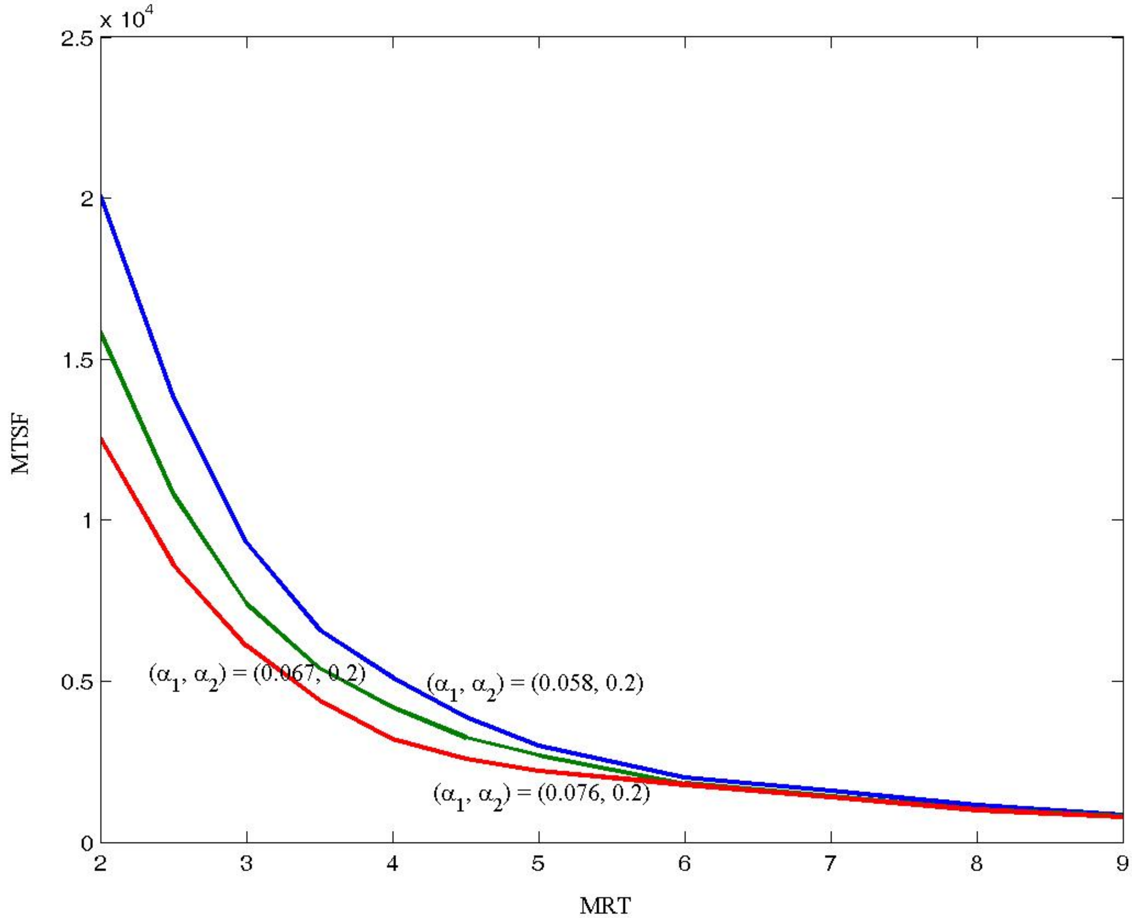

Figure 5.2 (Model 1)

As the Mean Repair Time (MRT) increases the Mean Time to System Failure (MTSF) is a decreasing function of MRT (for different values of α_1 and α_2) with almost convergence of MTSF at MRT = 7.

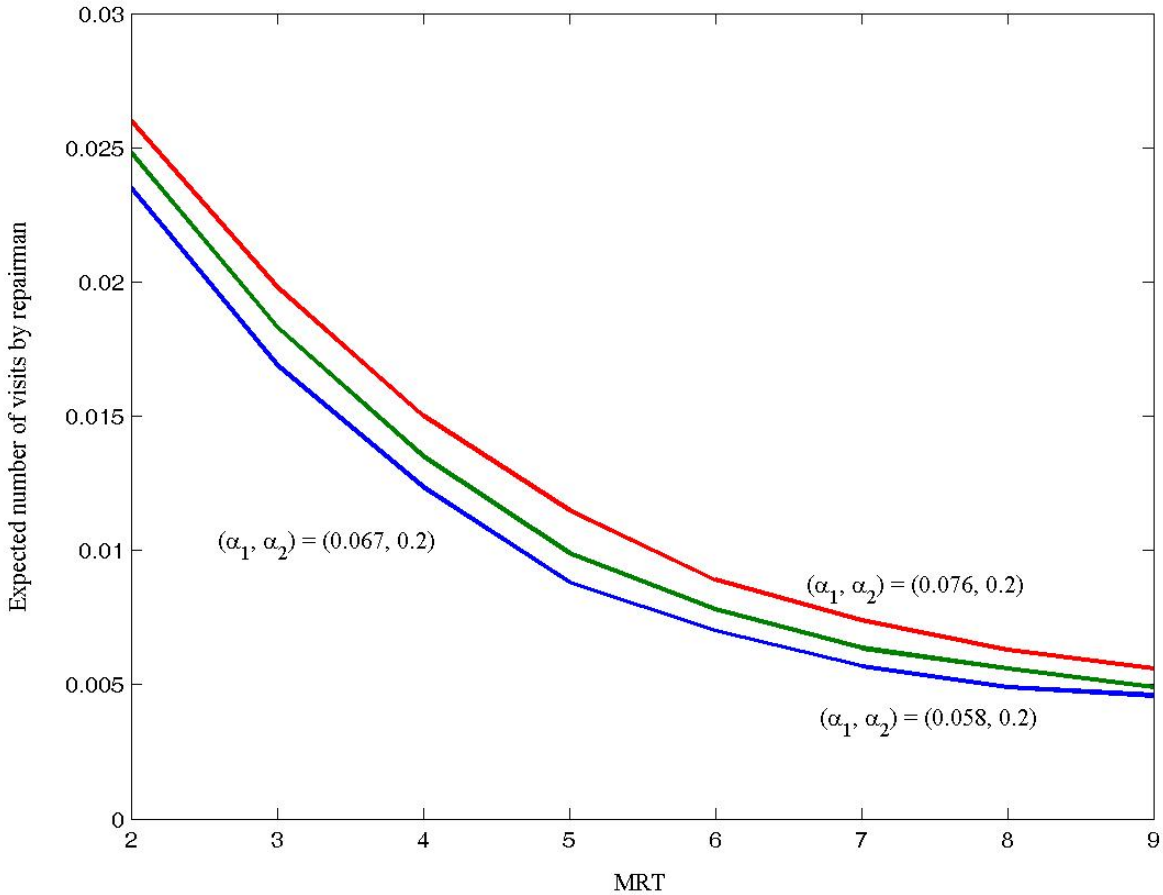

Figure 5.3 (Model 1)

As the Mean Repair Time (MRT) increases the Expected number of visits of the repairman is a decreasing function of MRT (for different values of α_1 and α_2).

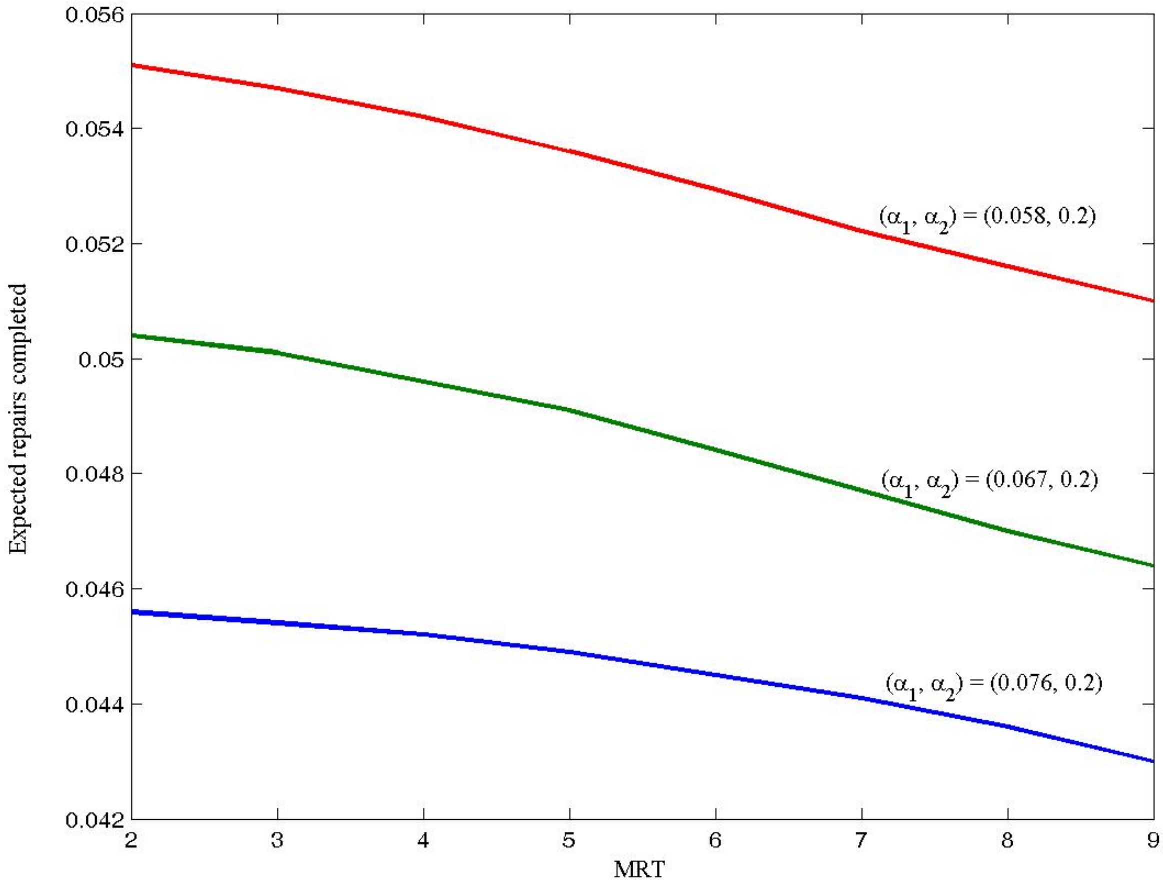

Figure 5.4 (Model 1)

As the Mean Repair Time (MRT) increases the Expected repairs completed is a decreasing function of MRT (for different values of α_1 and α_2).

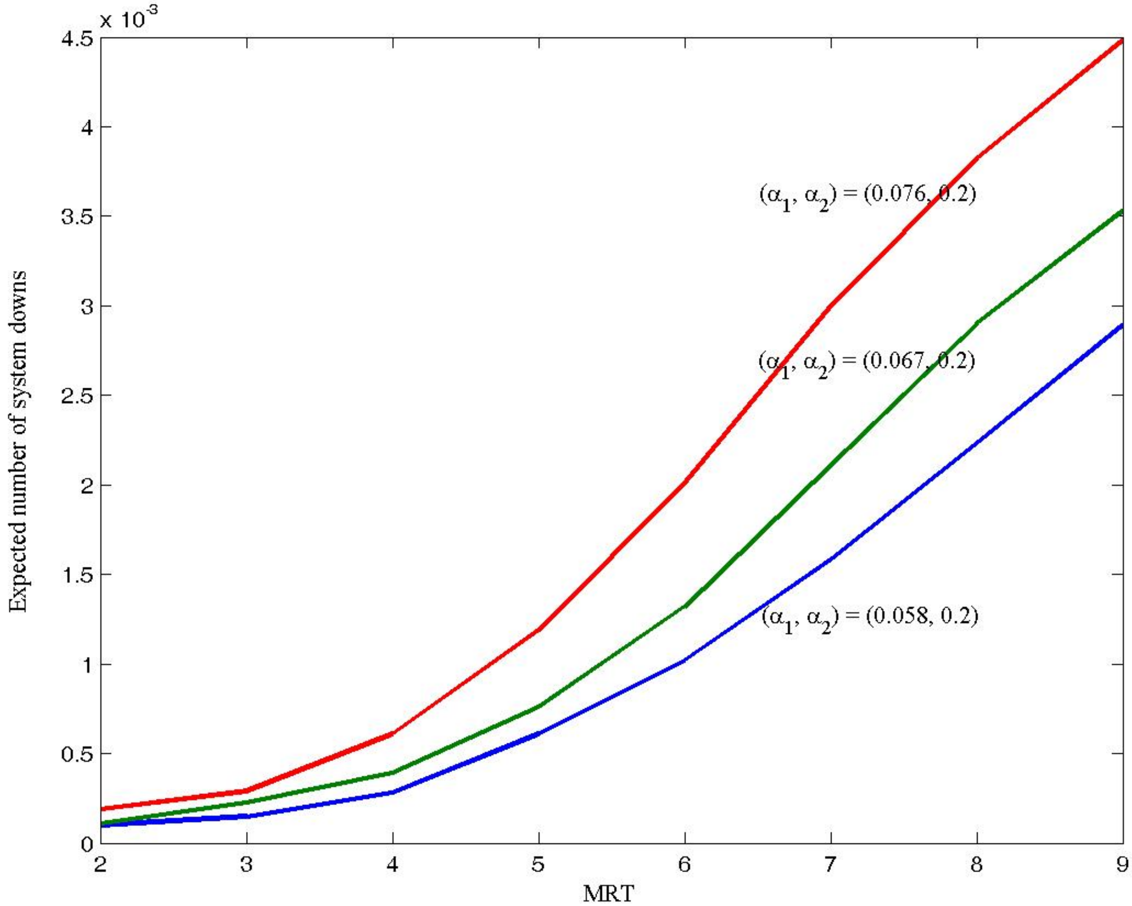

Figure 5.5 (Model 1)

As the Mean Repair Time (MRT) increases the Expected number of system downs is an increasing function of MRT (for different values of α_1 and α_2).

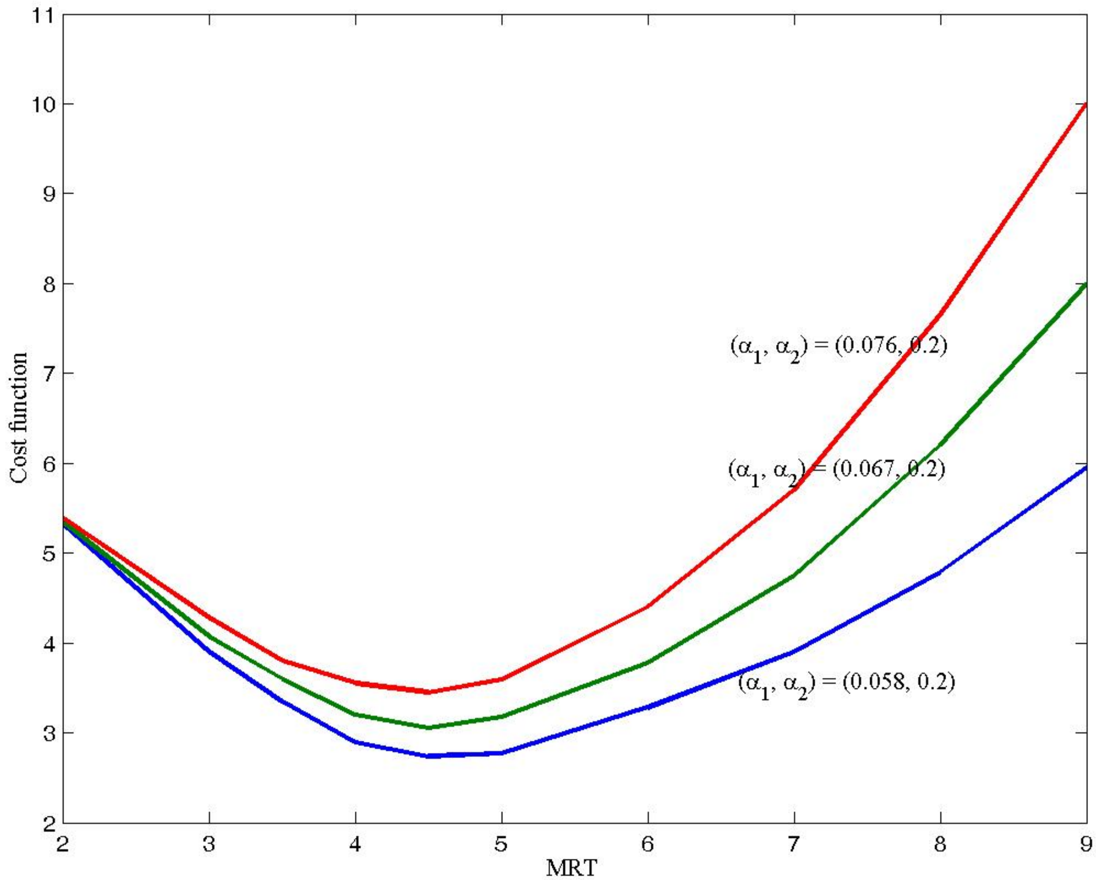

Figure 5.6 (Model 1)

As the Mean Repair Time (MRT) increases the Cost function first decreases but then increases after MRT = 4.5 (for different values of α_1 and α_2).

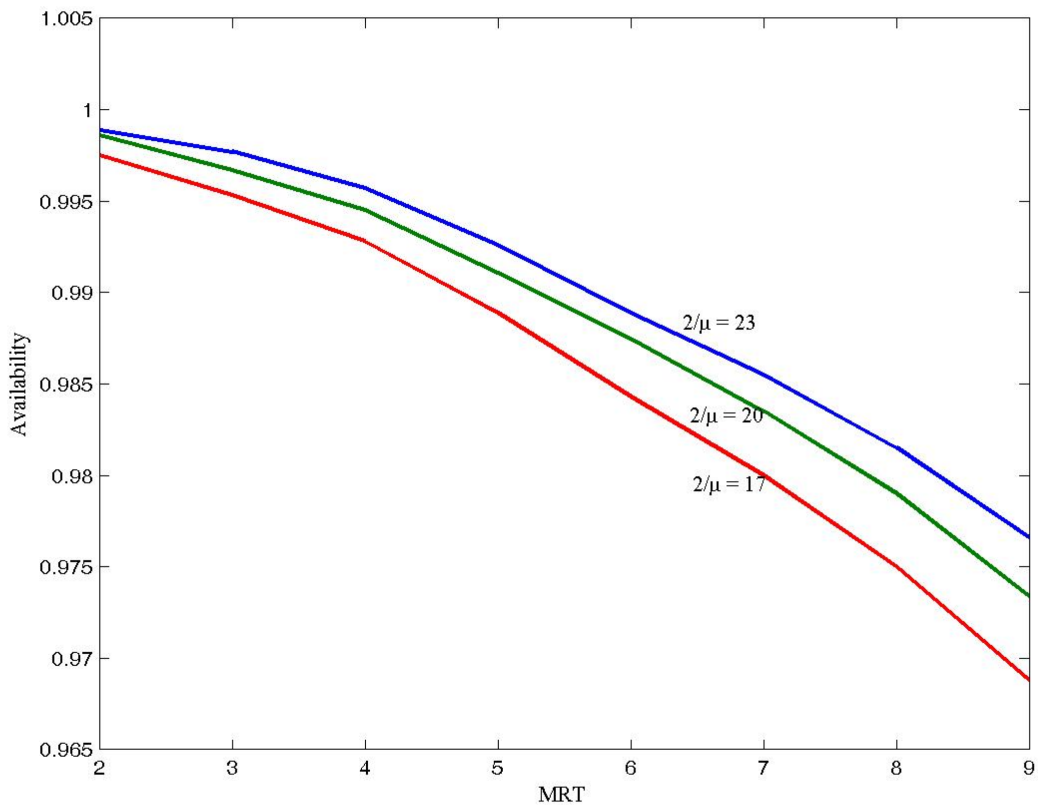

Figure 5.7 (Model 2)

As the Mean Repair Time (MRT) increases the steady-state availability is a decreasing function of MRT (for the different values of $2/\mu$).

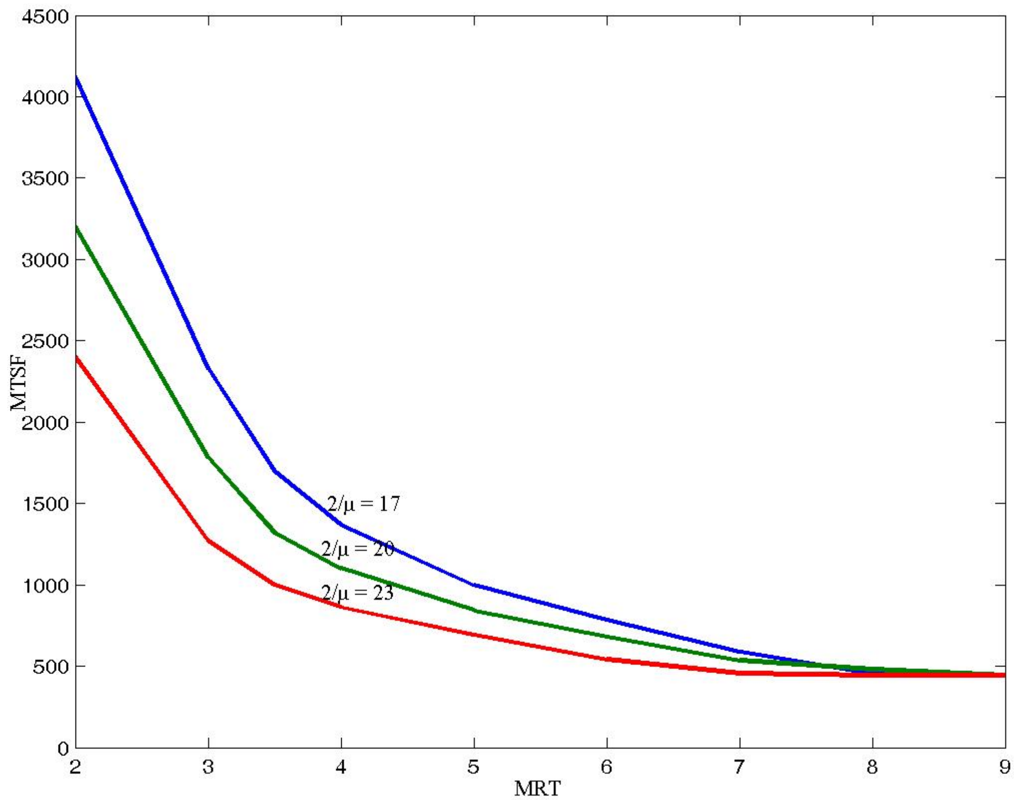

Figure 5.8 (Model 2)

As the Mean Repair Time (MRT) increases the Mean Time to System Failure (MTSF) is a decreasing function of MRT (for different values of $2/\mu$) with almost convergence of MTSF at MRT = 8.

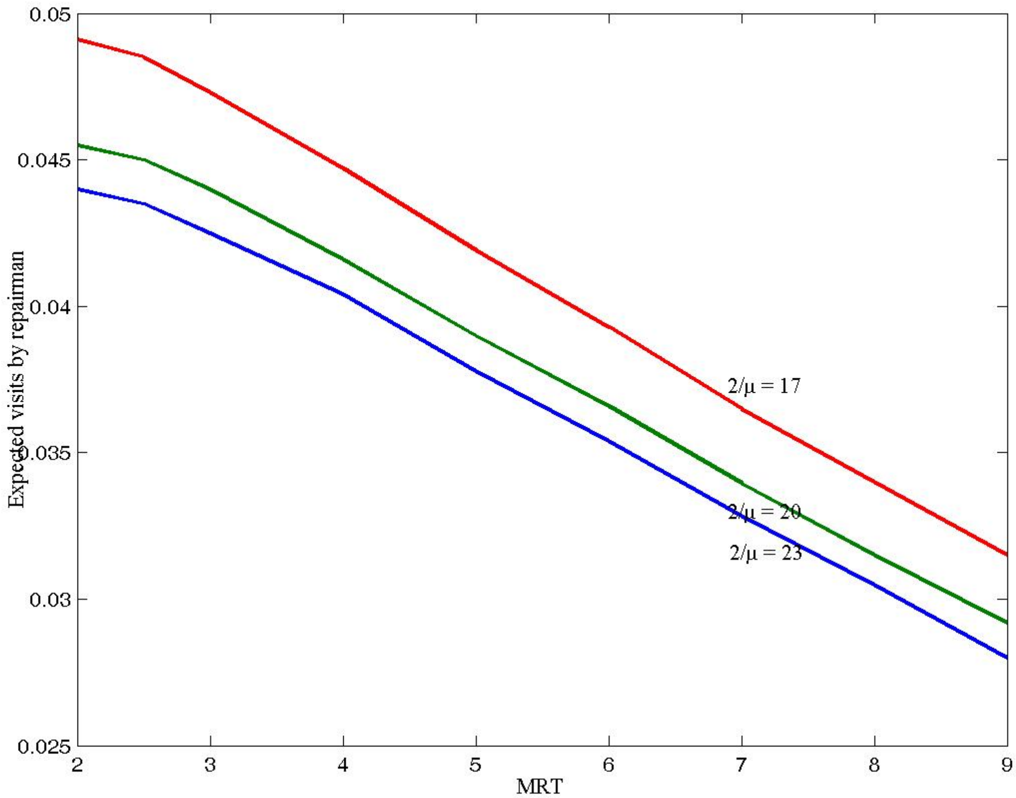

Figure 5.9 (Model 2)

As the Mean Repair Time (MRT) increases the Expected number of visits of the repairman is a decreasing function of MRT (for different values of $2/\mu$).

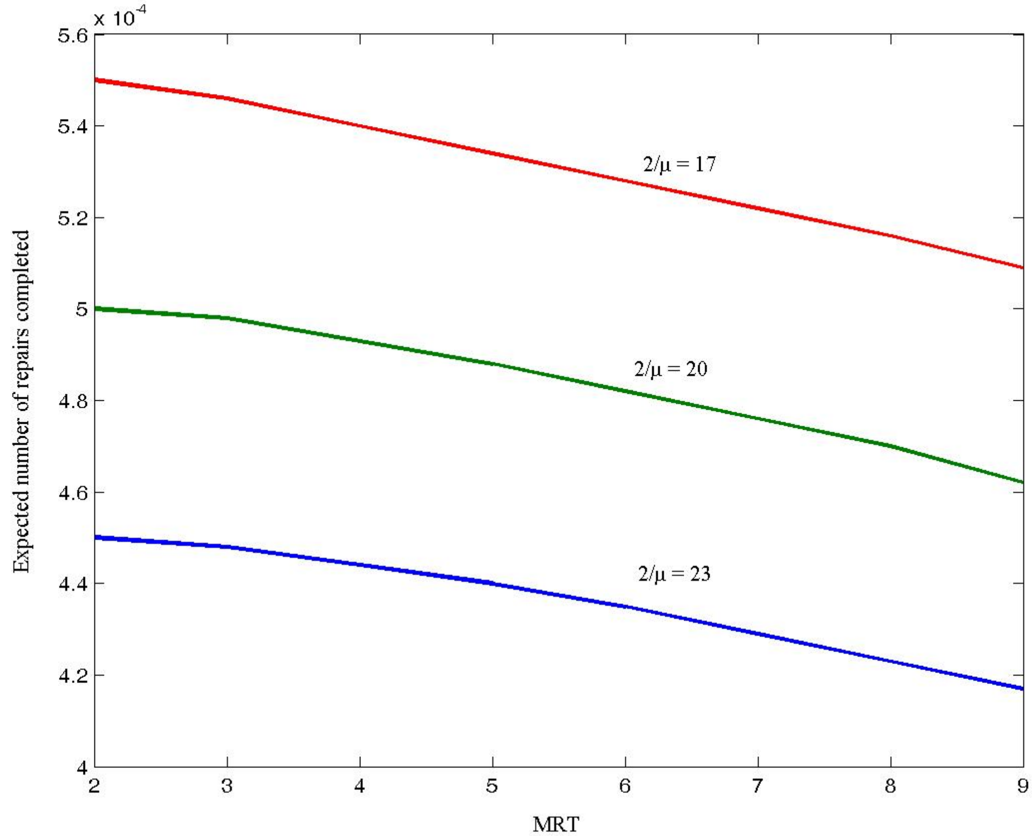

Figure 5.10 (Model 2)

As the Mean Repair Time (MRT) increases the Expected repairs completed is a decreasing function of MRT (for different values of $2/\mu$).

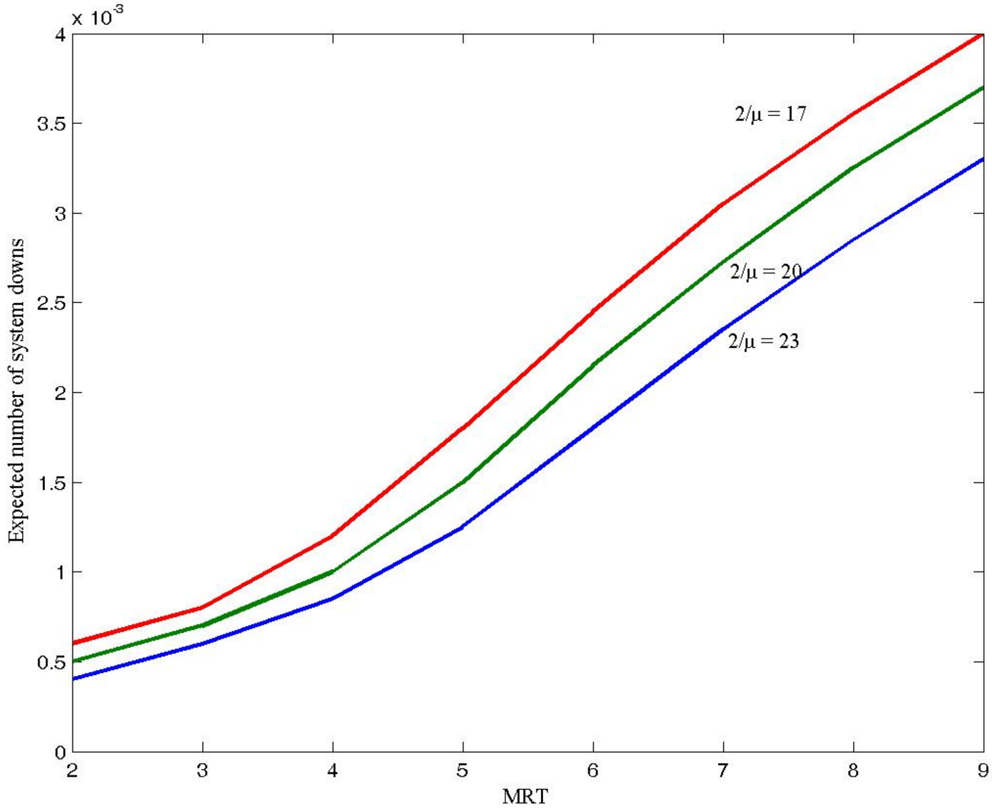

Figure 5.11 (Model 2)

As the Mean Repair Time (MRT) increases the Expected number of system downs is an increasing function of MRT (for different values of $2/\mu$).

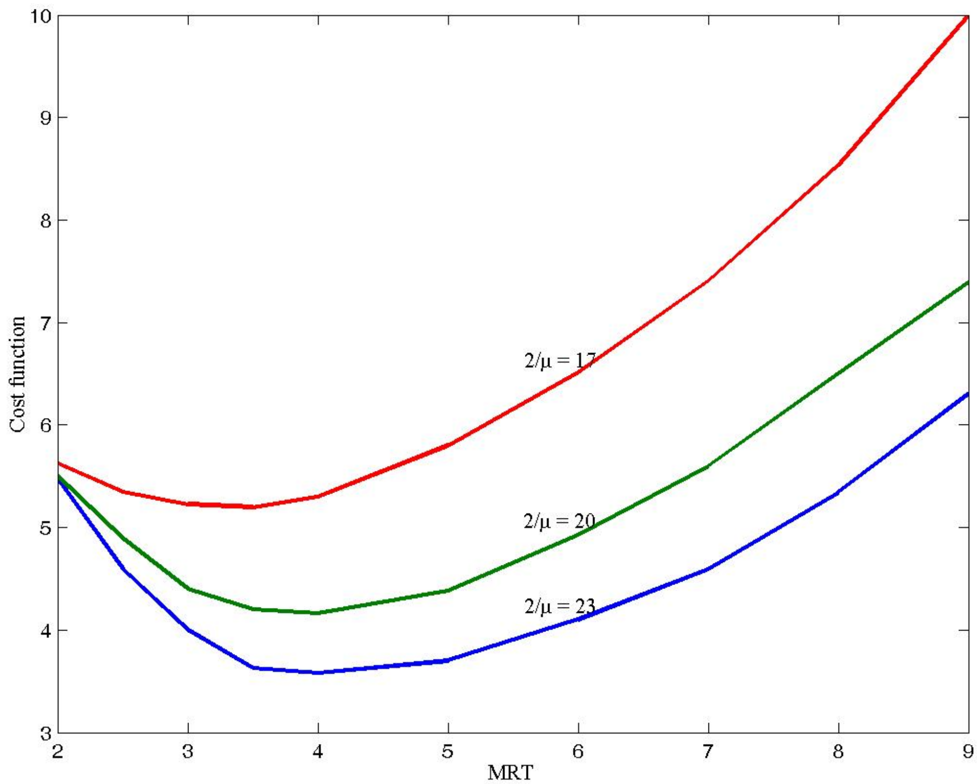

Figure 5.12 (Model 2)

As the Mean Repair time (MRT) increases the Cost function first decreases but then increases after MRT = 4.

5. 7 CONCLUSION

Contrary to the previous chapters a three unit system is considered in this chapter. The life time distributions are all assumed to be non-Markovian. The problem is very challenging when we assume that all the distributions are arbitrary. The system measures, like expected number of transitions from different states, expected number of repairs commenced, expected number of repairs completed, expected number of system breakdowns, expected number of recoveries, are obtained. Results are shown in Figures 5.1 – 5.12.