CHAPTER 4

CONFIDENCE LIMITS FOR A TWO-UNIT COLD STANDBY PRIORITY SYSTEM WITH VARYING PHYSICAL CONDITIONS OF THE REPAIR FACILITY AND WITH IMPERFECT SWITCHING DEVICE

4.1 INTRODUCTION

In the literature of reliability extensive studies have been made on different types of two-unit standby systems owing to their frequent use in modern business and industrial systems. Nakagawa and Osaki (1974) have studied the behaviour of a two-unit (priority and ordinary) standby system with two modes for each unit. They have taken exponential failure and repair time distributions for the ordinary unit, while the distributions for the priority unit are arbitrary. Much work related to the switching device in standby systems has been done by various authors including Goel and Gupta (1984a, b). The cost analysis of such systems has also been discussed by Murari and Goel (1984) and Goel et al. (1985).

Goel et al (1985) have discussed a man-machine system considering the physical conditions of the repair facility, namely poor and good. The physical conditions of the repair facility also affect the operation of the system. However, no previous work has considered the physical conditions of the repair facility. It is reasonable to expect the repair facility to work with a higher repair rate if it is in a poor physical condition. Consequently the repair time distribution will be different in these two situations. The purpose of the present chapter is to analyze such a system. The system under consideration is a two dissimilar unit cold standby system with an imperfect switch. Initially, one unit is operative and is called a priority unit (p) and the other is a cold standby or ordinary unit (o). The p-unit gets priority for both operation and repair (Shi and Liu (1996)). When the p-unit fails the standby unit is switched to operate with the help of a switching device. The switch may be available at the time of need with known probability $p\ (1 - q)$.

The distribution of random variables denoting time to failure and time to repair are taken to be arbitrary. Depending on the physical conditions (good or poor) of the repair facility, there are two different repair time distributions to be considered. The probability that at any time the repairman's condition will be good is $p_1(1-q_1)$. We analyze the system by using the regenerative point technique and obtain various operating characteristics. The confidence limits for the standby state availability and the busy period in steady-state are obtained.

The organisation of this chapter is as follows: Section 4.1 is introductory in nature, and the notation of this chapter is discussed in section 4.2. Various auxiliary functions (transition probabilities and sojourn times) are derived in section 4.3. The reliability analysis is discussed in section 4.4. In section 4.5, availability analysis is discussed. The busy period analysis and the cost benefit analysis have been studied in sections 4.6 and 4.7 respectively. The confidence limits, for the steady state availability, are studied in section 4.8, under the assumption that all the underlying distributions are exponential, with different parameters. In section 4.9, the system is illustrated numerically.

4.2 NOTATION

E_0	State of the system at t=0
E	Set of regenerative states
$\overline{E}$	Set of non-regenerative states
p_1	P[the switch is good at the time of need]; $p_1 = 1 - q_1$
$f_1(t)$, $F_1(t)$	The p.d.f. and c.d.f. of the life time of the *p*-unit
$f_2(t)$, $F_2(t)$	The p.d.f. and c.d.f. of the life time of the *o*-unit
$g_i(t)$, $G_i(t)$	The p.d.f. and c.d.f. of the repair of the *p*-unit ($i = 1, 2$)

$k_i(t)$, $K_i(t)$	The p.d.f. and c.d.f. of the repair of the *o*-unit ($i = 1, 2$)
$h_i(t)$, $H_i(t)$	The p.d.f. and c.d.f. of the time to repair of the switching device; $i = 1, 2$
i	$= \begin{cases} 1 & \text{if the repair facility is in good condition} \\ 2 & \text{if the repair facility is in bad condition} \end{cases}$
p_2	P[the repair facility's condition is good]; $p_2 = 1 - q_2$
$q_{ij}(t)$, $Q_{ij}(t)$	The p.d.f. and c.d.f. of direct transition time from one regenerative state S_i to another regenerative state S_j
p_{ij}	P[the system transits from regenerative state S_i to regenerative state S_j] $= Q_{ij}(\infty)$
$q_{ij}^{(k)}(t)$, $Q_{ij}^{(k)}(t)$	The p.d.f. and c.d.f. of transition time from regenerative state S_i to S_j via non-regenerative state S_k
$p_{ij}^{(k)}$	Steady-state probability that the system transits from state S_i to S_j via non-regenerative states S_k; $Q_{ij}^{(k)}(\infty)$
$\pi_i(\cdot)$	The c.d.f. of the time to system failure when the starting state $E_0 = S_i \in E$
$A_i(t)$	P[System is up at time t $\mid E_0 = S_i \in E$]
$B_i(t)$	P[System is under repair at time t $\mid E_0 = S_i \in E$]
μ_i	Mean sojourn time in states $S_i \in E$
$\tilde{Q}_{ij}(s)$	$\int_0^\infty e^{-st} dQ_{ij}(t)$
$q_{ij}^*(s)$	$\int_0^\infty e^{-st} q_{ij}(t) dt$

$\mu_i \qquad = \sum_j \int_0^\infty t\,dQ_{ij}(t) = -\sum_j \tilde{Q}'_{ij}(0) = -\sum_j q_{ij}^{*'}(0)$

© Symbol for ordinary convolution

Ⓢ Symbol for Stieltjes convolution

4.3 AUXILIARY FUNCTIONS

For the reliability and unavailability analyses, and the busy period analysis, we need to derive various auxiliary functions (transition probabilities and sojourn times). We need to define first the following states (see EL-Said & EL-Sherbeny (2005)):

Up states: S_0 (N_0, N_s) ; S_2 (F_r, N_0); S_4 (N_0, F_r),

Down states: S_1 (F_w, N_s, S_r) ; S_3 (F_r, F_w),

where

N_0 : unit in normal mode and operative

N_s : unit in normal mode and standby

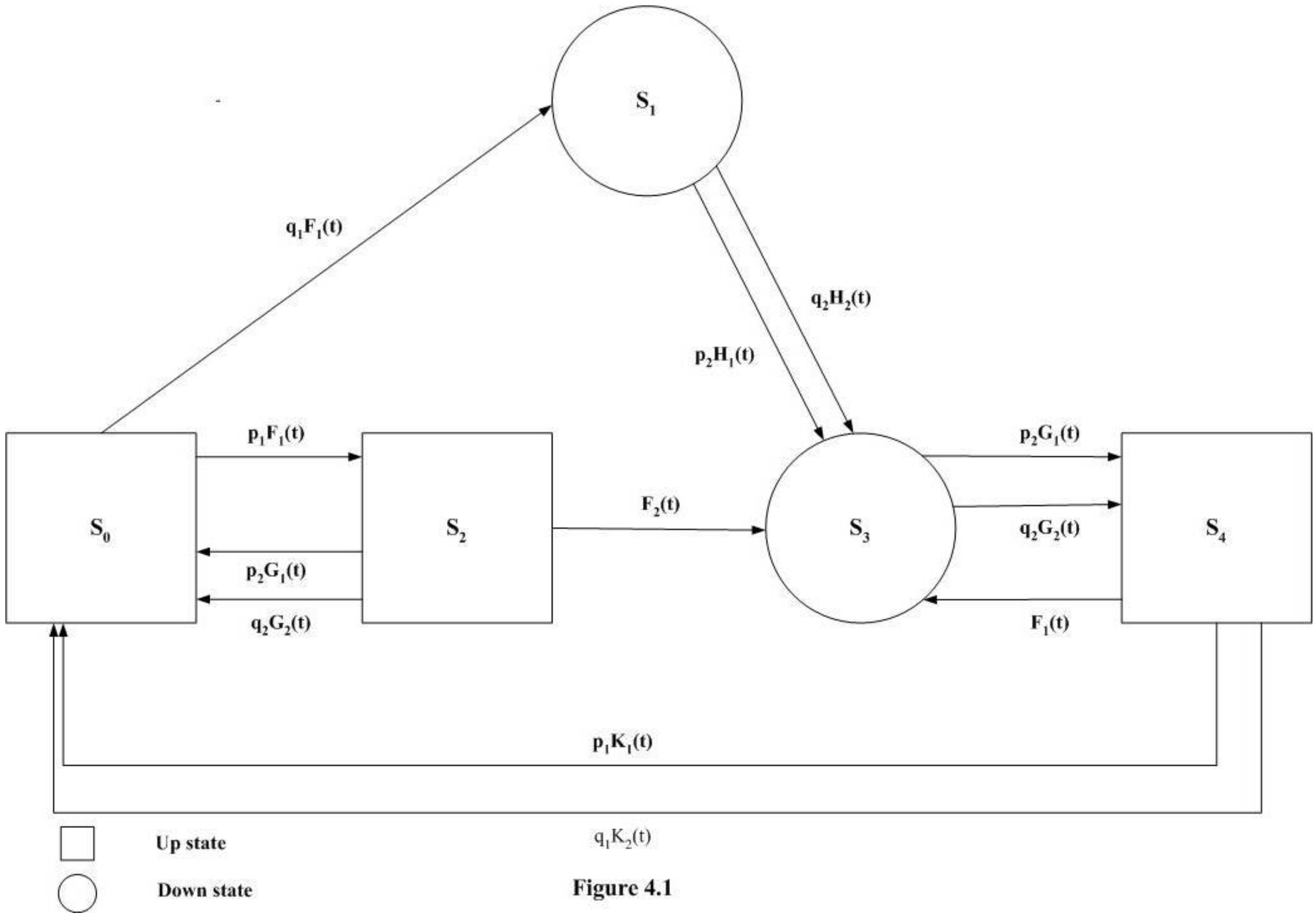
F_r : unit in failure mode and repair from the epoch of entry into the state

F_w : unit in failure mode and waiting for repair

S_r : switching device under repair

F_r : unit in failure mode and under repair with the repair continued from the earlier state.

The order of the position of units in the states specifies the type of unit. Possible transitions between states, with the failure and repair time c.d.f's, are shown in Figure 4.1 .



S1
q1F1(t)
q2H2(t)
p2H1(t)
S0
p1F1(t)
S2
F2(t)
S3
p2G1(t)
S4
p2G1(t)
q2G2(t)
q2G2(t)
F1(t)
p1K1(t)
q1K2(t)
Up state
Down state

Figure 4.1

It is observed that the epoch of entry into the states S_1, S_2 and S4 are degenerative points and therefore these states are regenerative states. E denotes the set of these states. Furthermore, the epochs of entry into the states S_3 from S_4 and S_0 from S_2 are regenerative and the epoch of entry into S_3 from S_2 and S_0 from S_4 are non-regenerative. Therefore these states will behave as regenerative states only with respect to S_4 and S_2 respectively.

Let 0 = T_0, T_1, ... denote the epochs of entry into the states $S_i \in E$ and X_n denote the state visited at epoch T_n^+, i.e. just after the transition at T_n. Then $\{X_n, T_n\}$ is a Markov renewal process with state space E.

Further

$$Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \le t | X_n = i]$$

where

$$Q_{01}(t) = q_1 \int_0^t f_1(u)du = q_1 F_1(t)$$

$$Q_{02}(t) = p_1 \int_0^t f_1(u)du = p_1 F_1(t)$$

$$Q_{12}(t) = p_2 \int_0^t dH_1(t) + q_2 \int_0^t dH_2(t)$$

$$= p_2 H_1(t) + q_2 H_2(t)$$

$$Q_{20}(t) = p_2 \int_0^t \bar{F}_2(t)dG_1(u) + q_2 \int_0^t \bar{F}_2(u)dG_2(u)$$

$$Q_{24}^{(3)}(t) = p_2 \int_0^t F_2(u)dG_1(u) + q_2 \int_0^t F_2(u)dG_2(u)$$

$$= Q_{23}(t)$$

$$Q_{34}(t) = p_2 G_1(t) + q_2 G_2(t)$$

$$Q_{41}^{(0)}(t) = q_1[p_2 \int_0^t k_1(u)dF_1(u) + q_2 \int_0^t k_2(u)dF_1(u)]$$

$$Q_{42}^{(0)}(t) = p_1[p_2 \int_0^t k_1(u)dF_1(u) + q_2 \int_0^t k_2(u)dF_1(u)]$$

and
$$Q_{43}(t) = p_2 \int_0^t \bar{K}_1(u)dF_1(u) + q_2 \int_0^t \bar{K}_2(u)dF_1(u).$$

Letting $t \to \infty$ and using $p_{ij} = Q_{ij}(\infty)$, we get the transition probability matrix $P = \left[p_{ij}\right]$ with the following non-zero elements

$$p_{01} = q_1\,;\; p_{02} = p_1\,;\; p_{12} = p_{34} = 1 \tag{4.1}$$

$$p_{20} = p_2 \int_0^\infty \bar{F}_2(t)dG_1(t) + q_2 \int_0^\infty \bar{F}_2(t)dG_2(t)$$

$$p_{24}^{(3)} = p_2 \int_0^\infty \bar{F}_2(t)dG_1(t) + q_2 \int_0^\infty \bar{F}_2(t)dG_2(t)$$

$$p_{41}^{(0)} = q_1[p_2 \int_0^\infty k_1(t)dF_1(t) + q_2 \int_0^\infty k_2(t)dF_1(t)]$$

$$p_{42}^{(0)} = p_1[p_2 \int_0^\infty k_1(t)dF_1(t) + q_2 \int_0^\infty k_2(t)dF_1(t)]$$

and
$$p_{43} = p_2 \int_0^\infty \bar{K}_1(t)dF_1(t) + q_2 \int_0^\infty \bar{K}_2(t)dF_1(t).$$

We can easily verify that

$$p_{01} + p_{02} = 1 \tag{4.2}$$

$$p_{20} + p_{24}^{(3)} + p_{23} = 1 \tag{4.3}$$

and

$$p_{41}^{(0)} + p_{42}^{(0)} + p_{43} = 1\,. \tag{4.4}$$

To calculate the mean sojourn time μ_0 in state S_0, we observe that so long as the system is in S_0, there is no transition on to S_1 or S_2 . Hence if T denotes the sojourn time in state S_0, then

$$\mu_0 = \int_0^\infty P[T > t]dt$$

$$= \int_0^\infty p_1 \overline{F}_1(t)dt + \int_0^\infty q_1 \overline{F}_1(t)dt$$

$$= \int_0^\infty \overline{F}_1(t)dt \tag{4.5}$$

$$\mu_1 = p_2 \int_0^\infty \overline{H}_1(t)dt + q_2 \int_0^\infty \overline{H}_2(t)dt \tag{4.6}$$

$$\mu_2 = p_2 \int_0^\infty \overline{G}_1(t)\overline{F}_2 dt + q_2 \int_0^\infty \overline{G}_2(t)\overline{F}_2 dt \tag{4.7}$$

$$\mu_3 = p_2 \int_0^\infty \overline{G}_1(t)dt + q_2 \int_0^\infty \overline{G}_2(t)dt \tag{4.8}$$

and

$$\mu_4 = p_2 \int_0^\infty \overline{K}_1(t)\overline{F}_1 dt + q_2 \int_0^\infty \overline{K}_2(t)\overline{F}_1 dt\,. \tag{4.9}$$

4.4 RELIABILITY ANALYSIS

The time to system failure (TSF) can be regarded as the first passage time to either of the failed states S_1 or S_3. To obtain it we regard these states as absorbing. Employing the arguments used for regenerative processes we obtain the following

$$\pi_0(t) = Q_{01}(t) + Q_{02}(t) \,ⓢ\, \pi_1(t) \tag{4.10}$$

$$\pi_2(t) = Q_{23}(t) + Q_{20}(t) \,ⓢ\, \pi_0(t) \tag{4.11}$$

and

$$\pi_4(t) = Q_{41}^{(0)}(t) + Q_{42}^{(0)}(t) \,ⓢ\, \pi_2(t) + Q_{43}(t)\,. \tag{4.12}$$

Taking the Laplace-Stieljes transform of the equations (4.10) to (4.12), the solution of $\pi_i(s)$, $(i = 0,2,4)$ can be written in the following form

$$\begin{bmatrix} \widetilde{\pi}_0 \\ \widetilde{\pi}_2 \\ \widetilde{\pi}_4 \end{bmatrix} = \begin{bmatrix} 1 & -\widetilde{Q}_{02} & 0 \\ -\widetilde{Q}_{20} & 1 & 0 \\ 0 & -\widetilde{Q}_{42} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \widetilde{Q}_{01} \\ \widetilde{Q}_{23} \\ \widetilde{Q}_{41} + \widetilde{Q}_{43} \end{bmatrix} \tag{4.13}$$

We have omitted the argument '*s*' for simplicity from $\widetilde{Q}_{ij}(s)$ and $\widetilde{\pi}_{ij}(s)$. Simplifying (4.13), we get

$$\widetilde{\pi}_0(s) = \frac{N_1(s)}{D_1(s)} \tag{4.14}$$

where

$$N_1(s) = \widetilde{Q}_{01} + \widetilde{Q}_{02}\widetilde{Q}_{23}$$

$$D_1(s) = 1 - \widetilde{Q}_{02}\widetilde{Q}_{20}\,.$$

Making use of relations (4.1) – (4.4), it can be shown that $\widetilde{\pi}_0(0) = 1$, which implies that $\pi_0(t)$ is a proper distribution. Now, the mean time to system failure, given that the system started from S_0,

$$E(t) = -\frac{d}{ds}\widetilde{\pi}_0(s)\Big|_{s=0}$$

$$= \frac{\mu_0 + p.\mu_2}{1 - p.p_{20}}\,. \tag{4.15}$$

4.5 AVAILABILITY ANALYSIS

Let $M_i(t)$ be the probability that the system, having started from S_i, is up at time t, without making any transition to any other regenerative state belonging to E.

By simple probabilistic arguments we have

$$M_0(t) = p_1\overline{F}_1(t) + q_1\overline{F}_1(t) = \overline{F}_1(t) \tag{4.16}$$

$$M_2(t) = \overline{F}_2(t)[p_2\overline{G}_1(t) + q_2\overline{G}_2(t)] \tag{4.17}$$

and
$$M_4(t) = \overline{F}_1(t)[p_2K_1(t) + q_2K_2(t)]\,. \tag{4.18}$$

From the arguments used in the theory of regenerative process, the pointwise availabilities $A_i(t)$ are seen to satisfy the following relations:

$$A_0(t) = q_{01}(t) \copyright A_1(t) + q_{02}(t) \copyright A_2(t) + M_0(t) \tag{4.19}$$

$$A_1(t) = q_{12}(t) \copyright A_2(t) \tag{4.20}$$

$$A_2(t) = q_{20}(t) © A_0(t) + q_{24}^{(0)}(t) © A_4(t) + M_2(t) \tag{4.21}$$

$$A_3(t) = q_{34}(t) © A_4(t) \tag{4.22}$$

and $$A_4(t) = q_{43}(t) © A_3(t) + q_{41}^{(0)}(t) © A_1(t) + q_{42}^{(0)}(t) © A_2(t) + M_4(t)\,. \tag{4.23}$$

Taking Laplace transforms of (4.19) – (4.23), the solution for $A_i^*(s)$ can be written in the matrix form

$$\begin{bmatrix} A_0^*(s) \\ A_1^*(s) \\ A_2^*(s) \\ A_3^*(s) \\ A_4^*(s) \end{bmatrix} = \begin{bmatrix} 1 & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 \\ 0 & 1 & -q_{12}^*(s) & 0 & 0 \\ -q_{20}^*(s) & 0 & 1 & 0 & -q_{24}^*(s) \\ 0 & 0 & 0 & 1 & -q_{34}^*(s) \\ 0 & -q_{41}^*(s) & -q_{42}^*(s) & -q_{43}^*(s) & 1 \end{bmatrix} \begin{bmatrix} M_0^*(s) \\ 0 \\ M_2^*(s) \\ 0 \\ M_4^*(s) \end{bmatrix} \tag{4.24}$$

Simplifying (4.24) for $A_0^*(s)$, the Laplace transform of pointwise availability when the system started operation from state S_0, we get

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)}$$

where $N_2(s) = (1 - q_{34}^* q_{43}^*)[M_0^* + M_2^*(q_{01}^* q_{12}^* + q_{02}^*)]$

$$-q_{24}^*[M_0^*(q_{41}^* q_{12}^* + q_{42}^*) + M_4^*(q_{01}^* q_{12}^* + q_{02}^*)]$$

and

$$D_2^*(s) = (1 - q_{34}^* q_{43}^*)[1 - q_{20}^*(q_{01}^* q_{12}^* + q_{02}^*)] - q_{24}^{(3)*}(q_{42}^{(0)*} + q_{41}^{(0)*} q_{12}^*)\,.$$

Here $q_{ij}^*(s) = q_{ij}^*$

The steady state availability A_∞, is given by

$$A_\infty = \lim_{s \to \infty} s A_0^*(s) = \frac{N_2}{D_2}$$

where

$$N_2 = (1 - p_{43})[p_{20}\mu_0 + \mu_2)] + p_{24}^{(3)}\mu_0$$

$$D_2 = (1 - p_{43})[p_{20}\mu_0 + q_1 p_{20}\mu_1 + m]$$

$$+ p_{24}^{(3)}[p_{41}^{(0)}\mu_1 + p_{43}\mu_3 + n]$$

$$m = p_2 \int_0^\infty t\,dG_1(t) + q_2 \int_0^\infty t\,dG_2(t)$$

and $$n = \int_0^\infty t\,dF_1(t)\ .$$

Now the expected up-time of the system in (0, *t*] is

$$\mu_u(t) = \int_0^t A_0(u)du$$

so that

$$\mu_u^*(s) = \frac{A_0^*(s)}{s}$$

and the expected down-time of the system in (0, *t*] is

$$\mu_d(t) = t - \mu_u(t)$$

so that

$$\mu_d^*(s) = \frac{1}{s^2} - \mu_u^*(s)\ .$$

Since $A_0^*(s)$ is known explicitly, the above quantities can be computed easily.

4.6 BUSY PERIOD ANALYSIS

Let $B_i(t)$ be the probability that the repair facility is busy given that the system entered state S_i at $t = 0$.

By probabilistic arguments, we have

$$B_0(t) = q_{01}(t) © B_1(t) + q_{02}(t) © B_2(t) \tag{4.29}$$

$$B_1(t) = q_{12}(t) © B_2(t) + v_1(t) \tag{4.30}$$

$$B_2(t) = q_{20}(t) © B_0(t) + q_{24}^{(3)}(t) © B_4(t) + v_2(t) \tag{4.31}$$

$$B_4(t) = q_{41}(t) © B_1(t) + q_{42}^{(0)}(t) © B_2(t) + q_{43}(t) © B_3(t) + v_4(t) \tag{4.32}$$

where

$$v_1(t) = p_2\overline{H}_1(t) + q_2\overline{H}_2(t)$$

$$v_2(t) = p_2\overline{G}_1(t) + q_2\overline{G}_2(t)$$

$$v_3(t) = p_2\overline{G}_1(t) + q_2\overline{G}_2(t)$$

and $$v_4(t) = \overline{F}_1(t)[p_2\overline{K}_1(t) + q_2\overline{K}_2(t)]. \tag{4.33}$$

Taking Laplace transforms of equations (4.29) – (4.33) and solving for $B_0^*(s)$,

$$B_0^*(s) = \frac{N_3(s)}{D_2(s)}$$

$$N_3(s) = v_1^*[q_{01}^*(1 - q_{43}^* q_{34}^*) + q_{24}^{(3)*} q_{41}^{(0)*} - q_{01}^*(1 - q_{43}^*)]$$

$$+(q_{01}^* q_{12}^* + q_{02}^*)[(1 - q_{43}^* q_{34}^*)v_2^* + q_{42}^* q_{24}^{(3)*} v_3^* + q_{24}^{(3)*} v_4^*].$$

In the long run, the fraction of time for which the system is under repair is given by

$$B_\infty = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} sB_0^*(s) = \frac{N_3}{D_2}$$

$$N_3 = \mu_1(1-p_{43})q_1[p_{20} + p_{24}^{(3)} p_4^{(0)}] + p_{24}^{(3)}\mu_4 + m(1-p_{43}p_{20})]$$

The expected busy period of the repair facility in (0, t] is

$$\mu_b(t) = \int_0^t B_0(u)du$$

so that

$$\mu_b^*(s) = \frac{B_0^*(s)}{s}.$$

4.7 COST ANALYSIS

We now obtain the cost function of the system considering the mean up-time of the system and the expected busy period of the repair facility.

Let us define C_1 as the revenue per unit-time and C_2 as the cost of repairs per unit time. Then the expected total profit earned in (0, t] is

G(t) = expected total revenue in (0, t] – expected repair cost in (0, t]

$$= C_1\mu_u(t) - C_2\mu_0(t).$$

The expected profit per unit time is

$$g(t) = \frac{G(t)}{t}$$

4.8 CONFIDENCE LIMITS

When failure and repair time distributions are exponentially distributed and the switch is perfect, i.e. $p_1 = p_2 = 1$; $q_1 = q_2 = 0$

$$f_1(t) = \alpha_1 e^{-\alpha_1 t} \;;\; f_2(t) = \alpha_2 e^{-\alpha_2 t}$$

$$g(t) = \beta_1 e^{-\beta_1 t} \;;\; k(t) = \beta_2 e^{-\beta_2 t}$$

then

$$MTSF = \frac{\beta_1 + \alpha_1 + \alpha_2}{\alpha_1 \alpha_2} \tag{4.34}$$

$$A_\infty = \frac{\beta_1[\beta_2(\beta_1 + \alpha_1 + \alpha_2) + \alpha_1\alpha_2]}{\beta_1\beta_2(\beta_1 + \alpha_1 + \alpha_2) + \alpha_1\alpha_2(\beta_1 + \beta_2 + \alpha_1)} \tag{4.35}$$

and

$$B_\infty = \frac{\alpha_1[\beta_1\beta_2 + \alpha_2(\beta_1 + \beta_2 + \alpha_1) + \alpha_1\alpha_2]}{\beta_1\beta_2(\beta_1 + \alpha_1 + \alpha_2) + \alpha_1\alpha_2(\beta_1 + \beta_2 + \alpha_1)}\,. \tag{4.36}$$

4.8.1 CONFIDENCE LIMITS FOR A_∞

Let $X_{i1}, X_{i2}, \ldots, X_{in}$; (*i* = *1*, *2*) be random samples of size *n*, each drawn from exponential populations with failure rates, (α_1, α_2) respectively.

Similarly $Y_{i1}, Y_{i2}, \ldots, Y_{in}$; (*i* = *1*, *2*) be random samples of size *n*, each drawn from exponential populations with repair rates (both *p*-unit and *o*-unit) (β_1, β_2) respectively.

If α_1 is the parameter of the exponential distribution, then an estimate can be found for either α_1, or for the parameter $\theta_1 = \frac{1}{\alpha_1}$, which is equal to the mean value of the time of failure-free operation of the *p*-unit.

For the sake of analysis, let

$$\theta_1 = \frac{1}{\alpha_1},\ \theta_2 = \frac{1}{\alpha_2},\ \theta_3 = \frac{1}{\beta_1}, \theta_4 = \frac{1}{\beta_2}.$$

The maximum likelihood estimator (MLE) of θ_1 is given by $\overline{X}_1 = \frac{1}{n}\sum_{i=1}^{n} X_{1i}$. Similarly $\overline{X}_2, \overline{X}_3$ and $\overline{X}_4$ are the MLE's of θ_2, θ_3 and θ_4 respectively.

$$\hat{A}_\infty = \frac{\overline{X}_1[(\overline{X}_1\overline{X}_2 + \overline{Y}_1\overline{X}_2 + \overline{X}_2\overline{Y}_2) + \overline{Y}_1\overline{Y}_2]}{\overline{X}_1(\overline{X}_1\overline{X}_2 + \overline{X}_1\overline{Y}_1 + \overline{Y}_1\overline{X}_2) + \overline{Y}_1(\overline{X}_1\overline{Y}_2 + \overline{X}_1\overline{Y}_1 + \overline{Y}_1\overline{Y}_2)}$$

$\hat{A}_\infty$ is a real-valued function in $\overline{X}_1, \overline{X}_2, \overline{X}_3, \overline{X}_4$, which is also differentiable.

By an application of the central limit theorem [Rao (1973)], it follows that

$$\sqrt{n}\,(\overline{X} - \theta) \xrightarrow{D} N_4\,(0, \Sigma) \text{ as n} \to \infty.$$

where

$$\overline{X} = (\overline{X}_{1,}\overline{X}_2, \overline{Y}_1, \overline{Y}_2)$$

$$\theta = (\theta_1, \theta_2, \theta_3, \theta_4).$$

The dispersion matrix $\Sigma = (\sigma_{ij})_{4\text{x}4}$ is given by

$$\Sigma = diag(\theta_1^{\,2}, \theta_2^{\,2}, \theta_3^{\,2}, \theta_4^{\,2}).$$

From (Rao (1973)), as n $\to \infty$

$$\sqrt{n}\,(\hat{A}_\infty - A_\infty) \xrightarrow{D} N(0, \sigma^2(\theta)) \text{ where}$$

$$\sigma^2(\theta) = \sum_{i=1}^{4} \left[\frac{\partial A_\infty}{\partial \theta_1} \right]^2 \sigma_{ii}$$

$$= \sum_{i=1}^{4} \left[\frac{\partial A_\infty}{\partial \theta_i} \right]^2 \theta_i^2 .$$

Replacing θ by its consistent estimator $\hat{\theta} = (\overline{X}_1, \overline{X}_2, \overline{Y}_1, \overline{Y}_2)$, it follows that

$\hat{\sigma}^2 = \sigma^2(\hat{\theta})$ is a consistent estimator of $\sigma^2(\theta)$ (see Wackerly et al (2002).

Then by Slutzky's theorem, (Slutsky (1928)),

$$\frac{\sqrt{n}\,(\hat{A}_\infty - A_\infty)}{\hat{\sigma}} \xrightarrow{D} N(0, D) \text{ as n} \to \infty.$$

This implies

$$P\left[-k_{\frac{\alpha}{2}} \le \frac{\sqrt{n}\,(\hat{A}_\infty - A_\infty)}{\hat{\sigma}} \le k_{\frac{\alpha}{2}} \right] = 1 - \alpha$$

where $k_{\alpha/2}$ is obtained from normal tables, i.e. the 100(1 – α)% confidence interval is given by

$$\hat{A}_\infty \pm k_{\alpha/2} \frac{\hat{\sigma}}{\sqrt{n}} .$$

4.8.2 CONFIDENCE LIMITS FOR B_∞

The procedure is identical to section 4.8.1 except

$$\hat{B}_\infty = \frac{\overline{Y}_1[(\overline{X}_1\overline{X}_2 + \overline{Y}_1\overline{X}_2 + \overline{X}_1\overline{Y}_1) + \overline{Y}_1\overline{Y}_2]}{\overline{X}_1(\overline{X}_1\overline{X}_2 + \overline{X}_1\overline{Y}_1 + \overline{Y}_1\overline{X}_2) + \overline{Y}_1(\overline{X}_1\overline{Y}_2 + \overline{X}_1\overline{Y}_1 + \overline{Y}_1\overline{Y}_2)} .$$

When we follow the procedure as in section 4.8.1, we get the confidence limits for $\hat{\beta}_\infty$.

The confidence limits for $\hat{\beta}_\infty$ are

$$\hat{B}_\infty \pm k_{\alpha/2} \frac{\hat{\sigma}}{\sqrt{n}}.$$

4.9 NUMERICAL ILLUSTRATION

Assuming that the units are identical, the switch is perfect and failure and repair rates are constant, that is

$$f_1(t) = f_2(t) = \alpha e^{-\alpha t}$$

$$g_1(t) = g_2(t) = \beta e^{-\beta t}$$

$$k_1(t) = k_2(t) = \gamma e^{-\gamma t}.$$

The expressions for MTSF and A_∞ reduce to the following forms:

$$MTSF = \frac{(\alpha+\beta)(\alpha+\gamma)+\alpha[p_2(\alpha+\gamma)+q_2(\alpha+\beta)]}{\alpha[(\alpha+\beta)(\alpha+\gamma)-p_2\beta(\alpha+\gamma)-q_2\gamma(\alpha+\beta)]}$$

and $$A_\infty = \frac{A}{B+C}$$

where $$A = \beta\gamma(\alpha+\beta)^2(\alpha+\gamma)^2$$

$$B = [\alpha\{(\beta+\gamma)-(p_2\gamma+q_2\beta)\}+\beta\gamma][\beta\gamma\{\alpha(p_2\gamma+q_2\beta)+\beta\gamma\}$$

$$+\alpha^3+\alpha^2(\beta+\gamma)+\alpha\beta\gamma(p_2\gamma+q_2\beta)]$$

and

$$C = [\alpha^2+\alpha(p_2\gamma+q_2\beta)][\{\alpha^3+\alpha^2(p_2\gamma+q_2\beta)\}(p_2\gamma+q_2\beta)+\beta\gamma(\alpha+\beta)(\alpha+\gamma)].$$

Taking β = 4, γ = 1 and β = 15, γ = 5, the values for MTSF and steady state availability corresponding to p_2 = 1, 0.5 and 0 and for different values of α can be calculated.

Figures 4.2 and 4.3 represent the values for A_∞ and MTSF respectively.

These graphs clearly indicate that the better the physical condition of the repair facility the better the performance of the system.

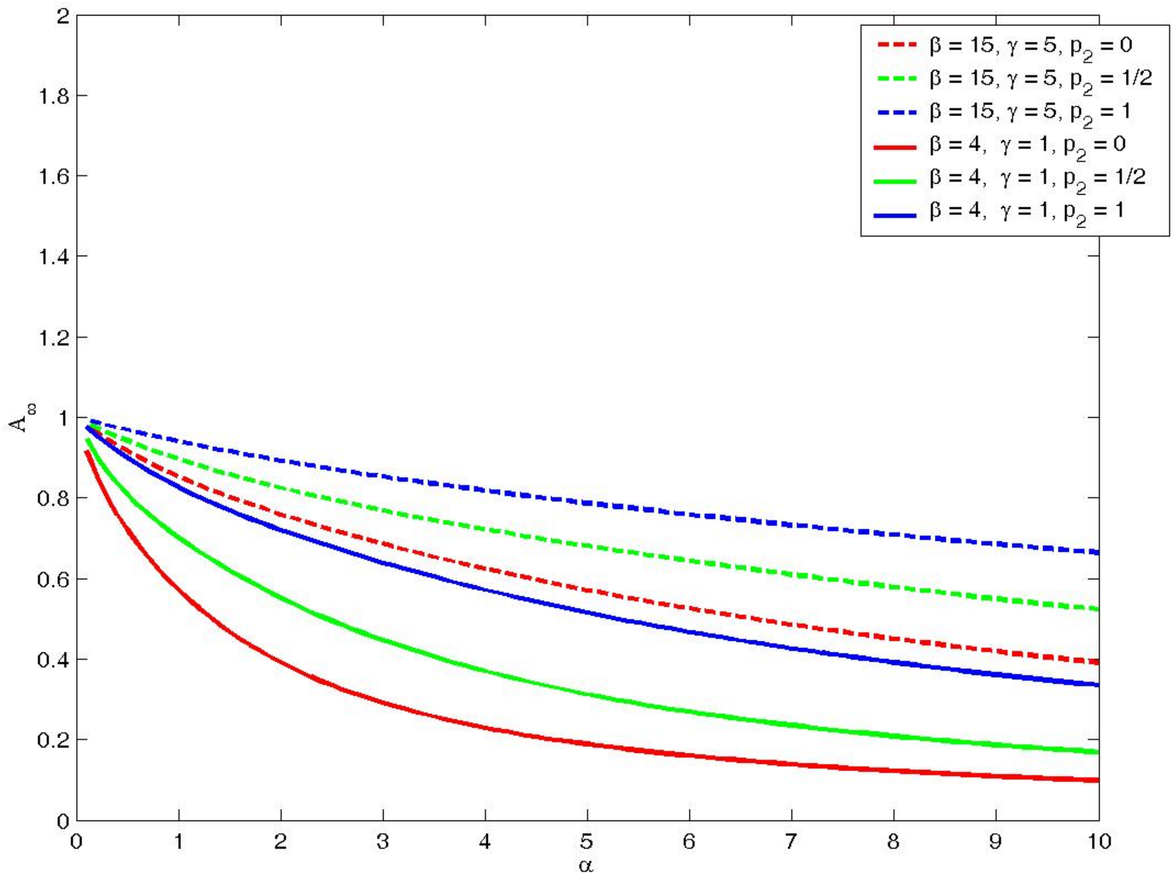


Figure 4.2

As α increases the steady-state availabity, A_∞, is a decreasing function of α (for different values of β, γ and p_2).

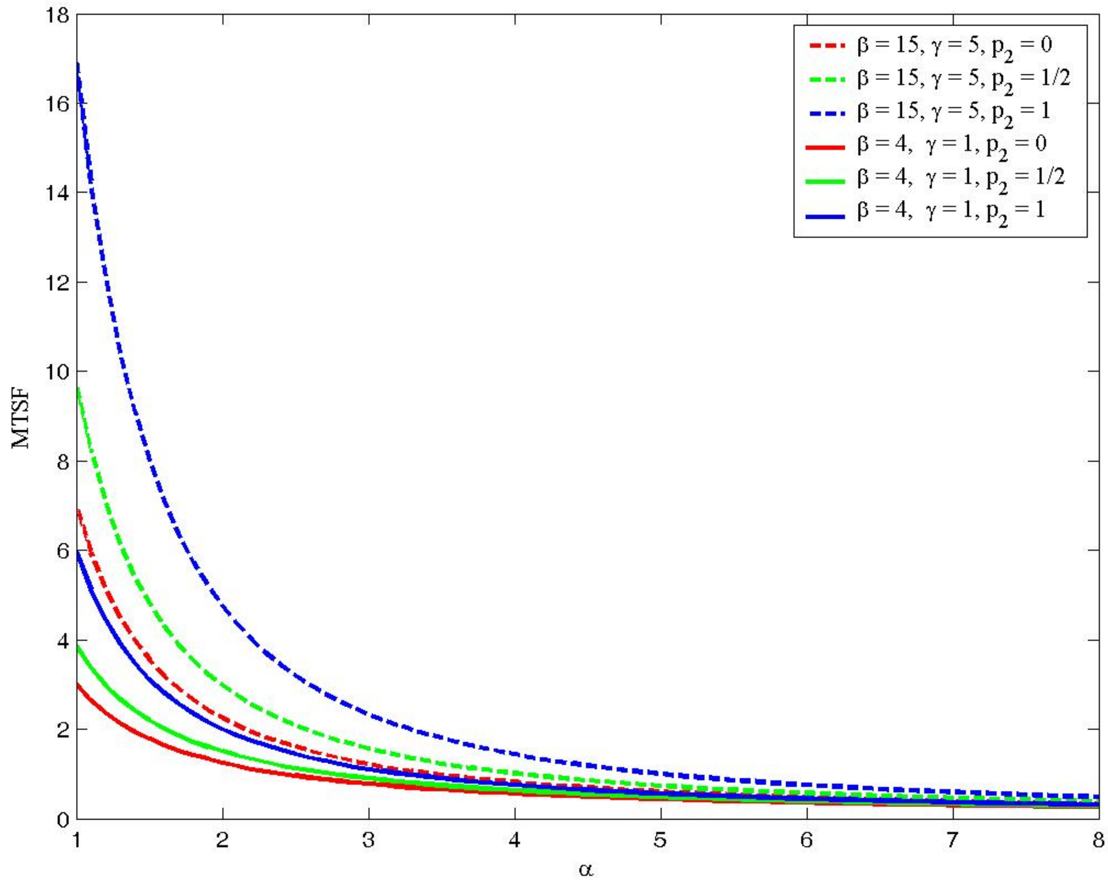


Figure 4.3

As α increases the Mean Time to System Failure (MTSF) is a decreasing function of α (for different values of β, γ and p_2).

4.10 CONCLUSION

A single server two-unit priority cold standby system is studied with varying physical conditions for the repairman, since the repair time's distribution is affected by such conditions. It is assumed that the switching device (the device which transfers the unit from cold standby state to operating online state) is not perfect, i.e. the switch can also fail. Identifying the regeneration points, various operating characteristics are obtained, both analytically and numerically. Explicit expressions for the steady state availability and the busy period in the steady state are obtained, when all underlying distributions are exponential. For these two measures, the asymptotic confidence limits are also obtained. These results were shown in Figure 4.2 (For an increasing α the steady-state availability (A_∞) decreases for different values of β, γ and p_2) and Figure 4.3 (For an increasing α the Mean Time to System Failure (MTSF) decreases for different values of β, γ and p_2).