CHAPTER 3

TWO-UNIT PRIORITY REDUNDANT SYSTEM WITH 'DEADTIME' FOR THE OPERATOR

3.1 INTRODUCTION

Two-unit standby redundant systems have attracted the attention of many applied probabilists and reliability engineers. A bibliography of the work done has been prepared by Osaki and Nakagawa (1976), Lie et al. (1977), Kumar and Agarwal (1980), Sarma (1982). Goel et al. (1985) analysed a two-unit cold standby system under the assumption that the operator of the system does not need rest, i.e. he is capable to work on the system without any rest. The literature available so far has the assumption that the operator is continuously available to repair the failed units. But it is reasonable to expect that a preparation time or rest period might be needed to get the operator ready before the next repair could be taken up. If this preparation is started only when a unit arrives for repair, it is easy to solve the problem, since the preparation time plus the actual repair time of the operator must be taken as the total repair time. But this preparation time usually starts immediately after each repair completion, so that the operator becomes available at the earliest. In our daily life the situations come about when a person needs such a preparation time. This preparation time of the operator is similar to the 'Dead time' in the counter models Ramakrishnan and Mathews (1953), Ramakrishnan (1954), Takács (1956, 1957). Yadavalli et al. (2002) studied several Markovian and non-Markovian models by introducing the 'Dead time'. Cold standby redundant systems in which the 'priority of units' and 'dead time' are introduced in this chapter.

The organisation of this chapter is as follows: Section 3.1 is introductory in nature describing the model considered in this chapter. In section 3.2, the basic assumptions and notation are presented. Various auxiliary functions (transition probabilities and sojourn times) are derived in section 3.3. The important system measures, Reliability and MTSF, are presented in section 3.4. The other important measures like mean up time in a particular interval, mean down time, expected number of visits by a repairman are studied in section 3.5. In sectin 3.6, the profit analysis is studied. Some special cases are presented in section 3.7. The system considered in this section is illustrated numerically in section 3.8.

3.2. SYSTEM DESCRIPTION AND NOTATION

1. The system consists of two dissimilar units each having two modes- Normal (N) and Total Failure (F).
2. Initially one unit of the system is operative, called the priority (P) unit and the other is kept as cold standby, called the non-priority or ordinary unit (O).
3. P-unit gets preference for both operation and repair over O-unit. When P-unit fails, the standby unit is switched to operate with a perfect switching device.
4. There is only one operator. Each unit is new after repair.
5. After each repair completion, the operator is not available for a random time. This corresponds to the 'dead time' in counter models and will be interpreted here as the 'rest time' or 'preparation time' needed before another repair could be taken up.
6. Switch is perfect and switchover is instantaneous. When the P-unit fails, it will be instantaneously switched over to the O-unit from standby state to online.

7. The lifetime of a unit, while online for P-unit and O-unit is arbitrarily distributed with pdf's $f_1(\cdot)$ and $f_2(\cdot)$.
8. The repair time of units (P-unit and O-unit) are exponentially distributed random variables with parameters β_1 and β_2 respectively.
9. The 'Dead time' of the operator is an arbitrarily distributed random variable with pdf $k(\cdot)$.

NOTATION:

$F_1(\cdot)$ and $F_2(\cdot)$	The c.d.f of the life time of P-unit and O-unit respectively
E	Set of regenerative events ≡ ($E_0, E_1, E_2, E_3, E_4, E_5, E_6$)
η	Constant rate of working time of the operator
$K(\cdot)$	The c.d.f of the 'dead time' of the operator
P_{ij}	Transition probability from regenerative event E_i to E_j
$q_{ij}(\cdot)$, $Q_{ij}(\cdot)$	The p.d.f. and c.d.f. of transition time from regenerative event E_i to E_j
ψ_i	Mean sojourn time in event E_i
$R_i(t)$	Reliability of the system when $E_i \in E$ (i = 0, 1, 2, 3, 4, 5, 6)
$U_i(t)$	Probability that the system is up when the events are E_0, E_1 or E_5 at epoch given that E_i;(i = 0, 1, 2, 3, 4, 5, 6)
$D_i(t)$	Probability that the system is down when the events are E_2, E_4 or E_6 at epoch given that E_i;(i = 0, 1, 2, 3, 4, 5, 6)
$B_i(t)$	Probability that the system is busy at epoch starting from $E_i \in E$.
$V_i(t)$	Expected number of visits by the repairman in (0,t] given that $E_i \in E$.

$\widetilde{Q}_{ij}(s) = \int_0^\infty e^{-st} dQ_{ij}(t)$, where ~ is the symbol for Laplace-Stieltjes transform

$q^*_{ij}(s) = \int_0^\infty e^{-st} q_{ij} dt$, the symbol * for Laplace transform

$\psi_i = \sum_j \int dQ_{ij}(t) = -\sum_j q^{*'}_{ij}(0) = \sum_j Q^{'}_{ij}(0)$

© Symbol for ordinary convolution

A(t) © B(t) = $\int_0^t A(t-u)B(u)du$

ⓢ Symbol for Stieltjes convolution

A(t)ⓢB(t) = $\int_0^t A(t-u)dB(u)$

Symbols for the Events of the System:

For the study of this system, we need to define the following states (see EL-Said & EL-Sherbeny (2005)). The reliability with dependent repair modes was also studied by Lim & Lie (2000).

N_a : unit in N-mode and operative
N_s : unit in N-mode and standby
F_r : unit in F-mode and under repair
F_w : unit in F-mode and waiting for repair
N_d : unit in N-mode when operator is in 'dead time'

We make use of the events given in Table 3.1 for the reliability analysis.

	State of	
Event	**P-unit**	**O-unit**
$E_0(N_o,N_s)$	operative	operable standby
$E_1(F_r,N_o)$	failed and under repair	operable
$E_2(N_o,N_s)$	not operating due to operator in 'dead time'	operable
$E_3(F_r,F_w)$	failed and under repair	failed and waiting for repair
$E_4(F_w,N_d)$	failed and waiting for repair	not operating due to operator in 'dead time'
$E_5(N_o,F_r)$	operative	under repair
$E_6(N_d,F_w)$	not operating due to operator in 'dead time'	failed and waiting for repair

Table 3.1

Transitions between events are shown in Figure 3.1

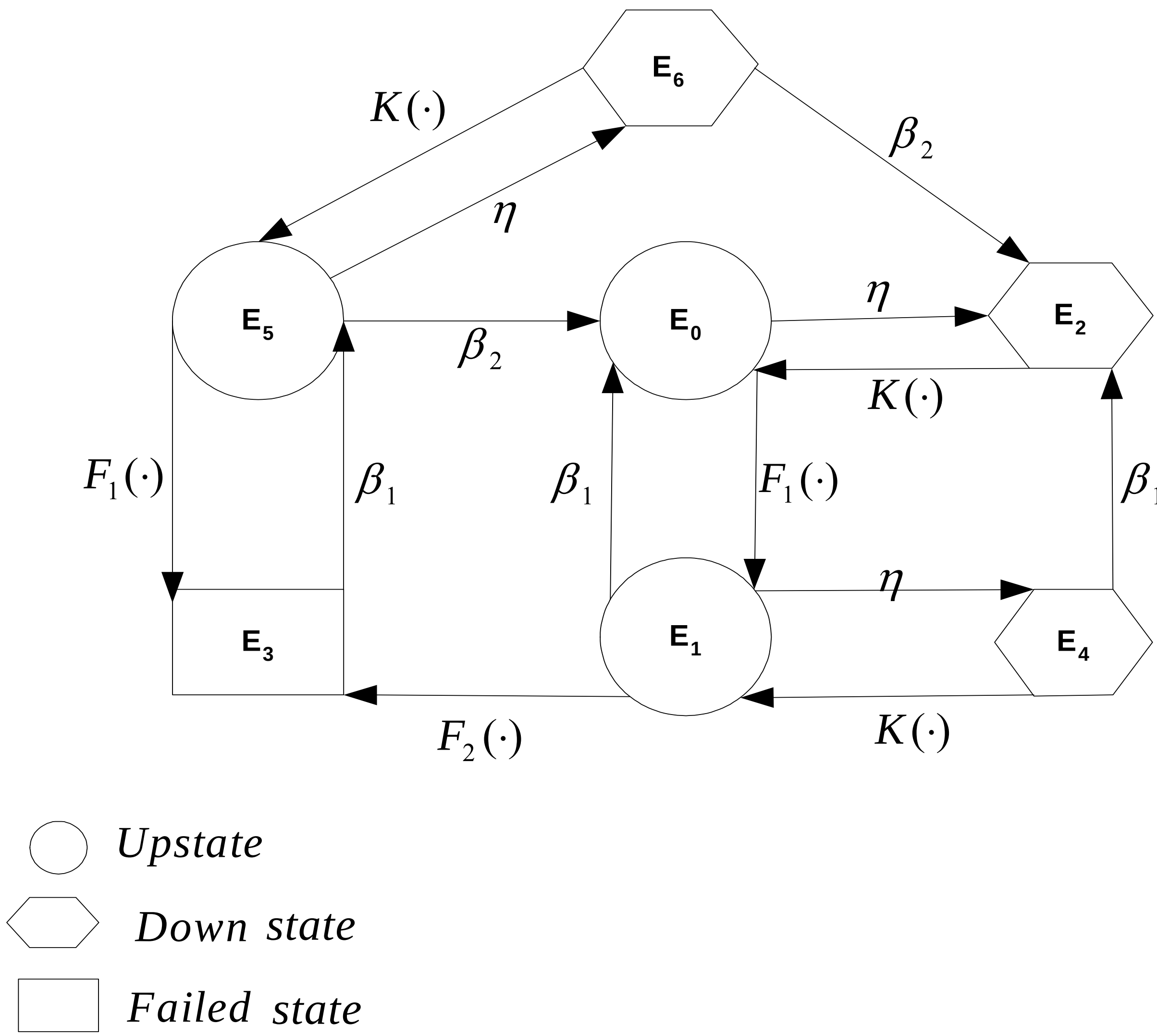
E6
K(·)
β2
η
E5
E0
η
E2
β2
K(·)
F1(·)
β1
β1
F1(·)
β1
η
E3
E1
E4
F2(·)
K(·)
Upstate
Down state
Failed state

Figure 3.1

3.3 AUXILIARY FUNCTIONS (TRANSITION PROBABILITIES AND SOJOURN TIMES)

Let O = T_0, T_1, … denote the epochs at which the system enters any state $E_i \in E$. Let X_n denote the state visited at epoch T_n+, i.e. just after the transition at T_n. Then

$$Q_{ij}(t) = P[X_{n+1} = j,\ T_{n+1} - T_n \leq t \mid X_n = i].$$

The transition probability matrix is given by

$P = [P_{ij}] = [Q_{ij}(\infty)] = Q(\infty)$ with non-zero elements.

Further,

$$P_{01} = \widetilde{F}_1(\eta)\,,\ P_{02} = 1 - \widetilde{F}_1(\eta)$$

$$P_{10} = \frac{[1-\widetilde{F}_2(\beta_1+\eta)]\beta_1}{\beta_1+\eta}$$

$$P_{13} = \widetilde{F}_2(\beta_1+\eta)\,,\ P_{14} = \frac{[1-\widetilde{F}_2(\beta_1+\eta)]\eta}{\beta_1+\eta}$$

$$P_{20} = p_{35} = 1,\ p_{41} = \widetilde{K}(\beta_1)\,,\ P_{40}^{(2)} = 1 - \widetilde{K}(\beta_1)$$

$$P_{53} = \widetilde{F}_1(\beta_2+\eta)\,,\ P_{51}^{(0)} = \widetilde{F}_1(\eta) - \widetilde{F}_1(\beta_2+\eta)$$

$$P_{56} = \frac{[1-\widetilde{F}_2(\beta_2+\eta)]\eta}{\beta_2+\eta}$$

$$P_{52}^{(0)} = \frac{1-\widetilde{F}_1(\eta)-[1-\widetilde{F}_1(\beta_2+\eta)]\eta}{\beta_2+\eta}$$

and $P_{60}^{(2)} = 1 - \widetilde{K}(\beta_2)\,,\ P_{65} = \widetilde{K}(\beta_2)\,.$

It can easily be verified that

$P_{01} + P_{02} = 1, \quad P_{10} + P_{13} + P_{14} = 1, \quad P_{20} = P_{35} = 1$

$P_{40}^{(2)} + P_{41} = 1, \quad P_{51}^{(0)} + P_{52}^{(0)} + P_{53} + P_{56} = 1$

$P_{60}^{(2)} + P_{65} = 1.$

To calculate mean sojourn time ψ_0 in state E_0, there is no transition to E_1 and E_2. Hence if T_0 denotes the sojourn time in E_0 then

$$\psi_0 = \int_0^\infty P[T_0 > t]dt = \frac{1-\tilde{F}_1(\eta)}{\eta}.$$

Similarly

$$\psi_1 = \frac{[1-\tilde{F}_2(\beta_1+\eta)]}{\beta_1+\eta}$$

$$\psi_2 = \int_0^\infty \overline{K}(t)dt = m_1 \text{ (say) where } \overline{K}(t) = 1 - K(t)$$

$$\psi_3 = \frac{1}{\beta_1}$$

$$\psi_4 = \frac{1-\tilde{K}(\beta_1)}{\beta_1}$$

$$\psi_5 = \frac{1-\tilde{F}_1(\beta_2+\eta)}{\beta_2+\eta}$$

and $$\psi_6 = \frac{1-\tilde{K}(\beta_2)}{\beta_2}.$$

3.4 RELIABILITY ANALYSIS

Let the random variable T_i denote time to system failure from event E_i

(i = 0, 1, …,6).

The reliability of the system is given by

$$R_i(t) = P[T_i > t]$$

To determine the reliability of the system we regard the failed state of the system (E_3) as absorbing. By probabilistic arguments

$$R_0(t) = e^{-\eta t}\,\overline{F}_1(t) + q_{01}(t) © R_1(t) + q_{02}(t) © R_2(t) \quad (3.4.1)$$

$$R_1(t) = e^{-(\eta+\beta_1)t}\,\overline{F}_2(t) + q_{10}(t) © R_0(t) + q_{14}(t) © R_4(t) \quad (3.4.2)$$

$$R_2(t) = \overline{K}(t) + q_{20}(t) © R_0(t) \quad (3.4.3)$$

$$R_4(t) = \overline{K}(t) + q_{40}^{(2)}(t) © R_0(t) + q_{41}(t) © R_1(t) \quad (3.4.4)$$

$$R_5(t) = e^{-(\eta+\beta_2)t}\,\overline{F}_1(t) + q_{51}^{(0)}(t) © R_1(t) + q_{52}^{(0)}(t) © R_2(t) + q_{56}(t) © R_6(t)$$

$$(3.4.5)$$

$$R_6(t) = \overline{K}(t) + q_{60}^{(2)}(t) © R_0(t) + q_{65}(t) © R_5(t). \quad (3.4.6)$$

Taking Laplace transforms for the equations (3.4.1) – (3.4.6) and simplify for $R_0^*(s)$ and omitting the argument 's' for brevity, we get

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} \quad (3.4.7)$$

where

$$N_1(s) = (1 - q_{56}^*\, q_{65}^*)\,[\,\overline{F}_1^*(\eta)(1 - q_{14}^*\, q_{41}^*) + \overline{K}^*(s)\, q_{02}^*\,(1 - q_{14}^*\, q_{41}^*)$$

$$+ \overline{F}_2^*(\eta+\beta_1)\, q_{01}^* + \overline{K}^*(s)\, q_{01}^*\, q_{14}^*]$$

and

$$D_1(s) = (1 - q_{56}^*\, q_{65}^*)\,[1 - q_{14}^*\, q_{41}^* - q_{01}^*\, q_{10}^* - q_{40}^{*(2)}\, q_{01}^* q_{14}^* - q_{02}^*\, q_{20}^*$$

$+ q_{02}^* q_{20}^* q_{14}^* q_{41}^*]$.

Note: For simplicity in this chapter, $q_{ij}^*(s)$ is written as q_{ij}^*.

From (3.4.7), the Mean Time to System Failure (MTSF) can be obtained

$$E(T_0) = \lim_{t\to\infty} R_0(t) = \lim_{s\to 0} sR_0^*(s)$$

$$= \frac{(1-p_{14}p_{41})(\psi_0 + \psi_2 p_{02}) + \psi_1 p_{01} + m_1 p_{01} p_{14}}{p_{01}p_{13}}. \qquad (3.4.8)$$

3.5 SYSTEM MEASURES

3.5.1 MEAN UP TIME IN (0, t]

As defined earlier $U_i(t)$ is the probability that the system is up in E_0, E_1 or E_5 at t given that $E_i \in E$. Hence we get

$$U_0(t) = e^{-\eta t}\ \overline{F}_1(t) + q_{01}(t) © U_1(t) + q_{02}(t) © U_2(t) \qquad (3.5.1)$$

$$U_1(t) = e^{-(\eta+\beta_2)t}\ \overline{F}_2(t) + q_{01}(t) © U_0(t) + q_{13}(t) © U_3(t) + q_{14}(t) © U_4(t) \qquad (3.5.2)$$

$$U_2(t) = q_{20}(t) © U_0(t) \qquad (3.5.3)$$

$$U_3(t) = q_{35}(t) © U_5(t) \qquad (3.5.4)$$

$$U_4(t) = q_{40}^{(2)}(t) © U_0(t) + q_{41}(t) © U_1(t) \qquad (3.5.5)$$

$$U_5(t) = e^{-(\eta+\beta_2)t}\ \overline{F}_1(t) + q_{51}^{(0)}(t) © U_1(t) + q_{52}^{(0)}(t) © U_1(t) + q_{53}(t) © U_3(t)$$

$$+ q_{56}(t) © U_6(t) \qquad (3.5.6)$$

and $U_6(t) = q_{60}^{(2)}(t) © U_0(t) + q_{65}(t) © U_5(t)$. (3.5.7)

Taking Laplace transforms for (3.5.1) – (3.5.7), we get

$$U_0^* = \frac{N_2(s)}{D_2(s)} \tag{3.5.8}$$

where

$$N_2(s) = \overline{F}_1^*(\eta)[(1 - q_{56}^* q_{65}^* - q_{35}^* q_{53}^* - q_{51}^{(0)} q_{13}^* q_{35}^* - q_{14}^* q_{41}^* - q_{14}^* q_{41}^* q_{56}^* q_{65}^*$$

$$+ q_{14}^* q_{41}^* q_{35}^* q_{53}^*] + \overline{F}_2^*(\eta+\beta_1)[q_{01}^* - q_{01}^* q_{56}^* q_{65}^* - q_{01}^* q_{35}^* q_{53}^*]$$

$$+ \overline{F}_1^*(\eta+\beta_1)[q_{01}^* q_{13}^* q_{35}^*]$$

$$D_2(s) = [1 - q_{56}^* q_{65}^*][1 - q_{14}^* q_{41}^* - q_{40}^{*(2)} q_{01}^* q_{14}^* - q_{02}^* q_{20}^*$$

$$+ q_{02}^* q_{20}^* q_{14}^* q_{41}^* - q_{01}^* q_{10}^*] - q_{35}^* q_{53}^* [1 - q_{14}^* q_{41}^*$$

$$- q_{40}^{*(2)} q_{01}^* q_{14}^* - q_{02}^* q_{20}^* + q_{02}^* q_{20}^* q_{14}^* q_{41}^* - q_{01}^* q_{10}^*]$$

$$- q_{13}^* q_{35}^* [q_{51}^{*(0)} + q_{52}^{*(0)} q_{20}^* q_{01}^* + q_{60}^{*(2)} q_{01}^* q_{56}^* q_{02}^* q_{20}^* q_{51}^{*(2)}].$$

The steady-state availability U_0 is given by

$$U_0 = \lim_{s\to 0} sU_0^*(s) = \frac{N_2(0)}{D_2'(0)} \tag{3.5.9}$$

where

$$N_2(0) = [(1-P_{14}P_{41})(1-P_{53}-P_{56}P_{65}) - P_{13}(P_{51}^{(0)} - P_{01})]\psi_0 + P_{01}(1-P_{53}-P_{56}P_{65})\psi_1$$

and

$$D_2'(0) = N_2(0) + [P_{01}P_{14}(1-P_{53}-P_{56}P_{65}) - P_{01}P_{13}P_{56}]m_1 + \psi_3[P_{01}P_{13}(1-P_{56}P_{65})]$$

$$+\psi_2[P_{02}(1-P_{14}P_{41})](1-P_{53}-P_{56}P_{65}) - P_{02}P_{13}(1-P_{53}-P_{56}) + P_{13}P_{52}^{(0)}].$$

Mean up time of the system during (0,t] is

$$\mu_{up}(t) = \int_0^t U_0(u)du \text{ so that}$$

$$\mu_{up}^{*}(s) = \frac{U_0^{*}(s)}{s}. \qquad (3.5.10)$$

3.5.2. MEAN DOWN TIME DURING (0, t]

To obtain mean down-time during (0, t], we consider $D_i(t)$ as the probability that the system is in state E_2, E_4 or E_6 at epoch t given that E_i has occurred at t = 0.

Here we have

$$D_0(t) = q_{01}(t) © D_1(t) + q_{02}(t) © D_2(t) \qquad (3.5.11)$$

$$D_1(t) = q_{10}(t) © D_0(t) + q_{13}(t) © D_3(t) + q_{14}(t) © D_4(t) \qquad (3.5.12)$$

$$D_2(t) = \overline{K}(t) + q_{20}(t) + D_0(t) \qquad (3.5.13)$$

$$D_3(t) = q_{35}(t) © D_5(t) \qquad (3.5.14)$$

$$D_4(t) = \overline{K}(t) + q_{40}^{(2)}(t) © D_0(t) + q_{41}(t) © D_1(t) \qquad (3.5.15)$$

$$D_5(t) = q_{51}^{(0)}(t) © D_1(t) + q_{52}^{(0)}(t) © D_2(t) + q_{53}(t) © D_3(t)$$

$$+ q_{56}(t) © D_6(t) \qquad (3.5.16)$$

and $$D_6(t) = \overline{K}(t) + q_{60}^{(2)}(t) © D_0(t) + q_{65}(t) © D_5(t). \qquad (3.5.17)$$

Taking Laplace transforms for the equations (3.5.11) - (3.5.17) and simplifying for $D_0^{*}(s)$ we get

$$D_0^{*}(s) = \frac{N_3(s)}{D_2(s)}$$

(3.5.18)

where

$$N_3(s) = \overline{F}_2^{*}(\eta+\beta_1)\,[q_{01}^{*}\, q_{13}^{*} q_{35}^{*}\ q_{52}^{(0)} + q_{02}^{*}(1 - q_{56}^{*}\ q_{65}^{*} - q_{35}^{*}\ q_{53}^{*}) - q_{02}^{*}\ q_{13}^{*} q_{35}^{*}\ q_{51}^{(0)}$$

- $q_{02}^* q_{14}^* q_{41}^* (1 - q_{56}^* q_{65}^* - q_{35}^* q_{53}^*)] + \overline{K}(s) q_{01}^* q_{14}^* (1 - q_{56}^* q_{65}^* - q_{35}^*$

$q_{53}^*)$

$+ \overline{K}(s) q_{01}^* q_{13}^* q\!\!/_{35}^* q_{56}^*$

The value of $D_0(t)$ can be obtained on taking the inverse Laplace transform of $D_0^*(s)$.

The steady-state probability of the system being down is given by

$$D_0 = \lim_{s\to 0} \frac{sN_3(s)}{D_2(s)} = \frac{N_3(0)}{D_2'(0)} \tag{3.5.19}$$

where

$$N_3(0) = m_1[1 - p_{01}p_{10} - p_{13}p_{51}^{(0)} - p_{02}p_{14}p_{41} + p_{56}p_{65}(1 - p_{14}p_{41}) + p_{01}p_{14}p_{56}p_{65}].$$

Now the mean down-time of the system during (0, t] is

$$\mu_{dn}(t) = \int_0^t D_0(u)du$$

$$\mu_{dn}^*(s) = \frac{D_0^*(s)}{s} \tag{3.5.20}$$

and the mean failed time in (0, t] is

$$\mu_f(t) = t - \mu_{up}(t) - \mu_{dn}(t)$$

so that

$$\mu_f^*(s) = \frac{1}{s^2} - \mu_{up}^*(s) - \mu_{dn}^*(s). \tag{3.5.21}$$

3.5.3 BUSY PERIOD ANALYSIS

$B_i(t)$ is defined as the probability that the system is busy at epoch t starting from state E_i, $E_i \in E$. We have the following recursive relations

$$B_0(t) = q_{01}(t) © B_1(t) + q_{02}(t) © B_2(t) \quad (3.5.22)$$

$$B_1(t) = e^{-(\eta+\beta_1)t}\,\overline{F}_2(t) + q_{01}(t) © B_0(t) + q_{13}(t) © B_3(t) + q_{14}(t) © B_4(t) \quad (3.5.23)$$

$$B_2(t) = q_{20}(t) + B_0(t) \quad (3.5.24)$$

$$B_3(t) = e^{-\beta_1 t} + q_{35}(t) © B_5(t) \quad (3.5.25)$$

$$B_4(t) = e^{-\beta_1 t}\,\overline{K}(t) + q_{40}^{(2)}(t) © B_0(t) + q_{41}(t) © B_1(t) \quad (3.5.26)$$

$$B_5(t) = e^{-(\eta+\beta_1)t}\,\overline{F}_1(t) + q_{51}^{(0)}(t) © B_1(t) + q_{52}^{(0)}(t) © B_2(t) + q_{53}(t) © B_3(t) + q_{56}(t) © B_6(t) \quad (3.5.27)$$

and $$B_6(t) = e^{-\beta_2 t}\,\overline{K}(t) + q_{60}^{(2)}(t) © B_0(t) + q_{65}(t) © B_5(t)\,. \quad (3.5.28)$$

Taking Laplace transforms for the equations (3.5.22) to (3.5.28) and simplifying for $B_0^*(s)$, we get

$$B_0^*(s) = \frac{N_4(s)}{D_2(s)} \quad (3.5.29)$$

where

$$N_4(s) = \overline{F}_2^*(\eta+\beta_1)\, q_{01}^*\, [1 - q_{56}^* q_{65}^* - q_{35}^*\, q_{53}^*] + \frac{1}{\beta_1 + s} q_{01}^*\, q_{13}^*\, [1 - q_{56}^*\, q_{65}^*]$$
$$+ \overline{K}^*(\beta_2+s)\, q_{01}^*\, q_{14}^*\, [1 - q_{56}^*\, q_{65}^* - q_{35}^*\, q_{53}^*] + \overline{F}_1^*(\eta+\beta_2+s)\, q_{01}^*\, q_{13}^*\, q_{35}^*$$
$$+ \overline{K}^*(\beta_2+s)\, q_{01}^*\, q_{13}^*\, q_{35}^*\, q_{56}^*\,.$$

The steady-state probability that the system is under repair starting from state E_0, i.e. probability that in the long run the repairman will be busy is given by

$$B_0 = \lim_{s\to 0} sB_0^*(s) = \frac{N_4(0)}{D_2'(0)} \quad (3.5.30)$$

where

$$N_4(0) = P_{01}[1 - P_{53} - P_{56}P_{65}](\Psi_1 + \Psi_4 P_{14}) + P_{01}P_{13}[\Psi_5 + \Psi_6 P_{56} + \Psi_3(1 - P_{56}P_{65})]\,.$$

The expected duration of busy time of repairman in (0, t] is

$$\mu_b(t) = \int_0^t B_0(u)du ,$$

so that

$$\mu_b^*(s) = \frac{B_0^*(s)}{s} \qquad (3.5.31)$$

and the expected idle time of repairman in (0, t] is

$$\mu_I(t) = t - \mu_b(t)$$

so that

$$\mu_I^*(s) = \frac{1}{s^2} - \mu_b^*(s) . \qquad (3.5.32)$$

3.5.4 EXPECTED NUMBER OF VISITS BY THE REPAIRMAN IN (0, t]

According to the definition of $V_i(t)$, by elementary probability arguments we have the following relations:

$$V_0(t) = Q_{01}(t) \circledS [1 + V_1(t)] + Q_{02}(t) \circledS V_2(t) \qquad (3.5.33)$$

$$V_1(t) = Q_{01}(t) \circledS V_0(t) + Q_{13}(t) \circledS V_3(t) + Q_{14}(t) \circledS V_4(t) \qquad (3.5.34)$$

$$V_2(t) = Q_{20}(t) \circledS V_0(t) \qquad (3.5.35)$$

$$V_3(t) = Q_{35}(t) \circledS V_5(t) \qquad (3.5.36)$$

$$V_4(t) = Q_{40}^{(2)}(t) \circledS V_0(t) + Q_{41}(t) \circledS V_1(t) \qquad (3.5.37)$$

$$V_5(t) = Q_{51}^{(0)}(t) \circledS [1 + V_1(t)] + Q_{52}^{(0)}(t) \circledS V_2(t) + Q_{53}(t) \circledS V_3(t) + Q_{56}(t) \circledS V_6(t) \qquad (3.5.38)$$

and $V_6(t) = Q_{60}^{(2)}(t) \circledS V_0(t) + Q_{65}(t) \circledS V_5(t)$. (3.5.39)

Taking Laplace-Stieljes transforms and simplifying $\widetilde{V}_0(s)$, we get

$$\widetilde{V}_0(s) = \frac{\widetilde{N}_5(s)}{\widetilde{D}_2(s)} \tag{3.5.40}$$

where

$$\widetilde{N}_5(s) = \widetilde{Q}_{01}(1 - \widetilde{Q}_{14}\widetilde{Q}_{41})[1 - \widetilde{Q}_{56}\widetilde{Q}_{65} - \widetilde{Q}_{35}\widetilde{Q}_{53}].$$

In the steady state, the number of visits per unit time is given by

$$V_0 = \lim_{t \to \infty} \frac{V_0(t)}{t} = \frac{\widetilde{N}_5(0)}{\widetilde{D}_2^{'}(0)} \tag{3.5.41}$$

$$\widetilde{N}_5(0) = P_{01}\ [1 - P_{14}P_{41}][1 - P_{35} - P_{56}P_{65}].$$

3.6 COST BENEFIT ANALYSIS

We are now in the position to obtain the profit function by the system considering mean up time, mean down time in (0, t], busy period and expected number of visits by the repairman in (0, t]. The next expected profit incurred in (0, t] is

C(t) = expected total revenue in (0, t] – expected total repair cost in (0, t]

– expected cost of visit by the repairman in (0, t]

$$= (C_0 - C_1)\ \mu_{up}(t) - C_1\mu_{dn}(t) - c_2\mu_b(t) - c_3V_0(t). \tag{3.6.1}$$

The expected total profit per unit of time in steady state is

$$C = \lim_{t \to \infty} \frac{C(t)}{t} = \lim_{s \to 0} s^2 C^*(s).$$

That is,

$$C = (C_0 - C_1)\ V_0 - C_1\ D_0 - C_2\ B_0 - C_3V_0 \tag{3.6.2}$$

where C_0 is the revenue per unit uptime, C_1 is the salary of the operator per unit time, C_2 is the cost per unit for which the system is under repair and C_3 is the cost per visit by the repairman.

3.7 SPECIAL CASES

CASE I

When the 'dead time' of the operator is zero, i.e. $\eta = 0$, then the results are as follows:

$$E(T_0) = \frac{n_1 + \phi_1}{P_{13}}$$

$$U_0 = \frac{n_1(1 - P_{10}P_{53}) + \phi_1 P_{51}^{(0)}}{X}$$

$$B_0 = \frac{P_{13}(\phi_3 + \phi_5) + \phi_1 P_{51}^{(0)}}{X}$$

and $$V_0 = \frac{P_{51}^{(0)}}{X}$$

where

$$X = P_{13}(\phi_3 + \phi_5) + n_1 P_{10} P_{51}^{(0)} + \phi_1 P_{51}^{(0)}$$

and

$$n_1 = \int \bar{F}_1(t)dt \,; \qquad \varphi_1 = \frac{1 - \tilde{F}_2(\beta_1)}{\beta_1}$$

$$\varphi_3 = \frac{1}{\beta_1} \,; \qquad \varphi_5 = \frac{1 - \tilde{F}_2(\beta_2)}{\beta_2}$$

$P_{10} = 1 - \tilde{F}_2(\beta_2)$; $P_{13} = \tilde{F}_2(\beta_2)$

$P_{53} = \tilde{F}_1(\beta_2)$, $P_{51}^{(0)} = 1 - \tilde{F}_1(\beta_2)$.

CASE II

When failure time distributions of both units in case I are negative exponential i.e.

$F_1(t) = 1 - e^{-\lambda_1 t}$; $F_2(t) = 1 - e^{-\lambda_2 t}$

then the results are as follows:

$$E(T_0) = \frac{\beta_1 + \lambda_1 + \lambda_2}{\lambda_1 \lambda_2}$$

$$U_0 = \frac{\beta_1(\beta_2(\lambda_1 + \beta_1) + \lambda_2(\lambda_1 + \beta_2))}{Y}$$

$$B_0 = \frac{\lambda_1(\lambda_1\lambda_2 + \beta_1\beta_2 + \beta_1\lambda_2 + \lambda_2\beta_2)}{Y}$$

$$V_0 = \frac{\lambda_1\beta_1\beta_2(\beta_1 + \lambda_2)}{Y}$$

where

$$Y = \beta_1\beta_2(\lambda_1 + \beta_1) + \lambda_1\lambda_2(\lambda_1 + \beta_1 + \beta_2).$$

3.8 NUMERICAL ANALYSES

Figure 3.2(i) shows graphically the change for β_1 versus $E(T_0)$

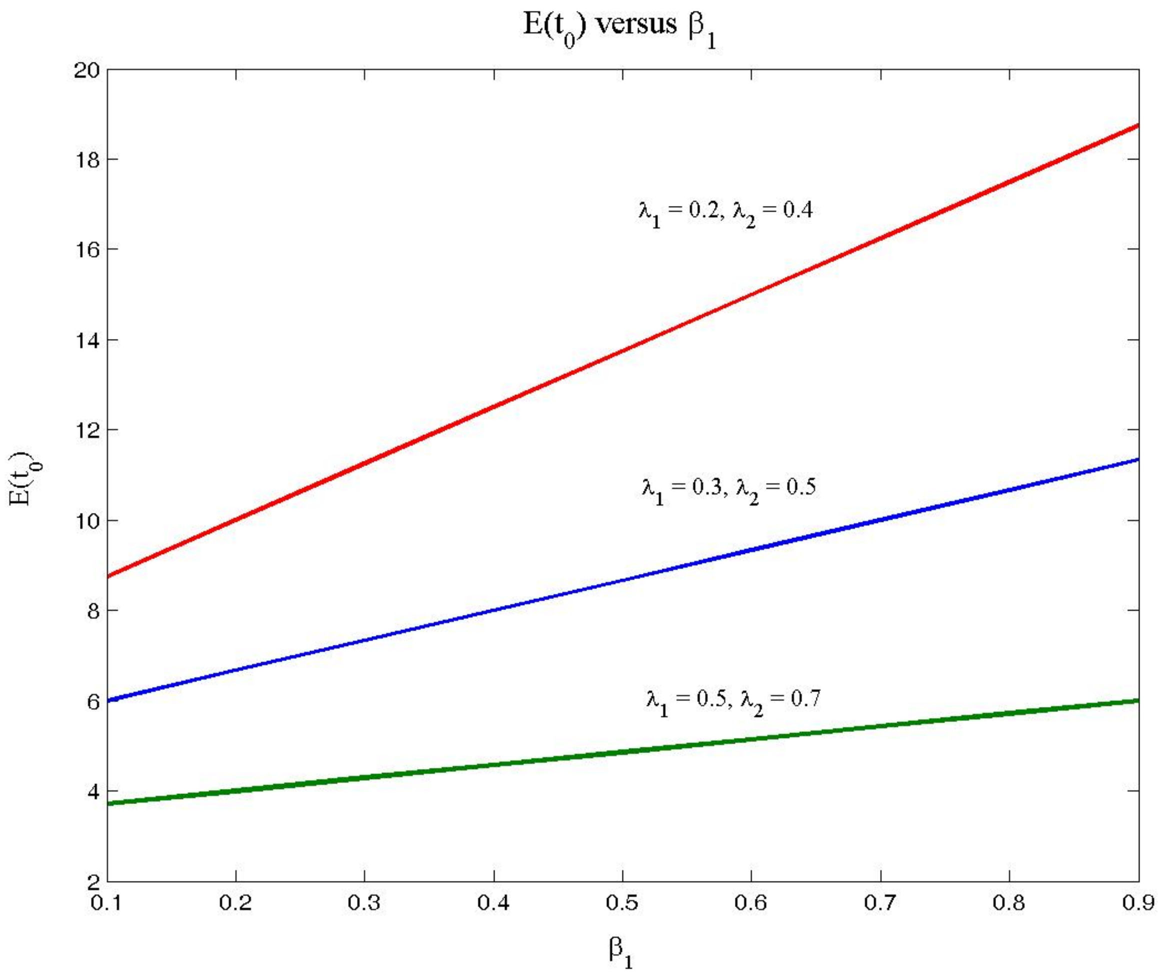

Figure 3.2

As the repair time of the priority unit, β_1, increases the mean expected time to failure $E(t_0)$ is an increasing function of β_1 (for different values of λ_1 and λ_2).

Figure 3.3 shows graphically the change for β_2 versus U_0

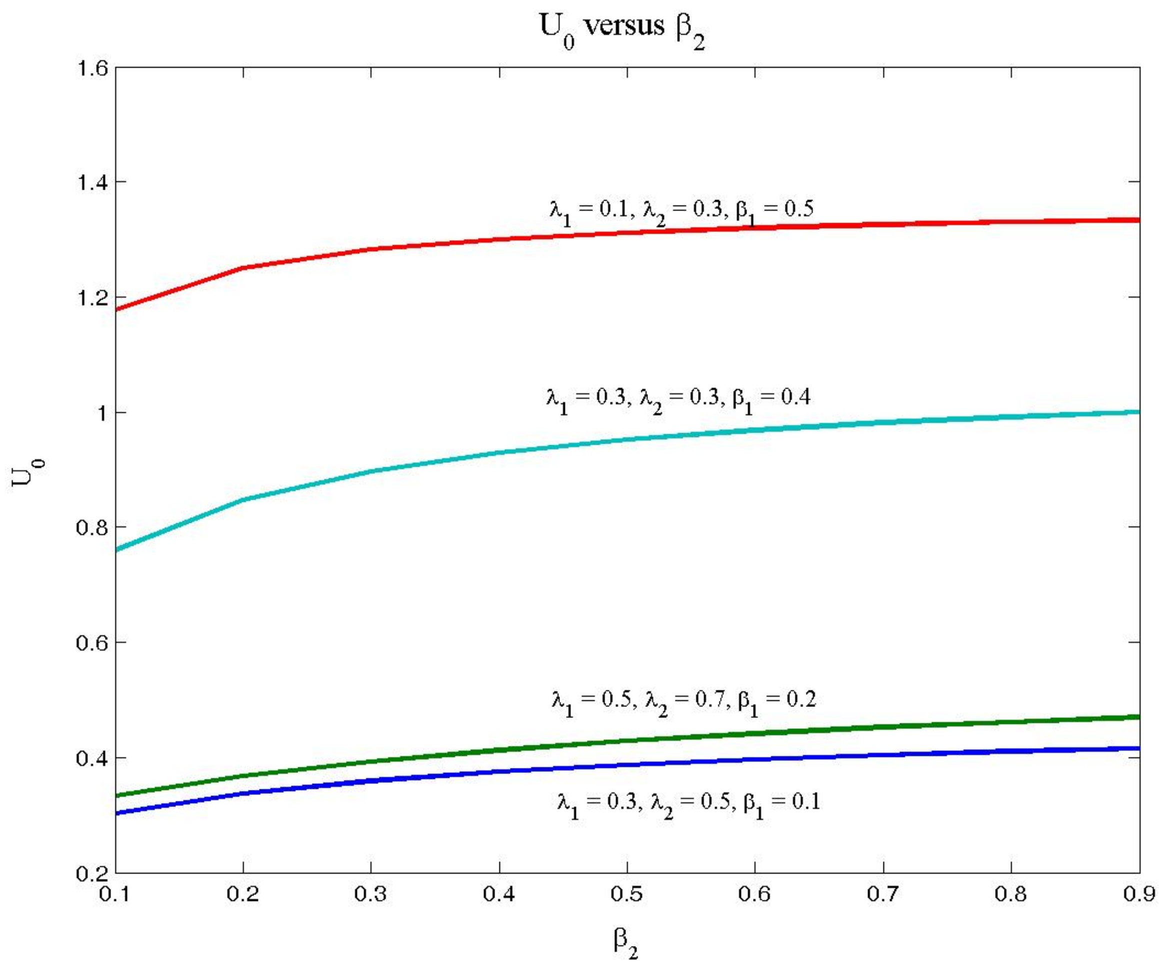

Figure 3.3

As the repair time of the ordinary unit, β_2, increases the steady-state availability U_0 is an increasing function of β_2 (for different values of λ_1, λ_2 and β_1).

Figure 3.4 shows graphically the change for β_2 versus B_0

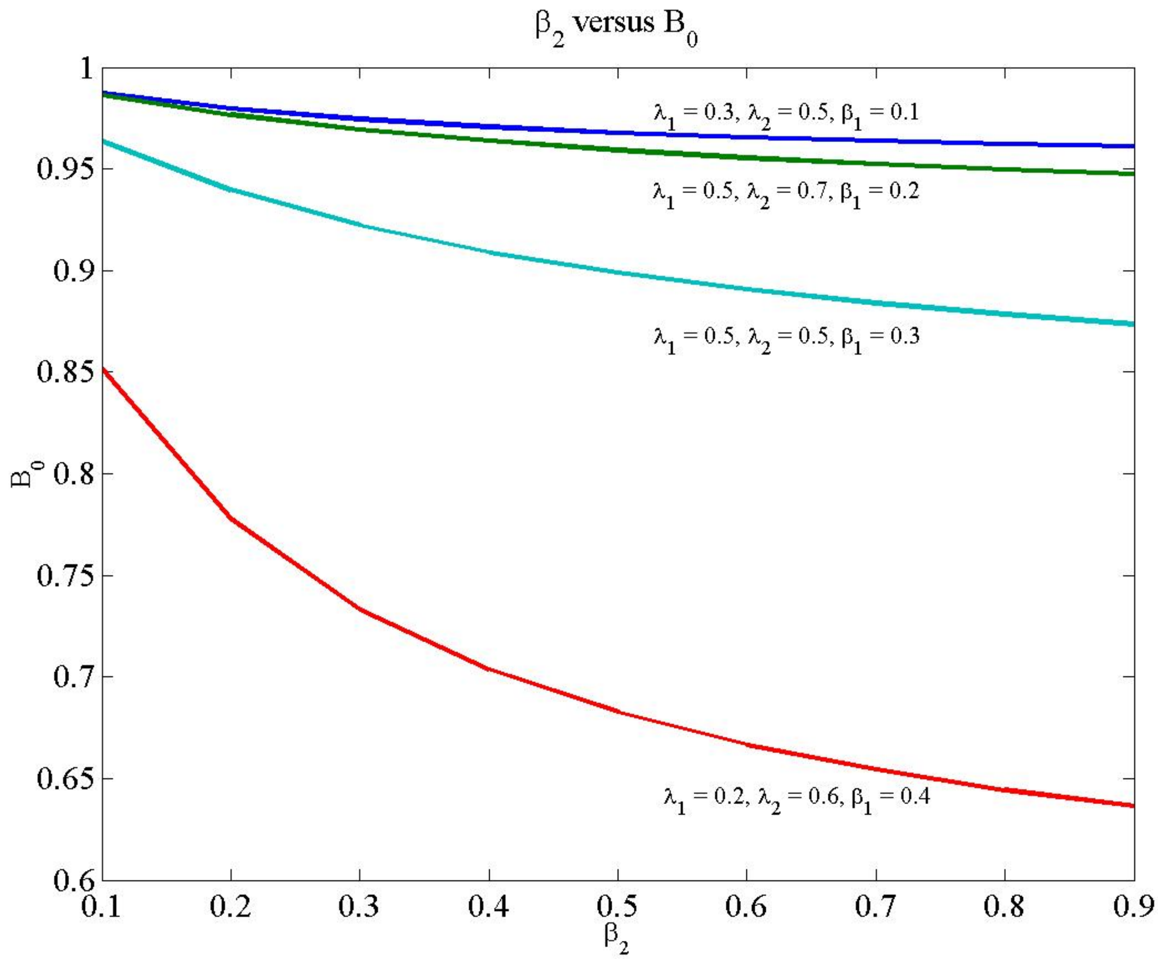

Figure 3.4

As β_2 increases the probability that the system is busy, B_0, is a decreasing function of β_2 (for different values of λ_1, λ_2 and β_1).

Figure 3.5 shows graphically the change for β_2 versus V_0

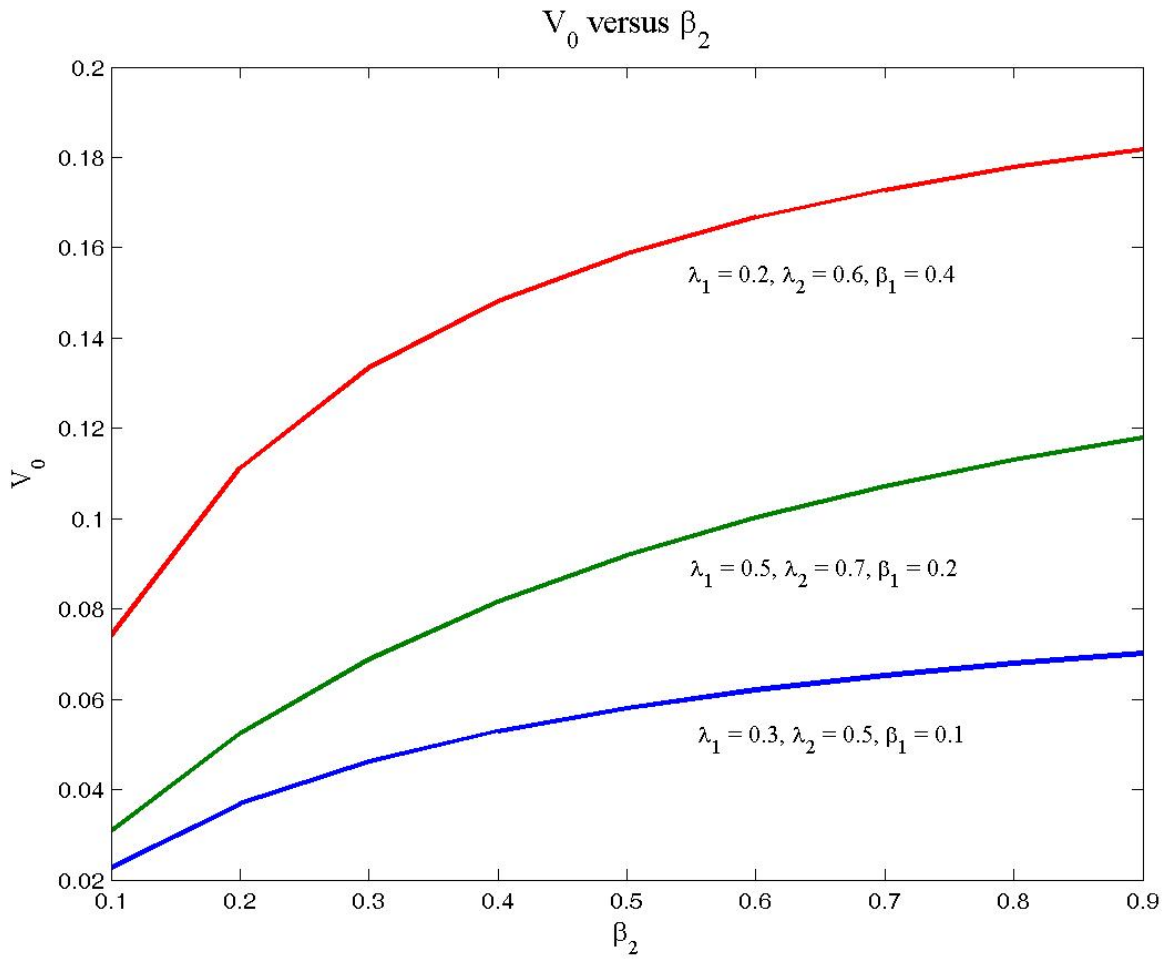

Figure 3.5

As β_2 increases the expected number of visits by the repairman, V_0, is an increasing function of β_2 (for different values of λ_1, λ_2 and β_1).

3.9 CONCLUSION

A two-unit single server priority redundant repairable system with two modes – normal and total failure has been studied. The priority unit got preference both in operation and repair. It is assumed that the repair facility is not available for a random time (Dead time). The system fails when both units are in total failure mode. Identifying the regeneration point technique, various operating characteristics of the system are obtained. The cost-benefit analysis is studied, and the results are illustrated numerically. The numerical results as shown in Figures 3.2 – 3.5 justify the results.