

## **CHAPTER 7**

### **ANALYSIS OF THE DYNAMIC CHARACTERISTICS OF PRACTICAL SYSTEMS OF CONGESTION USING CHAOS GENERATION METHODS**

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A modified version of this Chapter will be presented at a Southern African Institute for Industrial Engineering Conference, 2005.

## 7.1 INTRODUCTION

In pursuing the search for an alternative way of providing time-varying solutions for Systems of Congestion the thesis proceeds to examine a number of practical systems of a divergent and complex nature. **It only attempts to depict the transient operation of each practical system via chaos based system orbit generation and in so doing endeavours to furnish modelling techniques for use in achieving optimum dynamic operation.**

To eventually achieve optimum dynamic operation depends on the nature of operation of the System of Congestion i.e. that an operational objective be formulated against a background of economic, physical, social and other constraints of the system.

The various systems to be considered from the point of view of dynamic operation are

- System No. 1: Two single channel queues which alternatively make use of a single server and are combined to form a single System of Congestion.
- System No. 2: A multi-channel queue which serves a population of entities which arrive in a pattern which varies daily in time by orders of magnitude.
- System No. 3: A multi-channel queue (30 channels) each with a constant service rate combined to form a single System of Congestion.
- System No. 4: A multi-channel queueing system which serves an extensive population by communication when emergency conditions occur.

The various system configurations are explained in the following paragraphs:

## **7.2 SYSTEM NO. 1**

### **7.2.1 System scenario**

A crossing point over a river for vehicles in a rural area (Hartebeespoort dam wall in South Africa) consists of a single vehicle width bridge. The flow of vehicles over the bridge consists of eastbound and westbound traffic. Eastbound traffic cannot use the bridge while westbound traffic is using it and vice versa. The traffic flow is controlled by an existing automated signalling system which allows sequential periods of two minutes for traffic flow in a given direction. If sufficient entities are waiting to use the channel the service rate is approximately constant at 10 entities per two minute interval.

On certain days the traffic arrival rate increases (eastbound and westbound) over the period from 11h00 to 14h00 and then decreases over the period from 14h00 to 17h00.

### **7.2.2 The system model**

The Verhulst generation of eastbound and westbound arrivals are shown in Fig. 7.2.2.1 and Fig. 7.2.2.2 respectively and are based on fixed average arrival rates for sequential periods of 15 minutes. Each consecutive set of 15 minute temporally sequential periods of arrivals is further subdivided into 2 minute intervals for purposes of orbit generation. The arrivals generation process is based on actual observations on site.

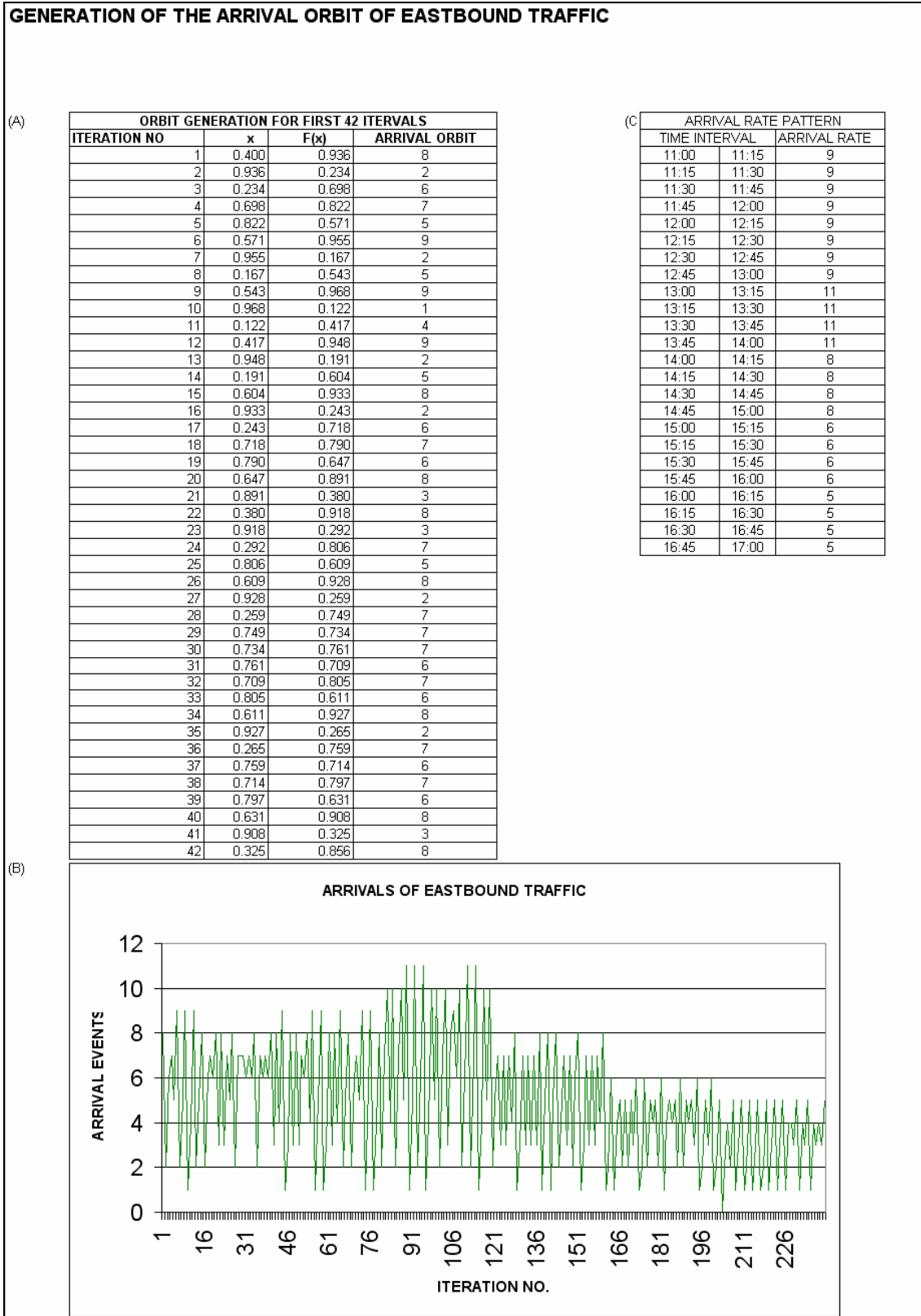


Fig. 7.2.2.1 GENERATION OF ARRIVALS OF EASTBOUND TRAFFIC

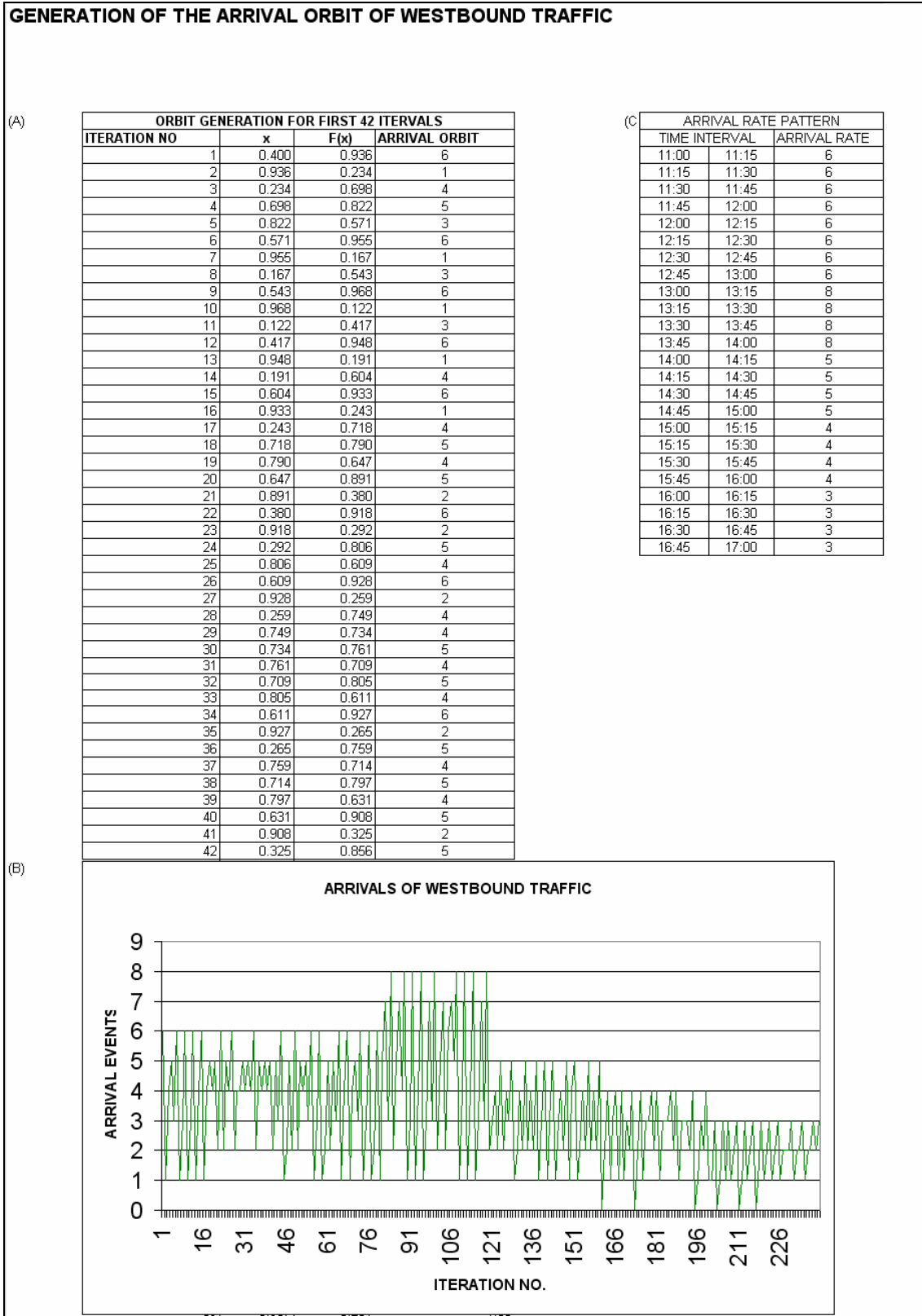


Fig. 7.2.2.2 GENERATION OF ARRIVALS OF WESTBOUND TRAFFIC

The Verhulst generation of service is shown in Fig. 7.2.2.3 and is based on an average service rate of 10 units per 2 minute interval with a standard deviation of 0,10 units. (i.e. an approximately constant service rate). The service event process is based on actual observations on site.

Combination of the eastbound traffic arrival orbit and interrupted service (each alternating 2 min. service time interval) results in a portrayal of the system event dynamics shown in Fig. 7.2.2.4 (C).

Pursuing the abovementioned combination method for the westbound traffic results in a portrayal of the system event dynamics shown in Fig. 7.2.2.5 (C).

Eventual combination (superposition) of the eastbound and westbound situations results in the portrayal of total system event dynamics as shown in Fig. 7.2.2.6 (D).

### **7.2.3 Diagnosis of the model results**

When viewing the system population values of Fig. 7.2.2.6 (A), (B) and (C), which closely match actual site conditions on a particular day, one could consider altering the service cycle pattern in an effort to decrease the system population values thereby improving the state of congestion.

Consequently several service cycle patterns have been considered as alternative patterns to the service cycle pattern employed in par. 7.2.2. The five service cycle patterns considered are shown in Fig. 7.2.2.7. Each of the situations of the system were analysed in the same way as described in par. 7.2.1 and par. 7.2.2.

The results of the analysis are shown in Fig. 7.2.2.8. A portrayal of the total system event dynamics for service cycle pattern No. 3 is shown in Fig 7.2.2.9.

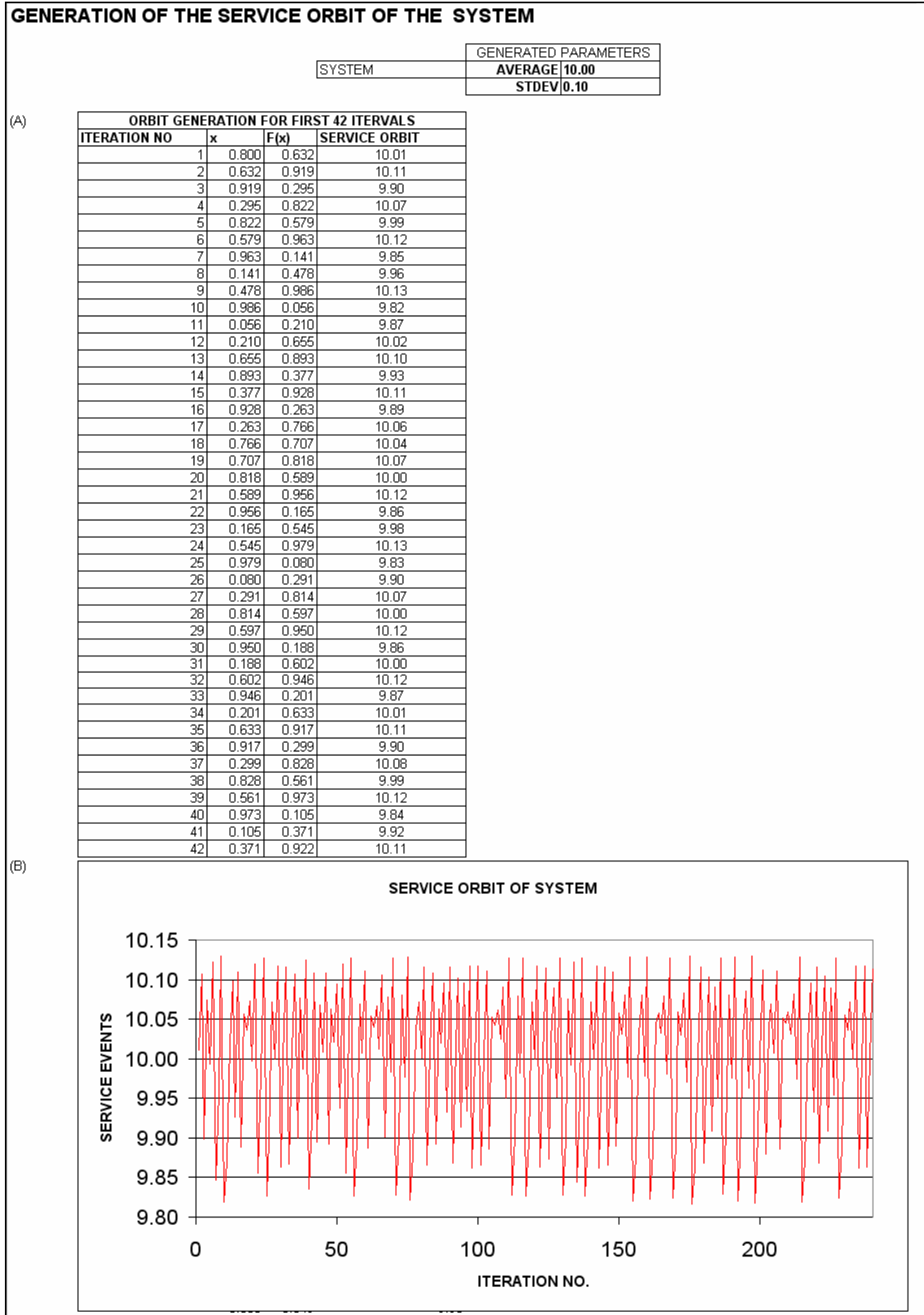
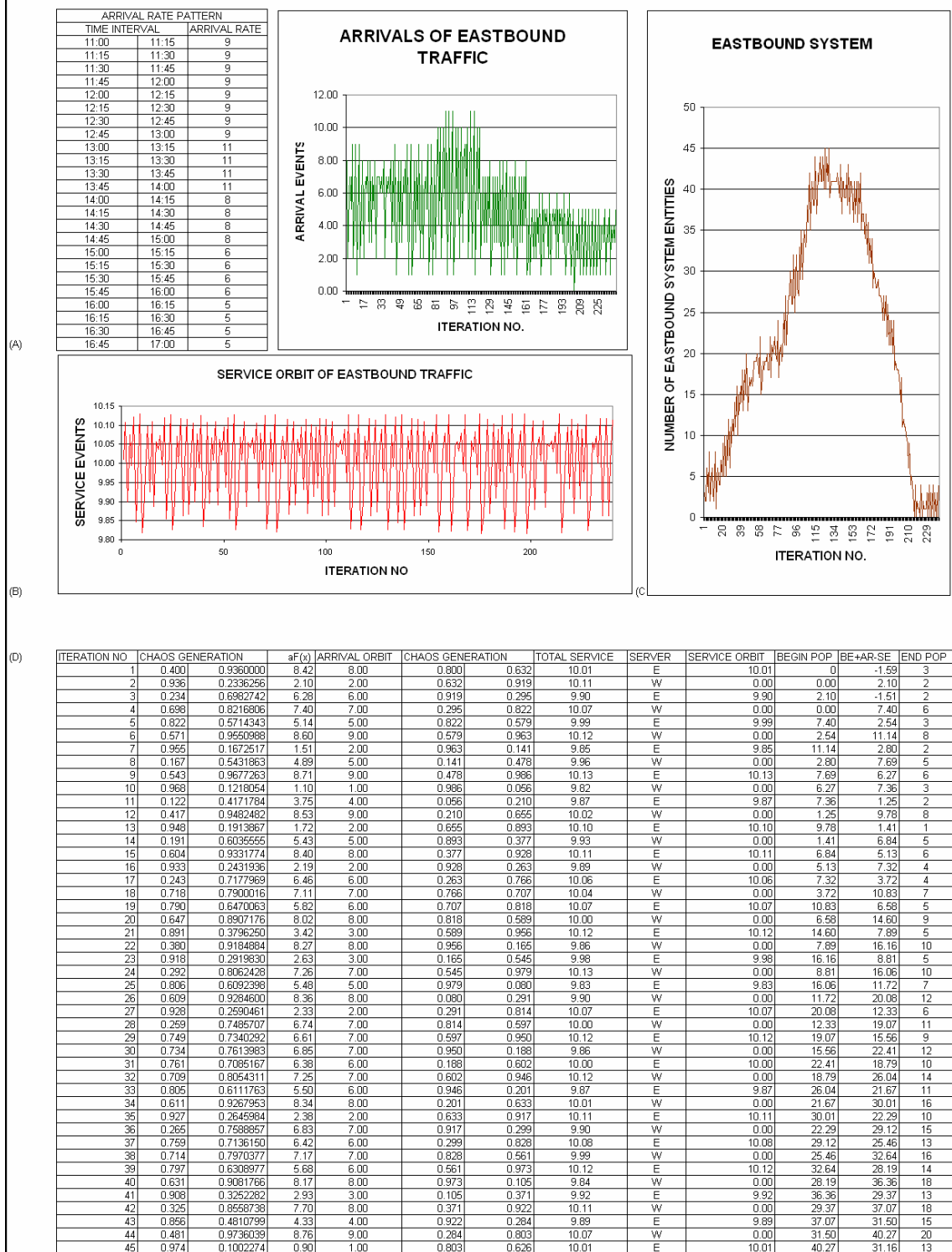


Fig. 7.2.2.3 GENERATION OF THE SERVICE ORBIT OF THE SYSTEM

**GENERATION OF THE EASTBOUND SINGLE CHANNEL SYSTEM ORBIT**



**Fig. 7.2.2.4 GENERATION OF THE EASTBOUND SINGLE CHANNEL QUEUE ORBIT**



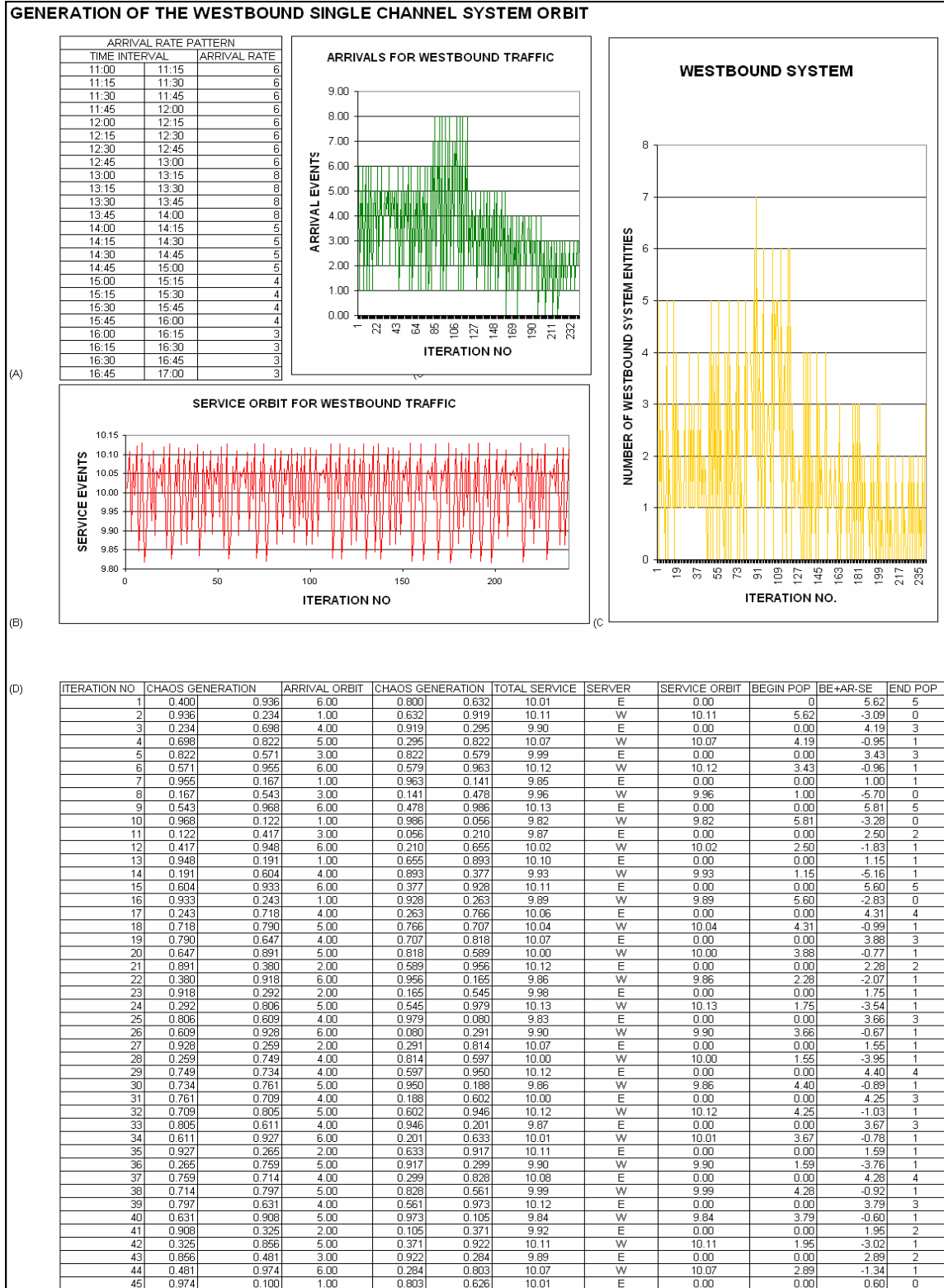


Fig. 7.2.2.5 GENERATION OF WESTBOUND TRAFFIC SINGLE CHANNEL QUEUE ORBIT

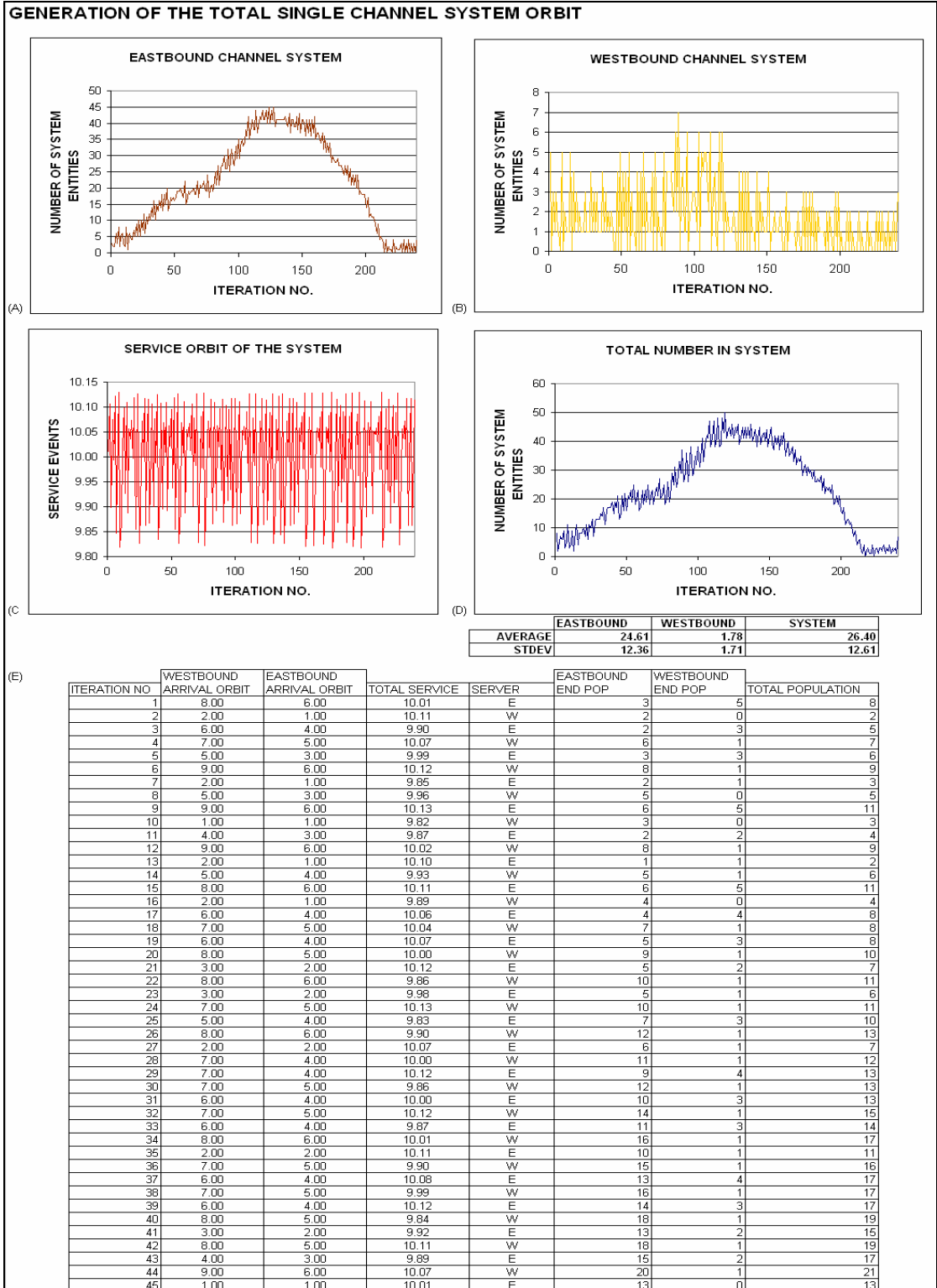


Fig. 7.2.2.6 GENERATION OF THE TOTAL SYSTEM ORBIT

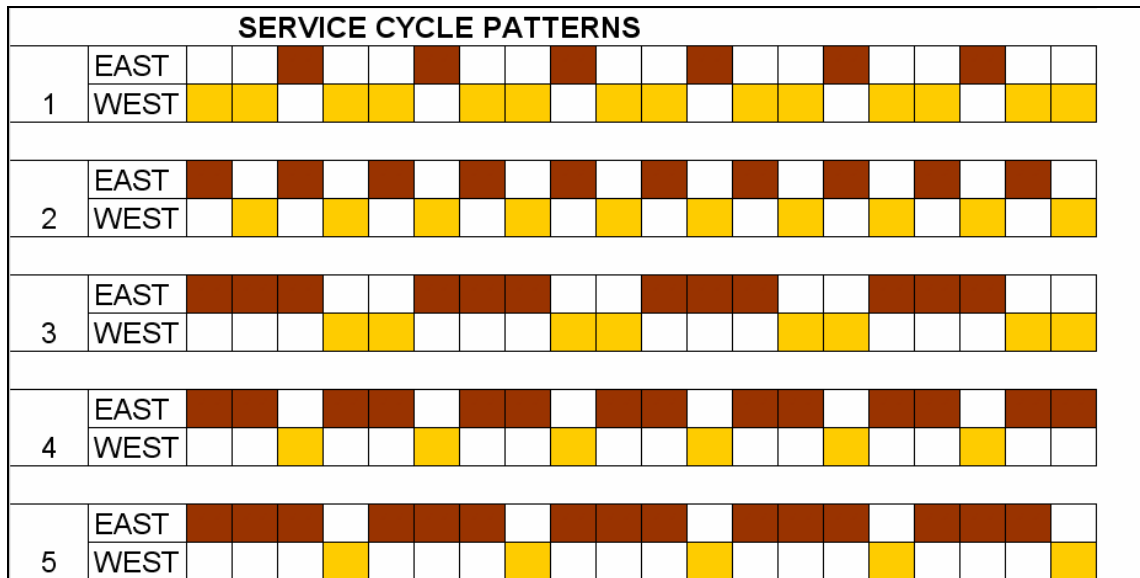


Fig. 7.2.2.7 ALTERNATIVE TEMPORAL SERVICE CYCLES FOR THE SYSTEM

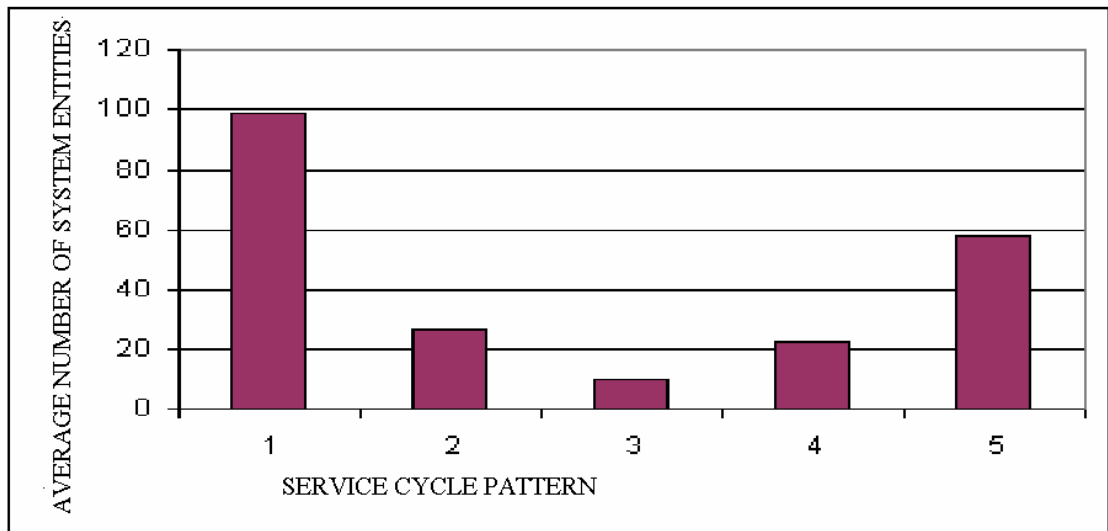
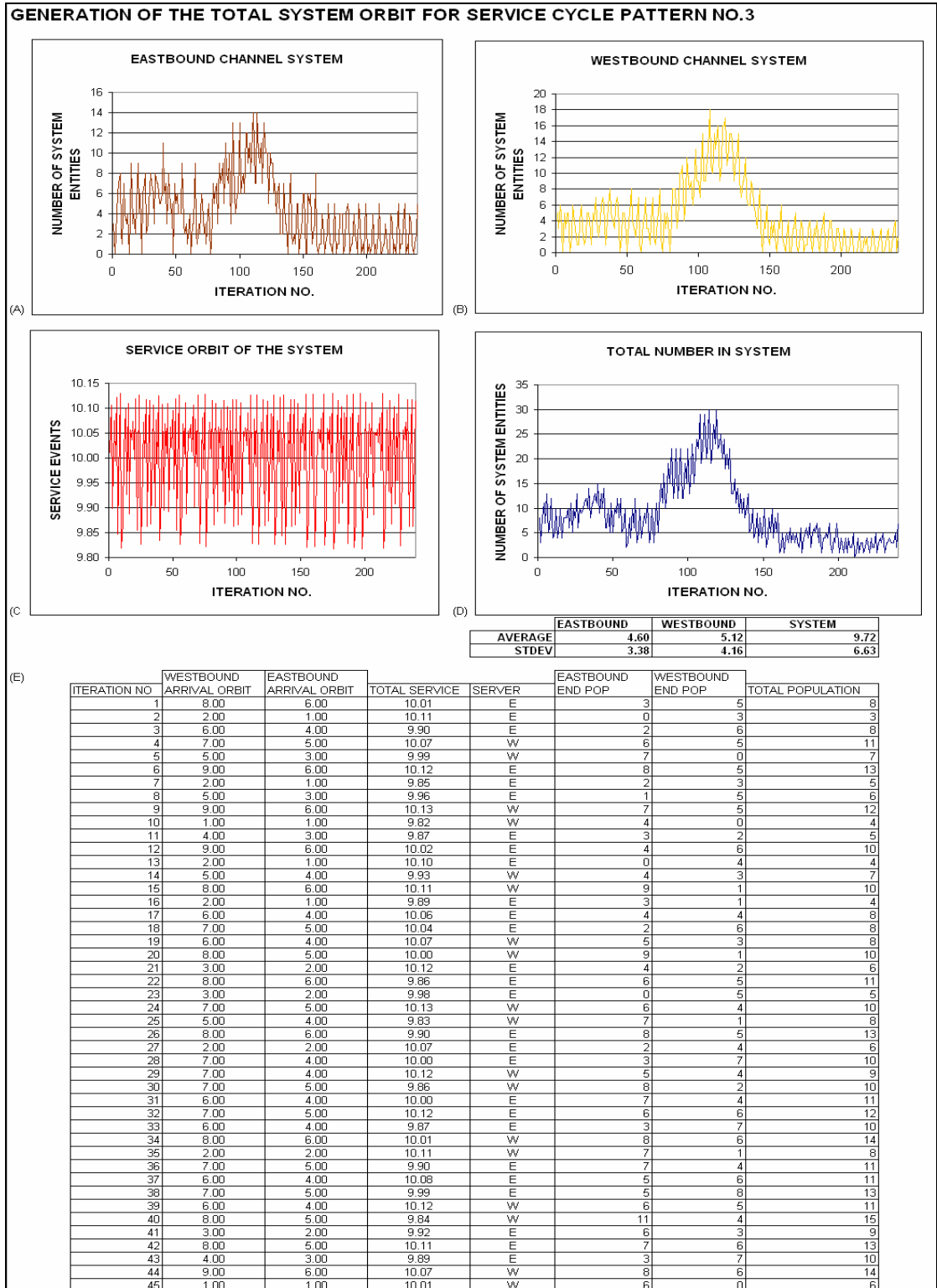


Fig. 7.2.2.8 AVERAGE SYSTEM POPULATION VALUES FOR SELECTED SERVICE CYCLE PATTERNS



**Fig. 7.2.2.9 GENERATION OF THE TOTAL SYSTEM ORBIT FOR SERVICE CYCLE PATTERN NO.3**

#### **7.2.4 Using realtime feedback to improve system performance**

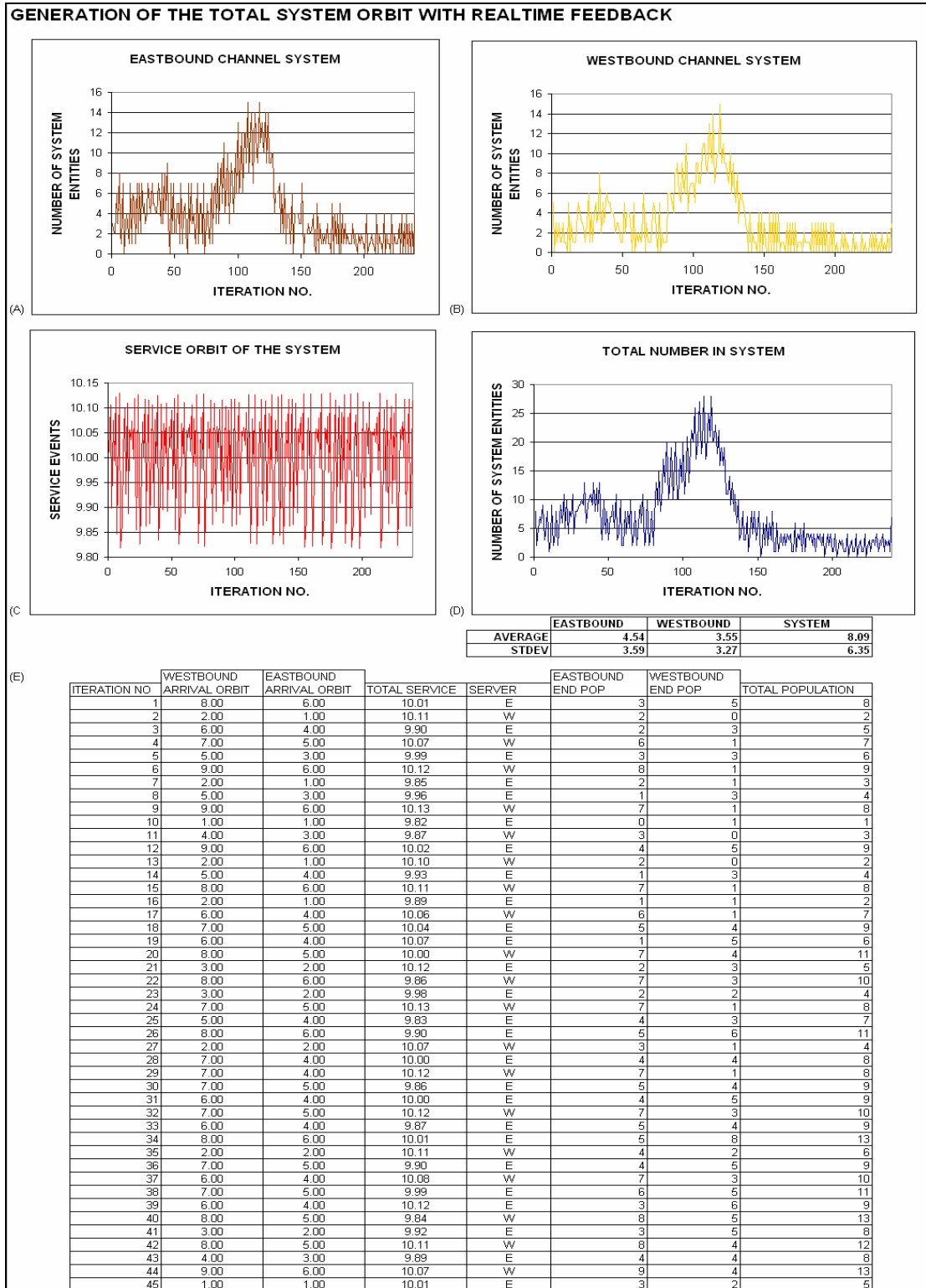
As an alternative to the foregoing attempt to minimize the total number of entities in the system by the use of service cycle pattern adjustment the use of a simplistic feedback system may be considered. Assume that the feedback system could influence the automated signalling system by comparing the number of eastbound and westbound entities in the system at the beginning of each 2-minute interval and then assigning the single service channel to the direction which contains the greater number of entities.

The results of analysing the system with realtime feedback are shown in Fig. 7.2.2.10 (D). When compared to the results of the system without feedback shown in Fig. 7.2.2.6 it is obvious that feedback can beneficially affect the degree of congestion by dramatically lowering the average number of entities in the total system from the initial value of 26.40 given in Fig 7.2.2.6 to 8.09 given in Fig 7.2.2.10.

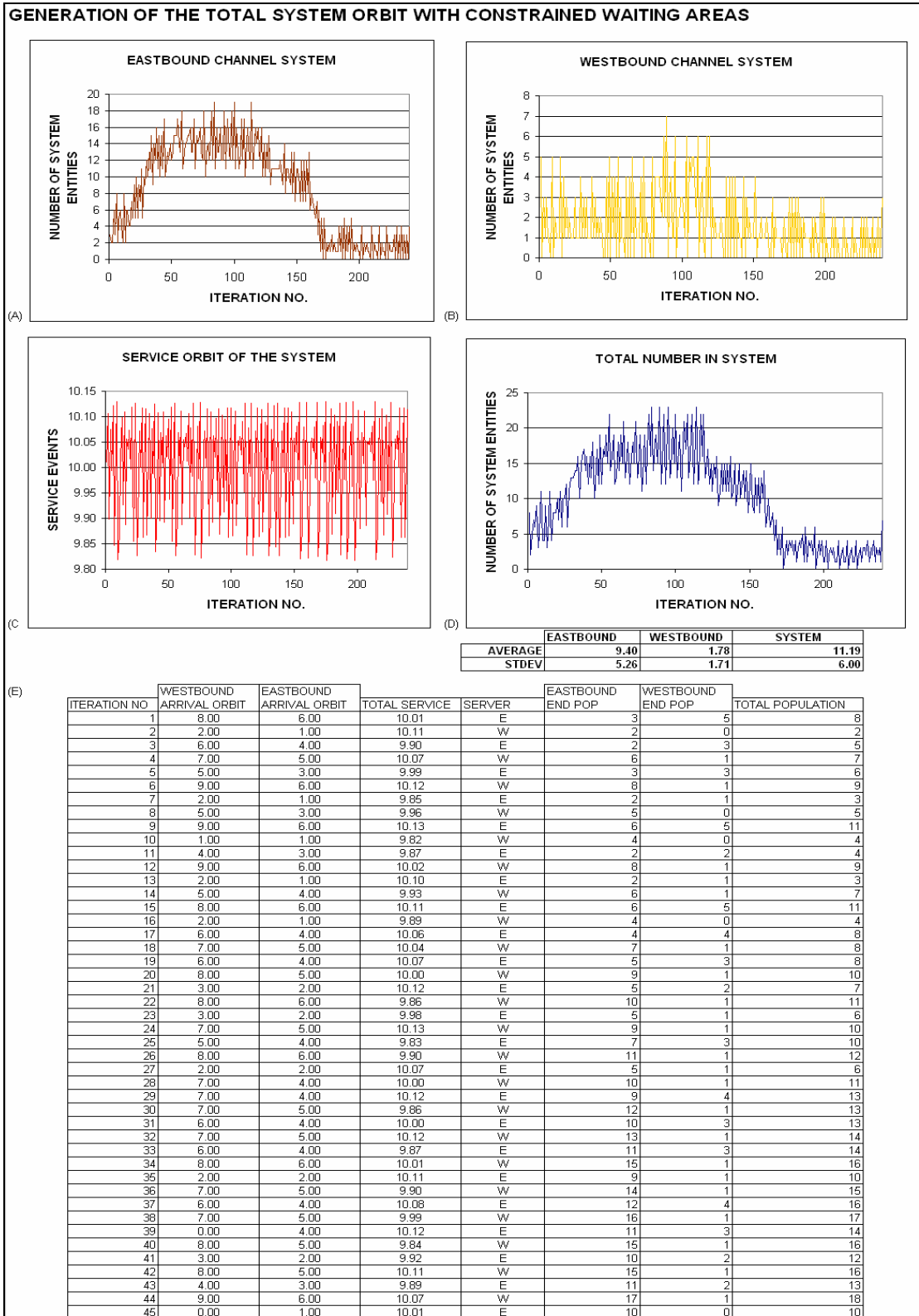
#### **7.2.5 The effect of the size of system waiting area on system performance**

A further measure which may be considered to improve congestion is to limit the total number of entities in the system for the eastbound and westbound traffic.

If one were to consider the system described in par. 7.2.2 and were to constrain the total eastbound population to 15 entities and impose the same limitation on the westbound population the results of analysing the system are shown in Fig. 7.2.2.11.



**Fig. 7.2.2.10 GENERATION OF THE TOTAL SYSTEM ORBIT WITH REALTIME FEEDBACK**



**Fig. 7.2.2.11 GENERATION OF SYSTEM ORBIT WITH CONSTRAINED WAITING AREAS**

#### **7.2.6 Concluding comments on System No. 1**

The foregoing analysis indicates that in attempting to depict the operation of System No. 1 one may investigate several alternative solutions and only be constrained by a lack of originality and imagination. Nevertheless one is often required to remain within the bounds of reality which implies that suggested improvement of the system under consideration must be practically feasible.



## 7.3 SYSTEM NO. 2

### 7.3.1 System scenario

System No. 2 is an example of a typical toll plaza (Pumalani Plaza South Africa) on a national highway serving vehicles in two directions between Pretoria and Polokwane. The specific system under investigation serves traffic in a North and Southbound direction. The system experiences congestion in the Northbound and Southbound direction on Fridays. At the end of the month the Northbound area of the toll plaza becomes heavily congested. The study will focus on this peak traffic situation.

The System of Congestion has four lanes for normal traffic and one lane dedicated to heavy vehicles. Lanes 1 and 2 serve more vehicles than lanes 3 and 4. The heavy vehicle lane serves a total of 14% of the total vehicle flow. The flow distribution pattern over the five lanes is shown in Fig. 7.3.2.3 (A). The traffic intensity increases over the period from 17h00 to 18h45 and then decreases over the period from 18h45 to 20h00.

### 7.3.2 The system model

Verhulst orbit generation of the northbound arrivals is shown in Fig. 7.3.2.1. The average arrival rate is determined for consecutive 15-minute intervals. The 15 minute intervals are divided into 1-minute intervals for purposes of orbit generation. The arrival orbit agrees with the actual observations gathered on site.

Service orbit generation of a single lane is shown in Fig. 7.3.2.2. The average service time was measured at 6 seconds per vehicle resulting in an average service rate of 10 vehicles per minute.

The total system was modelled using the arrival orbits and distributing the arrivals to different service lanes to match actual conditions on site. The

modelled result is shown in Fig. 7.3.2.3. It shows the arrival rate and the distribution of arrivals as percentages to the different service lanes in Fig. 7.3.2.3 (A). The queues that are generated ahead of each service lane and the total number of vehicles in the system are shown in Fig. 7.3.2.3 (B). The average number of vehicles in the system for the period from 17h00 to 19h00 is 53.

**GENERATION OF THE ARRIVAL ORBIT OF NORTHBOUND TRAFFIC**

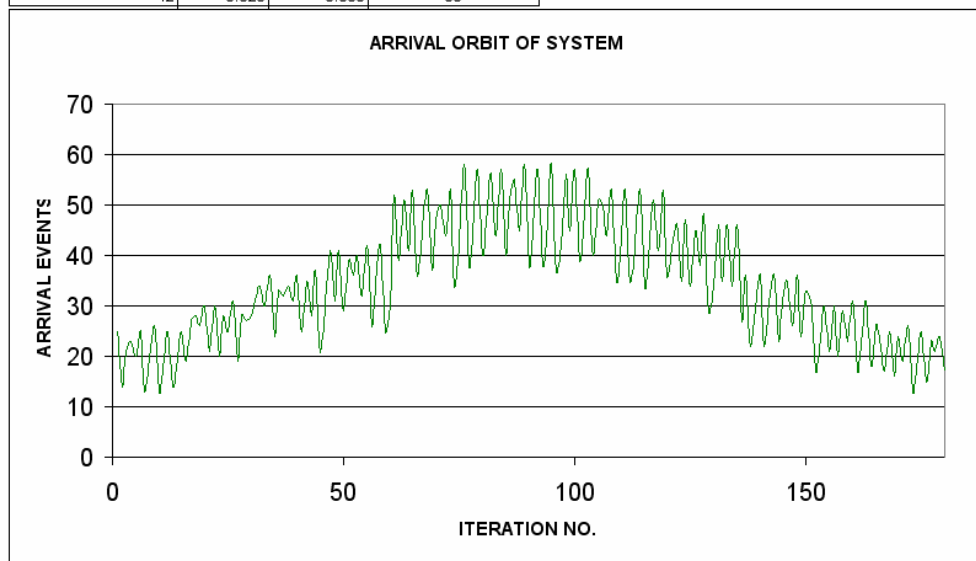
(A)

ORBIT GENERATION FOR FIRST 42 INTERVALS			
ITERATION NO	x	F(x)	ARRIVAL ORBIT
1	0.400	0.936	25
2	0.936	0.234	14
3	0.234	0.698	21
4	0.698	0.822	23
5	0.822	0.571	20
6	0.571	0.955	25
7	0.955	0.167	13
8	0.167	0.543	19
9	0.543	0.968	26
10	0.968	0.122	13
11	0.122	0.417	17
12	0.417	0.948	25
13	0.948	0.191	14
14	0.191	0.604	20
15	0.604	0.933	25
16	0.933	0.243	19
17	0.243	0.718	27
18	0.718	0.790	28
19	0.790	0.647	26
20	0.647	0.891	30
21	0.891	0.380	21
22	0.380	0.918	30
23	0.918	0.292	20
24	0.292	0.806	28
25	0.806	0.609	25
26	0.609	0.928	31
27	0.928	0.259	19
28	0.259	0.749	28
29	0.749	0.734	27
30	0.734	0.761	28
31	0.761	0.709	32
32	0.709	0.805	34
33	0.805	0.611	30
34	0.611	0.927	36
35	0.927	0.265	24
36	0.265	0.759	33
37	0.759	0.714	32
38	0.714	0.797	34
39	0.797	0.631	31
40	0.631	0.908	36
41	0.908	0.325	25
42	0.325	0.856	35

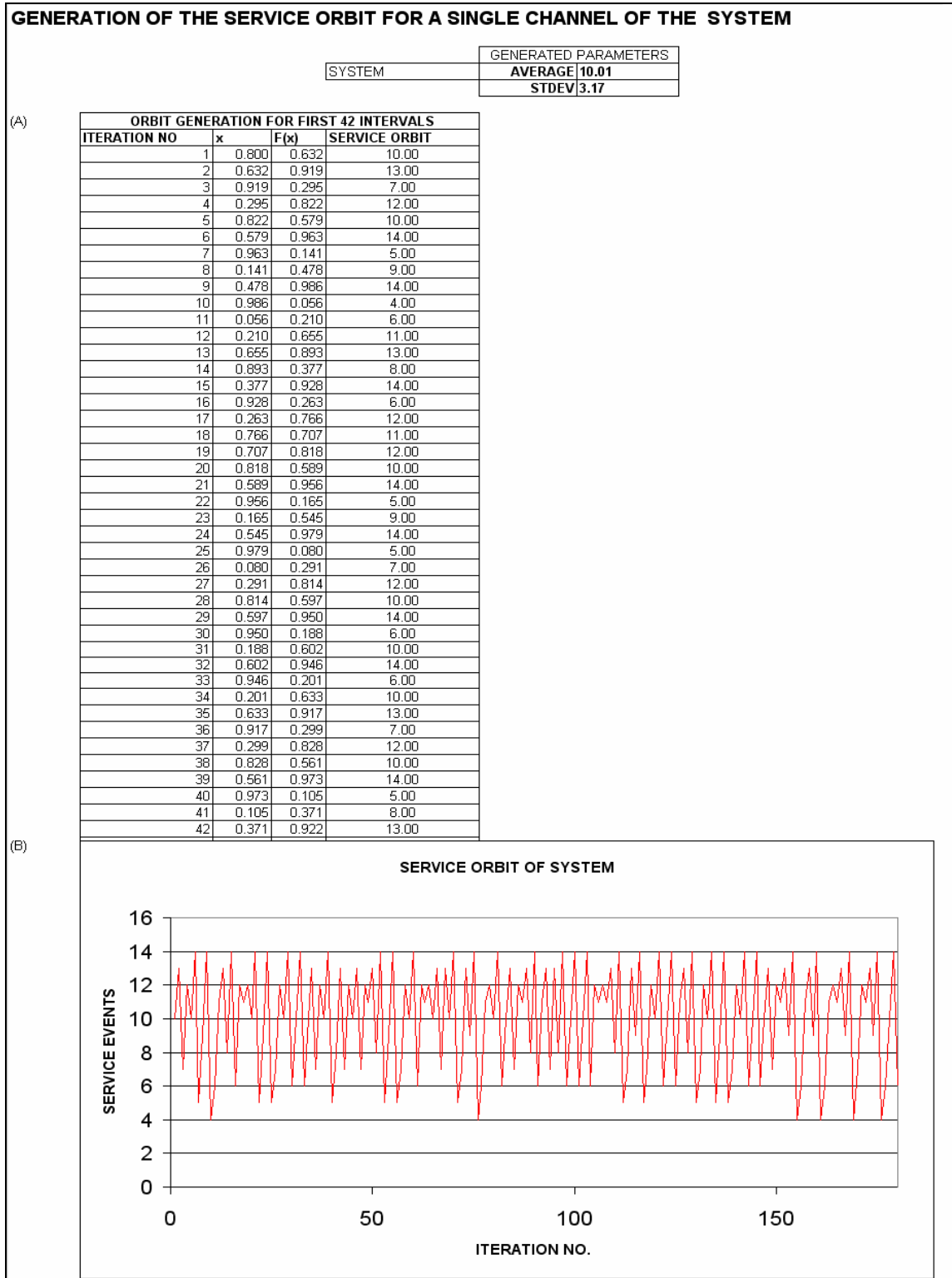
(C)

ARRIVAL RATE PATTERN		
TIME INTERVAL		ARRIVAL RATE
17:00	17:15	20
17:15	17:30	25
17:30	17:45	30
17:45	18:00	35
18:00	18:15	45
18:15	18:30	49
18:30	18:45	49
18:45	19:00	45
19:00	19:15	40
19:15	19:30	30
19:30	19:45	25
19:45	20:00	20

(B)



**Figure 7.3.2.1 GENERATION OF ARRIVALS OF NORTHBOUND TRAFFIC**



**Figure 7.3.2.2 GENERATION OF A TYPICAL SERVICE ORBIT FOR A SINGLE SERVICE LANE**

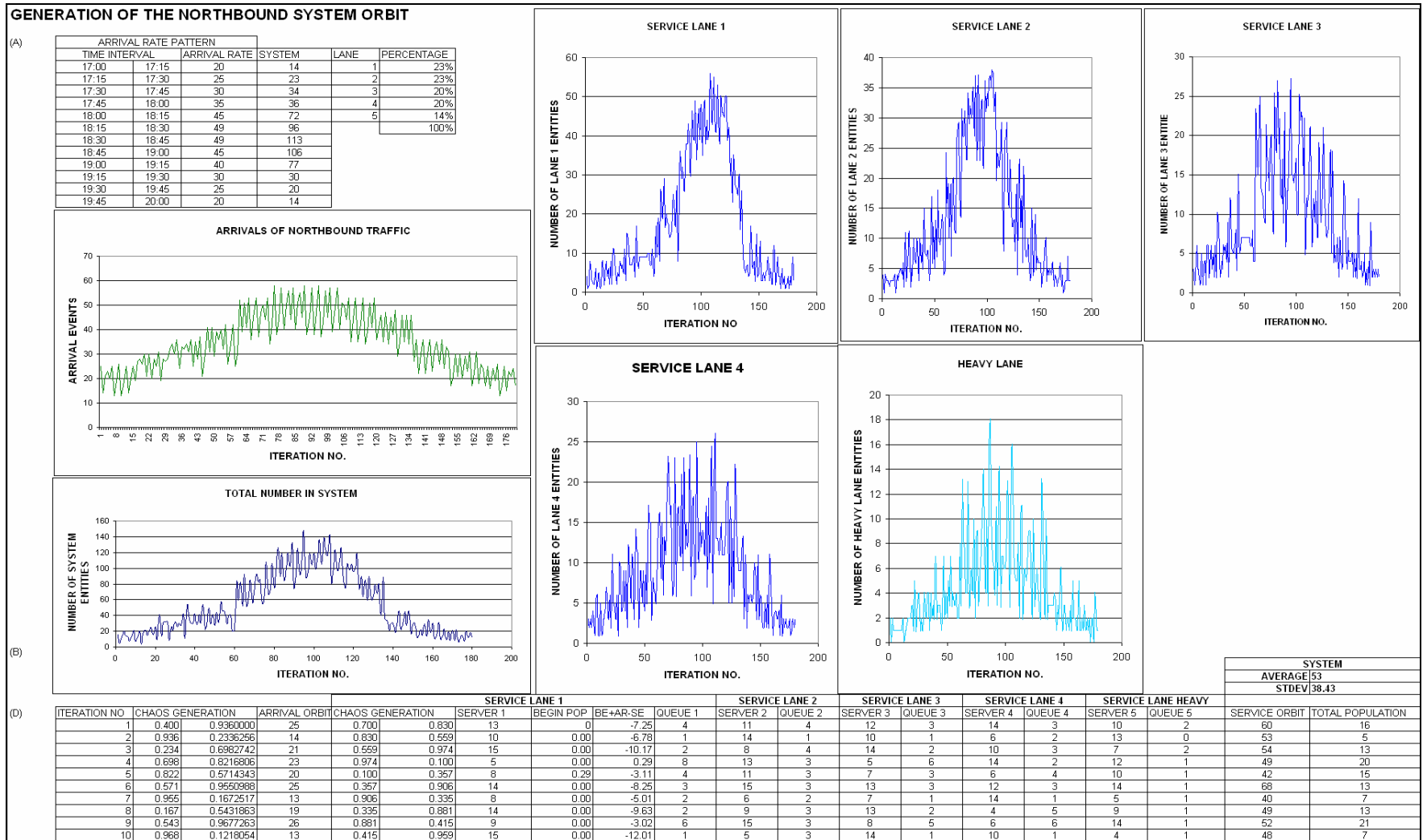


Figure 7.3.2.3 GENERATION OF THE TOTAL NORTHBOUND SYSTEM ORBIT

### **7.3.3 Diagnosis of the model results**

The results of Fig. 7.3.2.3 closely emulate the system behaviour. Lanes 1 and 2 have the greatest number of entities in their queues. The heavy vehicle lane peaks at 18 trucks in the queue. The maximum number of entities reaches a peak close to 150. The focus is on the average number in the system. One could attempt to improve the current system to reduce the average number in the system.

### **7.3.4 Using realtime feedback to improve system performance**

The physical system attributes suggest room for improvements such as urging drivers to choose the lane that is least congested. Such a solution is however not without physical hazard due to the jockeying (switching of lanes) that will take place.

A new technology that is available is the “e-tag” system that will improve the service. Certain of these electronic payment systems are already employed by some plazas. The designers of the system have made provision for multi-tasking by some of the lanes. This implies that a lane could be used to serve either northbound or southbound traffic as system conditions may dictate.

The use of a standby lane has been modelled and the results are shown in Fig 7.3.2.4. The model shows the use of the standby lane when the total number of vehicles in the system is above 80 vehicles. Fig 7.3.2.4 (A) shows the distribution of traffic during the peak congestion period. The standby lane only operates for periods of 70 consecutive iterations. These changes reduce the average number in the total system from 53 to 34 vehicles.

Other innovative improvement measures may be considered, the only limitation for improving the system being physical system constraints and financial implications.

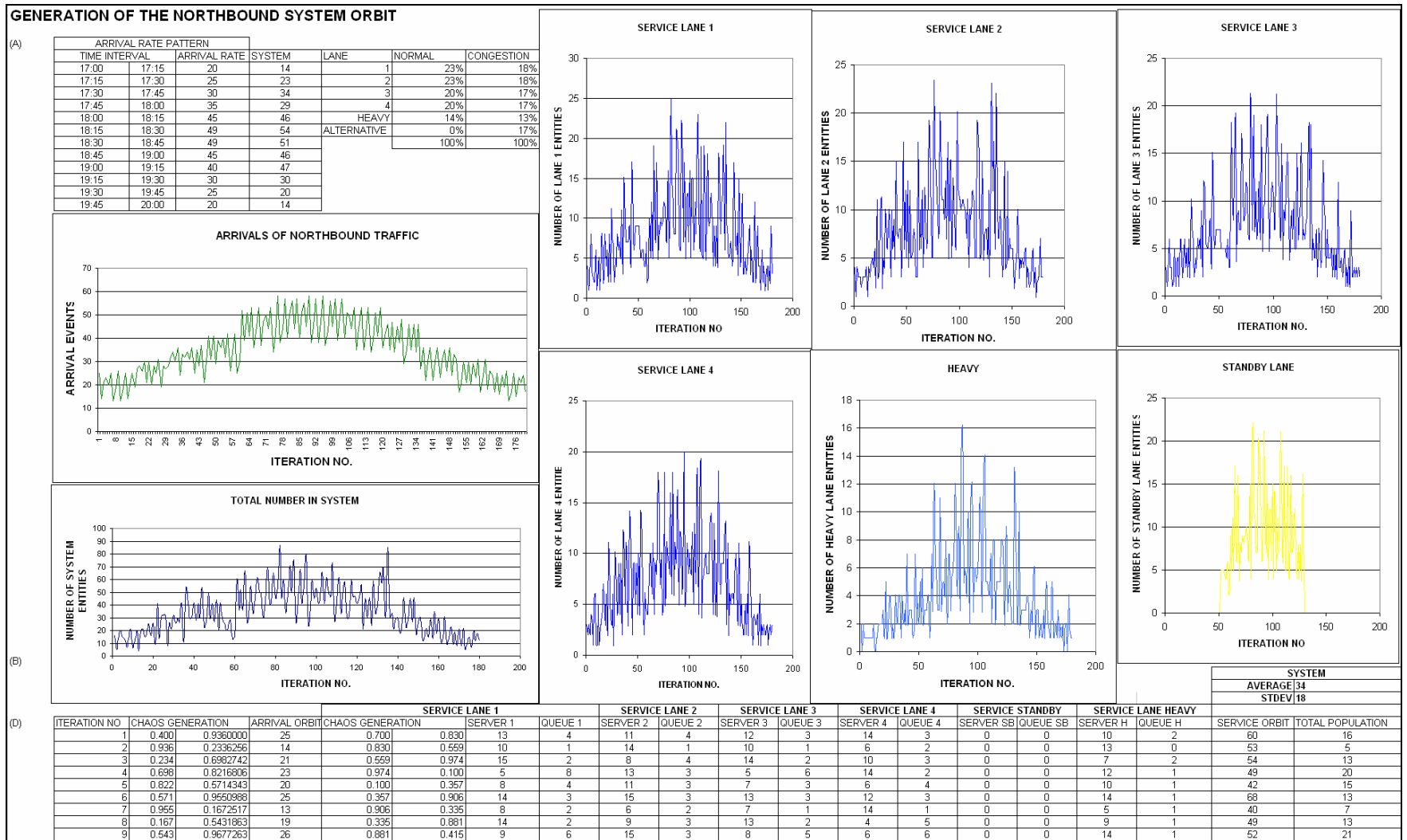


Figure 7.3.2.4 GENERATION OF SYSTEM ORBIT WITH REALTIME FEEDBACK

### **7.3.6 Concluding comments on System No. 2**

System No. 2 has been successfully modelled by means of the chaos orbit generation method. This implies that the basic building blocks which emanate from Chapter 6 may be used to model a multi-channel system. The modelling of the system has also demonstrated versatility in modelling improvements of the system via realtime feedback and other system adjustment methods.



## 7.4 SYSTEM NO. 3

### 7.4.1 System scenario

The system under consideration is a large *FERRIS*<sup>1</sup> wheel which is used for entertaining tourists in a large European city. The system is specifically designed to afford viewing of the entire city skyline. On clear days one can see up to 20km from the apex of the wheel. The wheel is equipped with 30 equally spaced cabins which can each accommodate 25 adult passengers.

The wheel makes 2 revolutions per hour. Each cabin completes a single revolution in 30 minutes and upon completion thereof discharges the passengers at ground level. The wheel diameter is 150 metres resulting in a peripheral speed of 0,26 metres per second.

The system described above is an “approximate” facsimile of the same order of magnitude as the actual system in respect of physical size and operational parameters. This has intentionally been done to avoid infringement of design copyright. The facsimile system is shown schematically in Fig.7.4.1.1. During certain periods of the day in the peak tourism season the system is a prime example of a System of Congestion.

### 7.4.2 The system model

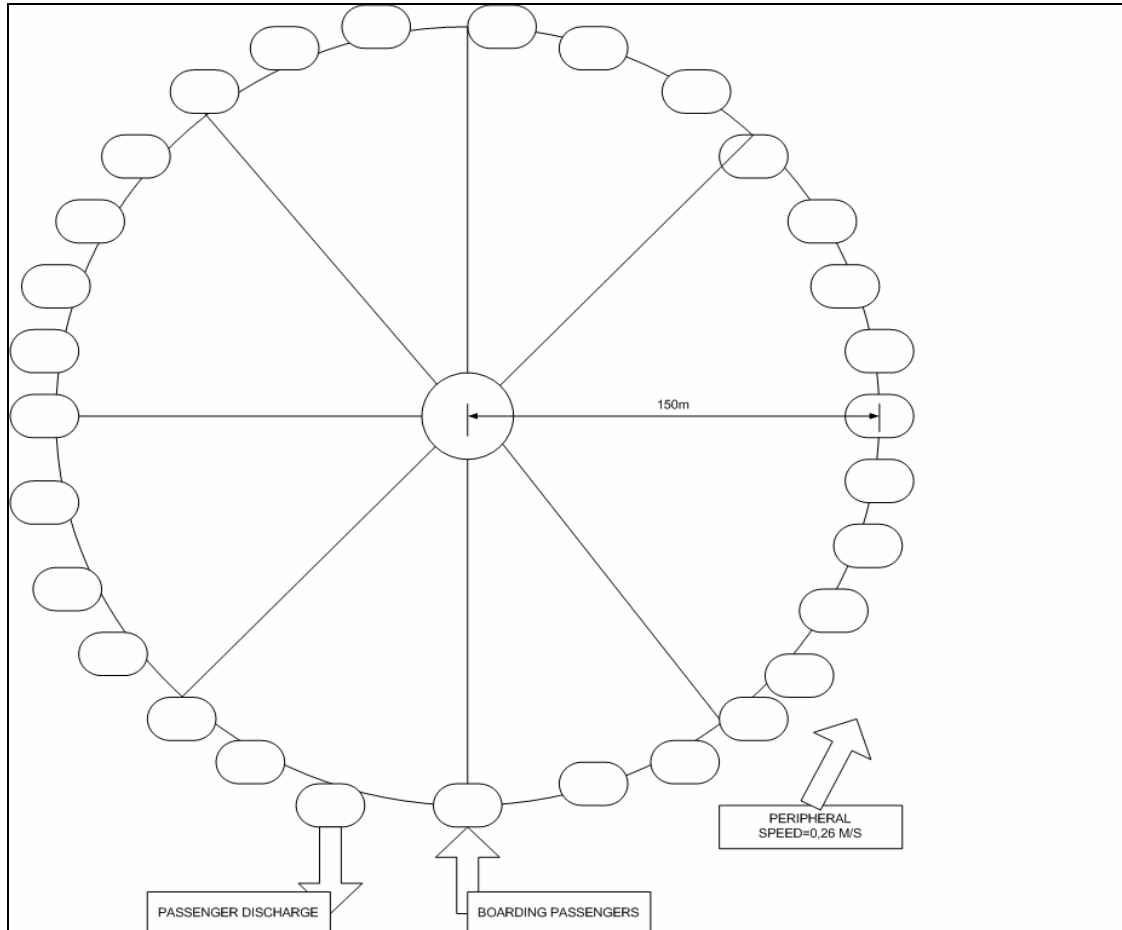
Whenever one attempts to model a System of Congestion it is wise to consider the simplest model which would generate credible dynamic operation. The use of “designer equations” is also facilitated in terms of extent and excessive complication of modelling. Consequently the system is modelled as a single channel queue which serves an arrivals process according to the average arrival rate pattern shown in Fig. 7.4.1.2. (A). The service rate is approximately 50 entities per two-minute interval. The

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<sup>1</sup>After George Washington Gale Ferris: American Engineer who designed a wheel for an Exposition in Chicago in 1893: An amusement device consisting of a large power-driven wheel having suspended seats which maintain a horizontal position while the wheel rotates in a vertical plane

generation of arrival and service orbits is based on actual observations on site.

The portrayal of the system event dynamics is shown in Fig. 7.4.1.2 (C) which agrees with “actual” observations on site.



**Figure 7.4.1.1 SCHEMATIC REPRESENTATION OF FERRIS WHEEL SYSTEM**

### **7.4.3 Diagnosis of the model results**

The system population values of Fig. 7.4.1.2 (C) which closely agree with observations on site indicate serious congestion, for example when the average system arrival rate shown in Fig. 7.4.1.2 (A) is 50 entities per 2 minute interval between 11h00 and 13h30 (fixed by conditions upstream of the Ferris wheel waiting area) the total system population is often 1500

entities which implies that approximately 750 entities are waiting for service on a FIRST COME FIRST SERVED (FCFS) basis for 30 minutes. At ground zero on site serious congestion occurs and is to be seen to be believed.

As is normally the case with systems suffering from congestion one should consider some-or-other action to improve system performance.

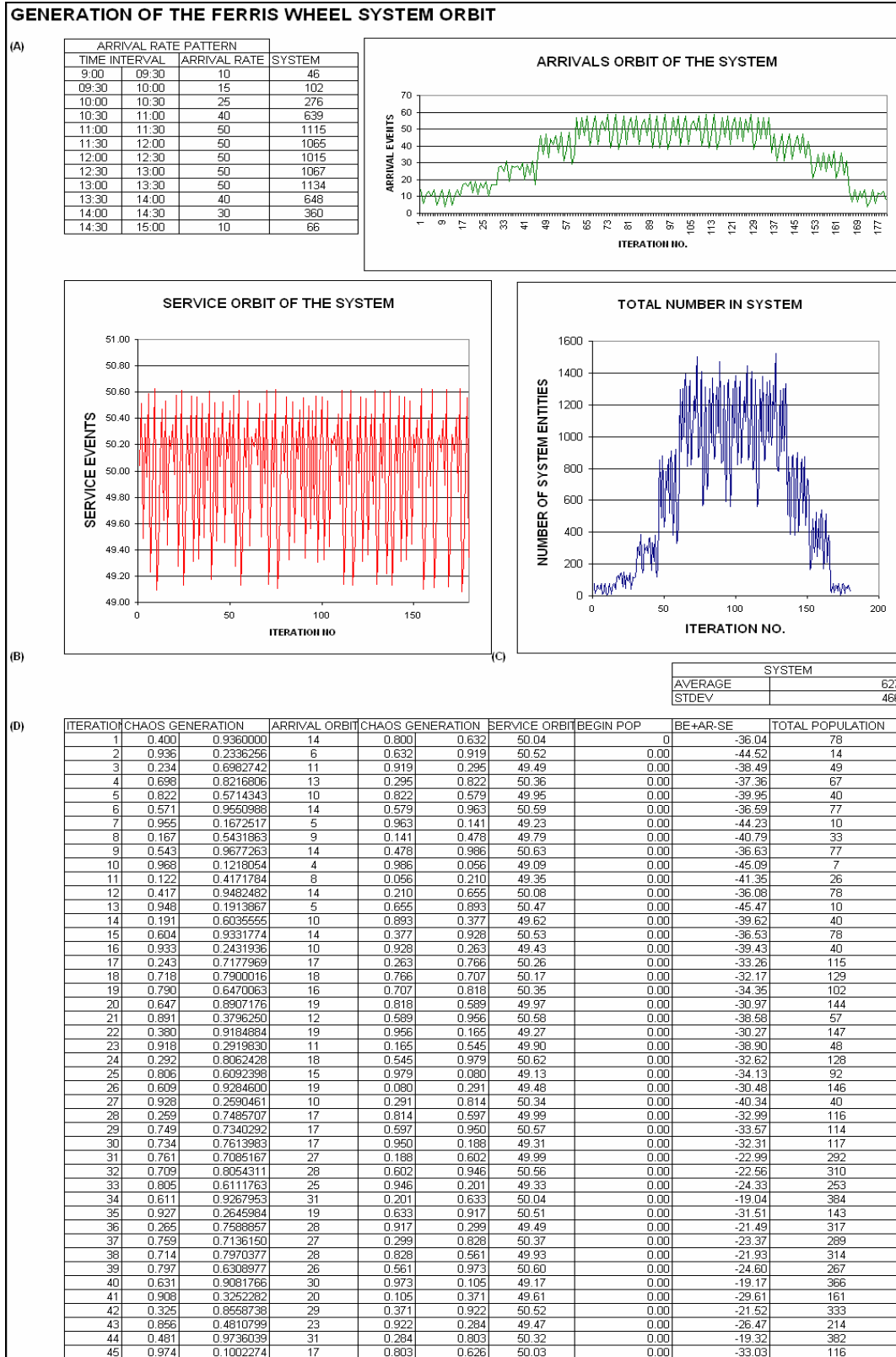


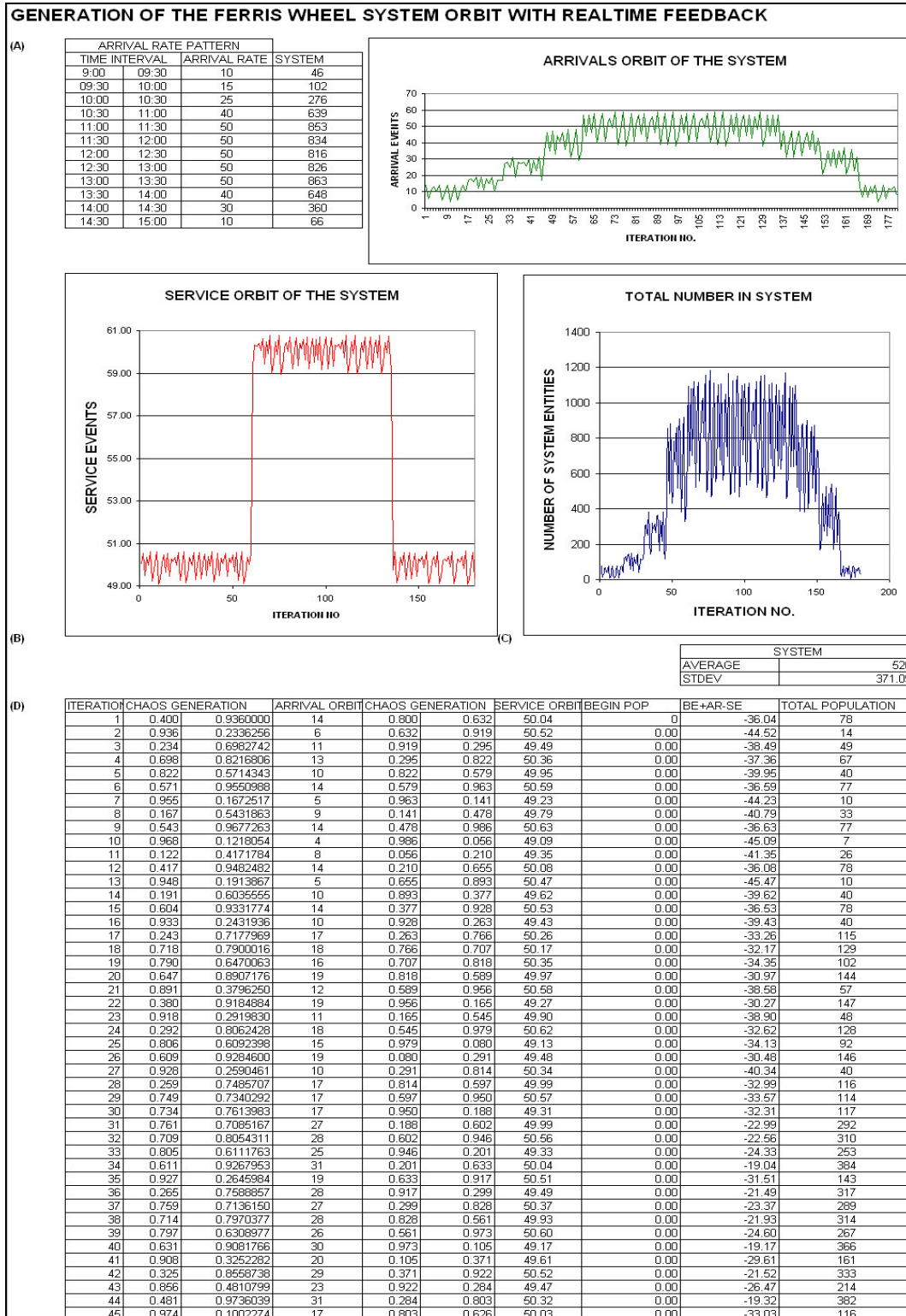
Figure 7.4.1.2 GENERATION OF THE FERRIS WHEEL SYSTEM ORBIT

#### 7.4.4 Using realtime feedback to improve system performance

To decrease the number of entities in the system one may consider increasing the service rate by increasing the peripheral speed of the Ferris wheel and/or by limiting the average arrival rate under peak demand conditions. To demonstrate the effect of realtime feedback the operation of the system could be geared to increase the service rate when the peak population exceeds a predetermined value.

The effect of such a realtime feedback arrangement could be tested by using the following feedback rule: “as soon as the system population exceeds 1200 alter the average service rate to 60 entities per 2 minute interval and reset the average service rate to 50 entities per 2 minute interval as soon as the system population becomes less than 600”.

The results of analysing the system with realtime feedback are shown in Fig. 7.4.1.3 which indicates that the degree of congestion is considerably diminished. One may pose the question whether one could not gain greater congestion improvement by a further increase of the service rate. The maximum feasible service rate is however 60 entities per 2 minute interval for physical (ergonomic) reasons of loading and unloading at ground level.



**Fig. 7.4.1.3 GENERATION OF THE FERRIS WHEEL SYSTEM ORBIT WITH REAL TIME FEEDBACK**

#### 7.4.5 Concluding comments on System No. 3

The analysis of the system confirms that attempting to improve the operation of System No. 3 does not afford much leeway in effort since one is constrained to optimize within bounds such as:

- structural reasons in respect of peripheral speed,
- ergonomic reasons relating to loading and unloading of entities while the Ferris wheel is in motion,
- passenger value-for-money by making use of the wheel for visual entertainment for a period of time, and last but not least
- specified economic performance of the system.

## **7.5 SYSTEM NO. 4**

### **7.5.1 System scenario**

System No. 4 is an example of a typical municipal call centre that handles enquiries and problem reporting by a given urban population. The call centre has replaced some previously existing service centres. The specific system under consideration handles general enquiries and is the reporting centre for interruptions of service. Occasionally it occurs that the centre becomes congested. The resulting peak conditions of congestion will be the focus of the study.

The call centre operates in the following way. Entities phone the centre, a computer answers the call and the entity has a range of options to choose from. The entity has the following options:

- Report a failure or general enquiries.
- Choose a region of failure.
- Listen to a scenario of reported failures.

The average delay is 1.6 minutes and 16% of entities abandon the call during this process. During the following process an operator serves entities on a FCFS basis. They form a queue if the operators are all busy. The specific congestion period that will be modelled is when an infrastructure failure occurs in one municipal region from 16h00 to 19h00.

### **7.5.2 The system model**

Verhulst orbit generation for the arrivals is show in Fig. 7.5.2.1. The average arrival rate is determined for consecutive 15-minute intervals. The arrivals orbit is similar to the conditions that prevail on site.

The service orbit for a single operator is shown in Fig. 7.5.2.2. The actual call duration is described by a general distribution with a mean service rate of 20 calls per hour and a standard deviation of 1.6. The total system was modelled



using the same multi-channel process as for System No. 2 of par. 7.3. The results are shown in Fig. 7.5.2.3 for two service lanes. It shows the arrivals, the abandonment rate and the number of operators in use. The average number of calls in the system is 6 for the period from 16h00 to 19h00.

**GENERATION OF THE ARRIVAL ORBIT AT THE CALL CENTRE**

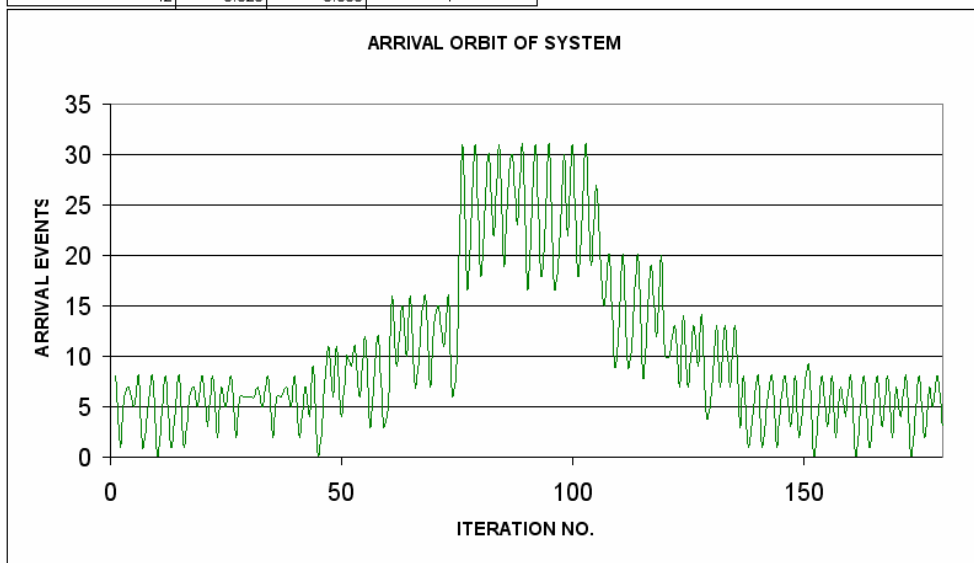
(A)

ORBIT GENERATION FOR FIRST 42 INTERVALS			
ITERATION NO	x	F(x)	ARRIVAL ORBIT
1	0.400	0.936	8
2	0.936	0.234	1
3	0.234	0.698	6
4	0.698	0.822	7
5	0.822	0.571	5
6	0.571	0.955	8
7	0.955	0.167	1
8	0.167	0.543	4
9	0.543	0.968	8
10	0.968	0.122	0
11	0.122	0.417	3
12	0.417	0.948	8
13	0.948	0.191	1
14	0.191	0.604	5
15	0.604	0.933	8
16	0.933	0.243	1
17	0.243	0.718	6
18	0.718	0.790	7
19	0.790	0.647	5
20	0.647	0.891	8
21	0.891	0.380	3
22	0.380	0.918	8
23	0.918	0.292	2
24	0.292	0.806	7
25	0.806	0.609	5
26	0.609	0.928	8
27	0.928	0.259	2
28	0.259	0.749	6
29	0.749	0.734	6
30	0.734	0.761	6
31	0.761	0.709	6
32	0.709	0.805	7
33	0.805	0.611	5
34	0.611	0.927	8
35	0.927	0.265	2
36	0.265	0.759	6
37	0.759	0.714	6
38	0.714	0.797	7
39	0.797	0.631	5
40	0.631	0.908	8
41	0.908	0.325	2
42	0.325	0.856	7

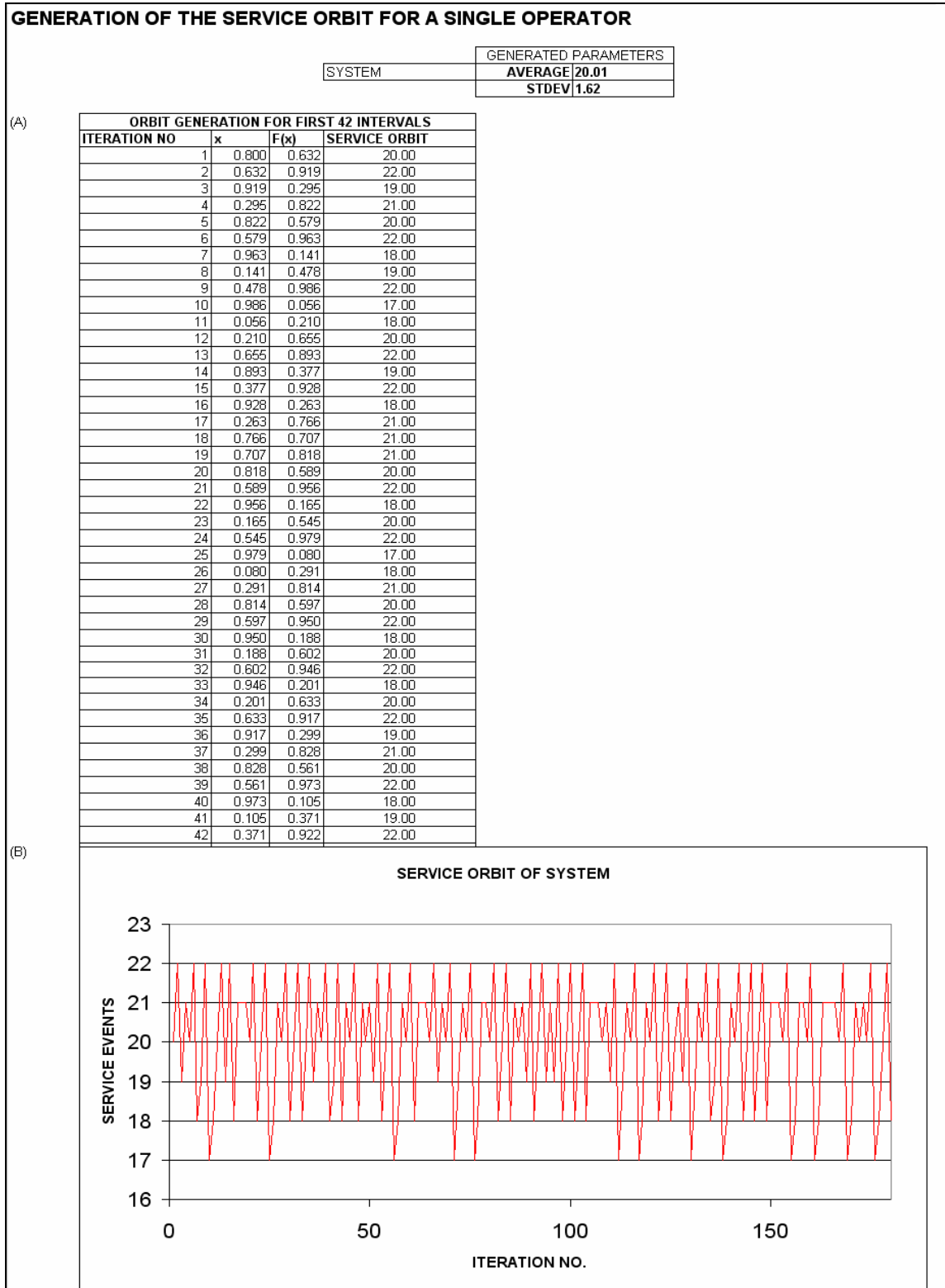
(C)

ARRIVAL RATE PATTERN		
TIME INTERVAL		ARRIVAL RATE
16:00	16:15	5
16:15	16:30	5
16:30	16:45	5
16:45	17:00	8
17:00	17:15	12
17:15	17:30	25
17:30	17:45	25
17:45	18:00	15
18:00	18:15	10
18:15	18:30	5
18:30	18:45	5
18:45	19:00	5

(B)



**Figure 7.5.2.1 GENERATION OF ARRIVALS AT THE CALL CENTRE**



**Figure 7.5.2.2 GENERATION OF A TYPICAL SERVICE ORBIT FOR A SINGLE OPERATOR**

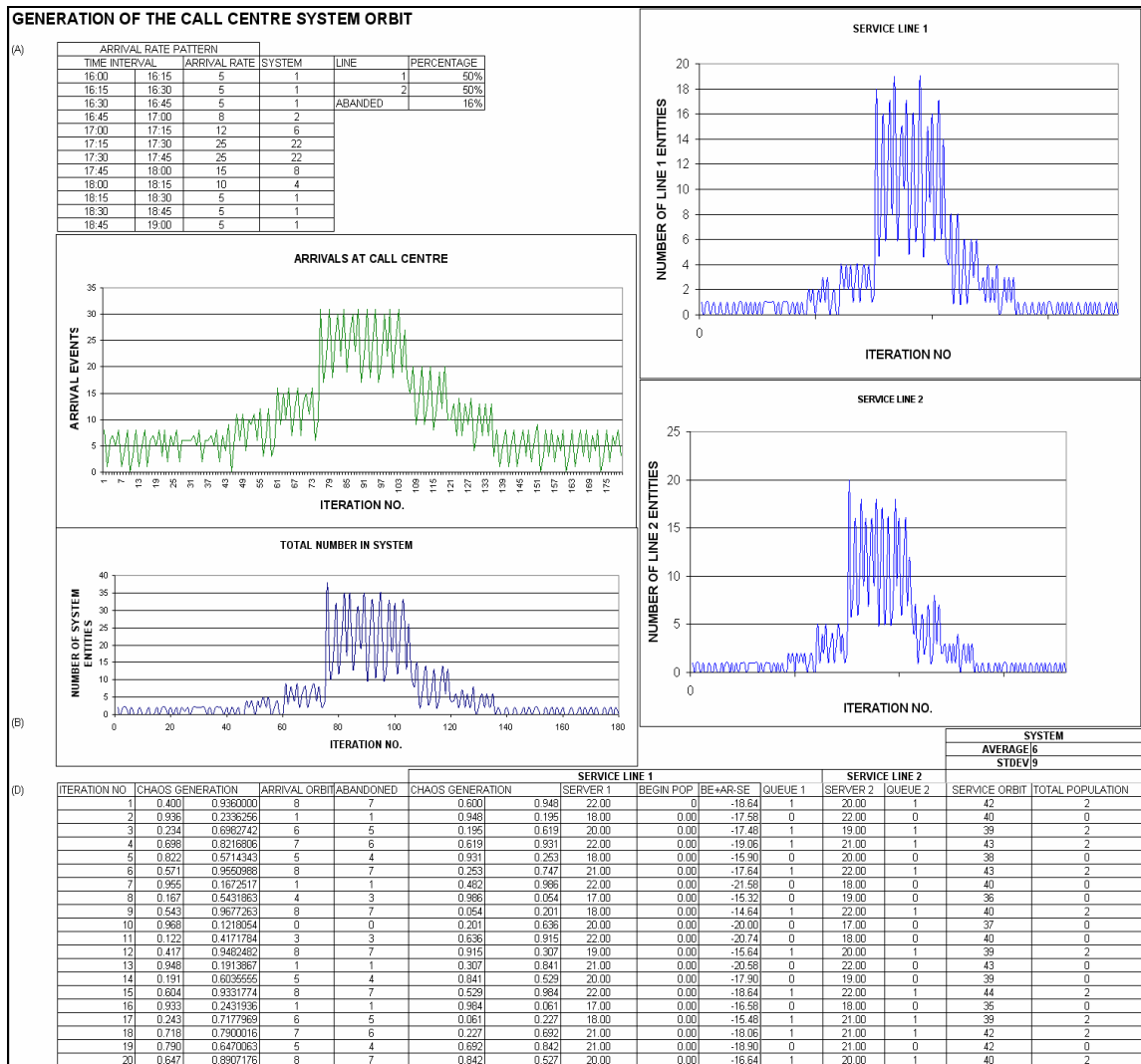


Figure 7.5.2.3 GENERATION OF THE CALL CENTRE SYSTEM ORBIT

### **7.5.3 Diagnosis of the model results**

The system population value of Fig 7.2.2.3 is closely related to the “actual” situation. This system arrival pattern differs from the previous models in that the sudden peak in arrival rate is more dramatic. If the system is operating under normal operating conditions the average number of entities in the queue is one. During the sudden increase the queue length increases to 36.

The next step is to improve the system by focussing on the average number in the system and the standard deviation. The latter value must also be decreased.

### **7.5.4 Using realtime feedback to improve system performance**

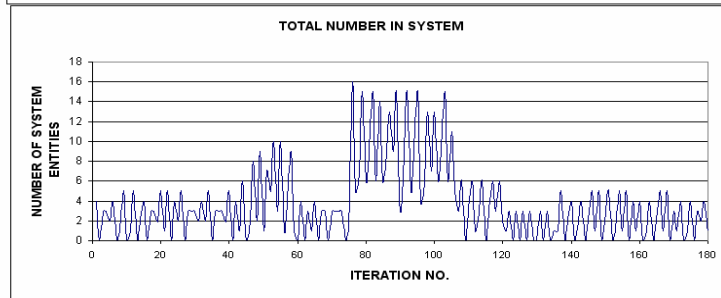
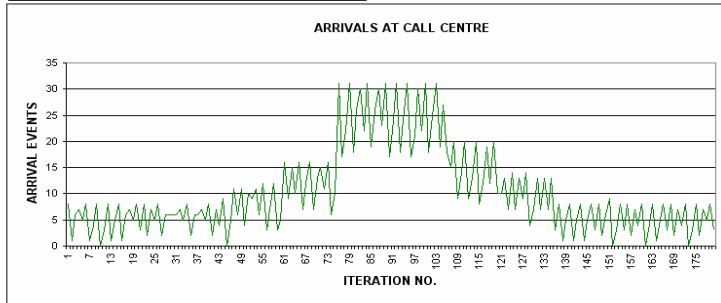
When one analyses the system one could attempt to decrease the peak load on the system by varying the number of service channels. The other concern is the idle time that system operators may have during normal uncongested operating conditions. The difficulty with the system is that one cannot predict when peak conditions will occur. It will also help when a major infrastructure failure of service occurs, that the computer communication menu be adapted to cause an increase of the call abandonment rate by offering the scenario of a history of reported failures first. The system management will have to use multi-tasking to limit the operating cost.

These improvements have been modelled and the results are shown in Fig. 7.5.2.4. This improvement has decreased the average number in the system and the standard deviation to 4 and 4 respectively.

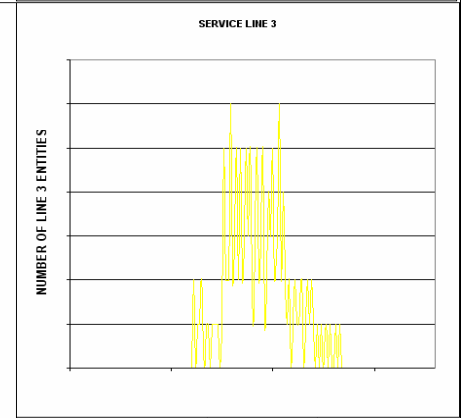
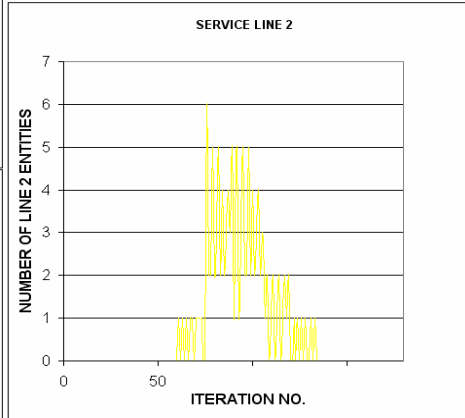
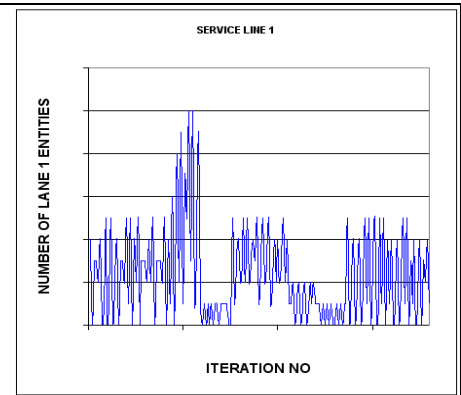
**GENERATION OF THE CALL CENTRE SYSTEM ORBIT**

(A)

ARRIVAL RATE PATTERN		SYSTEM	LINE	PERCENTAGE	CONGESTED
TIME INTERVAL	ARRIVAL RATE				
16:00	16:15	5	2	100%	33%
16:15	16:30	5	3	0%	33%
16:30	16:45	5	3	0%	34%
16:45	17:00	8	5	16%	20%
17:00	17:15	12	2		
17:15	17:30	25	9		
17:30	17:45	25	10		
17:45	18:00	15	4		
18:00	18:15	10	1		
18:15	18:30	5	2		
18:30	18:45	5	2		
18:45	19:00	5	2		



(B)



(D)

ITERATION NO	CHAOS GENERATION	ARRIVAL ORBIT	ABANDONED	NO. OF SERVER	CHAOS GENERATION	SERVICE LINE 1			SERVICE LINE 2		SERVICE LINE 3		SYSTEM		
						SERVER 1	BEGIN POP	BE+AR-SE	QUEUE 1	SERVER 2	QUEUE 2	SERVER 3	QUEUE 3	SERVICE ORBIT	TOTAL POPULATION
1	0.400	0.9360000	8	7	1	0.600	0.948	22.00	0	-15.28	4	0.00	0	18	4
2	0.936	0.2336256	1	1	1	0.948	0.195	18.00	0.00	-17.16	0	0.00	0	20	0
3	0.234	0.6982742	6	5	1	0.195	0.619	20.00	0.00	-14.96	3	0.00	0	22	3
4	0.698	0.8216806	7	6	1	0.619	0.931	22.00	0.00	-16.12	3	0.00	0	18	3
5	0.822	0.5714343	5	4	1	0.931	0.253	18.00	0.00	-13.80	2	0.00	0	21	2
6	0.571	0.9550988	8	7	1	0.253	0.747	21.00	0.00	-14.26	4	0.00	0	22	4
7	0.955	0.1672517	1	1	1	0.482	0.986	22.00	0.00	-21.16	0	0.00	0	17	0
8	0.167	0.5431863	4	3	1	0.986	0.054	17.00	0.00	-13.64	1	0.00	0	18	1
9	0.543	0.9677263	8	7	1	0.054	0.201	18.00	0.00	-11.28	5	0.00	0	20	5
10	0.968	0.1218054	0	0	1	0.201	0.636	20.00	0.00	-20.00	0	0.00	0	22	0

**Figure 7.5.2.4 GENERATION OF CALL CENTRE SYSTEM ORBIT WITH REALTIME FEEDBACK**

#### **7.5.5 Concluding comments on System No. 4**

System No. 4 was modelled successfully. The system orbit depicts useful results. The uniqueness of the system is clearly shown in the results. Realtime feedback will definitely improve the system congestion.

## 7.6 EVALUATION OF THE MODELLING METHODS AND ACHIEVEMENT OF DYNAMIC OPERATION RESULTS OF COMPLEX SYSTEMS OF CONGESTION

This chapter of the thesis demonstrates the application of chaos orbit generation methods to Systems of Congestion which are more complex in nature than those studied in Chapter 6. The orbit generation methods are deployed in each case in a system wide fashion that matches/ serves the accuracy of modelling of each of the individual systems studied.

In each instance the actual dynamic performance of the system was acceptably replicated by the chaos orbit generation method. These encouraging results consequently paved the way for improvement of the congested conditions by fashioning feedback bouquets. The use of chaos generation methods is therefore supported to such an extent that the **initial conjecture** that these methods could **possibly be used effectively** has achieved the status of **an assertion**.