

**Stochastic models of steady state and dynamic operation  
of systems of congestion**

By

**Gert Botha Erasmus**

Submitted in partial fulfilment of the requirements for the degree

**Philosophiae Doctor**

(Industrial Engineering)

In the

Faculty of Engineering, Built Environment and Information Technology

At the

UNIVERSITY OF PRETORIA

Pretoria

June 2005

## **Acknowledgements**

I would like to express my sincere thanks and appreciation to the following persons:

- a) Prof. Kris Adendorff and Prof. Sarma Yadavalli that acted as my supervisors and mentors, not only on academic level, but especially personally.
- b) My father and mother, I thank God each day for the good parents that I received. Thank you for providing the resources and motivation to complete my studies.
- c) Marnella, Andreas, H.P., Marelee, Martinus and Nicolaas for your devoted time and love that you have given me during my studies.
- d) All the personnel of the Department of Industrial and System Engineering, especially Anne-Marie and Christa for their motherly care and patience.
- e) To all my friends that have supported me mentally and physically during these studies.
- f) The University of Pretoria and all other personnel for providing me with the resources and research environment to complete my studies.

***“Soli deo Gloria”***

To the best two Mentors, Supervisors and friends in the whole world,  
my Family that stood by me and all my colleagues.

# **Stochastic models of steady state and dynamic operation of systems of congestion**

**Gert Botha Erasmus**

Supervisor: Prof. K. Adendorff

Co-supervisor: Prof. V.S.S. Yadavalli

Department of Industrial and Systems Engineering

**Philosophiae Doctor** (Industrial Engineering)

## **ABSTRACT**

Key terms: Systems of Congestion, Queueing Theory, Chaos Theory, Steady state, Transient state, System Dynamics, Waiting time, Bulk system with interruption.

- (i) The thesis sets out to address the problematic phenomenon of Systems of Congestion via Basic Queueing Theory. The theory, and its application in practice, appears to be a field of study which is the common domain of “theorists” and “practitioners”.
- (ii) This professional dichotomy has come about due to diverging interests in that one group is mainly interested in the purity of mathematical modelling, and the other group is motivated to use modelling, which conveniently employs applications oriented solutions.
- (iii) The schism between the groups has been accentuated by the “practitioners” who in addition to having an interest in steady state system behaviour make use of methods of modelling of the transient operation of complex Systems of Congestion.
- (iv) At the outset the thesis demonstrates how closed form solutions are obtained for steady state and transient state operation of a selection of Systems of Congestion. The attendant mathematical derivations are elegant and intricate.

- (v) Having revealed the limited utility of closed-form solutions the thesis proceeds to investigate the feasibility of using dynamical systems theory to study the transient behaviour of complex Systems of Congestion.
- (vi) The creation of Chaos Theory in recent decades suggests that it may be employed as a useful tool in analysing Systems of Congestion. Iterative Chaos Theory methods of orbit generation for complete Systems of Congestion are therefore examined. The use of such orbit generation methods is found to be satisfactory for simple Systems of Congestion. More than a perfunctory knowledge of chaos mapping is however required. The simplicity of modelling is emphasized.
- (vii) Based on the results of benchmarking the creation of dynamic system orbits against an existing simulation method, the research advances to modelling of the transient operation of complex systems. Once again the iterative method of orbit generation displays the ease of modelling while simultaneously unfolding system dynamics graphically.
- (viii) One may hopefully contend that a tool of eminent utility has been developed to aid practitioners in studying and optimizing Systems of Congestion.

## TABLE OF CONTENTS

<b>CHAPTER 1 .....</b>	<b>5</b>
<b>INTRODUCTION .....</b>	<b>5</b>
1.1 INTRODUCTION .....	6
1.1.1 <i>A General Description of the proposed Research</i> .....	6
1.1.2 <i>Exploring novel approaches to the modelling of Congestion</i> .....	8
1.2 LITERATURE STUDY.....	9
1.2.1 <i>Queues</i> .....	9
1.2.1.1 <i>Description of Queues</i> .....	9
1.2.1.1 <i>Historical perspective</i> .....	10
1.2.1.3 <i>Review of Queueing Models and their Modelling Approaches</i> .....	12
1.2.1.4 <i>Confidence limits</i> .....	16
1.2.2 <i>Chaos theory</i> .....	17
1.2.2.1 <i>Historical Perspective</i> .....	17
1.2.2.2 <i>Modelling Approach</i> .....	19
1.2.3 <i>The need for a new theory</i> .....	22
<b>CHAPTER 2 .....</b>	<b>24</b>
<b>CONFIDENCE LIMITS FOR EXPECTED WAITING TIME OF TWO QUEUEING MODELS .....</b>	<b>24</b>
2.1.1 <i>Introduction</i> .....	25
2.1.3 <i>The ML and CAN estimators for expected waiting time</i> .....	29
2.1.3.1 <i>The ML Estimator</i> .....	29
2.1.3.2 <i>An application of the multivariate central limit theorem</i> .....	30
2.1.3.3 <i>The CAN Estimator</i> .....	31
2.1.4 <i>Confidence limits for the expected waiting time</i> .....	32
2.2 STATISTICAL ANALYSIS FOR A TANDEM QUEUE WITH BLOCKING .....	36
2.2.1 <i>Introduction</i> .....	36
2.2.2 <i>System description and assumptions</i> .....	38
2.2.3 <i>Analysis of the system</i> .....	38
2.2.3.1 <i>Transient Solution</i> .....	40
2.2.3.2 <i>The Steady state solution</i> .....	41
2.2.3.3 <i>Expected service time per entity in the system</i> .....	42
2.2.4 <i>MLE and CAN estimator for the expected service time per entity in the     system</i> .....	42
2.2.4.1 <i>The ML estimator</i> .....	42
2.2.4.2 <i>An application of the multivariate central limit theorem</i> .....	43
2.2.4.3 <i>The CAN Estimator</i> .....	44
2.2.4.4 <i>Confidence limits for the expected waiting time</i> .....	45

<b>CHAPTER 3 .....</b>	<b>47</b>
<b>A SINGLE CHANNEL QUEUEING MODEL WITH OPTIONAL SERVICE AND SERVICE INTERRUPTION.....</b>	<b>47</b>
3.1 <i>Introduction.....</i>	48
3.2 <i>Model description.....</i>	48
3.3 <i>Assumptions and notation.....</i>	49
3.4 <i>Time dependant solution.....</i>	51
3.5 <i>Some special Cases.....</i>	55
3.6 <i>The steady state solution.....</i>	57
3.7 <i>Some special Cases.....</i>	59
3.8 <i>Concluding remark.....</i>	61
<b>CHAPTER 4 .....</b>	<b>62</b>
<b>AN M/M/1 QUEUEING SYSTEM WITH BATCH ARRIVALS OF VARYING SIZE, SERVICE OF FIXED BATCH SIZE AND TWO MODES OF FAILURE OF SERVICE FACILITY.....</b>	<b>62</b>
4.1 <i>Introduction.....</i>	63
4.2 <i>Model description.....</i>	63
4.3 <i>Assumptions and notation.....</i>	64
4.4 <i>Equations describing the system.....</i>	65
4.5 <i>Time dependant solution.....</i>	66
4.6 <i>The steady state solution.....</i>	71
4.7 <i>Some special cases.....</i>	72
4.8 <i>Concluding remark.....</i>	75
<b>CHAPTER 5 .....</b>	<b>76</b>
<b>AN M/G/1 QUEUEING SYSTEM WITH TWO MODES OF FAILURE....</b>	<b>76</b>
5.1 <i>Introduction.....</i>	77
5.2 <i>Model description.....</i>	77
5.3 <i>System description.....</i>	78
5.4 <i>Equations governing the system.....</i>	79
5.5 <i>Time dependant solution.....</i>	81
5.6 <i>Steady state solution .....</i>	87
5.7 <i>Some special cases.....</i>	88
5.8 <i>Concluding remarks.....</i>	89
<b>CHAPTER 6 .....</b>	<b>90</b>
<b>CHAOS THEORY BASED MODELS OF SIMPLE SYSTEMS OF CONGESTION.....</b>	<b>90</b>
6.1     INTRODUCTION .....	91
6.1.1 <i>The classical Poisson arrival system.....</i>	91
6.1.1.1 <i>The general modelling approach.....</i>	91

6.1.2	<i>The classical exponential service system</i> .....	93
6.1.2.1	<i>The general modelling approach</i> .....	93
6.1.3	<i>The classical M/M/1 queue</i> .....	95
6.1.3.1	<i>The general modelling approach</i> .....	95
6.2	INTRODUCTION TO CHAOS GENERATION .....	99
6.2.1	<i>The Verhulst generated arrival system</i> .....	99
6.2.1.1	<i>The general modelling approach</i> .....	99
6.2.	<i>The Verhulst generated service system</i> .....	101
6.2.2.1	<i>The general modelling approach</i> .....	101
6.2.3	<i>The Verhulst generated single channel queue</i> .....	103
6.2.3.1	<i>The general modelling approach</i> .....	103
6.2.4	<i>Benchmarking the Verhulst generated single channel queue model</i> .....	105
6.2.5	<i>Extending the Verhulst generated single channel queue model to deal with variable traffic intensity</i> .....	108
6.3	FURTHER EXAMPLES OF CHAOS GENERATION .....	111
6.4	CONCLUDING REMARKS ON SINGLE CHANNEL ORBITS RESULTING FROM A MENU OF METHODS OF GENERATION. ....	119
<b>CHAPTER 7 .....</b>		<b>120</b>
<b>ANALYSIS OF THE DYNAMIC CHARACTERISTICS OF PRACTICAL SYSTEMS OF CONGESTION USING CHAOS GENERATION METHODS.....</b>		<b>120</b>
7.1	INTRODUCTION .....	121
7.2	SYSTEM NO. 1 .....	122
7.2.1	<i>System scenario</i> .....	122
7.2.2	<i>The system model</i> .....	122
7.2.3	<i>Diagnosis of the model results</i> .....	125
7.2.4	<i>Using realtime feedback to improve system performance</i> .....	132
7.2.5	<i>The effect of the size of system waiting area on system performance</i> ....	132
7.2.6	<i>Concluding comments on system no. 1</i> .....	135
7.3	SYSTEM NO. 2 .....	136
7.3.1	<i>System scenario</i> .....	136
7.3.2	<i>The system model</i> .....	136
7.3.3	<i>Diagnosis of the model results</i> .....	142
7.3.4	<i>Using realtime feedback to improve system performance</i> .....	142
7.3.6	<i>Concluding comments on system no. 2</i> .....	144
7.4	SYSTEM NO. 3 .....	145
7.4.1	<i>System scenario</i> .....	145
7.4.2	<i>The system model</i> .....	145
7.4.3	<i>Diagnosis of the model results</i> .....	146
7.4.4	<i>Using realtime feedback to improve system performance</i> .....	149
7.4.5	<i>Concluding comments on system no. 3</i> .....	151
7.5	SYSTEM NO. 4 .....	152
7.5.1	<i>System scenario</i> .....	152
7.5.2	<i>The system model</i> .....	152



7.5.3	<i>Diagnosis of the model results</i> .....	157
7.5.4	<i>Using realtime feedback to improve system performance</i> .....	157
7.5.5	<i>Concluding comments on System No. 4</i> .....	159
7.6	EVALUATION OF THE MODELLING METHODS AND ACHIEVEMENT OF DYNAMIC OPERATION RESULTS OF COMPLEX SYSTEMS OF CONGESTION.....	160
<b>CHAPTER 8</b>	.....	<b>161</b>
<b>CONCLUSION</b>	.....	<b>161</b>
<b>APPENDIX A</b>	.....	<b>165</b>
	FLOW DIAGRAM FOR THE DESIGN OF AN ARRIVALS/SERVICE ORBIT GENERATING FUNCTION	165
<b>APPENDIX B</b>	.....	<b>166</b>
	FLOW DIAGRAM FOR THE DESIGN OF A SYSTEM ORBIT GENERATING FUNCTION .....	166
<b>REFERENCES</b>	.....	<b>167</b>