Stochastic models of steady state and dynamic operation of systems of congestion

By

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ABSTRACT

Key terms: Systems of Congestion, Queueing Theory, Chaos Theory, Steady state, Transient state, System Dynamics, Waiting time, Bulk system with interruption.

(i) The thesis sets out to address the problematic phenomenon of Systems of Congestion via Basic Queueing Theory. The theory, and its application in practice, appears to be a field of study which is the common domain of "theorists" and "practitioners".

(ii) This professional dichotomy has come about due to diverging interests in that one group is mainly interested in the purity of mathematical modelling, and the other group is motivated to use modelling, which conveniently employs applications oriented solutions.

(iii) The schism between the groups has been accentuated by the "practitioners" who in addition to having an interest in steady state system behaviour make use of methods of modelling of the transient operation of complex Systems of Congestion.

(iv) At the outset the thesis demonstrates how closed form solutions are obtained for steady state and transient state operation of a selection of Systems of Congestion. The attendant mathematical derivations are elegant and intricate.

(v) Having revealed the limited utility of closed-form solutions the thesis proceeds to investigate the feasibility of using dynamical systems theory to study the transient behaviour of complex Systems of Congestion.

(vi) The creation of Chaos Theory in recent decades suggests that it may be employed as a useful tool in analysing Systems of Congestion. Iterative Chaos Theory methods of orbit generation for complete Systems of Congestion are therefore examined. The use of such orbit generation methods is found to be satisfactory for simple Systems of Congestion. More than a perfunctory knowledge of chaos mapping is however required. The simplicity of modelling is emphasized.

(vii) Based on the results of benchmarking the creation of dynamic system orbits against an existing simulation method, the research advances to modelling of the transient operation of complex systems. Once again the iterative method of orbit generation displays the ease of modelling while simultaneously unfolding system dynamics graphically.

(viii) One may hopefully contend that a tool of eminent utility has been developed to aid practitioners in studying and optimizing Systems of Congestion.

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