Appendix The smoothness condition for a Vold-Kalman filter

Howell (2001) defines the smoothness condition as follows:

To be smooth over an interval (α, β), a function $x(t)$ must satisfy two conditions:

- $x(t)$ must be differentiable (and hence continuous) everywhere on (α, β), and
- $x^{'}(t)$ must also be a continuous function on (α, β)

This requires further definition of the terms continuous and differentiable (Howell, 2001):

A function $x(t)$ is continuous at a point t_0 if $x(t_0)$ and $\lim_{t \to t_0} x(t)$ both exist and $\lim_{t \to t_0} x(t) = x(t_0)$.

A function $x(t)$ is differentiable at a point t if and only if $\lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t}$ exist. If $x(t)$ is differentiable at every point in a given interval (α, β), then $x(t)$ is said to be differentiable on the interval (α, β).

These concepts can now be applied to the structural equation for a 2-pole filter as given by Tůma at equation (2.3) in chapter 2,

$$x(n) - 2x(n+1) + x(n+2) = \varepsilon(n)$$

Illustration:

For a 2-pole Vold-Kalman filter, the sequence as filtered according to the structural equation may be written as $x(n) = 2x(n+1) - x(n+2) + \varepsilon(n)$ (equation

(2.4) in chapter 2). If adjusting it to be continuous form, it becomes $x(t) = 2x(t+\Delta t) - x(t+2\Delta t) + \varepsilon(t)$. Since $\varepsilon(t)$ is the error which is minimized by global solution of the data and structural equations (Tůma, 2005), it may be neglected for the purposes of the illustration, and the equation becomes $x(t) = 2x(t+\Delta t) - x(t+2\Delta t)$.

From the above, smoothness now requires that $x(t)$ must be differentiable (and hence continuous) everywhere on certain interval (α, β) and $x^{'}(t)$ must also be a continuous function on (α, β).

Differentiability

$$\lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{2x(t+\Delta t+\Delta t) - x(t+\Delta t+2\Delta t) - 2x(t+\Delta t) + x(t+2\Delta t)}{\Delta t}$$

$$\lim_{\Delta x \to 0} \frac{0}{0} = 1$$

The limit will always exist. Thus, this function is differentiable (and hence continuous).

Continuity of $x^{'}(t)$

Since $x(t)$ is differentiable, then $x^{'}(t) = 2x^{'}(t+\Delta t) - x^{'}(t+2\Delta t)$, thus

$$\lim_{t \to t_0} x^{'}(t) = \lim_{t \to t_0} 2x^{'}(t+\Delta t) - x^{'}(t+2\Delta t) = \lim_{t \to t_0} 2x^{'}(t+\Delta t)) - \lim_{t \to t_0} x^{'}(t+2\Delta t)$$

$\Rightarrow$

$$\lim_{t \to t_0} x^{'}(t) = \lim_{t \to t_0}\left[\lim_{\Delta t \to 0} 2\frac{x(t+\Delta t+\Delta t) - x(t+\Delta t)}{\Delta t}\right] - \lim_{t \to t_0}\left[\lim_{\Delta t \to 0} \frac{x(t+2\Delta t+\Delta t) - x(t+2\Delta t)}{\Delta t}\right]$$

Swap the limits and notice that $x(t)$ is differentiable and hence continuous, then $\lim_{t \to t_0} x(t) = x(t_0)$, thus,

$$\lim_{t \to t_0} x'(t) = \lim_{\Delta t \to 0}\left[\lim_{t \to t_0} 2\frac{x(t+\Delta t+\Delta t)-x(t+\Delta t)}{\Delta t}\right] - \lim_{\Delta t \to 0}\left[\lim_{t \to t_0}\frac{x(t+2\Delta t+\Delta t)-x(t+2\Delta t)}{\Delta t}\right]$$

$$\Rightarrow$$

$$\lim_{t \to t_0} x'(t) = \lim_{\Delta t \to 0} 2\frac{x(t_0+\Delta t+\Delta t)-x(t_0+\Delta t)}{\Delta t} - \lim_{\Delta t \to 0}\frac{x(t_0+2\Delta t+\Delta t)-x(t_0+2\Delta t)}{\Delta t}$$

$$\Rightarrow$$

$$\lim_{t \to t_0} x'(t) = 2x'(t_0+\Delta t) - x'(t_0+2\Delta t) = x'(t_0)$$

So $x'(t)$ is continuous.

From the above the filtered signal from 2-pole Vold-Kalman filter is continuous and smooth. Similar procedures may be applied to 1 and 3 pole filters.

References

Albright M. F. and Qian Sh. (2001), A comparison of the newly proposed Gabor order tracking technique vs. other order tracking methods. *SAE paper*, No.2001-01-1471.

Blough J. R. (2003), A survey of DSP methods for rotating machinery analysis, what is needed, what is available. *Journal of Sound and Vibration*, 262, pp707-720.

Bossley K. M., Mckendrick R. J., Harris C. J. and Mercer C. (1999), Hybrid computed order tracking, *Mechanical Systems and Signal Processing* 13(4), pp627-641.

Brandt A., Thomas L., Ahlin K. and Tůma J. (2005), Main principles and limitations of current order tracking methods. *Sound and Vibration*, March 2005.

Brüel and Kjær (2007), Product Data: Vold-Kalman order tracking filter - Type 7703 for Pulse. Available at: http://www.bksv.com/pdf/bp1760.pdf/
[Accessed: 11 Sep 2007.]

Chen H. G., Yang Y. J. and Jiang J. S. (2007), Vibration-based damage detection in composite wingbox structures by HHT, *Mechanical Systems and Signal Processing*, 21, pp307-321.

Chen J. Sh., Yu. D. J., Tang J. Sh. and Yang Y. (2009), Local rub-impact fault diagnosis of the rotor systems based on EMD, *Mechanism and Machine Theory*, 44, pp784-791.

Cohen L. (1995), *Time-frequency analysis*. Englewood Cliffs, NJ: Prentice-Hall.

Daetig M., and Schlurmann T. (2004), Performance and limitations of the Hilbert-Huang transformation (HHT) with an application to irregular water waves, *Ocean Engineering*, 31, pp1783–1834.

Eggers B. L., Heyns P. S. and Stander C. J. (2007), Using computed order tracking to detect gear condition aboard a dragline. *The Journal of Southern African Institute of Mining and Metallurgy*, 107, pp1-8.

Feldman M. (2008), Theoretical analysis and comparison of the Hilbert transform decomposition methods, *Mechanical Systems and Signal Processing*, 22, pp509-519.

Feldman M. (2009), Analytical basics of the EMD: Two harmonic decomposition, *Mechanical Systems and Signal Processing*, 23, pp2059-2071.

Feldbauer C. and Holdrich R. (2000), Realisation of a Vold-Kalman Tracking Filter – A Least Square Problem, *Proceedings of the COST G-6 Conference on Digital Audio Effects*, DAFX 1-4, Verona Italy, December 7-9.

Flandrin P., Rilling G. and Gonçalvés P. (2004), Empirical mode decomposition as a filter bank, *IEEE Signal Processing Letters*, 11(2).

Fyfe K. R. and Munck E. D. S. (1997), Analysis of computed order tracking. *Mechanical Systems and Signal Processing*, 11(2), pp187-205.

Rilling G., Flandrin P. and Goncalves P. (2003), On empirical mode decomposition and its algorithms, in: IEEE-EURASIP. *Workshop on Nonlinear Signal and Image Processing NSIP-03*, Grado (I).

Gade S., Herlufsen H., Konstantin-Hansen H. and Vold H. (1999), Characteristics of the Vold-Kalman order tracking filter, *Brüel and Kjær Technical review.*

Gao Q., Duan C., Fan H. and Meng Q. (2008), Rotating machine fault diagnosis using empirical mode decomposition, *Mechanical Systems and Signal Processing*, 22, pp1072-1081.

Guo D. and Peng Z. K. (2007), Vibration analysis of a cracked rotor using Hilbert-Huang transform, *Mechanical Systems and Signal Processing*, 21, pp3030-3041.

Guo Y., Chi Y. L. and Zheng H. W. (2008), Noise reduction in computed order tracking based on FastICA, Proceedings of the 2008 IEEE/ASME, *International Conference on Advanced Intelligent Mechatronics*, July 2-5, 2008, Xi'an, China.

Guo Y., Tan K. K., Huang S. N. and Zhang Y. (2008), Noise removal in Vold-Kalman order tracking based on independent component analysis, *Proceedings of the IEEE International Conference on Automation and Logistics* Qingdao, China September 2008.

He L., Zhang Y. and Wen B. (2007), Experiment Research on Gear wearing Using order tracking, *12th IFToMM World Congress*, Besançon (France), June 19-21, 2007.

Herlufsen H., Gade S. and Konstantin-Hansen H. (1999), Characteristics of the Vold/Kalman order tracking filter. *Proceedings of the 17th International Modal Analysis Conference*, Kissimmee, Florida.

Howard I., Jia S. X. and Wang J. D. (2001), The dynamic modelling of a spur gear in mesh including friction and a crack, *Mechanical Systems and Signal Processing*, 15(5), pp831 -853.

Howell K. B. (2001), *Principles of Fourier analysis*, Chapman & Hall/CRC.

Huang N. E., Shen Z., Long S. R., Wu M. C., Shin H. H., Zheng Q., Yen N. C., Tung Ch. Ch. and Liu H. H. (1998), The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis, in: *Proceedings of the Royal Society, London*, 454, pp903-995.

Huang N. E., Wu Z. H. and Long S. R. (2006), On instantaneous frequency, in: Workshop on the recent developments of the Hilbert-Huang Transform methodology and its applications, Taipei, 15-17 March 2006.

Li H. (2007), Gear Fault Monitoring Based on Order Tracking and Bi-spectrum under Running-up Condition, Proceeding of the Fourth International Conference on Fuzzy Systems and Knowledge Discovery(FSKD 2007), 4, pp379-383.

Li H. L., Deng X. Y. and Dai H. L. (2006), Structural damage detection using the combination method of EMD and wavelet analysis, *Mechanical Systems and Signal Processing*, 21, pp298-306.

Li H. G. and Meng G. (2005), Detection of harmonic signals from chaotic interference by empirical mode decomposition, *Chaos, Solutions and Fractals,* 30, pp930-935.

Liu Y. and Jiang S. (2007), A simple method of computing signal order tracking fast in time domain, *Second International Conference on Innovative Computing, Information and Control*, 5-7 Sep. 2007, pp523-526.

Liu B., Riemenschneider S. and Xu Y. (2006), Gearbox fault diagnosis using empirical mode decomposition and Hilbert spectrum, *Mechanical Systems and Signal Processing,* 20, pp718-734.

Pan M. Ch. and Lin Y. F. (2006), Further exploration of Vold-Kalman filtering order tracking with shaft-speed information-I: Theoretical part, numerical implementation and parameter investigations. *Mechanical Systems and Signal Processing* , 20(5), pp1134-1154.

Pan M. Ch., Liao Sh. W. and Chiu Ch. Ch. (2007), Improvement on Gabor order tracking and objective comparison with Vold-Kalman filtering order tracking. *Mechanical Systems and Signal processing*, 21(2), pp653-667.

Pan M. Ch., Li P. Ch. and Cheng Y. R. (2008), Remote online machine condition monitoring system, *Measurement*, 41(8), October 2008, pp 912-921.

Pan M. Ch. and Wu Ch. X. (2007), Adaptive Vold-Kalman filtering order tracking. *Mechanical Systems and Signal Processing*, 21(8), pp2957-2969.

Potter R. (1990), A new order tracking method for rotating machinery, *Sound and Vibration*, 37(6), pp30-34.

Rai V. K. and Mohanty A. R. (2007), Bearing fault diagnosis using FFT of intrinsic mode functions in Hilbert-Huang transform, *Mechanical Systems and Signal Processing*, 21(6), pp2607-2615.

Rato R. T., Ortigueira M. D. and Batista A. G. (2008), On the HHT, its problems, and some solutions, *Mechanical Systems and Signal Processing*, 22, pp1374-1394.

Saavedra P. N., Rodriguez C. G. (2006), Accurate assessment of computed order tracking, *Journal of Shock and Vibration*, 13(1), pp13-32.

Schwartz M., Bennett W. R. and Stein S. (1966), *Communications Systems and Techniques*. New York: McGraw-Hill.

Stander C. J. and Heyns P. S. (2006), Transmission path phase compensation for gear monitoring under fluctuating load conditions. *Mechanical Systems and Signal Processing*, 20, pp1511-1522.

Schön P. P. (2006). Unconditionally convergent time domain adaptive and time-frequency techniques for epicyclic gearbox vibration. Master's thesis at University of Pretoria.
Available at: http://upetd.up.ac.za/thesis/available/etd-08282007-142010/
[Accessed: 30 August 2009]

Sharpley R. C. and Vatchev V. (2006), Analysis of the intrinsic mode functions, *Constructive Approximation*, 24 (1), pp17–47.

Tůma J. (2005), Setting the pass bandwidth in the Vold-Kalman order tracking filter. *Twelfth International Congress on Sound and Vibration*, 11-14 July 2005, Lisbon.

Tůma J. (undated study notes), *Vold-Kalman Order Tracking Filtration*. Faculty of Mechanical Engineering, Department of Control Systems and Instrumentation.

Vibratools (2005), MATLAB toolbox. Axiom EduTech, Sweden,

Vold H., Herlufson H., Mains M. and Corwin-Renner D. (1997), Multi axle order tracking with the Vold-Kalman tracking filter. *Sound and Vibration Magazine*, 13(5), pp30-34.

Vold H. and Leuridan J. (1993), High Resolution Order Tracing Using Kalman Tracking Filters – Theory and Application. SAE Paper, No. 931288.

Vold H., Mains M. and Blough J. (1997), Theoretical foundations for high performance order tracking with the Vold-Kalman tracking filter. *SAE Paper* 972007, pp1083–1088.

Wang K. S. (2008), Vibration monitoring on electrical machine using Vold-Kalman filter order tracking. Dissertation submitted for the degree of MSc at the University of Pretoria.
Available at:
http://upetd.up.ac.za/thesis/available/etd-08282008-171945/unrestricted/dissertation.pdf
[Accessed: 30 August 2009]

Wang K. S. and Heyns P. S. (2008), Inspecting FFT order components through the joint use of computed order tracking and Vold-Kalman filter order tracking. Comadem 2008, Prague, June 2008.

Wang K. S. and Heyns P. S. (2009), Vold-Kalman filter order tracking in vibration monitoring of electrical machines. *Journal of Vibration and Control*, 15(9), pp1325-1347.

Wu F. J., Qu L. S. (2009), Diagnosis of subharmonic faults of large rotating machinery based on EMD, *Mechanical Systems and Signal Processing*, 23, pp467-475.

Yang Zh. J., Yang L. H., Qing Ch. M. and Huang D. (2008), A method to eliminate riding waves appearing in the empirical AM/FM demodulation, *Digital signal processing*, 18, pp488-504.