Chapter 2 Logic developments of three novel improved order tracking approaches

In this chapter, three novel improved order tracking approaches are developed based upon available order tracking methods, namely Vold-Kalman filter order tracking and computed order tracking, Intrinsic mode function and Vold-Kalman filter order tracking and intrinsic cycle re-sampling. The logic of the discussions on three improved approaches is outlined below to simplify understanding of the thesis:

1. The discussions on the Vold-Kalman filter and computed order tracking (VKC-OT) method emphasize the benefits that each order tracking method (VKF-OT and COT) brings to the subsequent Fourier analysis so that the method can provide clearer order spectra.

2. The discussions on the intrinsic mode function and Vold-Kalman filter order tracking (IVK-OT) method emphasize the relationship between an intrinsic mode function and an order wave so that the sequential use of the two methods is developed to distinguish useful information in an IMF in terms of rotational speed.

3. The discussions on intrinsic cycle re-sampling (ICR) method emphasize the logic of exclusion frequency variation effects in an IMF and the interpretation of the resultant reconstructed IMF through empirical re-sampling. So that the rationale of the method to approximate order tracking effects as well as how to use ICR spectra results can be clarified.

2.1 Vold-Kalman filter and computed order tracking

The first technique is a novel technique that combines the use of Vold-Kalman filter order tracking and computed order tracking to improve the subsequent Fourier analysis and therefore to achieve a clear and focused order spectrum. It is called Vold-Kalman filter and computed order tracking (VKC-OT). Combining the use of the two order tracking methods to improve the subsequent Fourier analysis requires an understanding of the nature of these techniques and how their characteristics affect the Fourier analysis. Therefore, in the following each order tracking method will be discussed in terms of its characteristics for the subsequent Fourier analysis.

2.1.1 Discussions on Vold-Kalman filter order tracking

Herlufsen et al. (1999) describe order tracking as the art and science of extracting the sinusoidal content of measurements, with the sinusoidal content or orders/harmonics at frequencies that are multiples of the fundamental rotational frequency. To this end, VKF-OT relies on two equations to complete the filtering, namely the data equation and the structural equation. These equations define local constraints, which ensure that the unknown phase assigned orders are smooth and that the sum of the orders should approximate the total measured signal. This implies that the order components extracted from the Vold-Kalman filter should be harmonic and smooth waves. To explore the reason that these arguments are valid, the analytical form of data and structural equations described by Tůma (2005) are considered here for discussion.

Data equation

Assuming a second-generation Vold-Kalman filter for single order filtering, the data equation is defined as

$$y(n) = x(n)e^{j\Theta(n)} + \eta(n) \tag{2.1}$$

where $y(n)$ is the measured data, $x(n)$ is a complex envelope of filtered signal, $e^{j\Theta(n)}$ is a complex carrier wave, and

$$\Theta(n) = \sum_{i=1}^{n} \omega(i)\Delta t \tag{2.2}$$

where $\omega(i)$ is the discrete angular frequency, and $\eta(n)$ is the random noise and other order components, or error term.

From equation (2.1), it clearly shows that the order component $x(n)e^{j\Theta(n)}$ is a harmonic natured waveform. The frequency modulation of the signal is determined by $e^{j\Theta(n)}$. Further, equation (2.2) indicates that $\Theta(n)$ is the sum of $\omega(i)\Delta t$ from $i = 1,\ldots,n$, and $\omega(i)$ may vary from time to time, or be non-stationary, consequently $x(n)e^{j\Theta(n)}$ may also be non-stationary. It follows from the above analysis that the order component from a Vold-Kalman filter is a harmonic natured wave which may be non-stationary.

Structural equation

The structural equation provides the smoothness of successive digital points of filtered data, by fitting a low-order polynomial to the sequence $x(n)$. This condition is enforced through the structural equation with the unknown non-homogeneity term $\varepsilon(n)$ on the right-hand side of the equation. The polynomial order designates the number of the filter poles. By way of example, the structural equation for a two-pole filter is given by,

$$x(n) - 2x(n+1) + x(n+2) = \varepsilon(n) \tag{2.3}$$

Rearranging

$$x(n) = 2x(n+1) - x(n+2) + \varepsilon(n) \tag{2.4}$$

It can be seen that with the two-pole filter, for any three immediately adjacent points of the sequence $x(n)$ are constrained through the structural equation. This smoothes the filtered order data from the raw data. To demonstrate this idea, the smoothness condition for such a two-pole filter is illustrated in Appendix by rearranging equations (2.3) to (2.4).

Considering the data and structural equations presented above, one may conclude that the order components extracted from the Vold-Kalman filter are smooth and harmonic waves, but they may be non-stationary.

2.1.2 Discussions on computed order tracking

Computed order tracking is a very commonly performed and effective order tracking technique. Although inevitably errors will be introduced during the re-sampling process and its artificial assumptions (Fyfe and Munck, 1997), the technique still renders very useful results, and effectively transforms non-stationary time domain data to stationary angular domain data for rotating machinery. Blough (2003) uses a graphic representation to explain this transformation process on a simple sine wave. This is illustrated in chapter 1 Figure 1.3. It clearly demonstrates that the re-sampled data has the same properties as a stationary frequency sine wave sampled at uniform time intervals. This uniformly spaced re-sampled data or stationary re-sampled data can be effectively processed by using traditional Fourier transform to obtain clear estimates of the orders of interest. This implies a clearer analysis of the signal using the Fourier transform. However, COT does not address the quality of the raw data. Imperfections, such as distorted harmonic waves and noise, continue to exist. Besides, COT can only deal with the raw data as a whole and therefore loses the ability to separate each different order signal from the raw signal.

2.1.3 Development of Vold-Kalman filter and computed order tracking

The main ideas from above discussions about two techniques may be summarized as follows:

- Equation (2.1) indicates that the order components from the Vold-Kalman filter are clearly harmonic in nature.

- Equations (2.1) and (2.2) show that these order components may be harmonic waves of varying frequency due to the possibility of the varying fundamental frequency $\omega(i)$

- Equation (2.3) can be further demonstrated that the filtered order components from the Vold-Kalman filter are smooth waves.

- It can be seen from discussion of computed order tracking that the re-sampling process can transform varying frequency harmonic waves to stationary frequency harmonic waves. A Fourier analysis is then used to transform the re-sampled time domain data to the order domain.

Based upon the discussion of Vold-Kalman filter order tracking above, it is argued that the Vold-Kalman filter enforces the smoothness as well as the harmonic nature of the filtered data. The harmonic nature does not, however, ensure a stationary harmonic wave, although the re-sampling process can transform data from a non-stationary harmonic wave to a stationary harmonic wave in frequency. This suggests the possibility of using a Vold-Kalman filter to obtain smooth but possibly varying frequency harmonic waves and then transforming them to become stationary in frequency by using the re-sampling process of computed order tracking.

Therefore, if data are obtained from a non-stationary and noisy real machinery system and the data are then processed through a Vold-Kalman filter followed by the re-sampling process of COT, one may obtain order waves that are smooth, stationary frequency harmonic waves. Under these conditions the stringent requirements of Fourier analysis are largely satisfied. One may therefore expect clear and focused order spectra by means of this process. Based upon the above reasoning it follows that if the two order tracking methods are applied in sequence (VKF-OT and then COT), the restrictions of Fourier analysis can be largely satisfied to render clean order spectra. This combined use of order tracking techniques may be referred to as Vold-Kalman filter and computed order tracking (VKC-OT). Figure 2.1 describes graphically the logic of the combined use of the two order tracking techniques in sequence.

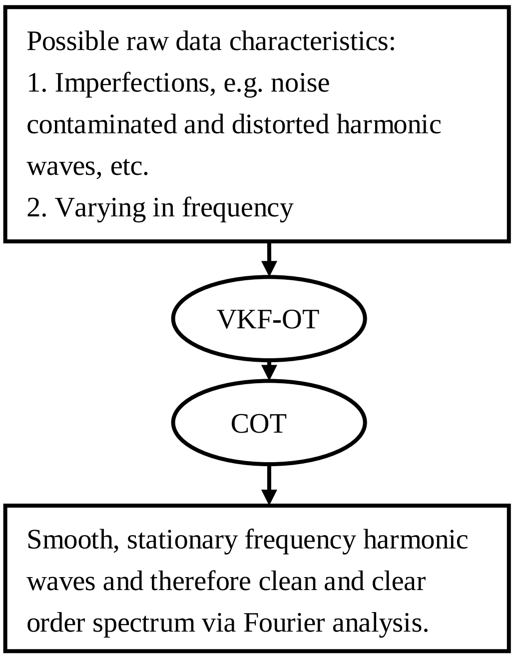


Figure 2.1 Logic of VKC-OT

2.2 Intrinsic mode function and Vold-Kalman filter order tracking

The literature indicates that both IMFs and order tracking techniques are effective in diagnosing faults in rotating machinery, e.g. Eggers et al. (2007); Gao et al. (2008); Wu et al. (2009). This suggests investigating the relationship between IMFs and order waves. However, this has not been explored further in the literature. To this end, the following will firstly exploit the relationship between an intrinsic mode function (IMF) and an order wave in rotating machinery and then develop the Intrinsic mode function and Vold-Kalman filter order tracking (IVK-OT) technique of combining abilities of two kinds of methods.

2.2.1 Discussions on the relationship between an intrinsic mode function and an order waveform in time domain

a. An order waveform

As has been discussed in the literature survey, there are several types of order tracking techniques. Some cannot extract time domain data (e.g. COT) whereas some are capable of extracting time waveforms by using additional information, usually rotational frequency (e.g. VKF-OT). In spite of these distinctions, in essence an order can generally be described as 'a time varying phasor that rotates with an instantaneous frequency related to the rotational frequency of the reference shaft', as shown in equation (2.5) (Blough, 2003).

$$x(t) = A(k,t)\sin\left(2\pi i\left(\frac{k}{p}\right)t + \phi_k\right) \tag{2.5}$$

where $x(t)$ is the order time series, t is time, $A(k.t)$ is the amplitude of the order k which is being tracked as a function of t, p is the period of the primary order in seconds, and ϕ_k is the phase angle of order k. Equation (2.5) defines an order as a time series combining amplitude modulation (AM) and frequency modulation (FM). Both amplitude and frequency modulations are functions of the order of interest k and time t. It should also be noted that equation (2.5) actually enforces an order wave of a sinusoidal nature.

b. An intrinsic mode function from empirical mode decomposition

A discussion about an IMF should start with EMD. For a given signal $x_g(t)$, EMD ends up with a representation of the form (Flandrin et al., 2004):

$$x_g(t) = \sum_{k=1}^{K} d_k(t) + m_K(t) \tag{2.6}$$

where $\{ d_k(t), k = 1,....K \}$ are the modes that are constrained to be zero-mean amplitude modulation frequency modulation waveforms and $m_K(t)$ represents a residue signal. These modes are called intrinsic mode functions (IMFs).

This methodology obviously does not give precise mathematical definitions of each $d_k(t)$ and $m_K(t)$. Flandrin et al. further point out that this makes it difficult to evaluate the performance of EMD. However, researchers (Huang et al., 2006 and Yang et al., 2008) have developed an empirical AM/FM demodulation technique for the purpose of resolving many of the traditional difficulties associated with instantaneous frequency calculations, giving a simple description of an IMF in terms of a normalized frequency modulation part and an amplitude modulation part. Accordingly, any IMF from $\{ d_k(t), k = 1,....K \}$ can be written as:

$$d(t) = A(t)\cos\phi_e(t)\,. \tag{2.7}$$

The suffix k omitted means $d(t)$ can be any one of IMFs where $A(t)$ is the amplitude modulation part, $\cos\phi_e(t)$ is the normalized empirical frequency modulation part and t is time.

In the equation, $A(t)$ is determined by the empirical envelope obtained through the spline fitting of the maxima points of the IMF signal. Both $A(t)$ and $\cos\phi_e(t)$ are dependent on the data itself and are functions of time t. However, it should be noted that the term $\cos\phi_e(t)$ actually enforces the oscillatory nature, though at each time instant, the phase angle and frequency of the carrier wave will not both be defined.

From the basic discussions above regarding an order and an IMF, as well as equations (2.5), (2.6) and (2.7), it can now be observed that:

- An order can be extracted from the original signal as a modulated signal in both amplitude and frequency, as shown in equation (2.5). Similarly, EMD can also decompose the original signal into IMFs, which are amplitude and frequency modulation waveforms plus a residue signal.

- Both an order waveform and an IMF can be treated as two parts, namely amplitude parts $A(k,t)$ and $A(t)$ as well as phase parts $\sin\left(2\pi i\left(\frac{k}{p}\right)t+\phi_k\right)$ and $\cos\phi_e(t)$.

- The amplitude part $A(k,t)$ of the order waveform is a function of time t and order of interest k. The amplitude part $A(t)$ of an IMF is only a function of time t and is determined by the data itself. This implies that the order waveform amplitude can be part of an IMF amplitude or $A(k,t)\in A(t)$.

- Similarly, the phase part $\sin\left(2\pi i\left(\frac{k}{p}\right)t+\phi_k\right)$ of the order waveform and $\cos\phi_e(t)$ of an IMF can also have the relationship $\sin\left(2\pi i\left(\frac{k}{p}\right)t+\phi_k\right)\in\cos\phi_e(t)$.

It can therefore be inferred from these observations that an IMF may include order waveforms plus other relevant information. The combination of order waveforms and other relevant information must satisfy the definition of an IMF. The definition of an IMF was originally stated by Huang et al. (1998) and has been referred to in Chapter 1 paragraph 1.2.3. This definition guarantees that amplitude and frequency modulation signals can both be extracted as IMFs.

2.2.2 Discussions on the relationship between an intrinsic mode function and an order waveform in order domain

The preceding discussions are based upon equations in the time domain. However, the order domain should also be considered so as to explore the characteristics of an order and an IMF. Firstly, equation (2.5) for an order may be written as an analytical signal:

$$x(\theta) = A_o(k,\theta)e^{jk\theta} \tag{2.8}$$

where $A_o(k,\theta)$ is the amplitude component of order k, $e^{jk\theta}$ is the unit sinusoidal wave of order k, k is the order of interest and θ is the angle.

Similarly, equation (2.7) for an IMF can also be written in the order domain as shown in equation (2.9). In this case, the amplitude and frequency modulations are both functions of angle θ instead of time t. Therefore, they may be considered as amplitude and order modulations in the order domain. Order domain analysis transforms non-stationary time-domain signals into stationary signals in angles for rotating machinery vibrations. Order signals are therefore periodic per revolution and Fourier expansion is suitable for the analysis. According to the Fourier expansion theory, a periodic signal can be approximated by Fourier expansions. Therefore any IMF $d(\theta)$ may also be expanded by Fourier expansion as in equation (2.9) in the order domain.

$$d(\theta) = A_o(\theta)\cos\phi_o(\theta) = \sum_{n=-\infty}^{\infty} C_n(\theta)e^{jn\theta} + R(\theta) \tag{2.9}$$

where $A_o(\theta)$ and $\cos\phi_o(\theta)$ are the amplitude and order modulations in the order domain, $C_n(\theta)$ are Fourier coefficients and $R(\theta)$ is a non-periodic signals in the order domain.

Comparing equations (2.8) and (2.9), one should note the following:

- The amplitude component $A_o(k,\theta)$ in equation (2.8) can be one of the amplitude components $C_n(\theta)$, $n \in (1,\infty)$, in equation (2.9), and

- A unit sinusoidal component $e^{jk\theta}$ in equation (2.8) can be one of the components in $e^{jn\theta}$, $n \in (1,\infty)$, in equation (2.9).

Clearly, an order wave can be a particular waveform contained in the IMF. However equation (2.9) indicates that an IMF can include signals other than orders. This discovery is in line with the previous time domain discussions.

2.2.3 Discussions on the resolution of an IMF

But considering the converse of equation (2.9), it however may not necessarily hold, i.e. a signal of the form of equation (2.9) may not necessarily constitute an IMF. This is because only signals that satisfy the definition of Huang et al. 1998, qualify as IMFs. If the composition of signals violates the definition, it will be further decomposed into different IMFs. This actually suggests that EMD as a filter bank is selective for each IMF. It is difficult to develop a general rule for this selective characteristic or resolution of each IMF, since in literature there is no universal mathematical equation reported for the EMD so far.

However, Feldman (2009) analyzed the special and useful case of the decomposition of two harmonics, demonstrating some of the important features of EMD, such as the nature of the resolution for each IMF. He describes an analytical basis for the EMD, and presents a theoretical limiting frequency resolution for EMD to decompose two harmonic tones. This helps to understand the resolution of EMD as filters.

Feldman shows that the frequency and amplitude ratios of two harmonics can be separated into three different groups, to evaluate the resolution of EMD for these harmonics:

- Harmonics with very close frequencies and a small amplitude, where $A_2 / A_1 \leq (\omega_1 / \omega_2)^2$ is unsuitable for EMD decomposition.
- Close frequency harmonics where $(\omega_1 / \omega_2)^2 \leq A_2 / A_1 \leq 2.4(\omega_1 / \omega_2)^{1.75}$ requires several sifting iterations for two harmonics to decompose.
- Distant frequencies and large amplitude harmonics where $A_2 / A_1 \geq 2.4(\omega_1 / \omega_2)^{1.75}$ can be well separated for a single iteration.

Based upon these criteria, one knows that if two harmonics have frequency and amplitude ratios of $A_2 / A_1 \leq (\omega_1 / \omega_2)^2$, EMD is incapable of separating them. This requirement is visually represented in Figure 2.2.

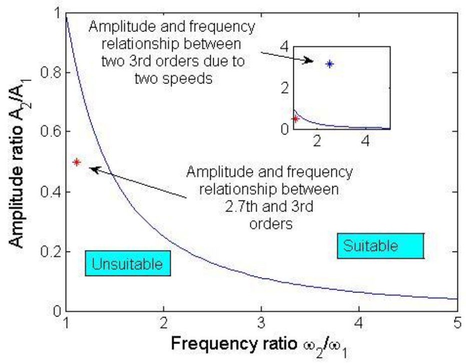


Figure 2.2 Theoretical boundary for the separation of two harmonics $A_2 / A_1 \leq (\omega_1 / \omega_2)^2$

It is clear from Figure 2.2 that two harmonic signals can be decomposed by EMD, depending upon the amplitude and frequency ratio. The limiting boundary determines the region to the right where EMD is able to separate harmonics, and the region to the left where EMD cannot separate two harmonics (Feldman, 2009). This criterion is very useful for determining the decomposition of harmonic vibration signals like orders in rotating machines. (This criterion will be used in the simulation studies in Chapter 3 therefore some extra arrows and descriptions are indicated in the figure for future use).

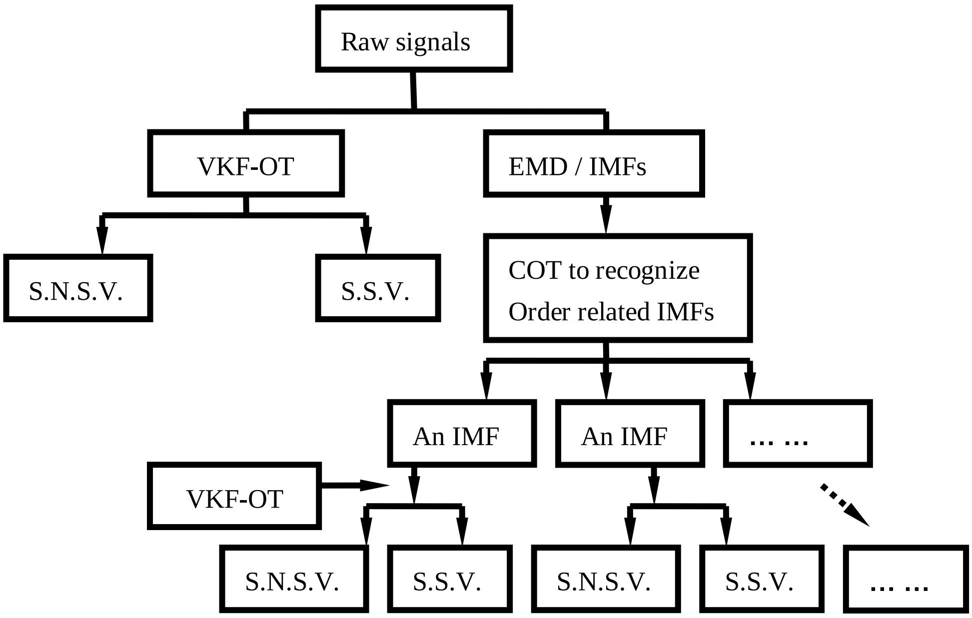
However, the general case of equation (2.9) is not as simple as two harmonic tones, as it is a combination of an order with several other harmonic tones, as well as some other signals, and this is a case where Feldman's theory cannot be applied directly. For rotating machinery, vibrations which are non-synchronous with rotational speed are often small. Therefore if one considers $R(\theta)$ as being negligible over a short period, an order signal and the combination of other harmonic tones may be treated as two quasi-stationary harmonics over that period. Feldman's theory may therefore approximate the resolution of an IMF at each instant. Though this is mathematically not rigorous, it does help to understand the behaviour of IMF.

In short, from all above discussion of the relationship between an IMF and an order in time and order domain as well as resolution of an IMF, it is clear that IMFs from EMD may include both orders and relevant vibrations. And relevant vibrations will modulate order signals so that, in rotating machine vibrations IMFs are usually order related oscillating waves.

2.2.4 Combined use of empirical mode decomposition and Vold-Kalman filter order tracking

From above studies, although EMD is limited in its ability to separate an order signal from other signals, it does have an edge over the traditional order tracking method, as it can capture signals that modulate the order waves. For a rotating machine, an order as defined in equation (2.5), only one specific AM and FM oscillatory waveform can be extracted. So, to a large extent, order tracking itself loses the capability of capturing signals that modulate the orders. But these signals are also critical for the vibration monitoring of rotating machinery. Vibration signals due to faults such as rotor cracks, looseness, worn-out parts, broken teeth or bearing problems are all closely related to the rotating speed or orders. These machine fault vibrations will usually modulate dominant order waves into modulated oscillating waves, which contain a rich source of machine fault information. As a matter of fact, detecting and separating this information from the dominant orders is of great importance. They are the key signatures of machine deterioration and closely related to the orders.

Besides, from the discussion of the relationship between an IMF and an order, it is clear that order tracking can solely focus on vibration signals that are strictly synchronous with the rotational speed and therefore lacks the ability to deal with speed non-synchronous vibrations. The information contained in the speed non-synchronous signals however may also be valuable in terms of machine conditions and deserve to be further utilised. Vibrations that modulate dominant orders, however, may be synchronous or non-synchronous with rotational speed and are therefore usually difficult to extract by traditional order tracking methods alone. In this sense, empirical mode decomposition may include these vibrations with the orders into different intrinsic mode functions. Considering the intrinsic nature of both traditional order tracking methods and EMD, the IMFs from EMD may be further decomposed in terms of rotational speed through order tracking methods so that order signals and vibrations that modulate orders in an IMF may be distinguished. Consequently, the sequential use of EMD and Vold-Kalman filter order tracking method to further decompose order related IMFs is introduced in this research as intrinsic mode function and Vold-Kalman filter order tracking (IVK-OT). Figure 2.3 graphically illustrates the process and as for comparison, traditional VKF-OT is also illustrated. And the logic of IVK-OT is graphically summarized in Figure 2.4.



S.N.S.V. → Speed Non Synchronous Vibration

S.S.V. → Speed Synchronous Vibration

Figure 2.3 Symbolic explanation of IVK-OT the process compared with VKF-OT

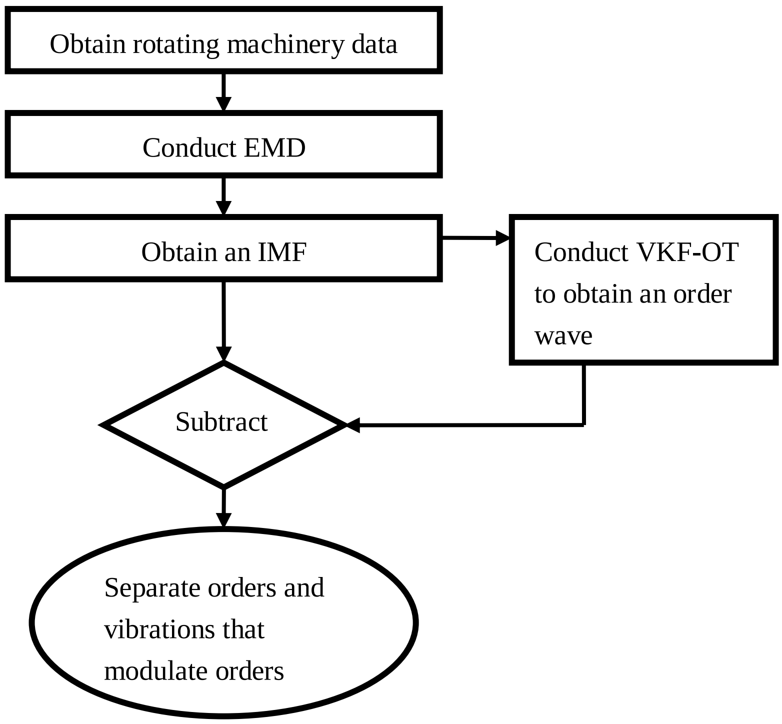


Figure 2.4 Logic of the combined use of EMD and VKT-OT

2.3 Intrinsic cycle re-sampling

The intrinsic cycle re-sampling (ICR) method is a novel way of reconstructing intrinsic mode function (IMF) from empirical mode decomposition (EMD) to approximate the effect of computed order tracking for rotating machine vibration signals. In stead of using traditional speed information to achieve the order tracking effects, an empirical re-sampling method is used on the IMF which approximates the order tracking effects that exclude frequency variations in an IMF. In the following, the logic of the technique to exclude frequency variation effects in an IMF is discussed so that the method can be developed.

2.3.1 Development of intrinsic cycle re-sampling

To begin with the ICR technique, it should repeatedly review the definition of an IMF from EMD. Huang et al. (1998) define an intrinsic mode function as a signal that satisfies two conditions:

- In the whole signal segment, the number of extrema and the number of zero crossings must be either equal or differ at most by one.
- At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

From this original definition of an IMF, it can be concluded that each IMF is a symmetric and zero mean oscillation wave. This excludes two or more peaks within two successive zero crossings. However, the definition does not ensure that the frequency content of this symmetric oscillation wave is constant. Since the important purpose of computed order tracking is to exclude frequency variation from the rotational speed, it is, therefore, worthwhile to further investigate IMF signals with regards to the effects of frequency variation.

a. Intrinsic cycle

The intrinsic cycle (IC) is now introduced. Based upon the idea of an IMF, one may consider a symmetric oscillation wave about a zero mean and define the IC as follows:

Start from the first zero crossing of an IMF and consider two successive zero crossings. The entire signal within these three zero crossings constitute one intrinsic cycle. In the same way, the signal from the last zero crossing of a previous intrinsic cycle and including the following two successive zero crossings, constitute another intrinsic cycle, and so on.

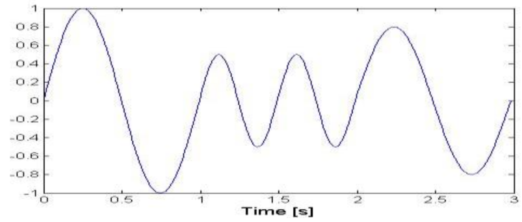
The above definition of an IC from an IMF implies that there are one maximum, one minimum and three zero crossings within each IC. Each IC roughly resembles one period of a sine wave. Considering frequency variations in terms of the newly introduced ICs in an IMF, frequency variations in these approximately sinusoidal natured ICs are not constrained. Variations may exist within and between ICs. If the frequency variation of a signal is solely due to the varying rotational speed, order tracking effects can be achieved by eliminating the frequency variations of the ICs. This is therefore discussed below by considering frequency variations within and between ICs in an IMF.

b. Frequency variation within ICs

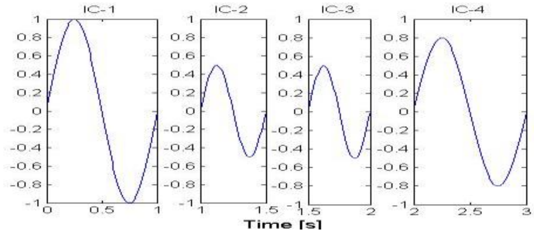
In computed order tracking (Fyfe and Munck, 1997) the assumption is usually made that the rotating shaft angular acceleration is constant or zero over one revolution, since large angular accelerations or decelerations are usually undesirable in practical machines. This is typically done in commercial software (Vibratools in Matlab, 2005). When there are several ICs within one revolution, one could also assume angular acceleration within an IC is zero so that frequency variations within the ICs may be considered negligible. If this assumption is made and a constant rotational frequency within an IC is therefore implied, the focus in dealing with frequency variation effects may then shift to the frequency variations between ICs.

c. Frequency variation between ICs

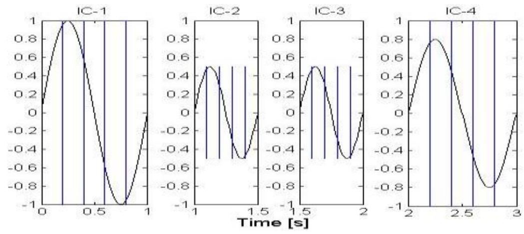
One can now get rid of the frequency variations between ICs by re-sampling with an equal number of intervals within every IC. The frequency variations between ICs may therefore be discarded and render re-sampled intrinsic cycle data. This process is illustrated in Figure 2.5 for an arbitrary intrinsic mode function - a non-stationary sine wave.



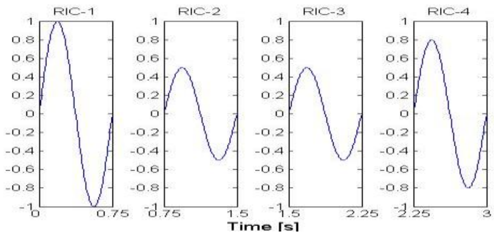
a. An Intrinsic Mode Function



b. Intrinsic Cycles



c. Re-sampling



d. Re-sampled Intrinsic Cycles

Figure 2.5 Illustration of re-sampled IC

For this illustration, an arbitrary IMF sine wave is separated into four individual ICs based upon the definition presented above. It can be seen that the periods of these ICs are different. The 1^{st} (IC-1) and 4^{th} (IC-4) ICs have the same period of 1s but different amplitudes and 2^{nd} (IC-2) and 3^{rd} (IC-3) ICs have the same period of 0.5s and the same amplitudes. This causes the non-stationarity of the signal. These signals are re-sampled into 100 equal intervals within each IC. (In order to clearly illustrate the process visually, only 6 lines are drawn in the figure (c) for each IC and within each line drawn there are 20 equal intervals.) Once the re-sampling is finished, the final wave is reconstructed and features the re-sampled ICs as are shown in (d) which have the same number of equal intervals in each re-sampled IC and each re-sampled IC has the same new period of 0.75s.

The re-sampled ICs have the same periods because each IC has been re-sampled with same number of intervals and a new sampling frequency can be obtained as,

$$f_{new} = \frac{t_{period}}{S_{resample}} \tag{2.10}$$

where f_{new} is the new sampling frequency, $S_{resample}$ is the number of samples of the re-sampled IMF and t_{period} is the time period of the original data.

Clearly, through this re-sampling process, the frequency variations between the ICs are eliminated. Subsequent to obtaining the re-sampled ICs, it can be seen that if frequency variations within ICs are negligible, which follows on the above zero angular acceleration assumption, then frequency variations of the overall signal are excluded in a way similar to eliminating frequency variations during computed order tracking re-sampling. For computed order tracking, the non-stationary time domain data is transformed into stationary angle domain data. In this method, a frequency varying IMF is transformed into a frequency stationary IMF. In this way, rotational speed variation effects in an IMF are eliminated. Fourier analysis can then be used to transform the re-sampled IMF into the frequency domain. Thus, similar computed order tracking effects are achieved through re-sampling the IMF. More importantly, though the present approach may achieve similar effects as to computed order tracking, it however neither requires a tacho signal, nor does it rely on interpolation of signals as is done in normal order tracking analysis.

2.3.2 Interpretation on the reconstructed intrinsic mode function result

From the above it is clear that ICR is a development of an IMF. To understand the ICR results it is therefore necessary to trace its analytical form from the basic definition of the IMF. An IMF $d(t)$ can be written in terms of a normalized amplitude modulation part $A(t)$ and an empirical frequency variation part $\phi_e(t)$, in the time domain as in equation (2.7), here repeat it again,

$$d(t) = A(t)\cos\phi_e(t) \tag{2.7}$$

The ICR method proposed here transforms the possible frequency varied IMF into a frequency stationary IMF (re-sampled IMF). The empirical frequency modulation carrier wave $\cos\phi_e(t)$ in equation (2.7) is therefore transformed into a stationary carrier wave as in equation (2.11)

$$d_{ICR}(t) = A_{ICR}(t)\cos(2\pi f_{ICR} t) \tag{2.11}$$

where $d_{ICR}(t)$ is the re-sampled IMF through ICR, $A_{ICR}(t)$ is the amplitude modulation part of the re-sampled IMF and f_{ICR} is the main frequency of the re-sampled IMF.

Specifically f_{ICR}, the main frequency of re-sampled IMF can be calculated through the ICs as,

$$f_{ICR} = \frac{N_{ICR}}{T_{ICR}} \tag{2.12}$$

where N_{ICR} is the number of intrinsic cycles of the calculated IMF and T_{ICR} is the time period of the calculated IMF.

Through the development of equation (2.11) from (2.7), the original empirical IMF becomes more specific than its original form. In equation (2.11) the parameters of the re-sampled IMF now become the fixed frequency carrier wave at f_{ICR} with amplitude modulation $A_{ICR}(t)$. As a result, the Fourier spectrum for this kind of signal is affected by only the two variables f_{ICR} and $A_{ICR}(t)$. This simplifies the interpretation of the ICR result. Once the calculated time period in equation (2.12) is selected, the number of intrinsic cycles will determine the main frequency component f_{ICR}. However $A_{ICR}(t)$ can still vary according to the nature of the signals but its variations will be reflected in the sidebands of the main frequency component at f_{ICR}. Thus, equations (2.11) and (2.12) lead to the following guidelines in examining the ICR results:

a) Considering a re-sampled IMF time waveform, when signal amplitude variations occur in the re-sampled ICs but the number of ICs remains the same, equation (2.11) implies that $A_{ICR}(t)$ changed due to the amplitude variations and f_{ICR} is invariant due to the unchanged number of ICs. Thus the final spectrum of ICR will exhibit sideband variations and a stationary main frequency peak.

b) When the number of ICs varies and the amplitude of re-sampled ICs in the time waveform remains constant, i.e. $A_{ICR}(t)$ is invariant and f_{ICR} changes in equation (2.11), the final spectrum will exhibit a shift of main frequency peak and stable sideband shapes.

c) When the signal variations influence both the number of ICs and amplitude of re-sampled ICs in the time waveform, according to equation (2.11) both $A_{ICR}(t)$ and f_{ICR} are varied. One may then expect a shift of the main frequency component as well as a variation in the sidebands.

d) Further, the more the variations of the amplitude modulation $A_{ICR}(t)$ in the re-sampled IMF, the more variations of sidebands will appear in the ICR spectrum. And the larger the number of ICs, the higher the value of the main frequency component f_{ICR} will be.

Firstly, considering rotating machinery faults, incipient machine faults will usually not severely influence the vibration signals, therefore in a re-sampled IMF, one may typically expect variations in the amplitude of the re-sampled ICs without changing the number of ICs. Introduction of a new IC requires at least one extra zero crossing in the signal. A small signal variation in a dominant vibration environment, especially for rotating machine vibrations where rotational speed harmonics are predominant, will not easy to introduce extra IC due to small variations of the signal. Thus, it may only change $A_{ICR}(t)$ and the main frequency component, f_{ICR}, will remain the same. In such a case, the sidebands of the ICR spectrum relative to the main frequency component amplitude can be used for condition monitoring purposes. This corresponds to case (a).

Secondly, if the measured response on the machine does not contain clear machine fault vibrations but only influences from the changes in rotational speed which leads to variations of ICs, f_{ICR} will however shift in the ICR spectrum but the sidebands will retain its original shape. This can be used to detect the influence of rotational speed on the measured signals. This corresponds to case (b).

Lastly, when severe changes in the sidebands and a clear shift of f_{ICR} occur, it usually indicates a machine fault occurred and is developing. This corresponds to case (c). In each condition mentioned in (a), (b) and (c), the severity of signal variations will influence the spectrum of ICR results differently in sidebands, main frequency component or both. This is relevant to case (d).

2.3.3 Discussions on intrinsic cycle re-sampling in terms of rotating machine vibration signals

For rotating machinery the order components will usually dominate the response. EMD can empirically decompose these orders into different IMFs. These characteristic orders in different IMFs usually have different physical meanings relating to machine conditions. Thus, each IMF is of great use in condition monitoring, and therefore ICR on IMF will also have advantages in this regard. Ideally, one IMF should capture one order signal and represent one single order component in the order spectrum, as implied by the word 'intrinsic'. However, the IMF may also include other components due to its empirical nature and it has been discussed in the previous IVK-OT technique in paragraph 2.2.2. And the more other components appear in the IMF, the more pronounced the deviations from the order signal will become. As such the final Fourier spectrum of this IMF may contain more variations. This is in fact extremely useful for fault diagnosis of rotating machines, since most of the machine fault vibrations would modulate the order signals. And the IMF has the ability to include this information together with the order of interest. However it should also be noticed that the IMF from EMD cannot get rid of frequency modulation effects due to the rotational speed variation, despite the fact that the rich information related to the machine faults has been decomposed into different IMFs, if Fourier analysis is applied to the signal. However the smearing effects in the frequency spectrum may also occur which could be an impediment for diagnostic decisions. For this reason, the empirical re-sampling intrinsic cycles through which frequency variation is excluded between intrinsic cycles, are useful for presenting better frequency spectrum and therefore beneficial for machine fault diagnostics.

As mentioned in previous IVK-OT technique, researchers such as Feldman, have discussed the resolution of the EMD method. They proved that one IMF may include more than one harmonic signals and signals with small amplitudes compared with the dominant harmonics, may easily be included in an IMF. While this is actually a disadvantage of IMF in extracting solely order signals, compared to conventional order tracking techniques, it does however provide a unique ability for capturing signals that modulate dominant order signals. Thus, ICR developed from IMFs can be used as a tool to reflect changes of vibration signals that modulate order signals. And it could be very useful for condition monitoring of rotating machinery. In the following chapter a simplified gear mesh model is used to demonstrate ICR. The logic of performing ICR is first schematically summarized in Figure 2.6.

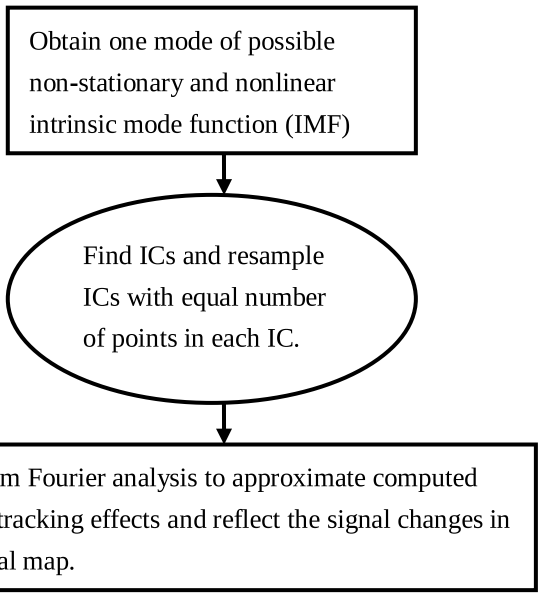


Figure 2.6 Logic of the ICR technique

2.4 Summary

In this chapter, three improved order tracking techniques are theoretically developed. Firstly, Vold-Kalman filter and computed order tracking (VKC-OT) is developed. The discussions are emphasised on their distinct characteristics for subsequent Fourier analysis. Secondly, intrinsic mode function and Vold-Kalman filter order tracking (IVK-OT) is developed. Time and order domain discussions that reveal the relationship between an IMF and an order are presented which fills the vacant of literature in this regard. Lastly, intrinsic cycle re-sampling method is formed through newly introduced term intrinsic cycle. Its unique interpretation method in terms of reconstructed IMF is also put forward which will bring benefits to condition monitoring rotating machines. In short, theoretical developments for three improved order tracking techniques have been made. In the following, these techniques will be further verified and validated in simulation studies.