The axial line placement problem

by

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In memory of my father,

Douglas Rutherford Sanders,

26 April 1927 – 18 October 1997

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Summary

Visibility, guarding and polygon decomposition are problems in the field of computational geometry which have roots in real world applications. These problems have been the focus of much research over a number of years. This thesis introduces a new problem in the field – The Axial line Placement Problem – which has some commonalities with these other problems. The problem arises from a consideration of the computational issues that result from attempting to automate the space syntax method. Space syntax is used for describing, quantifying and interpreting the spatial patterns in urban designs by analysing the relationship between the space through which one can move (roads, parks, etc.) and the buildings in the urban layout. In particular, this thesis considers the problem of the placing the axial lines, defining paths along which someone can move, to cross the shared boundaries between the convex polygons which represent the space through which someone can move in the town.

A number of simplifications of the original problem are considered in this thesis. The first of these is the problem of placing the smallest number of orthogonal line segments (orthogonal axial lines) to cross the shared boundaries (adjacencies) in a collection of adjacent orthogonal rectangles. This problem is shown to be NP-Complete by a transformation from the vertex cover problem for planar graphs. A heuristic algorithm which produces an approximation to the general solution is then presented. In addition, special cases of collections of orthogonal rectangles which allow polynomial time solutions are described and algorithms to solve some of these special cases are presented.

The problem where the axial lines, that pass through the adjacencies between orthogonal rectangles, can have arbitrary orientation is then considered. This problem is also shown to be NP-Complete and once again heuristic approaches to solving the problem are considered. The problem of placing axial lines to cross the adjacencies between adjacent convex polygons is a more general case of the problem of placing axial lines of arbitrary orientation in orthogonal rectangles. The NP-Completeness proof can be extended to this problem as well.

The final stage of the thesis considers real world urban layouts. Many urban layouts are regular grids of roads. Such layouts can be modelled as general urban grids and this thesis shows that it is possible to find the minimal axial line cover in general urban grids in polynomial time. Some urban layouts are less regular and the idea of a deformed urban grid is introduced to model some of these layouts. A heuristic algorithm that finds a partition of a deformed urban grid in polynomial time is presented and it is conjectured that the axial map of a deformed urban grid can be found in polynomial time. The problem is still open for more general urban layouts which cannot be modelled by deformed urban grids.

The contribution of this thesis is that a number of new NP-Complete problems were identified and some new and interesting problems in the area of computational geometry have been introduced.

Opsomming

Sigbaarheid, waghou en veelhoek-dekomposisie is probleme in berekeningsmeetkunde wat hulle oorsprong in reële toepassings het. Die probleme is sedert jare die onderwerp van vele navorsing. Hierdie tesis voeg 'n nuwe probleem by die navorsingsgebied – die Asselyn Plasingsprobleem – wat sekere gemeenskaplikhede met bogenoemde probleme het. Laasgenoemde probleem vloei voort uit 'n beskouing van die berekeningskwessies wat ontstaan wanneer pogings aangewend word om die ruimte-sintaksis metode te outomatiseer. Ruimte-sintaksis word gebruik vir die beskrywing, kwantifisering en interpretasie van ruimtelike patrone in stedelike ontwerpe en wel deur die verwantskap tussen die ruimte waardeur 'n mens kan beweeg (paaie, parke, ens.) en die geboue in die stedelike uitleg te ontleed. Hierdie tesis beskou, in die besonder, die probleem van die plasing van asselyne op sodanig wyse dat hulle gedeelde grense tussen konvekse veelhoeke kruis, waarby the lyne paaie waarlang mens kan beweeg en die veelhoeke die ruimte waardeur mens deur die stad kan beweeg, verteenwoordig.

'n Aantal vereenvoudigings van die oorspronklike probleem word in hierdie tesis beskou. Die eerste hiervan is die probleem om die kleinste moontlike aantal ortogonale lynsegmente (ortogonale asselyne) op so 'n wyse te plaas dat hulle die gedeelde grense in 'n versameling van aangrensende ortogonale reghoeke kruis. Daar word gewys dat hierdie probleem NP-volledig is, deur 'n transformasie van die nodus-dekkingsprobleem ("vertex cover problem") vir planêre ("planar") grafieke na die problem uit te voer. 'n Heuristiese algoritme wat 'n benaderde oplossing tot die algemene probleem bied, word dan voorgestel. Addisioneel word spesiale gevalle van versamelings van ortogonale reghoeke wat polinomiese tyd oplossings toelaat beskryf. Algoritmes wat sekere van hierdie spesiale gevalle oplos word aangebied.

Daarna word die probleem beskou waarvolgens asselyne wat deur aangrensende ortogonale reghoeke gaan, arbitrêre orientasie mag hê. Hierdie probleem word ook as NP-volledig bewys en weereens word heuristieke benaderings om die probleem op te los, beskou. Die probleem om asselyne te plaas sodanig dat hulle grense tussen aangrensende konvekse veelhoeke te kruis is 'n veralgemening van die probleem om asselyne van arbitrêre orientasie in reghoeke te plaas. Die NP-volledigheidsbewys kan ook na die meer algemene probleem uitgebrei word.

Die finale fase van die tesis beskou die uitleg van reële stede. In die geval van baie stede is die uitleg 'n reëlmatige rooster van paaie. So 'n uitleg kan as 'n algemene stedelike rooster gemodeleer word en hierdie tesis toon aan dat dit moontlik is om die minimum asselyn dekking van sulke roosters in polinomiese tyd te bepaal. Sekere stede se uitleg is minder reëlmatig en die konsep van 'n verwronge stedelike rooster word voorgestel om sommige daarvan te modeleer. 'n Heuristiese algoritme wat in polinomiese tyd 'n partisie van 'n verwronge stedelike rooster vind, word aangebied. Daar word gepostuleer dat die assekaart van 'n verwronge stedelike rooster in polinomiese tyd gevind kan word. Die probleem vir stedelike uitlegte wat nie deur verwronge stedelike roosters gemodeleer kan word nie, bly egter steeds onopgelos.

Die bydrae van hierdie tesis is dat 'n aantal nuwe NP-volledige probleme geïdentifiseer is, en sommige nuwe en interessante probleme tot die gebied van berekeningsmeetkunde toegevoeg is.

Preface

Some of the work in this thesis has been previously published.

- The NP-Completeness proof in Chapter 4 was published in the South African Computer Journal [Sanders *et al.*, 1999].

- The axial line placement problem in chains and trees of orthogonal rectangles presented in Chapter 4 was also published in the South African Computer Journal [Sanders *et al.*, 2000b]. Much of the work for this paper was done under my supervision by two Computer Science Honours students, Claire Watts and Andrew Hall, as the research component of their degrees.

- The axial line placement problem in urban grids and deformed urban grids in Chapter 7 was accepted for the South African Institute of Computer Scientists and Information Technologists 2000 research symposium as a full research paper. It was published in a special issue of the South African Computer Journal [Sanders, 2000].

- The NP-Completeness proof in Chapter 5 was presented at the 11th Canadian Conference on Computational Geometry – an extended abstract was published in a collection of such abstracts and the full paper is available electronically [Sanders, 1999].

- The heuristics proposed in Chapter 5 were presented at the 13th Canadian Conference on Computational Geometry – an extended abstract was published in a collection of such abstracts [Sanders and Kenny, 2001a]. The full paper is available as a technical report in the School of Computer Science at the University of the Witwatersrand [Sanders and Kenny, 2001b]. Some of the work for this paper was done under my supervision by a Computer Science Honours student, Leigh-Ann Kenny, as the research component of her degree.

- Various presentations were made of "work in progress" at the Southern African Computer Lecturers' Association annual conferences and the South African

Institute of Computer Scientists and Information Technologists annual research symposia [Sanders *et al.*, 1995, 1997; Sanders, 1998a,b; Bilbrough and Sanders, 1998].

- Some work has also been published as technical reports in the Department of Computer Science at the University of the Witwatersrand [Watts and Sanders, 1997; Sanders *et al.*, 2000a; du Plessis and Sanders, 2000; Sanders and Kenny, 2001b].
- My Honours students over the years have worked on some small parts of the overall research [Watts, 1997; Zarganakis, 1997; Soares, 1997; Wilson, 1997; Bilbrough, 1998; du Plessis, 1999; Hall, 1999; Ashman, 1999; Bukovska, 2000; Kenny, 2000; Soltész, 2000; Konidaris, 2001; Scott-Dawkins, 2001; Phillips, 2001; Hagger, 2001].

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