Entia non sunt multiplicanda praeter necessitate

"Entities must not be multiplied beyond necessity".

William of Ockham
(c. 1288 - c. 1348)

1

Introduction

1 INTRODUCTION

The object of pavement engineering is to design complex pavement structures based on as few assumptions as possible, while eliminating complexities that make little difference to the reliability of the performance prediction. It is vital that the simplifications in the design models are based on a sound understanding of the key mechanisms at play in the pavement system. With time, design models will be replaced by new simplifications that better reflect reality. Also, as innovative pavement solutions are introduced, the suitability of existing models for the design of these solutions needs to be verified.

In this introductory chapter questions are raised regarding the applicability of conventional concrete pavement design approaches to an innovative pavement system developed in South Africa. The high performance fibre reinforced concrete material used in the novel pavement system is known to behave distinctly different in fracture than plain concrete. A need exists to assess the validity of the assumptions in terms of the material behaviour underlying the conventional design methods, for application to this high performance material.

1.1 Background

South Africa boasts an extensive and mature road network. At present, the bulk of pavement design activities in the country are aimed at preserving and upgrading the existing road infrastructure. Innovative methods of pavement rehabilitation are required to increase the service life of wearing courses and to reduce the need for traffic hampering maintenance activities. To this aim, the South African National Road Agency Limited (SANRAL) has sponsored the development of the so-called Ultra Thin Continuously Reinforced Concrete Pavement (UTCRCP). UTCRCP is intended as an overlay strategy for existing roads. The technology comprises a high performance concrete layer with a nominal thickness of approximately 50 mm. The material incorporates fibres as well as mesh reinforcement and is characterised by its ability to withstand high deflections. The technology is discussed in more detail as part of Chapter 2. During the course of this study the UTCRCP technology progressed from the development phase to the implementation phase. The methodology is now being applied as part of major highway rehabilitation projects in South Africa. The design tools for the innovative UTCRCP system are currently based on conventional concrete pavement design methodologies.

The conventional Mechanistic-Empirical approach to concrete pavement design for fatigue makes use of Linear Elastic (LE) analysis. Both the stress in the pavement slab and the material strength are obtained assuming LE material behaviour. Non-linear, non-elastic post fracture behaviour is not taken into consideration. In these models the material strength is characterised by the Modulus of Rupture (MOR) obtained in monotonic Four Point Bending (FPB) test on beam specimens. The MOR is the stress in the extreme fibre of the specimen, calculated under the assumption of a LE stress distribution at the peak load condition. The ratio between the MOR and the stress in the pavement calculated through LE analysis, is used to predict the fatigue life of the pavement. Researchers have long established that flexural strength for concrete is not a true material property, because its value changes with specimen size (Reagel and Willis, 1931, Kellerman, 1932). The size effect phenomenon is caused by the fact that concrete is a quasi brittle material and at the peak load condition cracks will already have formed in the material. Due to the presence of a crack, the assumed LE stress distribution no longer exists in the beam. In different sizes of specimens, different amounts of energy are released into the crack front, giving rise to the observed size effect. Similarly, a LE stress distribution will not be present in a pavement slab loaded to failure, because it too will have cracked. Size effect in plain concrete has been well documented and can be predicted using fracture mechanics (Bažant and Planas, 1997). Notwithstanding these limitations, LE analysis remains the basis for fatigue prediction in state of the art concrete pavement design methods.

Studies at the University of Pretoria have shown the high performance fibre reinforced concrete material used in UTCRCP to have significantly increased post crack load carrying capacity when compared to plain concrete (Kearsley and Elsaigh, 2003, Elsaigh, 2007). The limitations of linear elastic design assumptions are expected to be magnified when applied to this material, due to its significant post crack stress capacity. The MOR only provides an indication of the peak load capacity of the material, it does not describe the post peak behaviour. The extent to which this impacts on the accuracy and precision of the design models requires investigation. It may also be possible to improve the predictive performance of the fatigue models through the use of size independent, fracture mechanics based approaches.

1.2 Problem statement

Current design methods for Ultra Thin Continuously Reinforced Concrete Pavements (UTCRCP) are based on conventional concrete pavement design theory. However, the fracture behaviour of the high performance fibre reinforced concrete material used in the UTCRCP technology is characterized by a significant post crack stress capacity. A need exists to determine to what extent assumptions with regards to material behaviour, underlying conventional design approaches, are applicable to this high performance material.

Fracture mechanics approaches can be used to simulate the post crack behaviour of both plain and fibre reinforced concrete. An investigation into the possible improvement of the predictive performance of the design methods for UTCRCP through the use of fracture mechanics concepts is required.

A prerequisite for the adoption of fracture mechanics based design approaches by the industry, would be the availability of practical and robust test methods for the determination of the relevant material parameters. The accuracy and precision of fracture mechanics based design models will have to be compared against the results obtained from conventional design methodology.

1.3 Objectives

The study takes a stepwise approach to the problem defined in the previous section. The first objective of the study is to quantify the size effect in the high performance fibre reinforced concrete material used for UTCRCP. The magnitude of the size effect will provide an indication of the suitability of linear elastically derived parameters in the design of UTCRCP.

The second objective is to determine the fracture properties of the material required for the numerical simulation of cracking in the material using a fracture mechanics model. The properties are to be used in fracture mechanics based numerical simulation of the experiments. The aim is to use existing, or develop new test methods that can readily be used by industry on a routine basis.

The final objective is to develop fatigue prediction models based on fracture mechanics concepts. The predictive performance of these models will be compared to the performance of predictive equations calibrated using the conventional design approach.

1.4 Thesis statement

The premise of this study is that current design methods do not include models that accurately describe the mechanisms of fracture damage formation in the UTCRCP material. The main hypothesis of this study is that:

The accuracy of design models for UTCRCP can benefit from the adoption of fracture mechanics concepts.

To build a case to validate or reject the main hypothesis, a set of hypothetical propositions will be tested in this work. These propositions represent the reasoning leading up to the main research hypothesis. These propositions are:

1. *The high performance fibre reinforced concrete material will exhibit a strong size effect due to its high post crack stress capacity. The size effect will limit the reliability of the Modulus of Rupture (MOR) obtained for a specific specimen size and geometry, as a predictor of the peak load of elements of a different size and or geometry.*

2. *In contrast to the MOR, fracture mechanics material parameters can be used to accurately and precisely, predict the peak load and importantly, the post-peak flexural behaviour of elements of a different size and geometry.,*
3. *The accuracy, and possibly the precision, of fatigue prediction models for the material can be improved through the use of fracture mechanics concepts.*

The work in this thesis is aimed at empirically testing each of these propositions and by doing so evaluating the main hypothesis.

1.5 Scope of the work

The scope of the study covers the experimental and numerical simulation work required to test the hypotheses formulated in the previous section. The work will include:

a) A literature survey on the state of the art in the design concrete pavements. The theoretical framework will also cover fracture mechanics theory and practice with regards to concrete,
b) Experiments to determine the fracture mechanics properties of several concrete mixes under monotonic loading using specimens of various geometries and for mix designs prepared with different steel fibre contents,
c) Numerical simulation of the monotonic experimental results using non-linear fracture mechanics,
d) Experiments to determine the fatigue performance of various concrete materials, under cyclic loading and tested in various geometries and at different steel fibre contents,
e) The development of a, fracture mechanics based, predictive model for the fatigue response observed in the experiments. This process will include the comparison of the precision and accuracy of this model against a model akin to conventional concrete design methodology.

The following topics will not be covered in this study:

a) The development of solid mechanics approach that allows for the accumulation of fatigue damage under dynamic loading and can completely replace Miner's linear cumulative damage hypothesis,
b) The calibration of fatigue damage models based on full scale field tests,
c) The evaluation of the predictive performance of existing design methods for UTCRCP.

1.6 Limitations

The experimental work in this thesis is limited to laboratory scale specimens. The assumption is that if the fracture mechanics based design methodologies outperform the conventional type damage models in terms of precision and accuracy with regards to the prediction of behaviour of laboratory specimens of different sizes and geometries, they will also outperform the conventional model for the prediction of fatigue in full size pavement structures.

1.7 Contribution to the state of knowledge

The present study will contribute to the state of knowledge in the following fields:

- Test methods to determine fracture mechanics properties of concrete are well developed. Many of these require advanced equipment. There is a need to develop test methods that can easily be implemented as part of pavement design practice. In addition to this, the test methods need to be adjusted to suit Fibre Reinforced Concrete (FRC). Key methodologies developed as part of the study are an adjusted tensile splitting tests to obtain an accurate measure of the true tensile strength (f_t) of FRC, and a methodology to determine the specific fracture energy (G_f) from flexural tests on beam specimens.
- The MOR of concrete is known to be subject to size-effect, this has been well documented by various researchers. Few studies have been done on the magnitude of the size-effect in fibre reinforced material. For this reason a study of size-effect in the UTCRCP material will contribute to the state of knowledge,

- The suitability of the MOR as a material parameter for use in the design of UTCRCP will be assessed, which is of importance for design practise,
- The development of a suitable fracture mechanics damage model, which can be used to reliably predict the flexural behaviour of UTCRCP under monotonic loading,
- The development of a fatigue prediction model for the high performance fibre reinforced concrete material based on fracture mechanics concepts, and
- As the literature survey in Chapter 2 will indicate, there is a relative limited amount of publications on fatigue behaviour of high performance fibre reinforced concrete. Fatigue tests on the UTCRCP material will therefore expand the available body of knowledge.

1.8 Thesis structure

This introductory chapter is followed by a discussion on the state of knowledge relevant to this study, in Chapter 2. The theoretical framework comprises a description of the UTCRCP technology, the mechanisms of fatigue damage, the state of the art in concrete pavement design and available fracture mechanics approaches.

The research methodology employed to test the hypotheses of the study is discussed in Chapter 3. The selection and development of test methods is also discussed in this chapter, as are the test matrix and the concrete mix designs. The methods used for the numerical simulation of fracture in the study are also introduced.

The results of the experiments and the analysis of the data are presented in Chapter 4. Size effect observations are discussed, as are the methods applied to obtain the fracture parameters for the material from the test data. Finally, the results of tests performed under monotonic loading conditions are compared to the results from cyclic loading (fatigue) tests.

Chapter 5 discusses the development of numerical models. Numerical simulation of the tests under monotonic loading is performed using two different fracture mechanics techniques. Models for fatigue prediction using a fracture mechanics based approaches are also developed in this chapter. The performance of these models is compared to the performance of a calibrated model of the type used in conventional concrete pavement design methods.

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The conclusions with regards to the empirical tests of the hypotheses of the study are presented in the final chapter. Recommendations for further work are also provided.

As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality

Albert Einstein (1922)

2

Theoretical framework

2 THEORETICAL FRAMEWORK

This chapter traces decades of research aimed at describing and predicting the development of fracture in concrete by means of mathematical modelling. As will become evident from the discussion, serious challenges still exist in bringing the mathematical prediction of fatigue fracture closer to the reality observed in concrete pavements.

The chapter starts with a section on the UTCRCP technology and its development. The discussion then shifts to the mechanism of fatigue cracking in concrete from its nucleus at an inherent material flaw to a full size crack. An overview of the current design methodologies for fatigue fracture in concrete pavements and their limitations is given. The advances in the modelling of crack formation in concrete by means of fracture mechanics both under monotonic and cyclic loading conditions are discussed in some detail.

2.1 Ultra Thin Continuously Reinforced Concrete Pavements (UTCRCP)

The South African Ultra Thin Continuously Reinforced Concrete Pavement is a further development of technology used in bridge deck rehabilitation in Europe. The technology comprises a high performance concrete layer with a nominal thickness of approximately 50 mm. A steel mesh is placed at the centre of the layer. The concrete, with an unconfined compressive strength (f_c) in the order of 100 MPa, incorporates both steel and synthetic fibres. The material is able to withstand high deflections and is intended as an overlay rehabilitation strategy for weakened pavement structures.

The technology was tested extensively under the Heavy Vehicle Simulator (HVS). As part of the test program, the amount of steel fibres, mesh reinforcement, and slab thickness was varied. A description of the development and testing of the technology is contained in Kannemeyer et al. (2008), and Du Plessis and Fisher (2008a, 2008b).

Figure 2-1 shows the typical damage evolution for UTCRCP in HVS experiments. Cracks are indicated with spray paint on the test section.

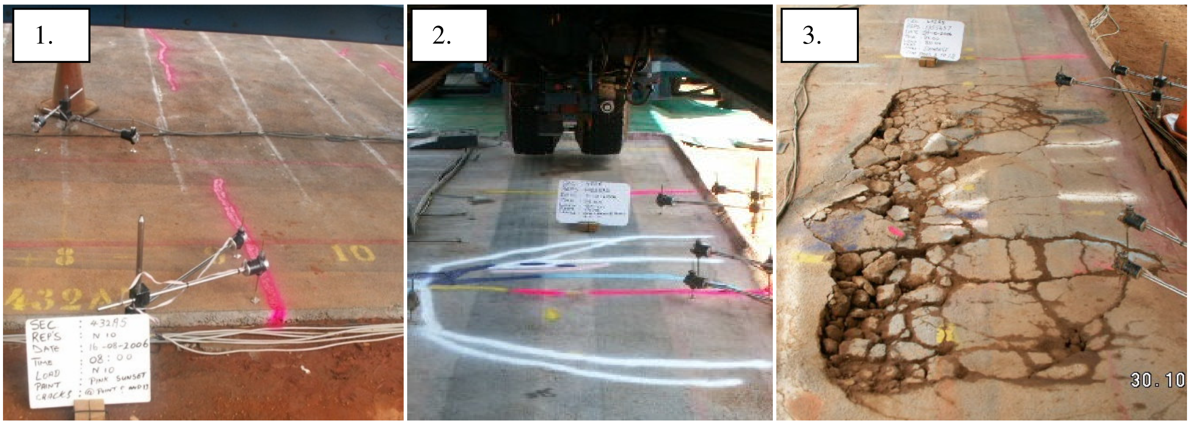


Figure 2-1: Typical damage evolution in UTCRCP under HVS testing (published earlier in Du Plessis and Fisher, 2008a)

The damage propagation in the material as observed during the HVS experiments is understood to occur as follows:

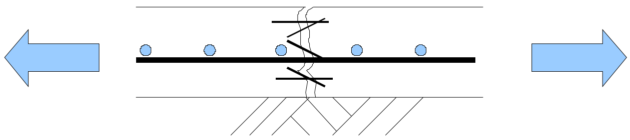
1. Before the start of trafficking, shrinkage cracks will be present as can be seen in the picture on the left. The final failure will always occur at the position of such a shrinkage crack.
2. As the section is being trafficked and water is added to the section, fines of the base material are pumped out through the growing shrinkage crack. Due to the loss of support the bending stresses in the UTCRCP increase in the affected area and at some stage secondary cracks occur as can be seen in the centre picture. These secondary

cracks typically occurred at a distance of approximately 300 mm from the original shrinkage crack (Kannemeyer et al., 2008).

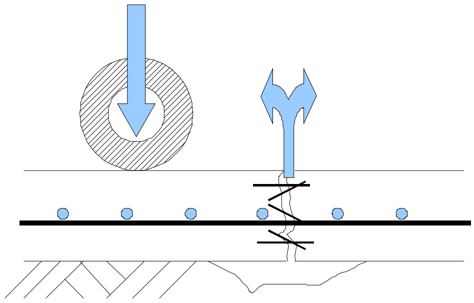
3. Once secondary cracking has occurred the various cracks are able to open wider and the process of water ingress and base erosion is accelerated, finally leading to total collapse of the system, as can be seen from the picture on the right. An observation relevant to the laboratory experiments to be carried out as part of the present study is that the steel mesh reinforcing did not fail during the experiments.

Figure 2-2 shows in schematic form the failure mode of the UTCRCP as observed in the HVS experiments. In the figure it is assumed the maximum stress under trafficking will occur at the top of the slab some distance from the initial shrinkage crack. Linear elastic Finite Element Method (FEM) analysis of the tests by Kannemeyer et al. (2008) indicated that the highest stress under trafficking may in reality occur at the bottom of the slab at a distance of 450 mm from the shrinkage crack. In this case, the mechanism would be similar to what is shown in the figure, with the secondary crack starting at the bottom of the slab.

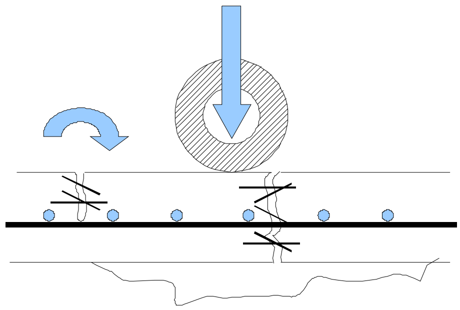
The HVS results were used to include the UTCRCP design option in cncPAVE (C&CI, 2009) the design software of the South African Cement and Concrete Institute (C&CI). The software and design methods were developed by Strauss et al (2001). The design models for UTCRCP were included in a later version of the software by Strauss et al. (2007). The design method for UTCRCP as implemented in cncPAVE makes use of a conventional approach to concrete pavement design. Stresses in the pavement are calculated using a Westergaard type equation, which is discussed in detail in Section 2.3.2. The ratio of the maximum tensile stress in the pavement to the MOR of the material is then used to predict the life of the pavement by applying Miner's law. Concerns about the MOR as a suitable material property for fatigue prediction are raised in Section 2.4.2, while limitations of Miner's linear damage hypothesis are discussed in Section 2.4.1. The main concern about current concrete pavement design methods is that they do not provide a mechanistic simulation of the fatigue fracture phenomenon discussed in the following section.



Stage 1: Formation of shrinkage crack shortly after construction



Stage 2: Water penetrates structure through the shrinkage crack, fines pump out under the action of traffic



Stage 3: secondary cracking occurs due to tensile strain caused by hogging moment.

Figure 2-2: Schematic representation of failure in UTCRCP

2.2 The mechanisms of fatigue in plain and fibre reinforced concrete

Material fatigue, is the growth of fracture damage in a material as a result of repeated loading at an amplitude lower than the ultimate stress limit of the material. The study of fatigue has its origins in mechanical metallurgy and in this field there is a well established understanding of the fatigue mechanism. In metals, fatigue damage starts at a molecular level with a crystallographic defect, or dislocation, resulting in slip bands followed by micro cracking. Under continued loading a micro crack in the weakest position of the affected area will eventually develop into a macro crack and propagate, resulting in failure.

Concrete is less homogeneous than metal, and rather than microscopic defects it contains relatively large imperfections in the form of air voids, shrinkage cracks and pockets of trapped water. It is at these inherent flaws in the material that micro cracking due to fatigue loading will initiate (Lee and Barr, 2004, Gao and Hsu, 1998). Based on a study of fatigue in concrete under uniaxial compressive loading, Gao and Hsu (1998) divide the fracture behaviour in three stages. In the first, or linear stage, the pre-existing flaws return to their original condition after loading. In the following non-linear hardening stage micro-cracks start to develop in the mortar and at the interface of mortar and aggregate. In the final stage the micro-cracks propagate to form macro-cracks and the material response shows softening under loading as a result.

Fatigue loading is often classified into low and high cycle fatigue, although various publications provide different brackets for the number of cycles per class. For pavement engineering the relevant fatigue classes are high cycle fatigue which is often used to describe the range from 10^4 to 10^7 load cycles and super high cycle fatigue with more than 10^7 load cycles to failure. Low cycle with less than 10^4 load repetitions is relevant for structures subjected to earthquakes. The loading of airport pavements may also fall within this range. Hsu (1984) points out that there is a difference in the damage mechanism for concrete under low-cycle and high-cycle loading regimes. In low cycle-fatigue cracks form in the mortar, whereas in high cycle fatigue it is the bond between aggregate and mortar that slowly deteriorates. Lee and Barr (2004) indicate that concrete does not appear to have a so-called fatigue limit. The fatigue limit is a stress level below which no fatigue damage occurs and the material is able to withstand an unlimited number of load cycles. This has the important implication that when the stress in a structure is kept lower than the fatigue limit, the structure has infinite life. The fatigue limit is a concept from mechanical metallurgy and has been reported to exist for various metals. With the advent of hypersonic testing however, doubts have been cast around the existence of a fatigue limit for ferrous metals as well (Miller and O'Donnel, 1999, Bathias, 1999).

The material under study is a Fibre Reinforced Concrete (FRC). FRC has a higher post-cracking load carrying capacity than plain concrete (Elsaigh, 2007). The ability of the fibres to transfer stresses across the cracks results in a larger ductile component in failure for FRC compared to plain concrete. The cracking mechanism in fatigue of FRC is determined by crack growth in the concrete matrix, the crack bridging action specific to the type of fibres, and the fatigue damage to bond between fibres and matrix (Li and Matsumoto, 1998, Zhang et al., 1999). Cracking is retarded through the energy dissipation required to fully pull the fibres out of the mastic. For this reason it would be expected that the fatigue performance of FRC in tension is also enhanced by the presence of fibres. The body of knowledge on fatigue behaviour of concrete relatively limited and even less information is available for fatigue in fibre reinforced concrete. Lee and Barr (2004) compared the fatigue behaviour of plain and fibre reinforced concrete available from literature and found conflicting information. The majority of publications however, were found to indicate that FRC exhibits better fatigue performance in flexure than plain concrete. Published results show that the addition of fibres does not increase the fatigue performance of concrete under compressive cyclic loading.

2.3 Design for fatigue in concrete pavements

Current concrete pavement design methods can be described as supported by three pillars. The first pillar is an empirical relationship for the fatigue performance of the material, which is expressed in terms of number of loading repetitions (N) to failure at a certain stress level (S) using a so-called S-N curve. The second pillar is the calculation of the stresses in the pavement under traffic loading using linear elastic analysis, and the final pillar is the calculation of damage accumulation under traffic loading using a linear cumulative damage hypothesis. The background of each of the pillars will be discussed to some detail in this section as this is key to understanding the need for improvement of design the methodology.

2.3.1 Relating pavement stress condition to fatigue life

Ever since their introduction by Wöhler (1870), S-N curves have been widely applied in different engineering disciplines. In mechanical metallurgy the S-N function is typically derived from a standard test method where a sinusoidal stress is introduced to the specimen. The test is repeated at different stress intensities and the results plotted with the number of repetitions to failure on a log scale. The method is notorious for the statistical scatter of results. The S-N functions in concrete pavement design methods generally are not the result of tests performed on laboratory specimen, but rather a statistical fit to data from full scale field tests or long term pavement performance test sections. The definition of failure may differ between design methods and the results are, amongst others, a function of the material characteristics, the structural support and vehicle types in the area where the method was developed. The number of repetitions to failure is predicted using a ratio between the tensile stresses in the concrete pavement, determined from linear elastic analysis, and the Modulus of Rupture (MOR) of the material. The MOR, also referred to as the flexural strength, is determined from the monotonic peak load in Four Point Bending (FPB) laboratory tests. The

MOR is the ultimate nominal stress (σ_{Nu}) in the extreme fibre of the specimen, calculated assuming a linear elastic stress distribution at failure. The nominal stress (σ_N) (at any load level) is obtained for FPB using Equation 2.1 For Three Point Bending (TPB) testing, a test configuration used extensively as part of this study, σ_N is obtained from using Equation 2.2.

$$\sigma_N = \frac{Ps}{bh^2} \tag{2.1}$$

$$\sigma_N = \frac{3Ps}{2b(h-a)^2} \tag{2.2}$$

Where P is the sum of external loading introduced to a beam, s is the span, b is the width of the beam and h the height, a is the depth of the notch, where applicable. Sketches of the FPB and TPB test configuration are shown in Figure 2-3a and Figure 2-3b respectively.

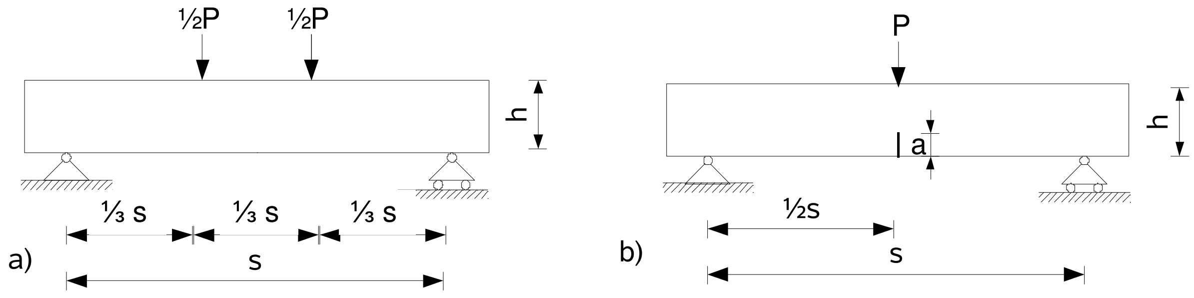


Figure 2-3: a) FPB test configuration, b) TPB test configuration

The calibrated S-N model to predict the fatigue life of a pavement from the results of the MOR tests is known as a transfer function and typically takes the form shown as Equation 2.3.

$$N = a_1 \left(\frac{\sigma_d}{MOR} \right)^{b_1} \tag{2.3}$$

Where N is the number of load repetitions to failure, σ_d is the linear elastically derived design tensile stress in the pavement for a defined load condition. Parameters a_1 and b_1 are the calibration constants which form the essence of the transfer function. The calibration constants provide the bridge between the statistical fatigue data from full scale observations on the one side and the ratio between the stress at peak load from monotonic FPB tests and the calculated tensile stress in the concrete slab on the other side, assuming LE material behaviour. The transfer function for fatigue in continuously reinforced concrete pavements developed as part of the new American Mechanistic Empirical Pavement Design Guide (MEPDG), NCHRP 1-37A (2004) by Selezneva et al. (2004) is plotted in Figure 2-4.

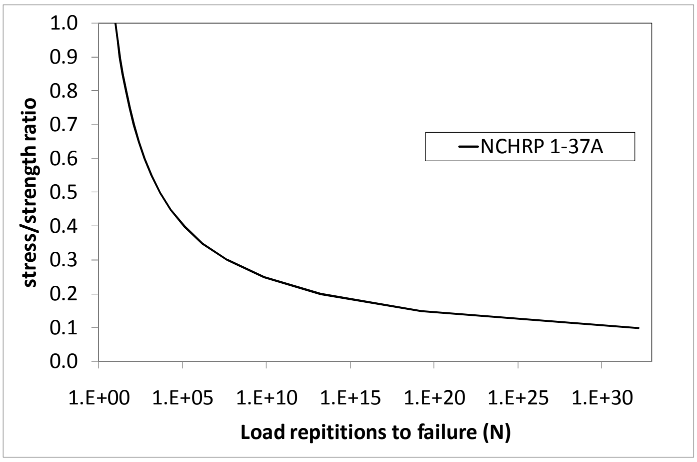


Figure 2-4: Fatigue curve at different stress / strength ratios for continuously reinforced concrete pavements according to NCHRP 1-37A (2004)

2.3.2 Calculation of stress condition in concrete pavements

Traditionally the maximum tensile stresses in the concrete pavement slabs have been calculated using the method introduced by Westergaard in 1923 for a plate on a bed of springs. The theory was first published in English in 1926 (Westergaard, 1926). The number of limiting assumptions of the original method has gradually been decreased over the years by various researchers including Westergaard himself (Ioannides, 2006). Many design methods in use today, including the latest version of the concrete pavement design method developed for South Africa by Strauss et al. (2007), still rely on some of the theory introduced by Westergaard in 1926. Using Westergaard theory, the stresses acting in a concrete slab are calculated based on the relative stiffness of the slab and the support. This linear parameter he named the radius of relative stiffness (l) is determined from the thickness of the slab (h), the modulus of elasticity of the concrete (E), Poisson's ratio (υ) and the linear elastic stiffness of the subgrade (k), using Equation 2.4

$$l = \sqrt[4]{\frac{Eh^3}{12(1-\nu^2)k}} \tag{2.4}$$

Using the factor *l,* Westergaard (1926) approximated LE stresses in the concrete pavement for three typical vertical load cases, i.e. a load at the corner of the slab, a load at an edge of the slab and a load at some distance from the edges of the slab. Examples of combinations of loads to cater for the influence of the different wheels of a truck were also included in the original paper. The equation to calculate the design value of the maximum tensile stress (σ_d) due to a load (P) at a distance (a_2) to the corner of a slab with thickness (h) is provided as an example in Equation 2.5.

$$\sigma_d = \frac{3P}{h^3}\left[1-\left(\frac{a_2}{l}\right)^{0.6}\right] \tag{2.5}$$

The paper by Westergaard did not include failure criteria, but he proposed that critical stresses be calculated in an empirical way, based on existing pavements that performed satisfactory. Before the introduction of suitable computer technology, influence charts developed from the general equations by Westergaard were typically used to calculate stresses under various load configurations (Yoder and Witczak, 1975). The wider use of computer technology made possible the application of Finite Element Method (FEM) in pavement design. Some of the newer design methods, such as the MEPDG (NCHRP 1-37A, 2004) use the results of large sets of pre-run FEM analyses to deduce stresses in the concrete pavement slab by means of neural networks, instead of Westergaard type analysis.

2.3.3 Fatigue damage accumulation

In conventional design methods, the relative damage of each load is calculated based on the calculated stress state in the pavement slab and the number of repetitions to failure at that stress ratio according to the transfer function. In pavement engineering the linear cumulative damage hypothesis first proposed by Palmgren (1924), but best known for the publication by Miner (1945) is generally used to calculate the accumulation of damage. The function shown as Equation 2.6, the hypothesis is that failure occurs when the number of stress cycles (n_i) applied at stress level (S_i) divided by the number of cycles to failure (N_i) at stress level (S_i) equals one.

$$\sum \frac{n_i}{N_i} = 1 \tag{2.6}$$

The linear cumulative damage concept provides a convenient tool to sum up the proportional damage caused by each load repetition over the life of a pavement.

The methodology described in this section has been widely applied in concrete pavement design. The approach does however contain a number of known limitations and presents scope for improvement.

2.4 Some concerns regarding the conventional concrete pavement design approach

The prediction of fatigue failure in concrete pavements as described in the previous section relies heavily on statistical calibration rather than an accurate model for the failure mechanism. The discussion in this section will highlight some of the limitations in various aspects of the methodology, but all of these may be viewed as consequences of the use of empirical models rather than a mechanistic model for fatigue fracture.

2.4.1 Limitations of Miner's linear cumulative damage hypothesis

Limitations of the cumulative damage hypothesis for prediction of fatigue in concrete are highlighted by a number of authors including Ioannides (1995, 1997a, 2006), Ioannides and Sengupta (2003), Ioannides et al. (2006), Roesler (2006) and Gaedicke et al. (2009). Some of the main limitations are discussed here. The evolution of concrete material response under cyclic loading is distinctly non-linear. The typical development of longitudinal strain of concrete in fatigue tests is schematically shown in Figure 2-5 based on results reported by Holmen (1979). The figure also shows the typical fit allowed by Miner's linear cumulative damage hypothesis, calibrated against empirical *S-N* end of life data. Miner's law will at best accurately forecast the number of load repetitions until the failure threshold is reached, but it cannot be used to reliably predict the state of the material at any other stage during the functional life. An important consequence of this for pavement engineering is that the remaining functional life of a pavement in service cannot be reliably back-calculated from deflection data using these models.

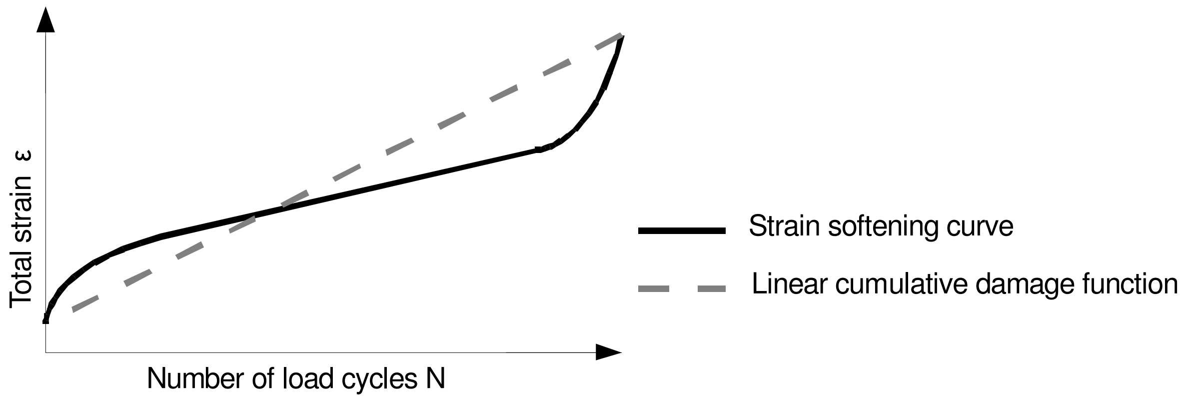


Figure 2-5: The evolution of strain in concrete under cyclic loading (after Holmen, 1979) versus the linear cumulative damage concept.

Another limitation of Miner's law is that it does not take the sequence of loading into account. In reality many small loads followed by a large load can have less of an impact on fracture propagation than the opposite sequence where a large load is followed by smaller loads, or vice-versa, due to the distribution of residual stresses and pre-formed cracks in the material.

A final limitation is that the hypothesis does not cater for the stochastic nature of fatigue damage propagation in concrete. Under Miner's hypothesis every load cycle has a probability of 1 to cause additional damage of a known size, where as in reality there is only a chance of additional damage occurring.

Notwithstanding the limitations, Miner's linear cumulative damage hypothesis has been applied with few adjustments in most concrete design methods. Ioannides (2005) argues that this is done because of practical expediency rather than reliability. Ioannides states that *"Studies to verify Miner's hypothesis often provide adequate information that could justify its abandonment instead. Data reported suggests that predictions of life using Miner can be expected only to approximate reality at best within two or even three orders of magnitude."*

2.4.2 Size-effect

Ioannides (2005) also points out that many of the fatigue relationships for concrete pavements are derived from repeated loading tests on small specimens. In the use of small lab specimens, typically beams, coupled with linear elastic mechanics for analysis, lies another weakness of current design methods. It has been known at least since the 1930s that the LE derived peak stress (MOR) in beams tested in bending depends on the size of the tested specimen (e.g. Reagel and Willis, 1931, Kellerman, 1932). Ward and Li (1991) showed that this also holds true for fibre reinforced mortars. It follows that the MOR, treated as a material property in concrete design method, is not a true material property as its value depends on the size of the specimen as well as boundary conditions of the test. To illustrate the size effect Figure 2-6 shows the typical load response of two beams with the same geometry, but of different sizes. According to conventional design approaches, these beams, if made from the same material, will have the same σ_{Nu} equal to the MOR determined from a standardized test. However, this is not the case. In general for quasibrittle materials, such as concrete, the smaller sample can be expected to have a higher σ_{Nu}.

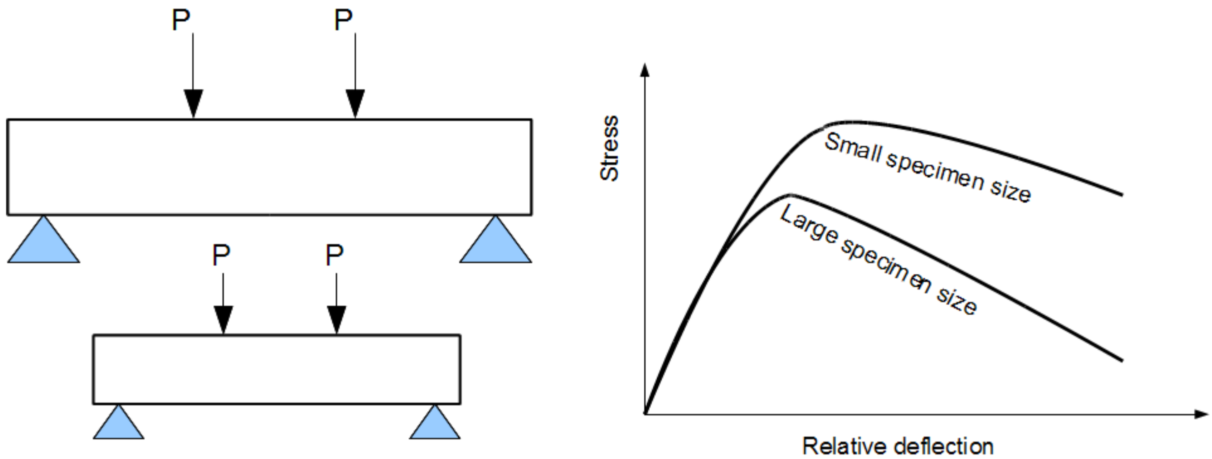


Figure 2-6: Size effect for beams in flexure (after Bažant and Planas, 1997)

There are several known sources of size-effect in concrete. An example is the boundary layer effect, caused by the higher density of fine particles at the side of the beam mould. Other sources are: diffusion phenomena, variation in hydration heat and the statistical size effect (the larger the specimen, the higher the chance of defects) (Bažant and Planas, 1997).

The main source of size-effect however, is the fracture mechanics size effect. The release of stored energy into the fracture front is higher for larger beams and this leads to a relatively lower strength. Ergo, if σ_{Nu} is determined for the small beam in Figure 2-6, it is not possible to use this parameter and accurately predict the peak load for the larger beam by means of linear elastic beam theory. Much less is it possible to predict the peak load of a slab produced of the same material. Based on this, the suitability of the MOR for the prediction of the fatigue performance of full scale pavements has to be questioned.

To clearly distinguish between the different parameters, the MOR, in this study will refer to the σ_{Nu} determined from a FPB test on a beam with a 150 mm x 150 mm cross section and a span of 450 mm as prescribed in most standard methods.

Roesler (2006) has shown that due to size-effects and boundary conditions, fatigue of full scale concrete slabs cannot be reliably predicted from tests on beam specimens. Slabs exhibit better fatigue performance than beams, which cannot be explained without the use of fracture mechanics. The difference between beam and slab fatigue response is greater for fibre reinforced concrete when compared to plain concrete (Roesler, 2006). The above underlines the necessity for a fracture mechanics based approach to fatigue prediction for concrete pavements in general and fibre reinforced concrete pavements in particular. The mechanics behind the fracture mechanics size effect are explained in more detail in the next section.

2.5 Fracture mechanics and its application to concrete

Fracture mechanics is a specialization within the field of solid mechanics which deals with the formation and growth of cracks in materials. It is increasingly looked upon to mitigate some of the limitations of current pavement design methods. This section contains a discussion on the general principles of fracture mechanics, followed by an introduction of the models most commonly applied to concrete.

2.5.1 Linear elastic fracture mechanics

Griffith (1921) famously introduced the energy balance approach for fracture in glass. His work forms the basis for what is now known as Linear Elastic Fracture Mechanics (LEFM). By studying the energy required to grow a crack by a unit volume he overcame a mathematical problem flowing from the work by Inglis (1913) concerning stress concentrations around elliptical holes. According to the theory developed by Inglis, a perfectly sharp crack in a material would lead to infinitely high stresses at the crack tip and the material would fail even if a small load was applied. Griffith proposed that energy should be used as a threshold for crack growth rather than stress alone. For a linear elastic material the Griffith equation for the strain energy (U^*) released per unit volume can be written as shown in Equation 2.7 (Roylance, 2001).

$$U^* = \frac{\sigma^2}{2E} \tag{2.7}$$

Where σ is the stress and E is Young's modulus. Griffith studied fracture in glass, a very brittle material, and proposed that the energy required to extend a crack with a unit area is equal to the surface energy that is released. Surface energy quantifies the disruption of intermolecular bonds. In a plane stress state the amount of surface energy (S_1) released in the formation of a crack is calculated using:

$$S_1 = 2\gamma a \tag{2.8}$$

Where γ is the specific surface energy for the material, it is factored by 2 because two surfaces are formed, a is the length of the crack. Figure 2-7a shows the approximately triangular area that is unloaded as a crack progresses. In LEFM the length of the side of the triangle perpendicular to the crack has been found to be π times crack length (a). Griffith's work can be used to determine the critical crack length. Up to the critical crack length an increase in stress is required to grow the crack, after the critical crack length has been reached the system becomes unstable and failure occurs, without additional stress being applied. The stress at fracture σ_f can be obtained from:

$$\sigma_f = \sqrt{\frac{2E\gamma}{\pi a}} \tag{2.9}$$

Surface energy alone is not enough to quantify the energy required to fracture less brittle, more plastic materials. Irwin (1957) argued that for plastic materials such as steels the majority of energy dissipation in crack growth is due to plastic flow at the crack tip. He further classified the two components of fracture energy i.e. the stored elastic strain energy released with crack propagation, and the dissipated energy which includes the surface energy used by Griffiths as well as the plastic flow dissipation and other possible sources. The equation for critical stress can be modified to incorporate the total specific fracture energy (G_f) by simply replacing γ with G_f in Equation 2.9.

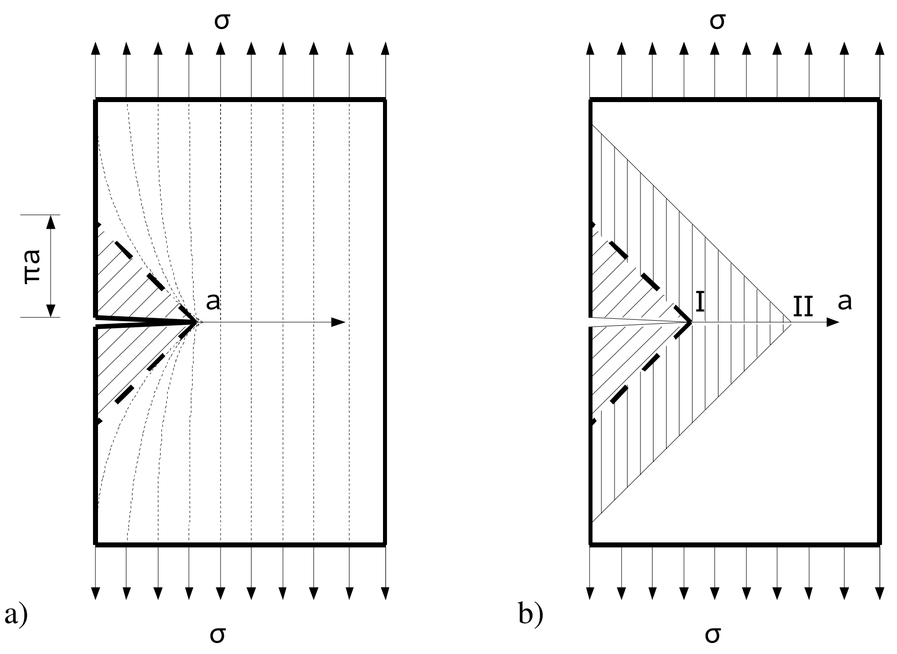


Figure 2-7: a) Approximation of unloaded area due to cracking, b) Change in amount of strain energy released as crack progresses.

2.5.2 The fracture mechanics size effect explained

The theoretical concepts described above can be used to explain the fracture mechanics size effect. Figure 2-7b shows how the amount of strain energy released increases per unit of crack length propagation. More strain energy is released as the crack grows from point I to point II than was initially released when the crack grew over an equal distance to point I. Similarly, if an equal stress state exists in two specimens of the same geometry, but different sizes, the amount of strain energy released into the crack is larger for the larger sized specimen. As a result the smaller specimen will fail at a larger nominal stress.

LEFM has limited applicability to concrete materials as first pointed out by Kaplan (1961). LEFM is suitable for the analysis of brittle or plastic materials where the Fracture Process Zone (FPZ) is small. LEFM is only valid if the size of the FPZ is negligible compared to the size of the specimen (Ioannides, 1997b). Concrete however, is a quasi-brittle material, with a relatively large FPZ. For plain concrete the length of the FPZ is in the order of a meter. LEFM is not the most suitable methodology for crack analysis in this material. The fracture properties determined for the material using LEFM theory would be size dependent (Zhang and Li, 2004). Therefore non-linear fracture mechanics models are typically applied to concrete.

2.5.3 Size effect equations

The size effect in concrete can be predicted without the use of complex fracture mechanics based numerical simulation. Bažant (1984) devised a size effect equation based on non-linear fracture mechanics that can be used to predict the size effect in terms of the value of σ_{Nu} in elements with the same geometry, but different sizes. The Bažant size effect model is shown as Equation 2.10.

$$\sigma_{Nu} = \frac{B_a f_t}{\sqrt{1+\frac{D}{D_0}}} \tag{2.10}$$

Where, B_a is a constant depended on the geometries and fracture properties of the material, as is D_0. The parameter D relates to the size of the specimen, usually the height is used for this parameter. f_t is the tensile strength of the material. This equation can readily be calibrated for a specific material and geometry. A distinct disadvantage of the model is that it cannot be generalized to predict the size effect for specimens with different geometries. Continuum domain based fracture mechanics models are required to be able to use the fracture properties obtained from tests on a certain specimen type and predict the fracture behaviour of structural elements of different geometries.

2.5.4 Cohesive crack model

One method for the analysis of crack propagation in concrete favoured by various researchers for implementation in finite element analysis is the Fictitious Crack Model (FCM). The FCM, nowadays commonly referred to as the cohesive crack model, was introduced by Hillerborg et al. (1976). These authors adapted the Barenblatt cohesive force model so that it would become suitable for concrete. Figure 2-8a provides a sketch of the model as proposed by Hillerborg and co-workers.

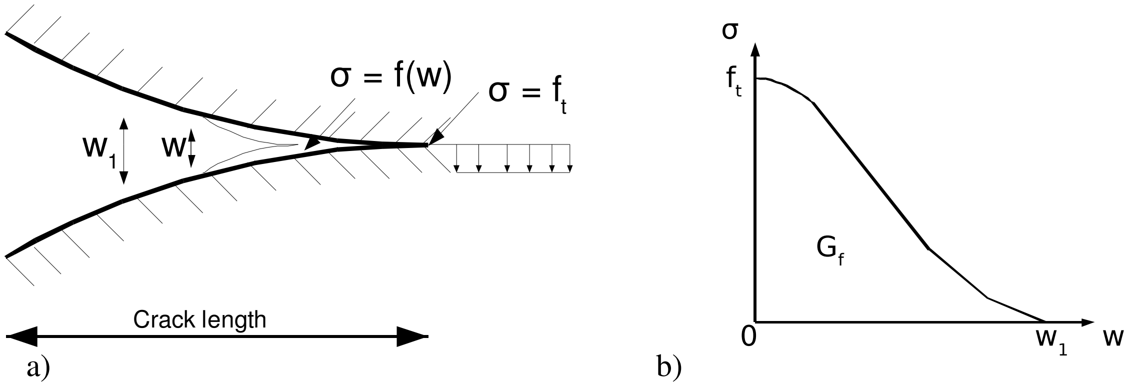


Figure 2-8: a) Sketch of fictitious crack model, b) shape of the softening curve for plain concrete (after Hillerborg et al., 1976)

According to the FCM model, the material behaves linear elastically until the principal stress reaches the tensile strength of the material. At this point a crack is induced. Stresses are transferred over the crack according to a softening function. The shape of the softening behaviour of concrete as put forward by Hillerborg et al. (1976) is shown in Figure 2-8b. The crack bridging stress (σ) is written as a function of the crack width (w):

$$\sigma = \mathrm{f}(w) \tag{2.11}$$

As the crack width approaches w_1 the crack bridging stress gradually falls away. Hillerborg et al., proposed that the zone between the crack tip and the critical crack width (w_1) represents a micro cracked zone where stress transfer is still possible over remaining ligaments, for instance through aggregate interlock. The area underneath the softening curve represents the specific fracture energy (G_f) for the material, therefore:

$$\int_0^{w_1} \mathrm{f}(w)\mathrm{d}w = G_f \tag{2.12}$$

To simplify the softening behaviour of concrete shown in Figure 2-8b, Hillerborg et al. (1976) assumed a linear function in their model. One of the co-authors of that early work later introduced a bi-linear softening function, to better represent the actual behaviour (Petersson, 1981). In literature the shape of the softening function for plain concrete is now often modelled using a bilinear function determined through various methods. For example Guinea et al. (1994), proposed a model using four parameters, i.e.: f_t, G_f and two parameters dependent on the shape of the function determined from experimental results. Recently, Park et al. (2008) proposed a two parameter model for bilinear softening. Exponential or quasi-exponential curves are used in literature as well. Figure 2-9 shows examples of different shapes of the softening curve. In the construction of the example the specific fracture energy (G_f), i.e. the area underneath the curve, was kept constant.

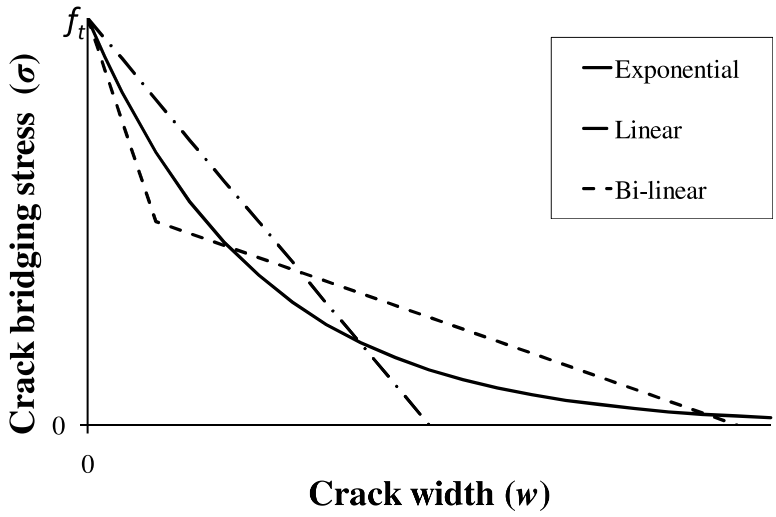


Figure 2-9: Typical assumed shapes of the softening curve

Hillerborg et al. (1976) used the fracture energy (G_f) combined with Young's modulus (E) and the tensile strength (f_t) to calculate a characteristic length (l_c) for the material as per Equation 2.13. Hillerborg (1983) proposed that the characteristic length is a pure material property without a direct physical correspondence, but often proportional to the fracture zone.

$$l_c = \frac{EG_f}{f_t^2} \tag{2.13}$$

Later research has shown that l_c is not a pure material property, it is subject to size-effect which, as discussed earlier, limits the reliability of laboratory testing to predict field performance. Bache and Vinding (1990) introduced the dimensionless brittleness number (B) to factor in the specimen size effect. The brittleness number is an expression of Hillerborg's characteristic length (l_c) in relation to a characteristic dimension usually taken as the thickness of the material (h). Bache and Vinding (1990) propose a model law that the test specimen should have the same brittleness number as the actual structure for which fracture is predicted. The brittleness number provides an indication of the ductility of the material. The smaller the brittleness number, the more ductile the material. The Brittleness number is defined as:

$$B = \frac{f_t^2 h}{EG_f} = \frac{h}{l_c} \qquad (2.14)$$

The FCM as described above has been used to analyse a single discrete crack by means of Finite Element codes. Examples of this approach applied to pavement structures, including the brittleness number refinement, can be found in Ioannides and Sengupta (2003) and Ioannides et al. (2006). The primary factors in the softening behaviour of the concrete, i.e. G_f and f_t are generally determined from beam bending tests and direct or indirect tensile tests on the material.
Some limitations of the FCM highlighted by Bažant (2002) include:

- The FCM is a uniaxial model and is applicable only to elements in bending. The model can not handle tri-axial stresses. The model in its original form also does not include shear stresses transferred across the crack and the influence of parallel compressive stresses,
- The FPZ is not a straight line, as assumed in the FCM, in reality the FPZ is formed by a wider area of material around the crack tip,
- The crack model assumes the fracture energy to be dissipated through micro cracking in the FPZ, while in reality a large part of this energy is dissipated through crack slipping,
- The FCM assumes that the stress at a point is only dependent on the crack width opening at that point and not on the continuum of displacements of the points around it.

Despite these limitations the FCM, often referred to as cohesive crack model remains popular among researchers. A number of the limitations have been addressed by new developments of the model.

2.5.5 Smeared crack or crack band models

A much used alternative to the discrete cohesive crack approach is to model the cracking as tension softening distributed over the material. This approach is known as smeared cracking, a concept introduced by Rashid (1968). The initial methodology had some limitations; it was sensitive to the size of the finite element mesh and allowed the development of volumeless fracture process zones. Bažant and Oh (1983) overcame these limitations by introducing localization limiter which forces the fracture zone to have a certain width (h_c). This width is treated as a material property. Bažant and Planas (1997) show that the response of strain softening crack band can be related to the width of the cohesive crack (w) of the FCM using:

$$w = h_c \varepsilon^f \tag{2.15}$$

where (ε^f) is the fracture strain. The stress-strain behaviour of the crack band model for linear softening is shown in Figure 2-10.

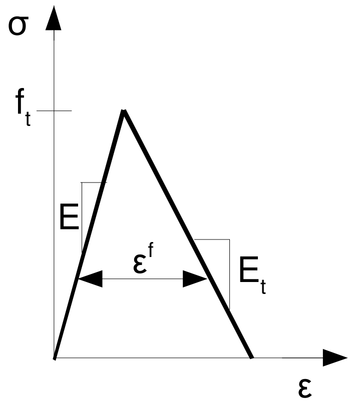


Figure 2-10:Stress-strain behaviour of crack band model after Bažant and Planas, 1997)

Similar to the cohesive crack model, the material response of the material is linear elastic under tensile loading until the tensile strength of the material is reached. The post cracking behaviour is characterized by the strain softening tangent modulus (E_t). For the linear softening behaviour shown in the figure the function defining E_t.is:

$$-\frac{1}{E_t} = \frac{2G_f}{h_c f_t^2} - \frac{1}{E} \tag{2.16}$$

where (E) is Young's modulus of the material in its undamaged state. The crack band model can readily be implemented in finite element software. An advantage over the cohesive crack approach is that tri-axial softening can be accommodated. The smeared crack approach was recently used in local work by Elsaigh (2007) to model the behaviour of steel fibre reinforced concrete pavements.

2.5.6 The Jenq-Shah two parameter fracture model

The last fracture mechanics method for concrete to be discussed in this section is the two parameter fracture model developed by Jenq and Shah (1985). The model has recently been applied extensively to concrete pavement materials by researchers at the University of Illinois (Roesler et al., 2007, Gaedicke et al., 2007, Roesler et al., 2007 and Gaedicke et al., 2009, Gaedicke and Roesler, 2010). The Jenq-Shah model makes use of LEFM in combination with an equivalent elastic crack concept to make the theory applicable to the quasi-brittle fracture behaviour of concrete. The model applies the concept of critical crack length of LEFM. As in LEFM, in the peak loading condition the stress intensity at the crack tip (K_I) is assumed equal to the critical stress intensity at failure (K_{Ic}), and the Crack Tip Opening Displacement (CTOD) is likewise assumed to reach a critical condition $CTOD_c$. These two conditions are formulated as follows:

$$K_I = K_{Ic} \tag{2.17}$$

and,

$$\frac{4K_I}{\pi E'}\sqrt{2\pi\Delta a_e} = CTOD_c \tag{2.18}$$

where a_e is the equivalent effective elastic crack length and E' is the effective modulus of elasticity in a plane strain condition, according to:

$$E' = \frac{E}{1-\nu^2} \tag{2.19}$$

The parameters for the Jenq-Shah model are typically obtained from Three Point Bending tests (TPB) on notched beams. The Crack Mouth Opening Displacement (CMOD) is used to control the test. The beams are loaded past the peak load. Then, as the load starts to drop the beam is unloaded and subsequently reloaded. The fracture parameters are calculated from the loading and unloading compliance.

Various studies have shown that the Jenq-Shah approach yields results equivalent to the cohesive crack method discussed in Section 2.5.4 (Tang et al., 1992, Bažant, 1994, Smith, 1995).

2.5.7 Application of fracture mechanics to conventionally reinforced and fibre reinforced concrete

The cohesive crack, smeared crack and Jenq-Shah models were all originally developed for plain concrete. Fracture simulation of concrete reinforced with steel bars is possible with the models by including the concrete and steel as separate entities in the numerical model (Hillerborg, 1990, Bažant and Planas, 1997). Fracture mechanics modelling of concrete with steel bar reinforcement has only limited application though, as the dominant mode of failure for reinforced concrete is yielding of the steel or compression failure. Quasi-brittle fracture may however, occur in lightly reinforced structures if the tensile capacity of the concrete section is higher than that of the reinforcement.

Importantly for this study, the models can also be applied to model the post crack behaviour of fibre reinforced concrete. In general there are two approaches to the modelling of steel fibre reinforced concrete.

The first involves the modelling of fibres as separate entities within the concrete matrix. This can be done by using a lattice model. Lattice models, also referred to as rigid body spring networks, like smeared crack models, have the advantage over cohesive crack models that they allow triaxial stresses. However, unlike smeared crack models, lattice models can in principle be used to simulate discrete cracks. The concrete material is modelled as consisting of three components, i.e. the mortar matrix (which includes the free water and the fine aggregate fraction embedded in the hardened cement gel), the coarse aggregate, and the Interfacial Transition Zone (ITZ) between the mortar and the coarse aggregate, which is more porous and less stiff than the general mortar matrix. Details of the methodology are contained in Yip et al. (2006), Lilliu and Van Mier (2007) and Schlangen et al. (2010). Because the different components of the concrete mix are modelled separately, their properties and interaction also need to be determined separately through testing. Pull-out tests are required for the fibres. The procedures to obtain the parameters are described by Redon et al. (2001).

The second approach to the modelling of fracture in FRC is by capturing the influence of fibres on the post crack softening material behaviour in cohesive crack, smeared crack or Jenq-Shah models. Hillerborg (1985) proposed the simplified representation of the post crack behaviour of steel fibre reinforced concrete shown in Figure 2-11. In the figure G'_f represents the fracture energy of the concrete matrix without the fibres. The crack needs to open by a certain width before the fibres are activated. Therefore the first part of the softening function is dominated by the softening behaviour of the concrete only. The fracture behaviour of fibre reinforced concrete has also been modelled using bi-linear softening functions e.g. Guo and Li (1999). Due to the extended post crack softening process more complex tri-linear shapes of the softening function have been proposed for fibre reinforced concrete by various researchers e.g. Lim et al. (1987), Pereira et al. (2004), RILEM (2003) and Roesler et al. (2007).

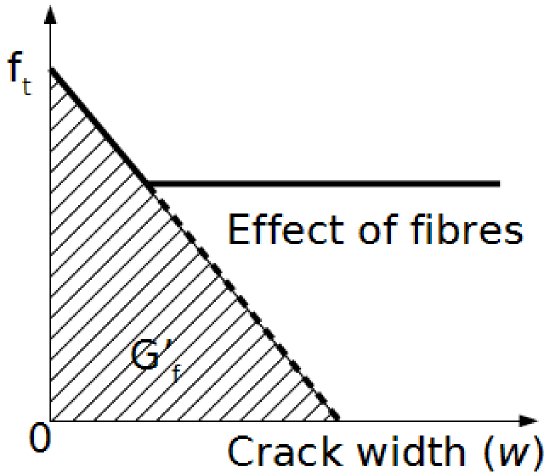


Figure 2-11: Softening function for steel fibre reinforced concrete (after Hillerborg, 1985).

Research on concrete with similar steel fibres from the same producer as used in this study has shown that when a crack forms, the stress transferred across the crack drops rapidly until it stabilizes at a lower stress level. After this, the material softens further at a slower pace (Lim et al., 1987). Based on this work, Lim et al. (1987) proposed a softening function with a crack tip singularity to simulate the behaviour of the material in tension. Such a curve with an initial spike at high strength followed by a long tail at a lower stress is suitable for fibre reinforced composites, as it simulates the initial failure of the matrix followed by the slow pull out of the fibres (Bažant and Planas, 1997).

2.6 Fracture mechanics for fatigue damage prediction

The previous sections described a number of well established methods for the simulation of fracture in concrete under monotonic loading. The models for simulation of fatigue fracture in concrete and in fibre reinforced concrete in particular, are less well developed.

2.6.1 Paris' law

Fatigue crack propagation in metals can be divided into four stages: Crack nucleation, followed by stage I crack growth, during which the crack propagates slowly, when a certain threshold is reached stage II crack growth commences, during this stage the crack propagates in a stable manner, and finally ductile failure. Stage II crack growth is important from a design perspective as it is much longer than the other stages. Fatigue crack growth of metals during phase II can be predicted using a fracture mechanics function shown as Equation 2.20 introduced by Paris and Erdogan (1963), which has become know as Paris' law.

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta K)^m \tag{2.20}$$

where da is the change in crack length, dN the change in the number of loading cycles, C and m are empirical material constants, and ΔK the change in amplitude of the stress intensity at the crack tip. The number of load repetitions to failure can be obtained by integrating Equation 2.20 over the domain from the initial crack length (a_0) to the critical crack length (a_c) at which the system collapses.

$$N = \int_{a_0}^{a_c} \frac{1}{C(\Delta K)^m} \mathrm{d}a \tag{2.21}$$

A number of researchers have adapted Paris' LEFM law for the quasi-brittle behaviour of concrete. The early work by Baluch et al. (1987) and Perdikaris and Calomino (1987), recognized the need for a non-linear fracture approach for fatigue in concrete. The studies by Bažant and Xu (1991) and Bažant and Schell (1993) show that the fatigue tests are subject to size effect and that Paris' rule can only be made applicable to a narrow range of sizes. To increase the applicability the models need to be adjusted using a size-effect law (Bažant and Planas, 1997).

Li and Matsumoto (1998) and Carpinteri et al. (2006) developed models based on Paris' law for the prediction of fatigue in fibre reinforced concrete. These models include additional factors to accommodate the crack bridging action of the fibres.

Slowik et al. (1996) devised the model for fatigue softening behaviour of concrete under variable amplitude loading shown in Equation 2.22. The model is rather complex to cater for loading history and size effect. Besides the fracture toughness (K_{Ic}) and the change in amplitude of the stress intensity (ΔK_I), the model includes the maximum stress intensity for the load cycle (K_{Imax}) and the maximum stress intensity reached during the life of the material (K_{Isup}). Factors m,n,p are calibration constants. F is a function that accounts for overloading of the system. The final parameter is Paris' constant (C) which is calibrated for the non-linear quasi-brittle crack growth per load cycle.

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\frac{K_{\mathrm{Im}ax}^{m}\Delta K_I^n}{(K_{Ic} - K_{I\,\mathrm{sup}})^p} + F(a,\Delta\sigma) \qquad (2.22)$$

The model by Slowik et al. (1996) was recently developed further in work by Sain and Chandrakishen (2007, 2008).

Gaedicke et al. (2009) made use of a Paris' law related non-linear fracture mechanics model originally developed by Subramaniam and co-workers (Subramaniam et al., 1998), (Subramaniam et al., 1999), (Subramaniam et al., 2000), to predict the fatigue response of concrete ground slabs. The model by Subramaniam et al., differs from Paris' approach in that it covers different stages of fatigue crack propagation. Where Paris' law was developed only for the stable phase II fatigue growth in metals, Subramaniam et al., proposed seperate equations for deceleration stage crack propagation and acceleration stage crack propagation. Figure 2-12 shows the deceleration and acceleration stages of the crack growth in concrete under cyclic loading.

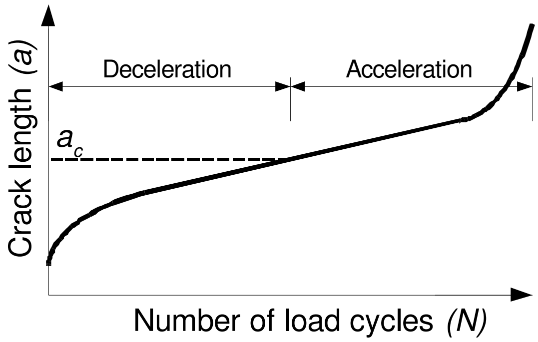


Figure 2-12: Stages of fatigue crack growth Subramaniam model

The crack growth per cycle is decelerating until the critical crack length (a_c) is reached, after this point crack growth starts to accelerate again. The crack growth in the deceleration stage is described by Equation 2.23, crack growth in the acceleration phase can be described using Equation 2.24.

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C_1(a - a_0)^{n_1} \qquad \text{for } a < a_c \tag{2.23}$$

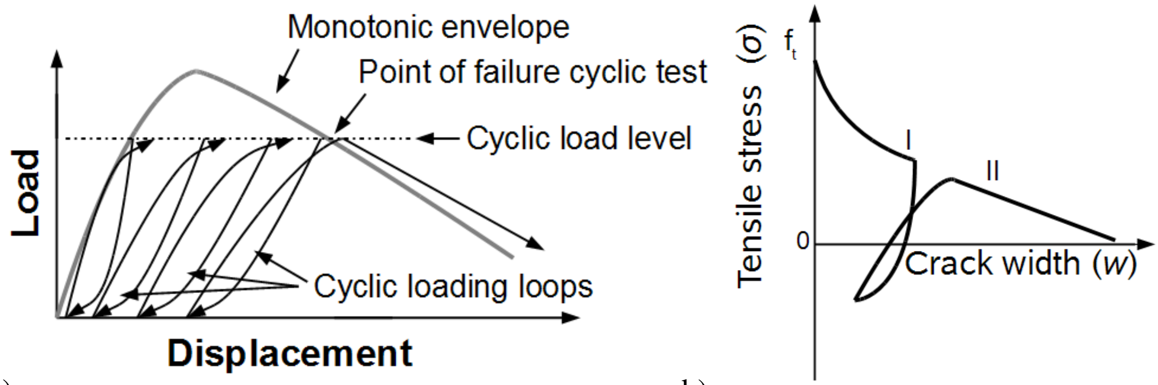
$$\frac{\mathrm{d}a}{\mathrm{d}N} = C_2(\Delta K)^{n_2} \qquad \text{for } a \geq a_c \tag{2.24}$$

where a_0 is the initial crack length, and C_1, C_2, n_1, n_2 are calibration constants.

Note that all fracture mechanics models based on Paris' law are valid only in situations with a pre-existing crack. The approach does not provide a threshold for crack nucleation. An additional limitation is that the calibrated models provide a prediction of fatigue crack growth, which is hard to implement in numerical software. The models can be used to predict the length of a fatigue crack after a certain number of load repetitions which can then be used to calculate the residual strength of the material as shown by Sain and Chandrakishen (2007). However, without numerical implementation the analysis of structures with more than one crack developing will be cumbersome.

2.6.2 Fatigue softening behaviour in cohesive crack model

Hordijk (1992) proposed a model for fatigue in concrete in which the load displacement curve under static loading represents the envelope within which evolution of the load displacement curve in fatigue takes place. Figure 2-13a shows a sketch of the concept. The cyclic load level is set at a percentage of the monotonic peak load. The material softens with every loading cycle until a cyclic load-displacement loop crosses the monotonic envelope, at which time unstable failure takes place. Hordijk used the focal points model, proposed by Yankelevsky and Reinhardt (1987), for the softening behaviour of the material in fatigue. A rather simplified sketch of the softening curve after crack induction for the cyclic model used by Hordijk is shown in Figure 2-13b.



a) b)

Figure 2-13: a) Load-displacement evolution according to Hordijk model, b) Sketch of cyclic cohesive softening function.

The figure shows that when the material is unloaded at point I and subsequently reloaded the function does not return to point I but joins the original softening curve at point II. The difference between point I and II is the fatigue damage. The softening function with damage degradation can be calibrated for a specific material and used to model the fatigue behaviour using FEM. The experimental study by Hordijk included direct tensile and Three Point Bending (TPB) tests on beam specimens. Although the model showed promise, the data obtained from the experiments to validate the simulation method showed considerable scatter.

Hordijk (1992) reported a better fit of the model to low cycle fatigue situations compared to high cycle fatigue situations. Other researchers later established that the monotonic curve does not provide a good fit for high cycle fatigue situations (Subramaniam et al., 2000), (Lappa et al., 2006). Lappa et al. (2006) propose that this difference is due to the fact that a single crack is formed in high cycle fatigue situations, while multiple cracks are formed under monotonic loading and low cycle fatigue.

Subramaniam et al. (2000) show that unlike the deflection, the crack length recorded in the monotonic tests on plain concrete does provide a suitable envelope for fatigue tests. There is some published research available that indicates that the monotonic load displacement curve does provide a suitable envelope for fibre reinforced concrete (Otter and Naaman, 1988, Zhang and Stang, 1998).

2.7 Discussion on theoretical framework

The aim of this chapter was to provide a concise overview of the state of the art in the prediction of fracture in concrete pavement materials. There is a sizeable body of previous knowledge available to take direction from in this study.

One of the limitations of conventional concrete pavement design methods is the assumption of LE material behaviour. Both the material strength and the stress in the pavement are characterized based on LE analysis. Concrete is known to have a distinctly non-linear, non-elastic component in fracture, giving rise to the size effect observed in experiments. The size effect phenomenon has been studied extensively for plain concrete by various researchers. From literature it is known that the MOR parameter, currently used in concrete pavement design to characterize the material in terms of fatigue performance, is subject to considerable size-effect. As a consequence of size effect the MOR obtained for a specimen of a certain size cannot be used to predict the peak load of a specimen with the same geometry, but a different size. This raises questions on the suitability of the MOR parameter obtained for a beam under monotonic loading to be used as an indicator of the fatigue performance of a full size concrete pavement under cyclic loading.

Compared to plain concrete, few studies on size effect in fibre reinforced concrete are available. It would be valuable to quantify the size effect in the high performance fibre reinforced concrete material used for UTCRCP.

Studies have indicated that fibre reinforced concrete has an improved resistance to fatigue fracture propagation in tension due to the post crack stress transfer capacity of the fibres. It is not known however whether the MOR is a suitable design parameter for this effect, as the MOR only provides an indication of the peak load capacity of a specimen. The available literature indicates that the there is a larger difference in the fatigue behaviour of FRC beams and slabs then there is for plain concrete beams and slabs.

The flexural behaviour of both plain and fibre reinforced concrete can be predicted using fracture mechanics models. Concrete pavement design approaches could benefit from the introduction of size independent fracture parameters.

Probably the most significant limitation to conventional design methods is the use of Miner's cumulative linear damage hypothesis. The hypothesis has been shown to be unsuitable for the prediction of fatigue damage evolution in concrete pavements. In future it may be possible to replace Miner's law with fracture mechanics based fatigue models.

A range of methods based on Paris' law are available, but they have found limited application. The challenge is to develop a model that can readily be adopted in general pavement engineering practice. This would require the model to use parameters that are easily determined from routine testing preferably under monotonic loading conditions. The relationship between the monotonic load displacement curve and the behaviour under cyclic loading should be explored.

Based on the findings of the literature survey, it is proposed that fracture mechanics properties obtained from monotonic tests are less susceptible to size effect and better capture the fracture toughness of the material, compared to the MOR parameter currently used in concrete pavement design. These parameters would allow a more accurate prediction of fracture propagation, both under monotonic and cyclic loading.