

Measuring counterparty credit risk: An overview of the theory and practice

by

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DECLARATION

I, the undersigned, hereby declare that the dissertation submitted herewith for the degree Magister Scientiae to the University of Pretoria contains my own, independent work and has not been submitted for any degree at any other university.

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ABSTRACT

The global over-the-counter derivatives market reached a staggering 14.5 trillion US dollars in gross market value at the end of December 2007. Although OTC derivatives are extremely useful and versatile in transferring risks, it appears to be a double-edged sword. For every derivative transaction concluded in the OTC market, there are two parties involved – each of which is exposed to the other defaulting on the agreed terms and conditions of the contract. Counterparty credit risk is defined as the loss that will be incurred in the event that a counterparty fails to honour its financial obligations.

This dissertation provides an overview of counterparty credit risk measurement from a theoretical point of view and puts an emphasis on the demonstration of the current solutions used in practice to address this problem. The author applies a bottom up approach to the problem by defining counterparty credit risk exposure on a contract (single-trade) level and expands this definition on a step-by-step basis to incorporate portfolio effects, such as correlation among underlying market variables as well as credit risk mitigation techniques, such as netting and collateral agreements, in measuring counterparty credit risk exposure on a counterparty level.

The author also discusses related concepts which impact counterparty credit risk such as wrong-way risk and proposes an enhancement to the framework introduced by Finger (2000) for incorporating wrong-way risk into existing measures of counterparty credit risk exposure. Finger's framework is enhanced by the introduction of a structural model approach which can be used in establishing a functional and intuitive relationship between the probability of default of the counterparty and the underlying market variable to the derivative contract under consideration. This approach is also applied to a typical South African situation through the use of Monte Carlo simulation. The topic of counterparty credit risk modelling is a very relevant topic in modern finance, especially since the advent of Basel 2 which this dissertation also touches on in terms of the applications of counterparty credit risk modelling and how this relates to the minimum regulatory capital requirements set by bank regulators.

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Dedicated to my parents, my family and my wife.



“...when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind: it may be the beginnings of knowledge but you have scarcely, in your thoughts, advanced to the stage of science...”

Lord Kelvin, 1883

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NOMENCLATURE

Collateral

Properties or assets that are offered to secure a loan or other credit. Collateral becomes subject to seizure on default. Collateral is a form of security to the lender in case the borrower fails to pay back the loan.

Confidence Interval

An interval estimate of a population parameter. Instead of estimating the parameter by a single value an interval likely to include the parameter is estimated. Confidence intervals therefore indicate the reliability of an estimate. The likelihood of an estimated confidence interval to contain a parameter is given by the confidence coefficient. Increasing the confidence coefficient will widen the confidence interval. For more information refer to any elementary text book on statistical inference.

Counterparty Credit Risk

Counterparty credit risk is the risk of loss as a result of a counterparty being unwilling or unable to fulfil their contractual obligations relating to some financial agreement, prior to the expiration of such financial agreement.

Credit Value Adjustment (CVA)

The difference between the risk-free market value of a derivative (or portfolio of derivatives) and the smaller value that results from taking credit risk into account.

Exposure at Default (EAD)

A total value that a bank is exposed to at the time of default. Each underlying credit exposure is given an EAD value and is identified within the bank's internal system. Using the internal ratings based (IRB) approach, financial institutions will often use their own risk management models to calculate their respective EAD estimates.

Exposure at default, along with loss given default (LGD) and probability of default (PD), is used to calculate the credit risk capital of financial institutions. The expected loss that will arise at default is often measured over one year.

Loss Given Default (LGD)

The amount of money that is lost by a bank or other financial institution when a borrower defaults on a loan. LGD is related to the recovery rate (RR) in the following manner:

$$LGD = 1 - RR.$$

Mark-to-Market (MTM)

The term refers to the practice of valuing derivative (or similar) financial contracts by using the most recent market-observable prices as inputs and in so doing obtaining a market-based value. MTM is a measure of the current market value of a derivative contract.

Netting Pool

A netting pool is a collection of contracts (or trades) covered by the same netting agreement. This collection of contracts can be legally offset in the event of default according to the specifications of the netting agreement.

Over-the-Counter (OTC) Derivatives

Over-the-counter derivatives are derivative contracts which are customised according to the client's needs in almost all respects. OTC contracts are therefore tailored in terms of size, underlying, maturity and pay-off profile.

Primary Risk Exposure (Traditional Credit Risk Exposure)

Traditionally, credit risk is defined, and understood in the context of lending products or loans. Primary risk exposure is straightforward to measure and is normally taken to be the outstanding balance of the loan.

Probability of Default (PD)

The degree of likelihood that the borrower of a loan or debt will not be able to make the necessary scheduled repayments. Should the borrower be unable to pay, they are then said to be in default of the debt, at which point the lenders of the debt have legal avenues to attempt obtaining at least partial repayment. Generally speaking, the higher the default probability a lender estimates a borrower to have, the higher the interest rate the lender will charge the borrower (as compensation for bearing higher default risk).

Wrong-Way Risk

Wrong-way risk arises where there is a significant unfavourable correlation between the value of a derivative contract and the likelihood of default of a counterparty.

LIST OF SYMBOLS

<i>Symbol</i>	<i>Description</i>
Ω_t^i	The state space containing the universe of all possible prices for all risk factors required to calculate the mark-to-market for derivative i at time t
$MTM_i(\omega_t^i)$	Denotes the mark-to-market value function for derivative i applied on the price(s) of the underlying market variable(s) with $\omega_t^i \in \Omega_t^i$.
Λ_t^i	The actual marked-to-market value of derivative i at time t .
A	The statistical level of confidence.
N	The notional amount of the underlying asset that the derivative is based on.
PV	The present value function, used to represent the value of a future cashflow discounted to time $t=0$.
F	The forward exchange rate.
K	The strike price of the derivative contract.
μ	The (local) drift term of the geometric Brownian motion stochastic process or the local mean rate of return of S_t . ¹
S_t	A random variable representing the value of the underlying asset (stock) of the derivative contract at time t .
σ	The diffusion term of the Geometric Brownian Motion process or the volatility of S_t . ²
W_t	A Wiener process.
a	Reversion speed in the CIR (Cox, Ingersol, Ross) model.
b	The mean parameter in the CIR model.
r	The short rate of interest.

¹ Often a subscript will be used to distinguish between the parameters of specific GBM processes.

² See footnote 1.

$PFE_{max}(\alpha, t_k)$	The maximum PFE at time t_k given a statistical confidence level of α .
$EE(t_k)$	The expected exposure at time t_k .
$EE_{eff}(t_k)$	The effective expected exposure at time t_k .
$EPE(t_k)$	The expected positive exposure at time t_k .
$EPE_{eff}(t_k)$	The effective expected positive exposure at time t_k .
CVA	The Credit Value Adjustment (CVA).
EM	The effective maturity.
$\mathbb{E}[x]$	Represents the function $max[x, 0]$.
v	An indicator variable. The variable is equal to 1 if a netting agreement is in place for the portfolio under consideration and 0 if a netting agreement is not in place.
$\Gamma_{\alpha}^H(t)$	Represents the portfolio-level MTM value at time t taking into account the effect of a netting agreement where applicable. The portfolio consists of H derivatives.
$\Pi^H(t)$	Denotes the actual (unknown) value of the portfolio of H derivative positions.
H	Denotes the number of derivatives in the portfolio under consideration.
ρ	Represents the linear correlation between two variables.
φ	A random variable from the $N(0, 1)$ distribution.
ζ	The mean of the risk factor in Finger's model.
ς	The standard deviation of the risk factor in Finger's model.
Y_M	The Merton-Model survival function.
Y_{CG}	The CreditGrades survival function.
\bar{R}	The mean of the lognormal distribution assumed to be followed by the average recovery rate of the CreditGrades model.
λ	The volatility of the average recovery rate process in the CreditGrades model.



E_0	The initial stock price in Finger's calibration method of the CreditGrades model.
E^*	The reference stock price in Finger's calibration method of the CreditGrades model.
σ_E^*	The volatility of the reference stock price in Finger's calibration method of the CreditGrades model.
D	The debt-per-share ratio of the company in Finger's calibration method of the CreditGrades model.
\bar{R}	The global debt recovery rate in Finger's calibration method of the CreditGrades model.
Θ_t	A stochastic variable denoting the price of the underlying market variable under the proposed enhancement to Finger's model.
E_t	A stochastic variable denoting the price of the equity price of the counterparty under the proposed enhancement to Finger's model.

1 Introduction

This chapter serves as an introduction to counterparty credit risk measurement and a general introduction to this dissertation. Counterparty credit risk is introduced on a conceptual level, and the need for measuring counterparty credit risk is motivated by an overview of the development and immense growth of the over-the-counter (OTC) derivatives market. This is followed by an overview of the role of counterparty credit risk in a typical investment banking organisation. The chapter concludes with an overview of the structure and scope of the rest of this dissertation.

1.1 Background

The development and proliferation of the over-the-counter (OTC) derivatives market has arguably been one of the most important events in finance over the last 25 years. Moreover, OTC derivatives, being negotiated directly between counterparties, can be tailored to the counterparties' specific needs and thus offer unlimited possibilities for risk transferral. Although a very powerful risk transference mechanism, each OTC derivative is in a way a double-edged sword in that it results in an increase in counterparty risk which, in turn, needs to be managed by each of the counterparties.

Counterparty credit risk is the risk that a counterparty to a financial contract will default prior to the expiration of the contract and will not make all the payments required by the contract (Pykhtin M., 2005). If the contract value for the surviving (non-defaulted) counterparty is negative this counterparty will experience no loss as it has to honour the contract regardless of the default status of its counterparty. If the contract value is positive for the surviving counterparty, it will receive nothing from the defaulted counterparty (in the worst case) and has to pay the contract value in order to replace the 'defaulted' contract with a similar contract involving some high credit quality counterparty. Counterparty credit risk is therefore bilateral and credit exposure (on a

single contract) is the maximum of the contract's risk-free value and zero. Since the contract value changes unpredictably over time, only the current exposure is known with certainty while future exposure is uncertain. Note that, throughout this document, we will be focussing on modelling credit exposure conditional on default as opposed to modelling credit exposure conditional on credit migration events which are often used in mark-to-market credit portfolio models such as CreditMetrics.³

Although counterparty credit risk is relatively easy to define, it is certainly not trivial to quantify or manage – especially when there are complex derivative instruments involved with different maturities and underlying assets which need to be aggregated, taking into account credit risk mitigants such as netting agreements and collateral agreements, in an attempt to measure a counterparty level exposure and make decisions relating to expected or unexpected losses. In practice, a significant amount of resources are spent annually by Banks and other financial institutions on developing and implementing extremely expensive systems to address this problem.

1.2 Motivation for counterparty credit risk management

There are various reasons why it is important for a financial institution such as an investment bank, to be able to record (calculate) and continually monitor and manage the credit risk exposure against its trading counterparties resulting from activities in the OTC derivatives market. The size of the OTC derivatives market alone should indicate the importance of managing counterparty credit risk accurately. The global OTC derivatives market reached a staggering 14.5 Trillion US Dollars in gross market value at the end of December 2007 according to the Bank of International Settlements' Semi-annual OTC Derivatives Statistics released in May 2008.

³ J.P. Morgan, (1997).

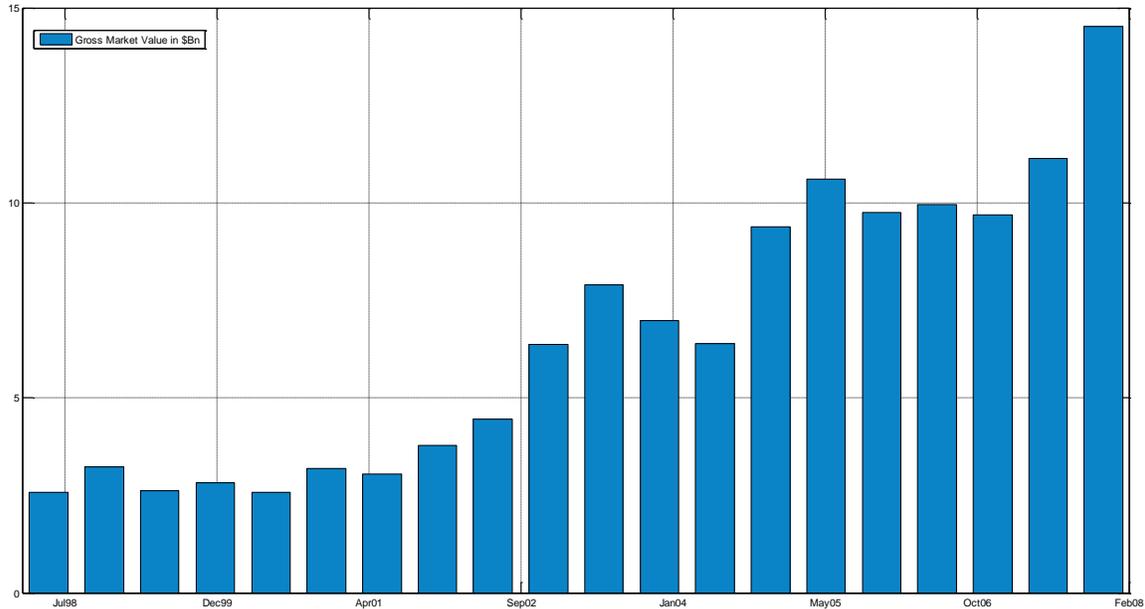


Figure 1.1 - Gross Market Values in Billions of US Dollars (BIS, May 2008)

A recent report by the Counterparty Risk Management Policy Group (CRPMG), comprised of senior officials from major financial institutions, identified counterparty risk as “...probably the single most important variable in determining whether and with what speed financial disturbances become financial shocks, with potential systemic traits” (CRMPG 2005). It is therefore quite clear that any active member in the OTC derivatives market needs to be aware, at all times, of the risk that it is exposed to as a result of counterparty credit risk through sound risk management processes. This financial institution should also be able to demonstrate to its regulator (in the case of regulated financial institutions like banks) that it is capable of determining what the level of capital is that it is required to hold in order to cover the unexpected losses which may result from counterparty credit risk.

This is not only important from a shareholder’s point of view but also for general stability in financial markets. In order for a financial institution to function optimally it is crucial to have a clear, accurate and up to date picture of all the major risks that it is exposed to. It should also be noted that it is of crucial importance that a bank measures these risks as accurately as possible since the measurement results directly impacts the return on capital from a bank’s perspective. Capital which is increasingly scarce.

1.2.1 The Credit Crisis in 2007, 2008

The current financial crisis which is wreaking havoc through financial markets worldwide originated in the United States as a result of the bursting of the US housing bubble and financial losses as a result of high default rates on sub-prime mortgages. The problem, although originating in the US spread globally as a result of the participation of global financial institutions in securitised assets backed by sub-prime mortgages. Traditionally, banks lent money to home owners for their mortgage and retained the risk of default (often referred to as credit risk) on their own balance sheet. However, through the use of financial innovations such as securitisation banks are now able to transfer these default risks to other financial institutions by repackaging these risks into debt instruments with a credit rating. This so-called originated and distribute model used by banks has consequential impacts on the global financial system as a result of the increased connectedness and inter-dependency of global financial institutions. Major banks and other financial institutions around the world reported losses of approximately US\$ 435 billion as of 17 July 2008⁴ as a result of losses from the sub-prime mortgage crisis. Some very well-known large financial institutions have had to file for bankruptcy as a result of this financial crisis including Bear Sterns and Lehman Brothers.

The above mentioned financial crisis highlights the importance for any participant in the global over-the-counter derivatives markets of being able to understand the credit risk which it is exposed to and measure this risk appropriately.

1.3 Counterparty Credit Risk Measurement in the Organisation

Counterparty credit risk measurement plays an important role in the more sophisticated players in the OTC markets, especially in investment banks. The information resulting from this risk measurement impacts the business from a micro⁵ to a macro⁶ level.

⁴See: <http://www.federalreserve.gov/newsevents/speech/bernanke20071015a.htm>

⁵ As in the case of business origination in the form of new deals.

⁶ As in the case of credit portfolio management.

1.3.1 Pre-Deal Analysis

When a bank originates new business it normally conducts a credit assessment of a potential client in order to assess the ability of the client to repay debt. This assessment involves the analysis of the financial statements of the client and sometimes, dependent on the size of the potential business, interviews with key management staff of the client. In the event of a successful credit application, the outcome of such a process is an approved credit limit which is an indication of the credit exposure which the bank is willing to take on against this counterparty reflective of the ability of the counterparty to repay or service such debt.

These activities are common in most financial institutions and in essence comes down to a process which determines a level at which the bank is comfortable that the counterparty will still be able to fulfill its financial obligations based on the strength of its balance sheet. It is specifically for this reason that it is often important that a bank is able to determine the impact of an additional trade or transaction on the credit exposure against a particular counterparty. In order to do this, it is obvious that the bank would need to be able to estimate the credit exposure on the specific proposed trade and the impact thereof on the current portfolio of trades already in place and active with the counterparty.

1.3.2 Credit Risk Management (Monitoring)

Once a trade is concluded the bank would need to monitor the credit exposure against the counterparty up to and including the maturity date of such a trade. The reason for this is that the credit risk (assuming a derivative transaction) could be quite volatile. If the market moves against a counterparty – i.e. the trade moves more and more into the money for the bank it could mean that there is an increased likelihood that the counterparty could not meet its obligation upon final settlement and (worst case) perhaps default.

Up to date monitoring and accurate estimation of the credit exposure could result in improved credit risk management. Normally, the portfolios are updated at least daily in order to take account of the impact of market movements on the current and potential future exposure of the different portfolios. This process also ensures that new transactions are added in a timely manner.

1.3.2.1 Regulatory Methodologies for Credit Risk Monitoring and Capital Adequacy

The Basel 2 regulations have been adopted by most regulators in the banking industries worldwide and contain different approaches to calculating regulatory capital in various degrees of sophistication. The most basic approach, called the standardised approach, has certain prescribed ‘risk weightings’ which are to be applied for calculating the regulatory capital required for different types of credit exposures. More advanced banks can apply for approval to use the more advanced “Foundation Internal Ratings Based Approach” according to which the bank uses its own estimate for the probability of default for the counterparty. The most advanced approach “Advanced Internal Ratings Based Approach” allows banks to use its own models to estimate PD, EAD and LGD in order to calculate its regulatory capital requirements. Some banks use the EAD estimates produced for regulatory capital purposes as the measure for their internal credit risk management needs. This is normally only done by relatively unsophisticated banks. The most sophisticated banks use a measure called Effective EPE (Expected Positive Exposure) which will be discussed later on in this document⁷.

1.3.2.2 Internal Methodologies for Credit Risk Monitoring

Although some of the regulatory approaches offered in the Basel accord are quite sophisticated, most banks still have their own internal credit risk measurement methodologies over and above the regulatory-prescribed methods. These methods are typically used for credit risk monitoring (i.e. monitoring credit exposure against credit

⁷ See section 5.4.3.

limits) as well as pre-deal analysis and are typically quite advanced from a mathematical point of view. Some banks have methodologies which they apply for exposure estimation only and which are not even used for capital adequacy purposes but merely for internal credit risk management purposes.

1.3.3 Credit Risk Distribution and Mitigation

Many banks manage their counterparty credit risk through various risk mitigation and distribution mechanisms. One very simple way of distributing credit risk is through credit derivatives such as Credit Default Swaps (CDS). A CDS is a financial contract under which the protection buyer pays a regular (typically quarterly) premium to the protection seller in return for the commitment by the protection seller to compensate the protection buyer for credit losses resulting from pre-defined credit events such as default. A CDS is therefore similar to an insurance contract with the insured risk being the event of default. In order to sell or transfer credit risk it is not only important to be able to know with a certain level of confidence what the current level of credit risk exposure is but also what the impact of the proposed 'hedge' is on the existing credit exposure. These different trades and associated hedges would therefore need to be reflected against the appropriate limits in order to have a complete picture of the most current state of affairs with regards to the credit exposure.

In practice, banks also have various risk mitigation techniques which serves to reduce counterparty credit risk. Standardised netting agreements and collateral support annexes (CSAs) are typically used between derivative trading counterparties in an attempt to minimise counterparty credit risk. It is therefore extremely important that such risk mitigation methods are taken into account when determining a consolidated counterparty credit risk exposure against a particular counterparty. Credit risk mitigation techniques and the effect thereof on counterparty credit risk exposure will be discussed in Chapter 3.

1.3.3.1 Dynamic Credit Risk Management

A recent development in the management of counterparty credit risk is what is referred to as “Dynamic Credit Risk Management”. The more sophisticated global investment banks which operate in well developed financial markets are able to manage their counterparty credit risk dynamically through the use of credit derivative technology. This practice enables such institutions to incorporate counterparty credit risk into the ‘pricing’ of their OTC derivative transactions. These prices are therefore counterparty specific since the counterparty credit risk charge is based on the costs involved to transfer or mitigate such risks through the use of credit derivatives.

1.4 Outline of the Dissertation

From the discussions above it is clear that counterparty credit risk measurement plays an important role in any major financial institution active within the global over-the-counter derivatives markets. This dissertation discusses the challenges involved in estimating counterparty credit risk on derivative counterparties and, specifically, the methodologies used in practice for addressing this issue. Related applications of these measures are also discussed, and where appropriate, practical examples or applications of such methods are provided.

In Chapter 2 we introduce the concept of potential future exposure (PFE) which is often used in practice for the measurement of counterparty credit risk. This measure is also important from a practical point of view since it is typically used for measuring counterparty credit risk against limits set by credit officers. The estimation of potential future exposure is demonstrated using practical examples of typical derivative instruments. These examples also yield PFE profiles which are significantly different in terms of shape. The differences are highlighted and explained through an intuitive argument. The Chapter concludes with a summary of other useful PFE profile statistics and related risk measures. These statistics and measures are referred to and applied extensively throughout this dissertation.

Chapter 3 builds on the discussions and definitions from Chapter 2. The concept of potential future exposure is enhanced to provide a measure of counterparty credit risk exposure on a counterparty level. This concept and the calculation thereof is demonstrated on a simple set of derivative contracts assumed to be traded with a single counterparty. Two different portfolios of derivative contracts are used in this demonstration. Firstly a portfolio with two derivatives on the same underlying market variable is used, followed by a more complex example of a portfolio of derivatives on two correlated underlying market variables. In the process the effect of correlation between underlying market variables on the resultant measurement of counterparty-level PFEs are discussed in detail. Related concepts which impact counterparty-level PFE estimation are introduced such as netting agreements, collateral agreements and other credit risk mitigation techniques. Practical examples are once again used to demonstrate the estimation of counterparty-level PFEs under these conditions. The chapter concludes with considerations around consistency in the estimation of counterparty credit risk on a counterparty-level which are often problematic in practice.

Chapter 4 introduces the concept of wrong-way risk. A detailed introduction to wrong-way risk is given followed by an overview of a Finger's framework for measuring wrong-way risk⁸. The structural model introduced by Merton is discussed and provides some background to more recent enhancements to in this field by Moody's KMV and CreditGrades, both of which are also discussed on a high level. The structural model provides an interesting link between the equity price of a company and its probability of default. This link is exploited and applied in a proposed enhancement to Finger's model for measuring wrong-way risk exposure. This proposed enhancement to Finger's approach is the author's contribution to this field of research and provides an intuitive method for measuring the impact of wrong-way credit risk exposure using information observable in the market. The model is demonstrated through examples which are typical in the South African market.

⁸ See Finger C. C., (2000).

Chapter 5 provides a brief overview of the Basel 2 capital accord focussing on the aspects relating to counterparty credit risk. A detailed overview of the treatment of counterparty credit risk measurement is given with a focus on the internal models method (IMM) and the current exposure method (CEM) used for the measurement of exposure at default (EAD). The majority of South African banks employ the CEM for measuring EAD and this approach is therefore compared to the more advanced IMM. Calculations of EAD under these two approaches are demonstrated. Observations relating to the differences in results are discussed in detail.

In Chapter 6 a brief overview of the pricing of counterparty credit risk is given. The concept of credit value adjustment (CVA) is introduced and discussed. This is followed by a short overview of the hedging and transferral of counterparty credit risk.

Chapter 7 focuses on some practical considerations and approximations relating the measurement of counterparty credit risk. The validation and calibration of counterparty credit risk models is discussed briefly followed by an approximation approach often used in practice: the add-on approach. The add-on approach is discussed in some detail followed by a demonstration of a practical application of this approach.

The dissertation concludes with Chapter 8. A summary of the dissertations main findings are presented as well as suggestions for potential areas of future research.

2 Contract-Level Counterparty Credit Exposure

2.1 Background

There is a significant difference between the measurement of credit risk resulting from normal lending activities by a bank and that resulting from derivative trading activities in the OTC derivative market. The former is relatively easy to measure or quantify (Pykhtin, 2005): the credit exposure is merely the amount outstanding on the loan with accrued interest. In the case of credit exposure on derivative contracts however, it is not as straightforward since the value of the contract (and therefore the credit exposure) is dependent on some underlying market (random) variable (De Prisco and Rosen, 2005). The exposure on such a derivative is also dependent on time. The further one looks into the future the more the underlying variable's potential dispersion (Wahrenburg, 1997). In other words, the range of possible values that the underlying market variable can assume becomes larger the further one looks into the future.

The quantification of the exposure at default on derivative contracts is clearly a non-trivial exercise and sometimes, in practice, quite subjective. In attempting to formalise an approach to solving this problem it is quite useful to note that typical derivative contracts could be seen to have a 'current exposure' and a 'potential future exposure'. This approach is widely used in practice (De Prisco and Rosen, 2005).

2.1.1 Current Exposure

The current exposure is defined as the amount at risk should the counterparty default now and is normally (for a single derivative trade) assumed to be the mark-to-market (MTM) value of that trade. The MTM value of the trade(s) is a measure of the replacement cost of the trade and is therefore appropriate for this purpose.

2.1.2 Potential Future Exposure

The potential future exposure, in turn, is much more difficult to quantify due to its stochastic or random nature and one would need to make quite a number of assumptions in order to derive such an estimate. The value of a derivative contract is per definition dependent on some underlying market variable. In order to estimate the future value of such a derivative contract we need to make some assumptions around the evolution of the underlying variable(s) over time over the life of the contract. In other words, we would need to derive (or make an assumption regarding) the probability distribution of the underlying risk factor(s) impacting the value of the trade over the life of the contract. Note that for some derivatives, such as interest rate swaps, the potential future exposure profile is not a strictly increasing function. Therefore it is crucial that the PFE is estimated at various points in time through the life of the contract and not just at the point before maturity in order to estimate a peak PFE. More formally, we refer to the graphical representation of the collection of the $PFE(t)$ values, estimated for various values of t , as a PFE profile. There are two main factors which determine the shape of a PFE profile⁹:

1. The combined effect of the volatility of the underlying risk factor and the future time point at which this estimate is calculated increases the estimated exposure. This is because there is greater variability and uncertainty of market variables the further one looks into the future. In the text that follows we will refer to this as the dispersion effect.
2. The effect of amortisation counteracts the previous force and has a decreasing effect on the exposure profile as periodic payments are realised. This reduces the remaining cash flows exposed to default risk. This effect is especially relevant in the case of interest rate swaps as will be seen in a later section.

The argument above is merely an intuitive discussion on the problem – a more rigorous mathematical approach follows later.

⁹ Pykhtin and Zhu, 2007.

2.2 PFE – An Estimate of Exposure at Default

2.2.1 Overview

In defining a measure for counterparty credit risk exposure we draw upon the notion of a confidence interval. Using this tool we are able to define the concept of Potential Future Exposure (PFE). This measure of counterparty credit risk exposure enables a risk manager to make statements such as¹⁰ “According to our model we can be 95% certain that the exposure to counterparty A will not *exceed* x rand in one year’s time – assuming that the portfolio of trades with such counterparty remains static (no new contracts are entered into).”

2.2.2 Definition of Contract-Level PFE

Potential future exposure is defined as being a time-dependent function, $PFE(\alpha, i, t)$. We present an adaptation from the definition of maximum peak exposure presented in De Prisco and Rosen, (2005).

Let

- Ω_t^i represent the state space containing the universe of all possible prices for all risk factors required to calculate the mark-to-market for derivative i at time t ;
- $MTM_i(\omega_t^i)$ denote the mark-to-market value function of derivative i calculated using the prices ω_t^i , where ω_t^i is one of an infinite number of possible risk factor states at time t in Ω_t^i - i.e. $\omega_t^i \in \Omega_t^i$;
- Λ_t^i be the *actual* mark-to-market price of derivative i at time t . (Note that this quantity is unknown);
- α be the level of confidence.

¹⁰ Wahrenburg, 1997.

Then:

$$PFE(\alpha, i, t) := \inf\{MTM_i(\omega_t^i) : \mathbb{P}[\Lambda_t^i > MTM_i(\omega_t^i)] \leq \alpha\}. \quad (1)$$

PFE is therefore defined as the smallest possible value for derivative i at time t such that the probability of the actual value of the derivative at time t exceeding this value is less than α .¹¹ Note that the mark-to-market function is essentially that function which gives the no-arbitrage price of the derivative given the prices of the underlying assets (and other relevant market information) required in determining the value of the derivative. Often practitioners insist that a credit exposure number should be strictly positive. In this document, however, we allow PFEs to be any real number.

The definition of the contract-level PFE given in (1) is an estimate of future market values for a specific transaction (or set of transactions in the case of counterparty-level PFEs). There is no mention of probability of default – in fact, the concept of PFE is an attempt to quantify what the market value of a derivative *could* be at some point in the future and therefore represents the potential loss that could be suffered *given* the counterparty defaults at that specific point in time. In other words, the exposure represented by the PFE measure at t assumes that default occurs at t .

2.2.3 Estimation of Contract-Level PFE: Some background

Potential Future Exposure as defined in (1) can be estimated using various techniques – some which are more complex than others and each with its own advantages and disadvantages (Wahrenburg, 1997). In general, more complex methods are not only more involved from a mathematical point of view, but also more resource intensive from a computational point of view.

Counterparty credit risk modelling has come a long way over the past few years and has recently demanded much more attention from financial institutions as a result of the

¹¹ Given that the assumptions relating to the statistical distribution of the underlying risk factors hold.

dramatic downfall of several notable institutions such as Barings, Long Term Capital, Enron, WorldCom and Parmalat which resulted in significant credit losses to other financial institutions (De Prisco and Rosen, 2005). The evolution of exposure measurement methodologies are summarised in the following diagram¹²:

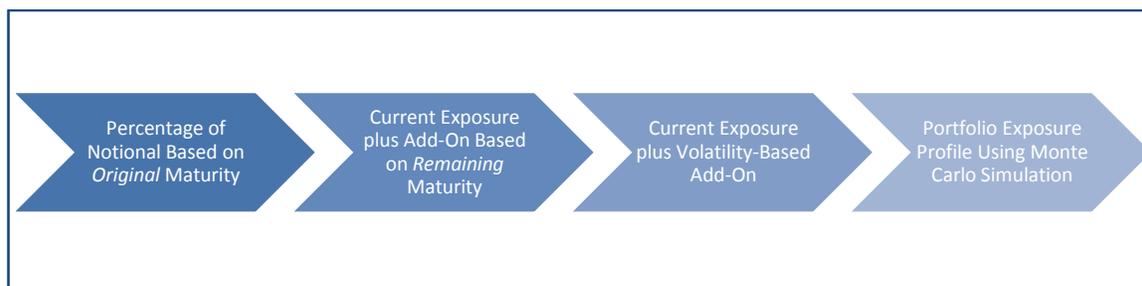


Figure 2.1 - Evolution of Counterparty-Risk Exposure Measurement Methodologies

Banks initially used the percentage of notional approach in estimating credit exposure. The results were not only very crude but also, due to them only being a number (as opposed to an exposure profile), failed to incorporate roll-off risk. Roll-off risk is the risk that a sudden increase in credit exposure occurs as a result of an exposure-reducing contract maturing (‘dropping off’).

The following example illustrates this point. Suppose that there are two OTC contracts with counterparty A. The first contract has an estimated credit exposure (using the ‘percentage of notional’ approach) of -500 rand and a maturity of 11 months. The second contract has an exposure of 450 with a maturity of 18 months. The exposure number that would have typically been calculated by banks using the ‘percentage of notional’ amount would have been -50.¹³ The following graph illustrates the information provided by an exposure profile. The exposure profile is forward-looking in that it shows sudden increases in exposure in the future and allows mitigating action to be taken by the responsible credit officer.

¹² Wahrenburg, 1997.

¹³ Note that this calculation assumes that there is a netting agreement in place and that the two contracts under consideration are both covered by this same netting agreement. For more information on netting agreements see chapter 3.

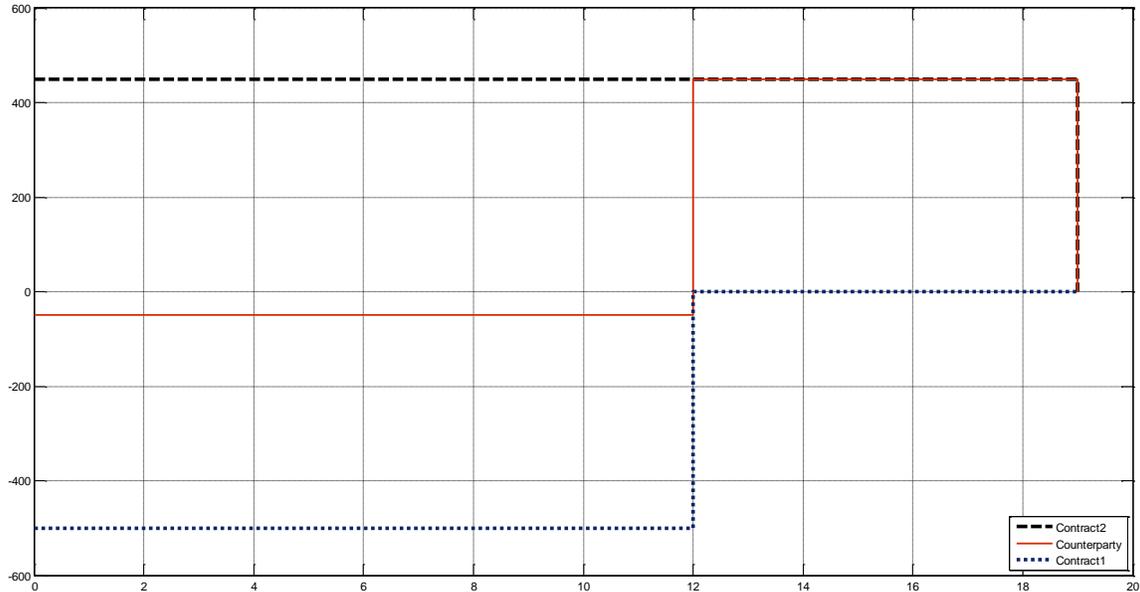


Figure 2.2 - Illustration of Roll-Off Risk

The exposure profile therefore gives insight into the exposure which lies ahead whereas the representation of credit exposure as one number (scalar) is very limited and may lead to unexpected ‘jumps’ in exposure as a result of offsetting trades maturing as illustrated in Figure 2.2.

The next step in the evolution of counterparty credit risk measurement was breaking down the credit exposure into current exposure and potential future exposure and treating the potential future exposure as an add-on based on remaining maturity. This is certainly more advanced than the ‘percentage of notional’ approach but failed to incorporate the fact that different underlying assets had different characteristics (such as volatilities) which should ideally be incorporated into the measurement methodology (Wahrenburg, 1997). The reasoning behind the “current exposure plus volatility-based add-on” was the principle from statistics which states that the largest value that a variable can take on can always be expressed as its mean value plus a multiple of its volatility (Wahrenburg 1997). This method yields results that are relatively accurate on a contract-level¹⁴ but all of the add-on approaches, irrespective of the degree of complexity, failed to encapsulate on very important portfolio effect: correlation. This resulted in grossly overstated and overly

¹⁴ When compared to results using Monte Carlo simulation. See section 7.2.2.2.

conservative counterparty-level exposures. This problem was solved by the introduction of PFE profiles estimated using Monte Carlo simulation techniques.

A number of financial institutions still use some form of “current exposure plus add-on” approach. It is commonly accepted that Monte Carlo simulation is the preferred approach due to the accuracy and flexibility that it provides. For certain complex derivative products it is impossible to estimate a PFE using add-ons. For these reasons we will focus on Monte Carlo simulation techniques throughout this dissertation. Section 7.2 discusses add-on approximation methods in some detail.

2.2.4 Example: Contract-Level PFE on FX Forward (1)

We now consider a simple example to illustrate the process of calculating a contract-level PFE profile:

A bank enters into a foreign exchange forward¹⁵ contract with counterparty A. The detail of the contract is as follows:

The bank agrees to take delivery of 1,000 USD (US Dollars) in 6 months time in exchange for 8,170 ZAR (South African Rand). The following table summarises the specific details of this contract:

<i>Underlying</i>	<i>Notional (USD)</i>	<i>Maturity (yrs)</i>	<i>Strike¹⁶</i>
USD/ZAR	1,000	0.5	8.17

Table 2.1 - Contract details: Example FX Forward 1

The first step is to determine the current value of the derivative contract, for which we require the latest 6 month forward USD/ZAR exchange rate. Let’s assume that the value

¹⁵ An FX forward contract is a contract whereby to parties agree to exchange a fixed amount of currency A for a fixed amount of currency B at a future point in time.

¹⁶ Quote in number of ZAR per USD.

for this variable is currently 8.17. In order to calculate the current value, we use the following formula:

$$MTM_{ZAR} = N_{USD} \times PV_{ZAR}(F_{USD/ZAR} - K_{USD/ZAR}), \quad (2)$$

where

- N_{USD} is the notional amount that the transaction is based on, measured in USD
- PV_{ZAR} is the present value function, in this case discounting cash flows using ZAR interest rates
- $F_{USD/ZAR}$ is the appropriate 6-month forward exchange rate, quoted USD/ZAR.
- $K_{USD/ZAR}$ is the strike price of the forward contract, quoted USD/ZAR – in this case 8.17

Let's assume the current value is zero. Note that the forward exchange rate is the main risk driver in the case of FX Forwards and this is therefore the variable which we will need to simulate in order to generate a PFE profile. It is crucial to note, however, that in order to calculate the PFE at time t ($t > 0$) one would need to know the value of the *then* prevailing $(T-t)$ forward exchange rate – T being the original maturity of the contract. In practice, forward exchange rates are determined using no-arbitrage assumptions and are calculated using the current spot exchange rate and the associated interest rates.

For simplicity we will assume that interest rates remain constant but that the current spot exchange rate follows a Geometric Brownian motion (GBM) process. This is equivalent to assuming that the forward-exchange rate curve will retain the same shape over time, but that there will be parallel shifts up and down driven by changes in the spot exchange. The spot exchange rate is assumed to follow a GBM process, with SDE¹⁷:

¹⁷ We have made the simplifying assumption that the volatility parameter is constant which normally fits the behaviour of spot prices well over the short term but yields long term volatilities that are too large. One way of addressing this problem is to use the GBM Term Structure of Volatility model which treats the volatility parameter as a deterministic function of time, typically $\sigma(t)$ is treated as a decreasing function of t . The SDE becomes:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \text{ with solution } S(t) = S(0) \exp\left(\mu t - \frac{1}{2} \int_0^t \sigma^2(s) ds + \int_0^t \sigma^2(s) dW_s\right)$$

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (3)$$

We now proceed to the actual PFE calculation. Let's divide the duration of the contract into 10 time steps. The first step is to simulate, for the ten time steps over the life of the contract, values for the USD/ZAR spot exchange rate. Note that, as mentioned before, the shape of the forward exchange curve will be assumed to be static. The volatility and drift parameters in the GBM process was estimated using a one year history of USD/ZAR spot exchange rates.

The duration of the contract (6 months) is divided into 10 equally-sized time intervals and at each one of the time steps, the MTM value is calculated using the MTM formula above. This procedure is repeated 10,000 times – i.e. there will be 10,000 MTM values for each of the 10 time steps. The final step is then to take the 95th percentile of the 10,000 values at each time step in order to yield the 95-percent contract-level PFE profile. Figure 2.3 shows the resulting 10,000 scenarios as well as the resulting 95% PFE profile indicated by the dashed line.

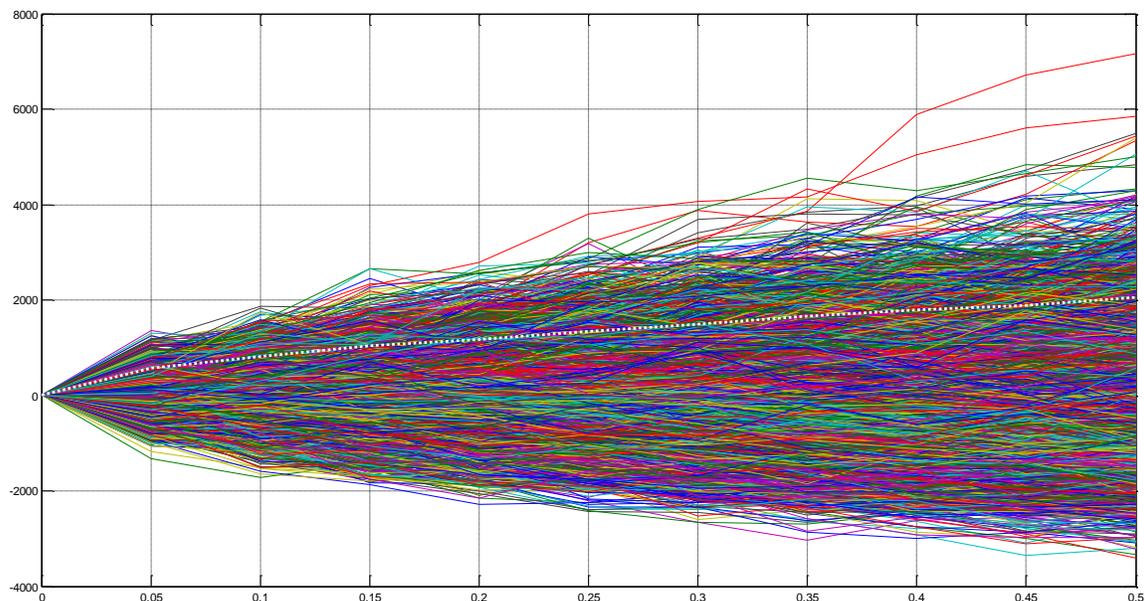


Figure 2.3 - Contract-Level PFE on FX Forward

The shape of the graph in Figure 2.3 makes intuitive sense, in that the resulting PFE profile is strictly increasing as one would expect from a forward contract. The reasoning behind this is that, during the life of the trade, the mark-to-market value could continually increase (as a result of the ‘dispersion/diffusion effect’) as the exchange rate, which drives the MTM, moves higher and higher. There is therefore no ‘amortisation effect’ (only one final payment at maturity). Obviously, since we are making use of a 95% confidence interval the extent to which this increase in mark-to-market can materialise is limited¹⁸.

2.2.5 Example: Contract-Level PFE on FX Forward (2)

Let’s now compare the PFE on two FX forward contracts that are slightly different. Let’s assume that FX forward 1 is as specified below, and FX forward 2 is different from FX forward 1 only in that the strike is higher, at 8.25.

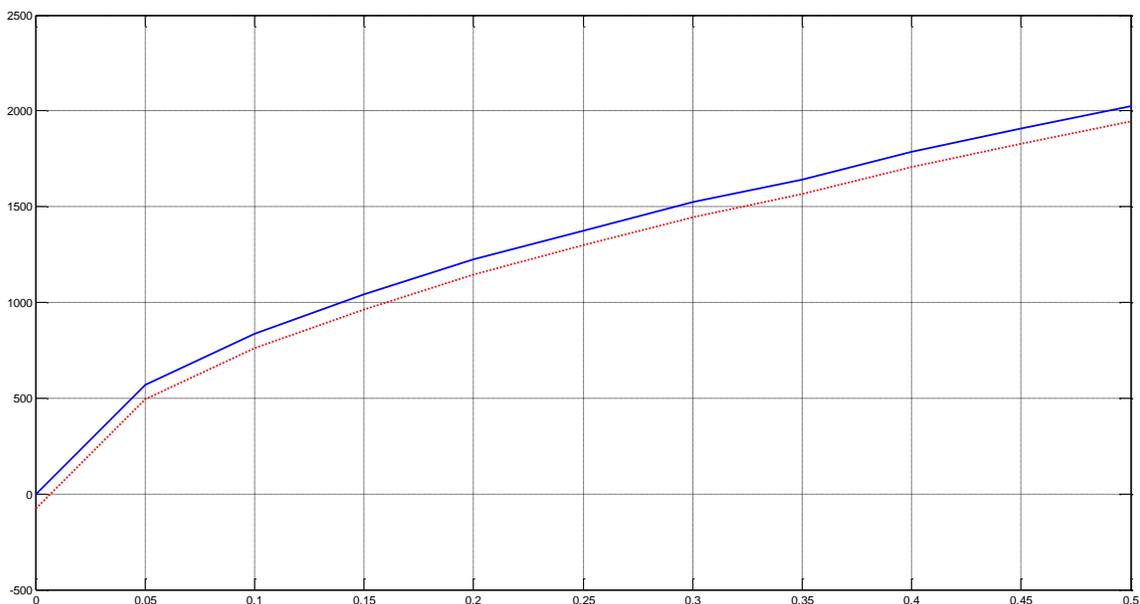


Figure 2.4 - Contract-Level PFE for FX Forwards with Different Strikes

¹⁸ If the assumed statistical distribution of spot prices holds true.

As expected, the current value (MTM) of FX forward 2 is lower than that of FX forward 1. The PFE of FX forward 1 is also higher than the PFE of FX forward 2 since the strike is lower in the 1st case and it is therefore more likely that the strike of FX forward 1 will be exceeded, compared to the strike of FX forward 2.

Note that both contracts have exactly the same underlying market variable and exactly the same maturity. We are therefore considering exactly the same probability distributions when we estimate the PFE profiles of these two contracts.

2.2.6 Contract-Level PFE Algorithm for Monte Carlo Simulation

We have looked at an example and performed the calculation of a PFE for a simple product using Monte Carlo Simulation. Let us now consider a more generic and formal approach to the problem of estimating a PFE on a contract level using Monte Carlo Simulation. We will also use the symbols as in the definition of PFE above. The steps are as follows:

1. Identify the risk driver(s) of the contract under consideration. In our example above this was the forward exchange rate. Note that it is sometimes possible to break a risk driver down into its constituent parts and that this may lead to simplified scenario-generation (e.g., using the FX Spot exchange rate instead of simulating the whole FX Forward curve). There may, however, be more than one risk driver which could significantly increase the complexity of PFE estimation.¹⁹
2. Divide the time interval between time zero (today) and the maturity date of the contract into M time steps. Note that these time steps need not be of equal size. In fact, it would be preferable to have time steps fall on so-called ‘significant dates’ of the particular contract.

¹⁹ The main objective of the simulation model is to project, as realistically as possible, the potential future state of the market being simulated. In that sense the model should operate under the real probability measure.

Significant dates are dates, during the life of the derivative contract, on which certain events pertaining to the cash flows of the contract take place – for example reset-dates on swap agreements. These significant dates typically have an impact on the shape of the PFE profile. In the estimation of the PFE on a fixed for floating interest rate swap with quarterly payments one would typically have the time steps be quarterly, just before each cash flow, in order to illustrate and capture the amortisation effect and to ensure that the PFE is not under-estimating the real exposure.

3. Using an appropriate stochastic process, simulate N scenarios of the underlying risk driver identified in step 1. For each scenario M values need to be generated – one value for each time step. This step will result in a grid of simulated risk driver values which represent the possible values that the underlying risk driver may take on in the future. Note that this step simulates values which will be used to estimate Ω_t^i in the context used in the definition of PFE above.

It is important to bear in mind that using Monte Carlo Simulation is computationally intensive and that one needs to weigh up the time taken to perform the calculations with the accuracy obtained through using the particular number of simulations. More specifically the error in Monte Carlo Simulation has the order \sqrt{N} convergence, and one would therefore need to quadruple the number of simulations (and typically need to wait four times longer) in order to increase the accuracy by a factor of two.

4. On each of the time steps for each scenario, calculate the *MTM* value of the contract at that point in time. This step therefore calculates $MTM(\omega_t^i)$ for various simulated states of ω_t^i . The resultant values are simulated values from the probability distribution for Λ_t^i , the actual yet unknown *MTM* of the contract at point t .

- For each of the time steps, calculate the α^{th} percentile of the calculated MTM values in step 4. This will yield the estimated contract-level PFE profile as defined above.

We now consider another example of a contract-level PFE calculation. The estimation of a PFE on a fixed-for-floating interest rate swap is slightly more complex than that of a simple FX forward. We will follow the steps set out above.

2.2.7 Example: Contract-Level PFE on IR Swap

We now consider a bullet²⁰ fixed-for-floating interest rate swap. Firstly, from an intuitive point of view one would expect the PFE profile on a vanilla fixed-for-floating interest rate swap with quarterly resets to be increasing initially but then decreasing after some t ; $t \in (t, T)$. This is as a result of the ‘amortisation effect’ discussed above. In fact, the MTM on an interest rate swap tends to zero as time tends to maturity because of the realised quarterly payments. We proceed to a formal and detailed step-by-step process for estimating the PFE on a ZAR fixed-for-floating interest rate swap.

- Firstly we need to identify the main risk driver in this contract. In order to value an interest rate swap, we need to determine the difference in the present value of the fixed-leg and the present value of the floating leg. The formulae below is typically used:

$$MTM_{fixed} = C \cdot \sum_{i=1}^M \left(N \cdot \frac{t_i}{T_i} \cdot df_i \right),$$

$$MTM_{float} = \sum_{i=1}^M \left(N \cdot f_i \cdot \frac{t_i}{T_i} \cdot df_i \right),$$

where C is the swap rate (the fixed rate), M is the number of fixed/floating payments, N is the notional amount and t_i is the number of days in period i and T_i

²⁰ The notional of the swap is constant over the life of the swap.

is the number of days according to the day count convention and df_i is the discount factor used to discount a cash flow from time i to the valuation date.

The MTM of an interest rate swap is therefore heavily dependent on the value of the floating leg of the swap, which in turn is dependent on the floating interest rate. The main risk driver is therefore the floating interest rate²¹.

- Let's assume that we are calculating the PFE on a 5 year interest rate swap from the point of view of the fixed payer. Let's assume that the payments are quarterly and that this is a ZAR interest rate swap. Due to the fact that the payments are quarterly, let's divide the duration of the contract into quarterly intervals. We will therefore be calculating the PFE on $4 \times 5 = 20$ time points.

<i>Instrument</i>	<i>Fixed Rate</i>	<i>Maturity</i>	<i>Notional in Rand</i>
Interest Rate Swap	10.5%	5 years	100,000,000

Table 2.2 - Contract Details: Example Interest Rate Swap

- We now need to simulate paths of the underlying risk driver using a 'suitable' stochastic process. In this example we will consider a CIR (Cox, Ingersoll and Ross) model, calibrated using historical zero-coupon ZAR interest rate data obtained from the Bond Exchange of South Africa (BESA)²². Note that the CIR model is a model for the short rate – i.e. the instantaneous interest rate. We will use this model and, analogous to the FX Forward example, shift the zero curve up and down in line with the up and down shifts of the short rate of interest. This model therefore does not in any way capture changes in the shape of the yield curve. More advanced models can be used for capturing these risks – this example is merely for illustrative purposes.

- Model Specification**

²¹ Typically, in this case, the 3 Month Jibar rate.

²² See www.bondexchange.co.za

The CIR model is a stochastic process for the short rate of interest of which the stochastic differential equation is:

$$dr = a(b - r)dt + \sigma\sqrt{r}dW. \quad (4)$$

It can be shown that the following closed form solutions can be obtained for the price, at time t , of a zero coupon bond that pays 1 at time T under the CIR model (Hull , 2002):

$$ZCB(t, T) = A(t, T)e^{-B(t, T)r(t)},$$

where

- $B(t, T) = \frac{2(e^{\gamma(T-t)} - 1)}{(\gamma + a)(e^{\gamma(T-t)} - 1) + 2\gamma}$,
- $A(t, T) = \exp \left[\frac{2\gamma e^{\frac{(a+\gamma)(T-t)}{2}}}{(\gamma + a)(e^{\gamma(T-t)} - 1) + 2\gamma} \right]^{\frac{2ab}{\sigma^2}}$,
- $\gamma = \sqrt{a^2 + 2\sigma^2}$.

Using the following discrete approximation, we apply Monte Carlo simulation in generating a set of simulated paths for the short rate of interest.

$$r_{t+1} = a(b - r_t)\Delta t + \sigma\sqrt{r_t}\varepsilon.$$

- **Model Calibration**

Using zero curve data obtained from BESA, it is possible to calibrate the CIR model²³ to fit the current market conditions. More specifically, we can observe prices of bonds and other similar instruments, with various maturities, in the market and use the formula for $ZCB(t, T)$ above in order to mimic the current market prices by changing the values for the model parameters. In derivative pricing it is possible to calibrate the model to the real-world measure and to the risk-neutral measure. For a more complete discussion on this topic see Section 7.1.2.

There are two possible approaches to follow in calibrating the model to the current market:

- Observed bond prices can be used in order to construct a zero-coupon yield curve which is representative of current market conditions. Using the formula for $ZCB(t, T)$ above it is possible to construct a zero-coupon yield curve with the CIR model. Using an error-minimising technique²⁴ one can then proceed to change the model parameters and minimise the difference between the market-observed zero-coupon yield curve and the model-derived zero-coupon yield curve by adjusting the parameters of the model.
- Similar to the above approach, one can use the observed bond prices directly and compare them to the model produced bond prices (using the formula for $ZCB(t, T)$ above) and minimise the error by adjusting the model parameters appropriately.

In this example, we use the second approach and obtained the following values for the model parameters.

²³ Change the parameters a , b and σ in order to have model-predicted values as close as possible to market-observed prices.

²⁴ In this example specifically, we have calculated the model parameters by minimising the squared error of the model-predicted zero-coupon bond prices and the actual observed zero-coupon bond prices.

<i>Parameter</i>	<i>Description</i>	<i>Value</i>	Source
a	Reversion Speed	0.2417	Calibrated
b	Mean	0.0809	Calibrated
σ	Volatility	0.0212	Observed 5yr Cap Volatility ²⁵
r_0	Start Rate	0.1186	From Zero-Coupon Yield Curve

Table 2.3 - CIR Model Parameters used in the PFE Estimates

- **Model Fit**

The graph below illustrates the calibrated CIR model and how the model-produced bond prices compare to the market-observed bond prices.

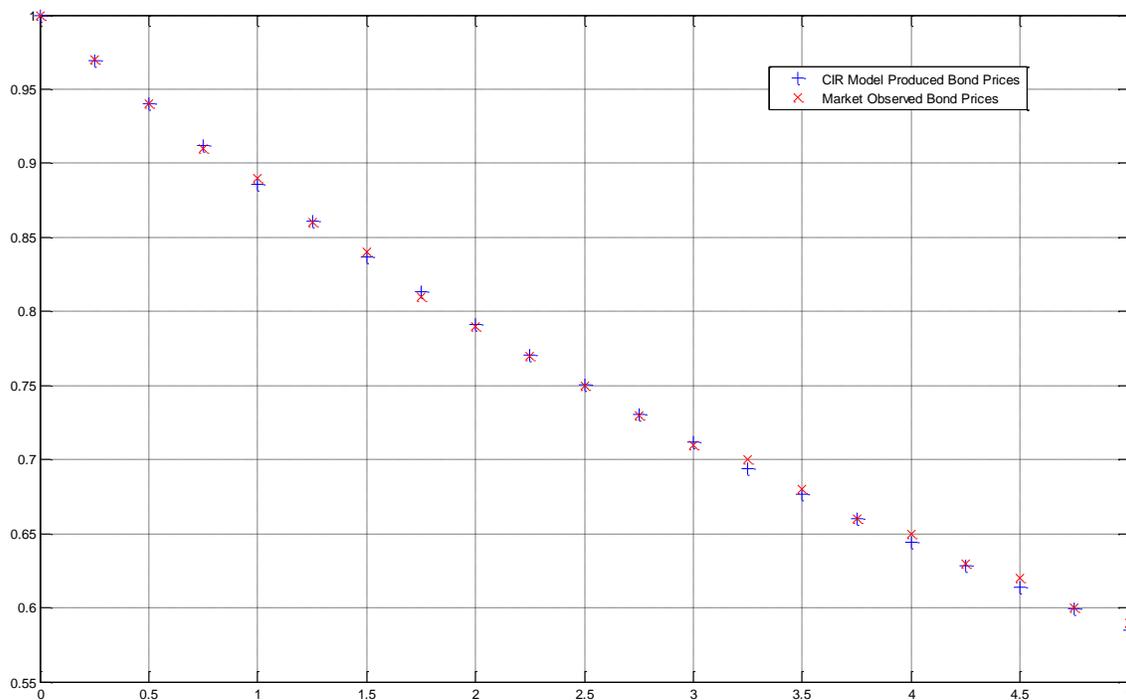


Figure 2.5 - CIR Model-Produced Bond Prices vs. Market-Observed Bond Prices

- Using the calibrated model it is possible to simulate the required paths of the short interest rate and then to calculate the simulated MTM values at various points in

²⁵ The cap volatility is assumed to be a forward-looking view of the volatility of the underlying floating interest rates.

time over the life of the swap. The graph below shows the simulated paths for the short rate produced by the CIR model using Monte Carlo Simulation.

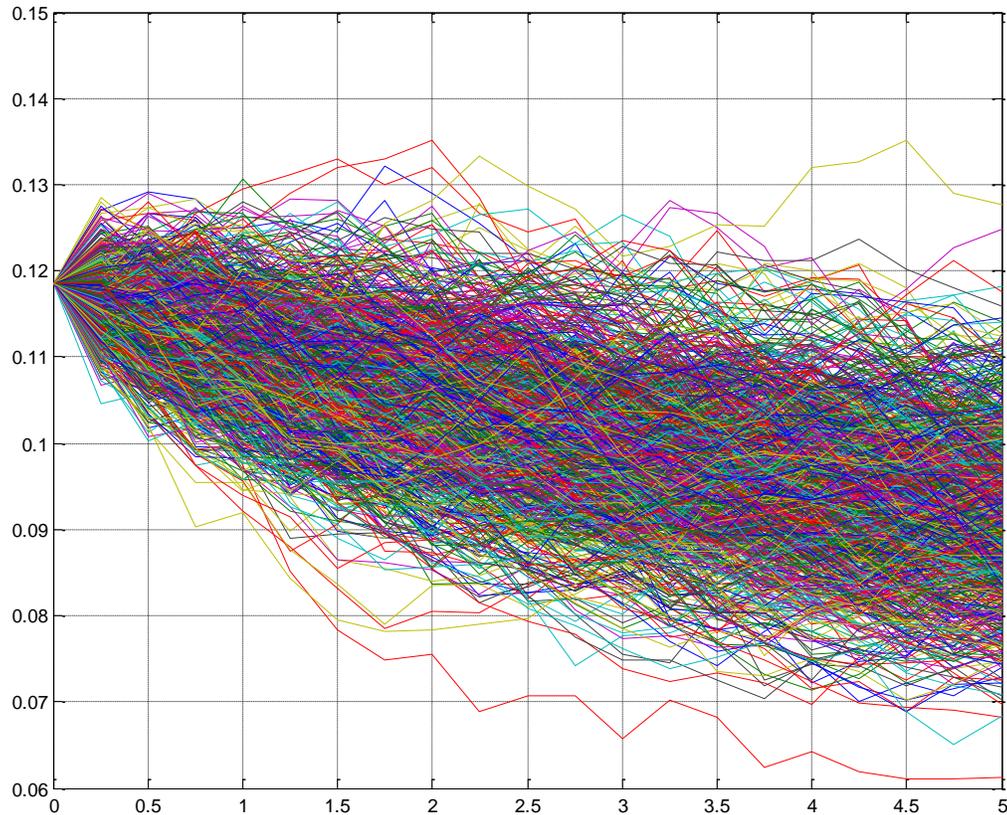


Figure 2.6 - Simulated Paths using CIR Process

The reason for the downward sloping effect on the above simulation profile is the fact that the model parameters are such that the model tends to revert to the long-term average rate of 8.09%. Figure 2.7 below shows the simulated MTM values of the swap.²⁶

²⁶ Note that the code used to generate the simulations for this example is given in the appendix.

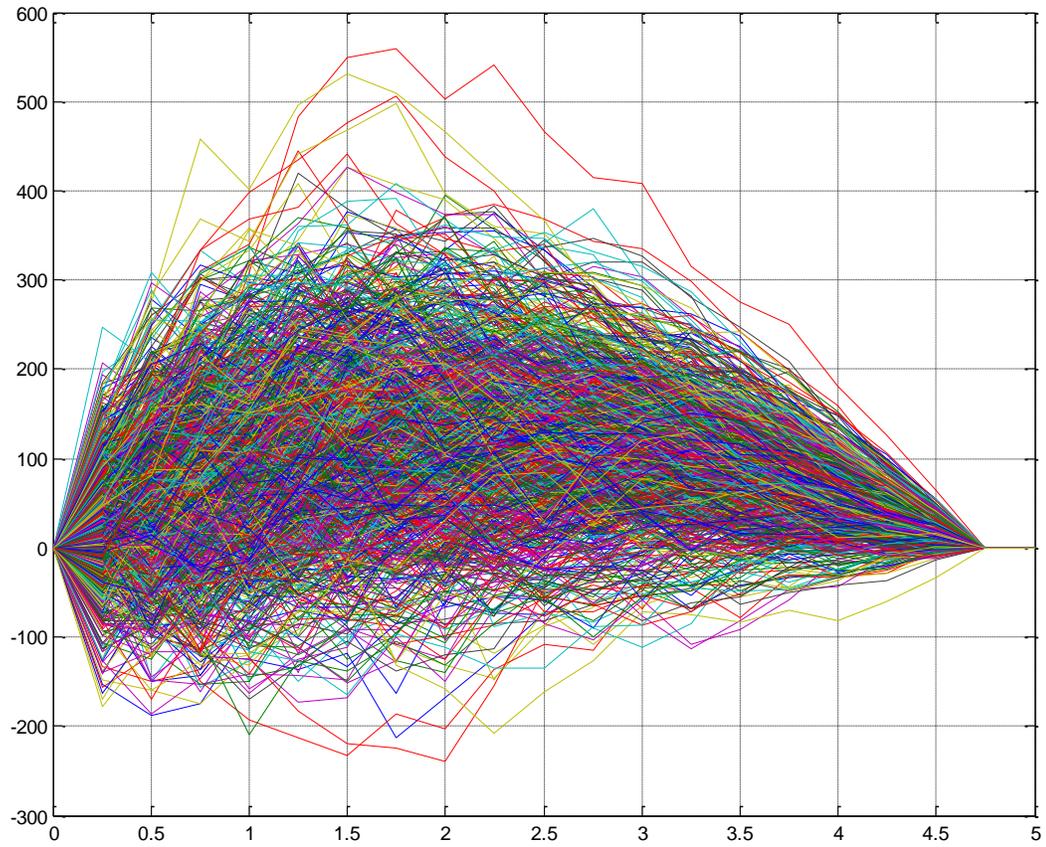


Figure 2.7 - Simulated MTM values in R'000 for Interest Rate Swap in Table 2.2.

5. Using the calculated MTM values calculated in step 4 it is possible to calculate the 95th percentile at each time point which yields the PFE profile of the swap below:

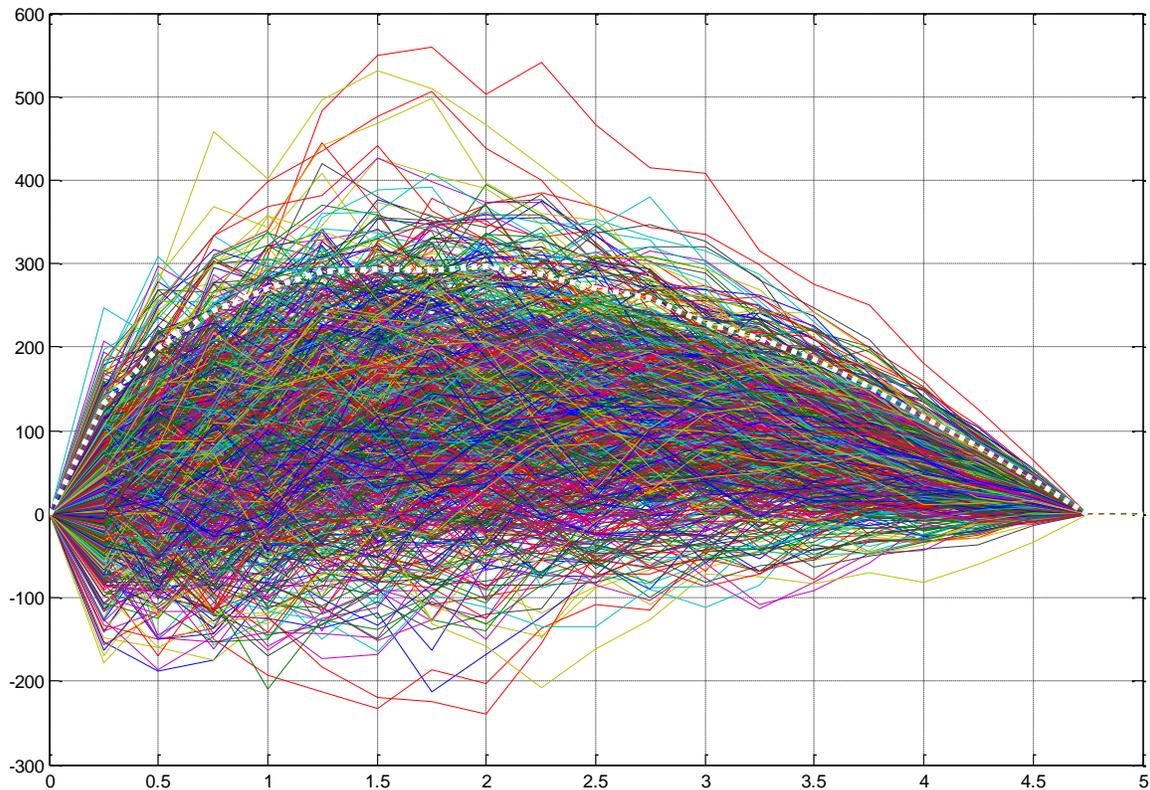


Figure 2.8 - Simulated MTMs of Interest Rate Swap in Table 2.2 and 95% PFE

As expected the profile in Figure 2.8 above clearly illustrates the amortisation effect present in the PFE profile of an interest rate swap. The CIR model used above works relatively well for capturing the credit exposure resulting from general changes in the level of interest rates, but fails to capture higher order risks such as changes in the shape of the yield curve. In practice, more advanced models are used for the evolution of interest rates, and specifically models which also incorporate changes in the shape of the yield curve²⁷.

2.2.8 Interpretation of and uses for Contract-Level PFE

For individuals with a mathematical or statistical background the concept of PFE is quite straightforward and easily understood. One should however always bear in mind that the individuals typically involved in the decision making which relies on such PFE numbers (typically Credit Officers) do not necessarily have the same quantitative or statistical

²⁷ The interested reader is referred to Jamshidian and Zhu (1996) for detail on the so-called principal components model which models several individual zero coupon rates on the rate curve and reduces the number of factors through principal component analysis.

background as those individuals developing these models. It is of utmost importance that the ‘users’ of information are appropriately trained not only to understand and interpret the results but also to understand its limitations.

2.3 Other useful PFE Profile Statistics and Risk Measures

There are other statistics that can be obtained from the same data that a PFE profile is estimated from which are used in, for example, regulatory and economic capital calculations. The following table²⁸ is a summary of these:

<i>Measure</i>	<i>Definition</i>	<i>Formula</i>
Maximum PFE	Maximum peak exposure that occurs at a given date or any prior date over all scenarios	$PFE_{max}(\alpha, t_k) = \max_{0 \leq j \leq k; \forall i} [MTM_i(\omega_{t_j}^i)]$
Expected Exposure	Average of the distribution of exposures at a particular future date	$EE(t_k) = \frac{1}{n} \sum_{i=1}^n MTM_i(\omega_{t_j}^i)$
Effective Expected Exposure	Maximum expected exposure at a given date or any prior date	$EE_{eff}(t_k) = \max_{0 \leq j \leq k} [EE(t_k)], \forall j$
EPE (Expected Positive Exposure)	Weighted average over time of expected exposures	$EPE(t_k) = \frac{1}{t_k - t_0} \sum_{r=1}^k (EE(t_r) \cdot (t_r - t_{r-1}))$
Effective EPE	Weighted average over time of effective expected exposure	$EPE_{eff}(t_k) = \frac{1}{t_k - t_0} \sum_{r=1}^k (EE_{eff}(t_r) \cdot (t_r - t_{r-1}))$
CVA	Credit risk premium of a counterparty portfolio	$CVA = spread \cdot EPE_{D(t_0)}(T) \cdot (T - t_0)$
Effective Maturity	Ratio of discounted EPE over the life of the portfolio divided by the EPE over one year	$EM = \frac{EPE_{D(t_0)}(t = T)}{EPE_{D(t_0)}(t = 1 \text{ year})}$

Table 2.4 - PFE Profile Statistics and Risk Measures

Example calculations in Chapter 5 will give more background to the meaning and application of the formulae in Table 2.4. Quantile PFE measures are typically used in risk

²⁸ Pykhtin (2005)

management and monitoring of credit exposure against credit limits. Measures of expected exposure on the other hand, are used for credit pricing²⁹ and calculations of regulatory capital requirements. This is part of the internal ratings based models discussed in the new Basel II Accord³⁰.

²⁹ As in the case of Credit Value Adjustment (CVA).

³⁰ See Chapter 5.

3 Counterparty-Level Credit Exposure

In practice, it is not only of importance to quantify the potential future exposure on a single contract but also on a collection or portfolio of contracts. Typically, banks set credit limits against trading counterparties and use PFE exposure estimates to monitor the exposures against these set limits. In order to monitor counterparty-level PFEs a bank needs to be able to apply the effects of netting (i.e. take account of netting agreements appropriately), collateral and other credit risk mitigation techniques successfully. This chapter looks at the measurement of counterparty credit exposure on a counterparty-level and related issues such as the incorporation of risk mitigation techniques into the measurement of exposure. In addition, we discuss related issues such as the impact of credit derivatives on counterparty risk measurement. A is also devoted to the issue of consistency in counterparty credit risk exposure models.

3.1 Aggregation of Exposures and Applying Netting

Potential Future Exposure on a portfolio of trades with a counterparty (counterparty-level PFE) is a very important concept and is not just a matter of aggregation of the PFEs of the underlying trades in the portfolio. In fact, the counterparty-level PFE will always be smaller than or equal to the sum of the underlying PFEs of the trades in the portfolio. The reasoning behind this is that, if one merely adds up the constituent parts of the portfolio one would not take into account the dependence³¹ that exist between some of the underlying market variables and therefore over-estimate the exposure on the counterparty level.³²

When we consider defining the potential future exposure on a counterparty-level one firstly would need to be familiarised with the concept of netting. In practice, derivative trading counterparties use netting agreements in order to mitigate counterparty credit risk.

³¹ Correlation is typically used as a measure of dependence.

³² See Gibson (2005).

Netting agreements are legal contracts which are standardised (generally, for example ISDA Master Agreements³³) and allow counterparties to offset amounts owed in the event of default through so-called close-out netting. As an example, assume that a Bank's counterparty defaults at time t and that at this time there are two derivative contracts between these two parties that have not yet matured and that the mark-to-market values of these derivatives, at time t , are -25 and 10 for the bank respectively (a positive mark-to-market means that the derivative has economic value for the bank and a negative mark-to-market means that the derivative has economic value for the counterparty). Now, in this default scenario, let's consider what the effect of a netting agreement is on the economic result for the bank:

- Assuming there is no netting agreement, the bank would need to pay the counterparty 25 for the value of the first derivative but will receive nothing on the second derivative immediately. In fact, the bank will have to wait in line with all other concurrent creditors of the counterparty for any recovered value from the bankruptcy proceedings – which could take a long time and also does not guarantee that the bank will in fact recover anything. There is therefore an outflow of 25 for the bank and an inflow of an amount less than or equal to 10 and potentially an amount of zero (at an unknown date).
- In the case where there is a netting agreement between the bank and the counterparty the situation is much different. The bank will now not pay over 25 to the counterparty but only $25-10=15$ since the bank is 'owed' 10 by the counterparty on the second derivative.

3.2 PFE on a Portfolio of Trades with a Counterparty

Bearing the introduction to netting in mind, let us now turn to defining the concept of PFE on a portfolio of derivatives with a single counterparty taking into account the effect of netting.

³³ See www.isda.org

Let

- $\Xi[x] = \max[x, 0]$
- $v = \begin{cases} 1 & \text{if a netting agreement is in place} \\ 0 & \text{otherwise} \end{cases}$
- $\Gamma_{\alpha}^H(t) = v \sum_{i=1}^H MTM_i(\omega_t^i) + (1 - v) \sum_{i=1}^H \Xi[MTM_i(\omega_t^i)]$ ³⁴
- $\Pi^H(t)$ denote the actual (unknown) value of the portfolio of N assets at time t .

Then we define the counterparty-level potential future exposure (PFE^{CP}) at time t , of a portfolio of H derivatives as:

$$PFE^{CP}(\alpha, H, t) := \inf \left\{ \Gamma_{\alpha}^H(t) : \mathbb{P} \left(\Pi^H(t) > \Gamma_{\alpha}^H(t) \right) \leq \alpha \right\}.$$

In other words, the counterparty-level PFE is the smallest value of $\Gamma_{\alpha}^H(t)$ such that $\Pi^H(t)$ is not larger than that value of $\Gamma_{\alpha}^H(t)$ with probability $(1 - \alpha)$. Furthermore, it is very important to note that the PFE^{CP} measure is not the sum of the PFEs of all the individual derivatives in the portfolio under consideration.

In fact, the PFE^{CP} is calculated using the distribution of possible portfolio values at t and then obtaining the $(1 - \alpha)^{\text{th}}$ percentile of this counterparty-level distribution. This means that, in order to calculate the PFE^{CP} , one would have to estimate the joint distribution function of the prices of the underlying portfolio of derivatives and then price the portfolio of derivatives on each of these scenarios (applying netting appropriately) and then proceed to determine the $(1 - \alpha)^{\text{th}}$ percentile at each calculation step.

³⁴ This formula assumes that all trades covered by a netting agreement is covered by the *same* netting agreement and therefore forms part of the same netting pool. This is not necessarily always the case, and in practice one would firstly apply netting to each netting pool individually and then apply netting across netting pools using the resultant netted exposures.

3.2.1 Example: Counterparty-Level PFE (Single Underlying)

Let us now consider a simple portfolio consisting of four derivatives. Let's, for simplicity, assume that the portfolio consists of only foreign exchange derivatives – two FX Forwards and two FX Options³⁵ with the following characteristics³⁶:

<i>No.</i>	<i>Contract Type</i>	<i>Underlying</i>	<i>Notional (USD)</i>	<i>Strike (ZAR)</i>	<i>Maturity (Yrs)</i>
1	Forward	USD/ZAR	1,000	8.17	0.75
2	Forward	USD/ZAR	-850	5.83	1.50
3	Call Option	USD/ZAR	1,000	7.77	2.00
4	Put Option	USD/ZAR	1,000	7.77	2.00

Table 3.1 - Example Portfolio 1

In order to be able to calculate the PFE on this portfolio of trades one needs to simulate scenarios for all the underlying risk drivers which impact the market value of the positions in the portfolio. In this example the main risk driver is the USD/ZAR exchange rate.

Let's assume that the spot exchange rate follows a GBM process. We proceed exactly as before (see section 2.2.4) in generating scenarios for the USD/ZAR Exchange rate and once again we assume that interest rates remain constant. In other words, we will be simulating values for the spot exchange rate and assume that the forward exchange rate curve moves up and down in parallel shifts as the spot exchange rate moves.

For the FX options, however, we will be requiring the following inputs (at each time step) in order to calculate the PFE profile:

- σ – The implied volatility. We make the simplifying assumption that this parameter remains constant throughout the life of the contract.

³⁵ The two options in this portfolio form a 'straddle' volatility trading strategy.

³⁶ For simplicity we assume that all the derivatives in the portfolio have the same underlying. This assumption will be relaxed in section 3.2.2 where the effects of dependency (measured by correlation) will be taken into account.

- S – The spot price for the USD/ZAR exchange rate. This is the variable which will be simulated using Monte Carlo simulation as in section 2.2.4.
- K – The strike price of the option as given in the table above.
- r – The appropriate interest rate used to discount the cash flows from the maturity of the option back to the mark-to-market date. Note that, in assuming that the relative shape of the forward exchange rate curve remains constant, we are implicitly assuming that interest rates remain constant and that in moving forward in time the zero-curve moves with us (as opposed to us moving into the curve).

Figure 3.1 shows the contract-level PFE (in ZAR) of each of the individual deals in the portfolio:

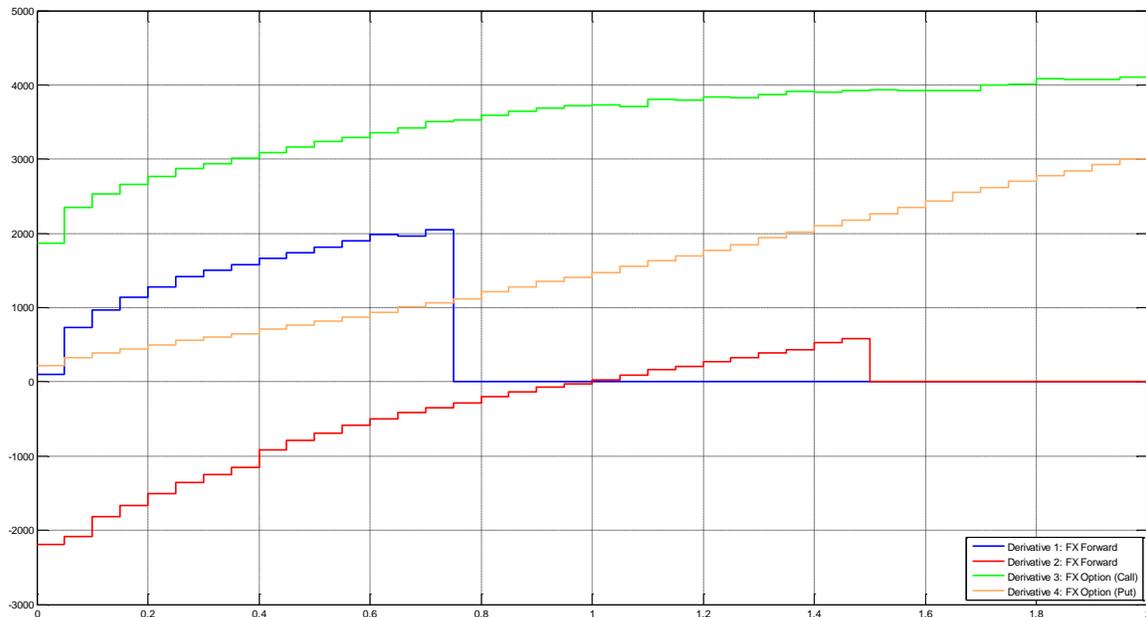


Figure 3.1 - Contract-Level PFE on Individual Deals

Figure 3.2 compares the aggregate PFE³⁷, measured in ZAR with the Netted Counterparty-Level PFE (Netted PFE^{CP}) and the Non-Netted PFE^{CP} .

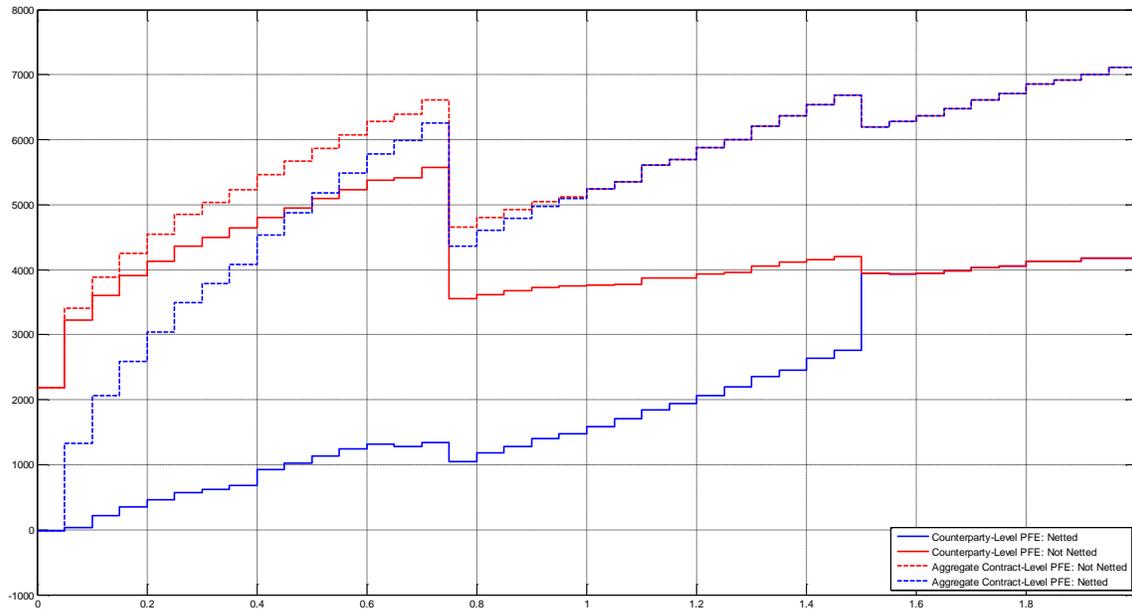


Figure 3.2 - Counterparty-Level PFEs: A Comparison

Let's briefly discuss each of the elements in Figure 3.2 above:

- Aggregate Contract-Level PFE: Not Netted vs. Counterparty PFE: Not Netted

The calculation of 'Aggregate Contract-Level PFE: Not Netted' is very simple. 'Aggregate Contract-Level PFE: Not Netted' is an aggregate of the contract-level PFEs illustrated in Figure 3.1 with the exception that negative PFE values are taken to be zero (i.e. netting is not applied). In contrast, 'Counterparty PFE: Not Netted' is calculated within each of the 20,000 scenarios used in calculating the $MTM(t)$ values at each of the t values during the life of the transactions by aggregating (again using zeros where negative MTMs are observed) the individual MTMs of each of the underlying deals forming part of this portfolio. Finally the 95th percentile of the resultant aggregate values at each of the time steps is calculated. It is clear from Figure 3.2 that the 'Aggregate Contract-Level

³⁷ The PFE yielded by adding each of the underlying PFEs illustrated in Figure 3.1.

PFE' grossly overstates the Counterparty-Level PFE (PFE^{CP}) assuming there is no netting agreement in place.

- Aggregate Contract-Level PFE: Netted vs. Counterparty PFE: Netted

The 'Aggregate Contract-Level PFE: Netted' is simply calculated by the addition of the contract-level PFEs shown in Figure 3.1. Again, the result is clearly an overestimate of the netted counterparty-level PFE as illustrated in Figure 3.2.

It is clear from Figure 3.2 how effective a netting agreement can be in reducing or mitigating counterparty credit risk. It should be noted, however, that a netting agreement is only effective if there are economically offsetting positions in a portfolio – for example long and short positions in derivatives with the same or highly correlated underlying assets. If the portfolio does not have economically offsetting positions, a more effective credit risk mitigant is a collateral or margining agreement which is discussed in detail in section 3.3. Figure 3.2 also clearly illustrates the potential over-estimation of exposure that could occur by adding the contract-level PFEs instead of calculating the PFE using the counterparty-level MTMs (calculated under each scenario).

3.2.2 Example: Counterparty-Level PFE (Two Correlated Underlyings)

We now move on to a more complex example. Let's consider a portfolio of trades with a counterparty consisting of the following contracts:

<i>No.</i>	<i>Contract Type</i>	<i>Underlying</i>	<i>Notional</i> ³⁸	<i>Strike</i> ³⁹	<i>Maturity (Yrs)</i>
1	Forward	USD/ZAR	1,000	8.17	0.75
2	Forward	GBP/ZAR	-490	20.75	1.50
3	Call Option	GBP/ZAR	-500	15.45	3.00
4	Call Option	USD/ZAR	1,000	7.50	3.00

Table 3.2 - Example Portfolio 2

³⁸ The notional is expressed in the primary currency. In other words, if the currency pair is ABC/DEF then the Notional is expressed in ABC. Note that a negative sign indicates a short position.

³⁹ The strike is represented in terms of DEF (using the above example) and therefore has the meaning: "how many DEF for one ABC".

Note that the above portfolio has more than one underlying variable and therefore, in estimating the PFE^{CP} profile we would need to incorporate a measure of dependence in order to take account of the fact that the USD/ZAR and GBP/ZAR exchange rates are not statistically independent. The most widely used measure of dependence, also used extensively in practice, is linear correlation usually represented by ρ .

In our calculations, we use linear correlation as our measure of dependence, and the value for the correlation between the USD/ZAR and GBP/ZAR exchange rates is estimated from historical daily USD/ZAR and GBP/ZAR spot exchange rates. The following graph shows the daily spot exchange rates for the last 3 years:

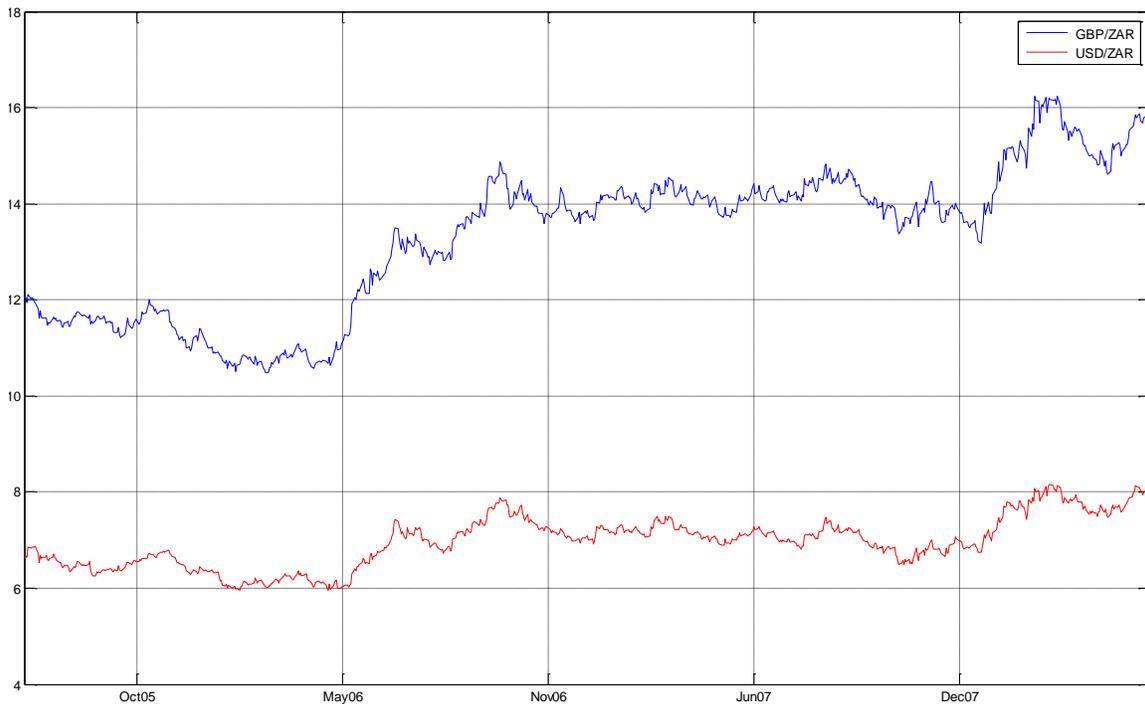


Figure 3.3 - USD/ZAR and GBP/ZAR Historical Spot Rates

Using this data, the estimate for ρ (the correlation between the USD/ZAR and GBP/ZAR exchange rates) is $\rho=92.89\%$ when using a 3 year history of spot prices. The estimated correlation value increases to 96.81% when based on only the last year's data.⁴⁰ We will

⁴⁰ The correlation estimates were calculated using the log-relative returns of the daily spot exchange rates. See Hull (2002).

estimate the counterparty-level PFE initially using $\rho=92.89\%$, but also in a separate graph illustrate the impact of the correlation estimate on the counterparty PFE.

We now turn to the simulation of the underlying correlated market variables. In order to estimate the counterparty PFE for this portfolio, which is dependent on correlated underlying market variables, we need to be able to simulate correlated market movements. This is achieved by the application of the following approach⁴¹:

Generating Two Correlated Random Variables

Let x_1 and x_2 denote independent samples from a univariate standardised normal distribution. Then one can obtain e_1 and e_2 which are correlated standard normal variables with correlation ρ using the following:

$$\begin{aligned}e_1 &= x_1 \\e_2 &= \rho x_1 + \sqrt{1 - \rho^2} x_2\end{aligned}$$

The Figure 3.4 shows the contract-level PFE profiles of the individual transactions that make up the portfolio described above.

⁴¹ The most widely used method for generating correlated random variables is the Cholesky decomposition approach.

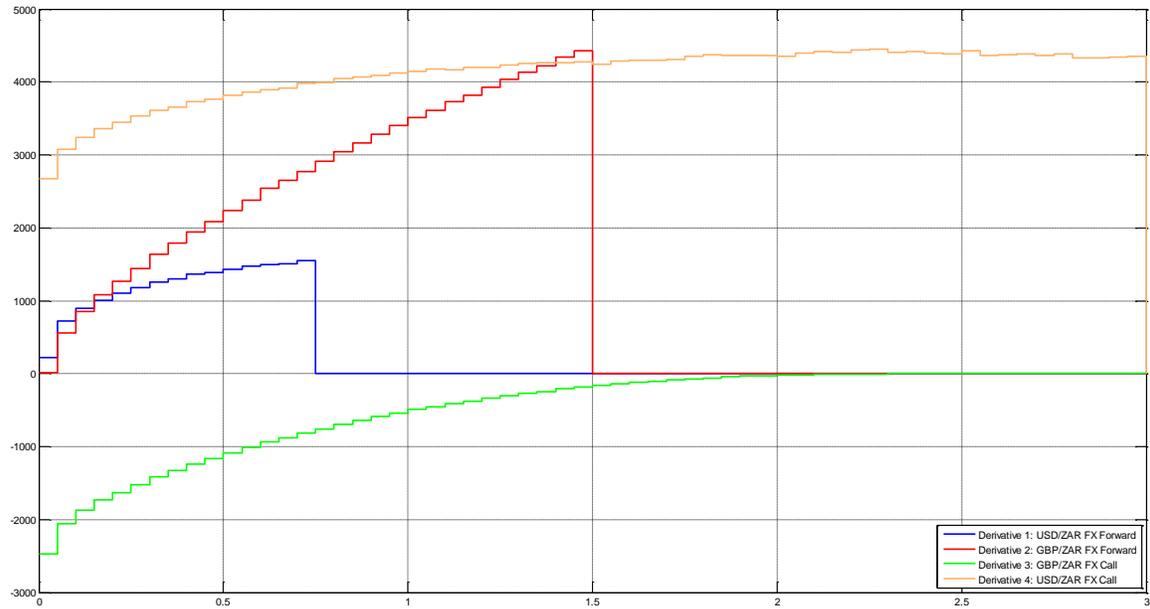


Figure 3.4 - Contract-Level PFEs of Individual Contracts

Using the correlation estimate above ($\rho=92.89\%$), it is now possible to estimate the portfolio-level PFE. The resultant netted counterparty PFE is illustrated in Figure 3.5, together with the aggregate contract-level PFE with netting appropriately applied (as specified in section 3.2.1 above).

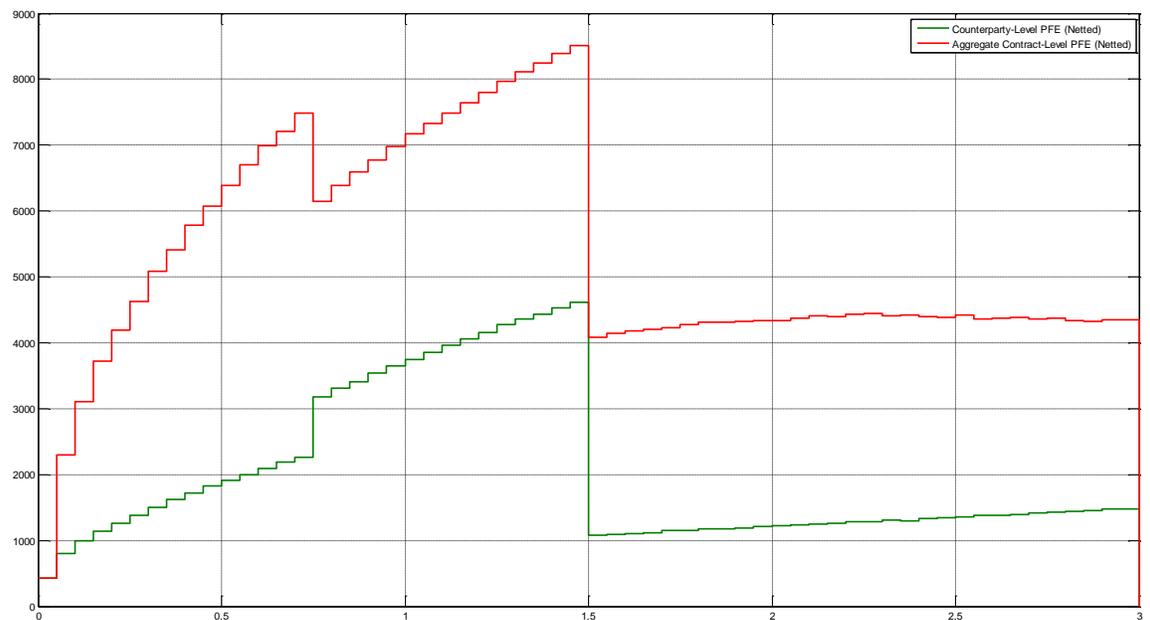


Figure 3.5 - Netted Counterparty Level PFE and Aggregate Contract-Level PFE (Netted)

In general, it is crucial to be able to interpret PFE estimates in order to make informed decisions. It is often important to understand under what economic circumstances a PFE profile would realise. In other words, it is important to ask the question: “Under what economic (market) conditions would the estimated 95% PFE occur or realise?” This is not only an important question on the contract-level, but also (if not more so) on a counterparty or portfolio level. For example, if one looks at the portfolio defined in Table 3.2 the value⁴² of derivative 1 will increase if the USD/ZAR exchange rate⁴³ decreases beyond the strike – in other words, the PFE will occur on a ZAR appreciation against USD. However, for derivative 2 the PFE will occur in the case of the Rand depreciating against the British Pound. Therefore, if one looks at these two positions, it would be quite unexpected, considering a correlation of more than 90%, to have the two positions not offsetting each other economically. The same holds for the two option positions.

The counterparty-level PFE estimate for the portfolio under consideration therefore has a very high dependency on the accuracy of the correlation estimate between the two underlying risk drivers – i.e. the USD/ZAR and the GBP/ZAR exchange rates. Figure 3.6 shows the PFE profile for various values of ρ and illustrates the importance of the correlation estimate in the calculation of PFEs on a counterparty level. The significant impact on the correlation estimate is evident from this illustrative example.

⁴² The value of a derivative contract is typically measured as the MTM.

⁴³ Technically speaking the T-day forward exchange rate determines the MTM value of a forward contract maturing in T days from today.

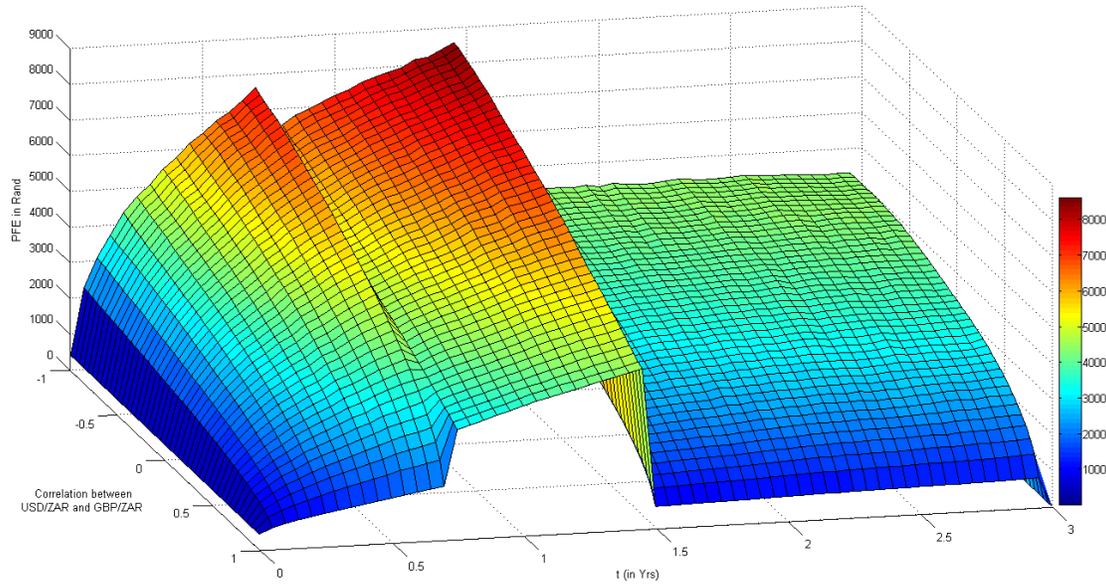


Figure 3.6 - Counterparty-Level PFE (Netted) using Various Correlations

It is interesting to note that, in Figure 3.6, the PFE profile corresponding to a very high correlation between the USD/ZAR and the GBP/ZAR exchange rates is very similar to the ‘Counterparty PFE (Netted)’ in Figure 3.5. Similarly, the PFE profile corresponding to a level of assumed correlation of $\rho=-1$ corresponds to the profile ‘Aggregate Contract-Level PFE (Netted)’ in the same figure. This result is not entirely unexpected, since the PFE calculated using a correlation of $\rho=-1$ assumes that the USD/ZAR and GBP/ZAR exchange rates will, throughout the simulated period⁴⁴, always move in opposite directions. Moreover, if one considers the individual contract-level PFEs in Figure 3.4, these profiles represent the ‘worst case PFE’⁴⁵ of each of the individual contracts based solely on the movement of the underlying market variable impacting that specific contract. The simulated exchange rate values that resulted in the estimated contract-level PFE profiles illustrated in Figure 3.4 are depicted below:

⁴⁴ Since we assume that ρ is constant throughout the life of the transaction.

⁴⁵ Strictly speaking this is not a worst case PFE but rather a worst case PFE assuming a 95% confidence level.

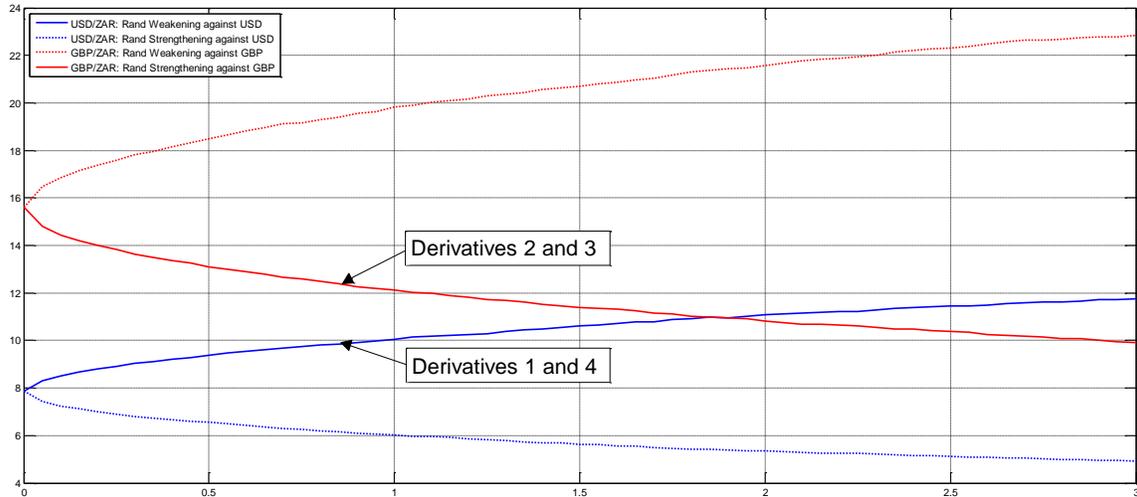


Figure 3.7 - Simulated Exchange Rates Resulting in Estimated Contract-Level PFEs

The different contracts each have their ‘worst case PFE’ occurring at scenarios which would normally not occur simultaneously⁴⁶. Previously, we have mentioned the fact that the different contracts constituting this portfolio are exposed to different movements of the underlying market variables. As is evident from Figure 3.7, the estimated contract-level PFEs materialise on scenarios where GBP weakens against the Rand (contracts 2 and 3) and where USD strengthens against ZAR (contracts 1 and 4) – and these two scenarios are only likely to materialise simultaneously if the two exchange rates exhibit a perfectly negative correlation.

Since a correlation assumption has such a significant impact on a counterparty PFE profile it is fair to question the accuracy and also the validity of the approach taken in estimating the PFE above. The first questionable assumption in the approach followed is the estimation of correlation from historical data – obviously the time period used plays an important role. Secondly, we have assumed that correlation is constant – which again is probably not a very realistic assumption when one considers the historical distribution of correlations.

⁴⁶ More specifically, we expect that these not occur simultaneously based on our observed historical correlation.

A natural way of extending the modelling technique used in terms of correlation is to assume that correlation is, in itself, a stochastic process⁴⁷. Although the idea of modelling correlation as a stochastic variable is a natural progression from what we have done so far it is far from a trivial exercise to implement and still an active area of research in the quantitative finance arena. The interested reader is referred to Van Emmerich, (2006).

3.3 Other Methods of Credit Risk Mitigation

We have discussed the impact of netting agreements as a form of credit risk mitigation. There are other risk mitigation techniques which also impact the measurement of counterparty credit risk (see Gibson, (2005) and Pykhtin M. ,(2005)). We will discuss two of these techniques: Collateral Agreements and Transaction-Specific Provisions.

3.3.1 Collateral Agreements

Often in the OTC derivatives markets, market participants agree to enter into ISDA Master Agreements which, among other details, contain provisions which specify close-out netting. In addition to this netting agreement certain counterparties also sign a Credit Support Annex (CSA). CSA agreements further permit parties to an ISDA Master Agreement to mitigate their credit risk by requiring the party which is ‘out-of-the-money’ to post collateral⁴⁸ corresponding to some agreed amount based on the current mark-to-market value of the agreements between the relevant counterparties. Normally, if the collateral is not cash its value will be reduced by a hair-cut to account for the risk inherent in the collateral itself⁴⁹.

In general, a collateral agreement contains the following parameters which each have a direct impact on the resultant counterparty credit risk mitigation effect:

⁴⁷ As opposed to assuming that the correlation parameter is constant.

⁴⁸ Normally cash, government securities or other highly rated bonds.

⁴⁹ For example, if non-government bonds are used as collateral, the party receiving the collateral is exposed to fluctuations in the market value of the collateral. This may result in the collateral decreasing in value and not covering the original risk the collateral was posted for.

- ***Threshold Amount***

This amount specifies the mark-to-market amount beyond which a collateral call will be made. For example, if this amount is 100 a collateral call will be made by counterparty A once the MTM value on its contracts with counterparty B exceeds 100. Let's denote the threshold amount by CT (short for Collateral Threshold).

- ***Minimum Transfer Amount***

This amount specifies the minimum amount by which the collateralised exposure can exceed the threshold amount before a collateral call is made. The reason for a minimum transfer amount is purely for cost purposes since in practice there is a cost associated with posting as well as receiving collateral.

Continuing with the above example, if this amount is 5, then counterparty A will only call for collateral if the collateralised exposure exceeds the threshold amount by at least 5 rand. For example, if the collateralised exposure is 104 no collateral call will be made but if the collateralised exposure is 105 a collateral call will be made. We denote the minimum transfer amount by MTA.

- ***Close-Out Period***

The close-out period specifies the number of days from the last collateral call until the positions with the defaulted counterparty will be closed out and the resultant market risk is re-hedged.

It is important to note that the longer this period the more risk the parties are exposed to, since the trades could potentially move more and more into the money for the counterparty which called for collateral increasing the exposure to loss. The close-out period generally covers the time it takes to recognise the default event, the decision to act and physically close out the positions and liquidate the collateral. Some agreements explicitly state waiting periods whereas some

jurisdictions require statutory periods. We will denote the close-out period by COP.

- ***Barrier Amount***

The barrier amount refers to the maximum amount of collateral that a party is willing to receive from its counterparty. This will impact the exposure, in that, once the collateral held exceeds this amount, no further collateral will be called and the resultant exposure will increase if the positions are not closed out. We denote the barriers amount by BA.

- ***Collateral Call Frequency***

The collateral call frequency specifies the frequency (i.e. daily, weekly etc.) at which collateral calls will be made. Practically, this refers to the frequency at which the parties will compare their collateralised exposure (i.e. current exposure *less* collateral held) with the threshold amount (TA). If the collateralised exposure exceeds the TA by the MTA then a collateral call will be made.

Incorporating the effects of collateral agreements into PFE estimation is relatively straightforward (in the case of cash-collateral) since it is logic which is applied to the Counterparty-Level PFE profile simulated as normal. The most basic approach is to assume that the exposure of a collateralised portfolio cannot exceed the threshold amount. The resultant collateralised counterparty-level, $PFE_{COL}^{CP}(\alpha, H, t)$, will therefore be calculated as follows:

$$PFE_{COL}^{CP}(\alpha, H, t) := \min[PFE^{CP}(\alpha, H, t), CT],$$

with PFE^{CP} as defined in 3.2.

The equation for $PFE_{COL}^{CP}(\alpha, H, t)$ above implicitly assumes that collateral is received instantaneously following a collateral call and that positions can be closed out immediately following the failure to post such collateral called. In reality, however, there

is a significant lag in the time between when collateral is called and when collateral is received. During this time lag, the collateralised exposure can increase significantly. There are, however, three possible approaches to incorporating the lag effects of collateral agreements into a PFE profile⁵⁰:

- Full Simulation
- Analytical Lag Add-Ons
- Post Simulation Sampling.

We demonstrate the full simulation approach in an example. The approach involves augmenting the original time steps of the simulation to also include the time steps at which collateral is received, taking account of a time lag between the time at which a collateral call is made and the time at which the collateral is assumed to have been received.

For example, if the close-out period is k days it is reasonable and prudent to assume that the collateral that was called at time t_j will only be received at time t_j+k . The effect of such an assumption is that the PFE profile will potentially continue to increase beyond the threshold amount after time t_j , and only be reduced by the collateral called at time t_j , at time t_j+k . The PFE profile will therefore have to include values for the portfolio on t as well as on t_j+k .

⁵⁰ De Prisco and Rosen (2005)

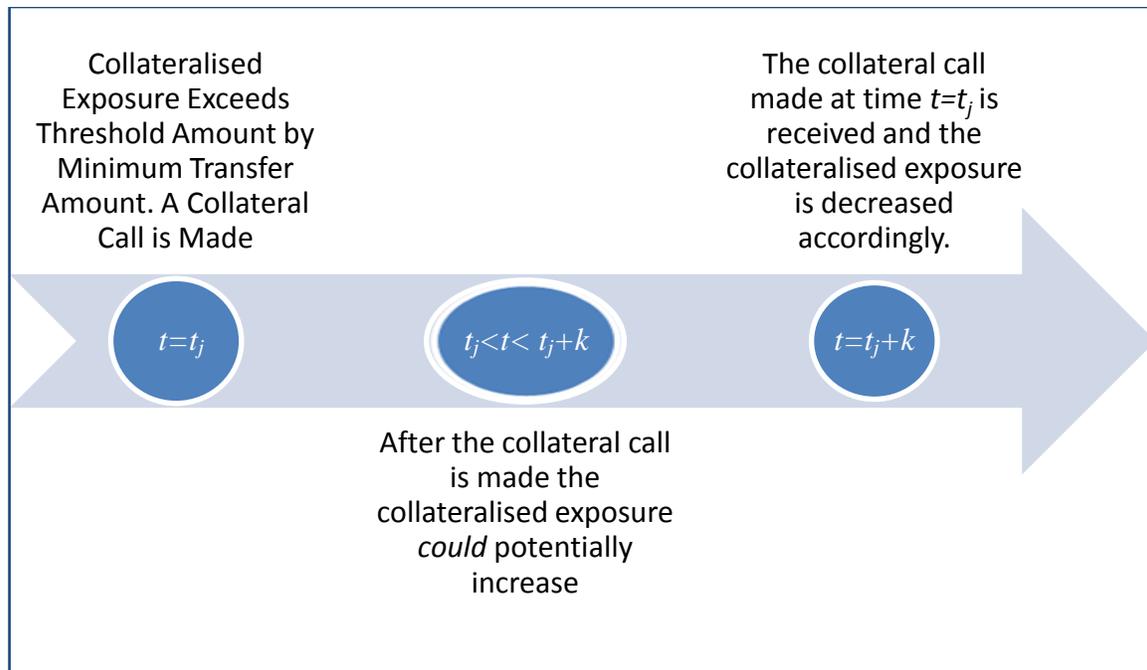


Figure 3.8 - The Lag Period in Collateral Agreements

3.3.1.1 Example: Counterparty-Level PFE, the effect of cash collateral

We now consider the effect of a collateral agreement on the portfolio discussed in Table 3.2 using the full simulation approach⁵¹. Firstly, let's assume that the parameters of the collateral agreement are as follows:

<i>Parameter</i>	Value
Threshold Amount	1,500
Minimum Transfer Amount	275
Close-Out Period (in days)	10
Barrier Amount	2,500

Table 3.3 - Assumed Parameters of Collateral Agreement

Considering the netted, uncollateralised PFE profile in Figure 3.5 it is clear that the threshold amount is significantly lower than the general level of the PFE profile. It is expected that the collateralised PFE profile should be very close to this threshold

⁵¹ See De Prisco and Rosen (2005) for details on the simulation approach.

amount⁵², since this is at the end of the day what the collateral agreement aims to achieve. In order to make the calculations in this example more practical⁵³, we have adjusted the calculation algorithms used in calculating the PFEs in Section 3.2.2 slightly so that the time steps in the PFE profile are in daily intervals.

Figure 3.9 demonstrates the effect of the collateral agreement specified in Table 3.3 on the counterparty-level PFE profile. The graph also illustrates the mechanics of the collateral agreement which is discussed in detail below.

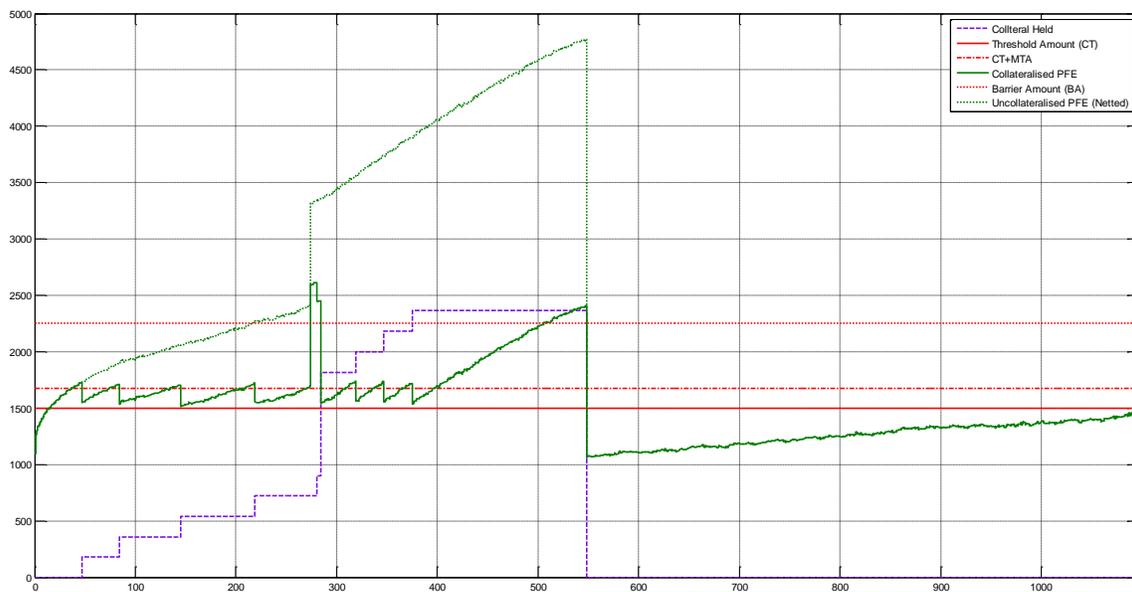


Figure 3.9 - Collateralised Counterparty-Level PFE

At first glance Figure 3.9 appears quite complex. There are, however, quite a number of very valuable insights that can be obtained from closer inspection. Firstly, let's discuss the three horizontal lines:

- the *CT* line represents the Threshold Amount,

⁵² The longer the COP, the more time allowed for the uncollateralised exposure to increase beyond the threshold amount.

⁵³ With the use of the word practical it is not intended to imply that in practice simulations are run using 1 day time steps, rather that the time steps are usually in multiples of one day (as opposed to the time steps of 0.05 years \approx 18.25 days used in Section 3.2.2 which make the application of collateral agreements unnecessarily complicated).

- *CT+MTA* line represents that barrier that the collateralised PFE needs to exceed before a collateral call is made, the
- Barrier Amount line indicates the collateral held amount beyond which no more collateral calls will be made.

These lines aid in the understanding of the collateralised PFE profile. We now discuss the other elements of the graph in detail:

- ***Uncollateralised PFE (Netted)***

This profile is the result from the example portfolio with correlated underlyings in 3.2.2 with netting and correlation taken into account.

- ***Collateral Held***

The collateral held represents the total collateral that is assumed to have been received at a point in time. Recall that we assume that collateral called at time t will only be received at time $t+COP$. In the calculations which produced Figure 3.9 we have also assumed that collateral refunds (i.e. when the collateral held is in excess of what is required, typically following a sudden decrease in exposure) are instantaneous (i.e. no lag).

- ***Collateralised PFE***

The most important element in Figure 3.9 is the collateralised PFE which is an estimate of what the counterparty credit exposure could be at each point in time over the life of the portfolio. The collateralised PFE profile in relation to the threshold amount and the *CT+MTA* line gives insight into the mechanics of the collateral agreement and its effectiveness in reducing the counterparty-level PFE.

In the event that the collateralised PFE exceeds the CT+MTA line⁵⁴ a collateral call is made.

Since the close out period is 10 days, the collateralised PFE could potentially increase from the date at which the collateral call was made until such time as the collateral is received. This situation is demonstrated in Figure 3.10 and Figure 3.11 below, followed by a discussion.

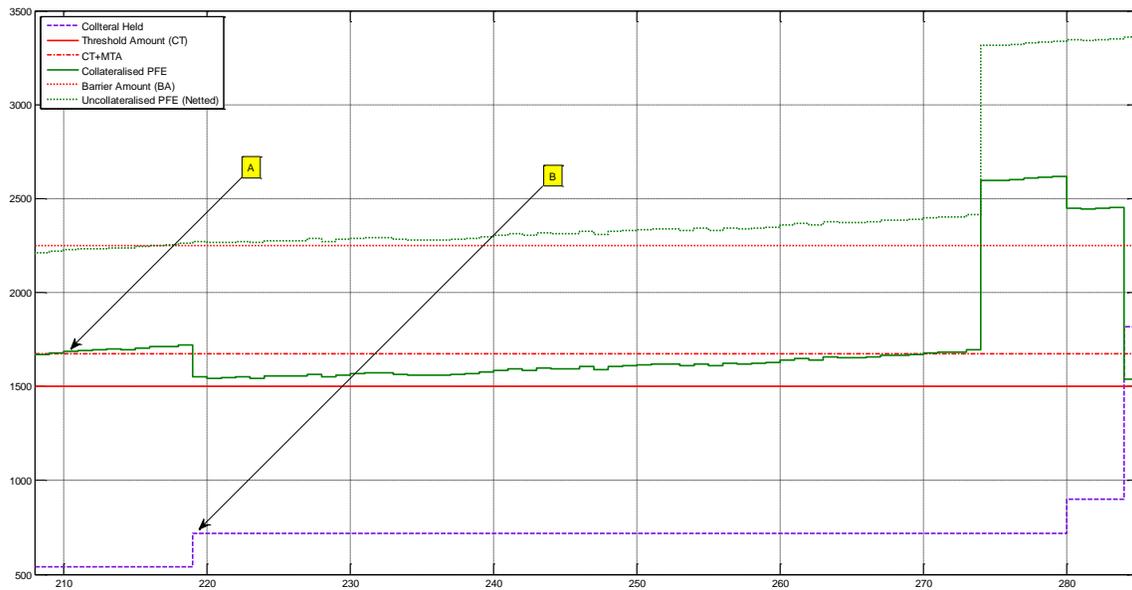


Figure 3.10 - Collateralised Counterparty-Level PFE, ($208 < t < 285$)

It is clear from Figure 3.10 that, at point $t=209$ (indicated by arrow A), the collateralised PFE exceeds the CT+MTA line, indicating that a collateral call is justified. From the assumptions regarding the close-out period, however, the collateral is only assumed to be received at time $t=219$ (indicated by arrow B). Therefore, between $t=209$ and $t=219$ the exposure has the potential to increase as illustrated by the collateralised PFE. Figure 3.11 illustrates the timing and extent of the collateral calls for the same interval as in Figure 3.10.

⁵⁴ When the current collateral held is not adequate and the collateralised exposure exceeds the threshold amount by an amount larger than the minimum transfer amount.

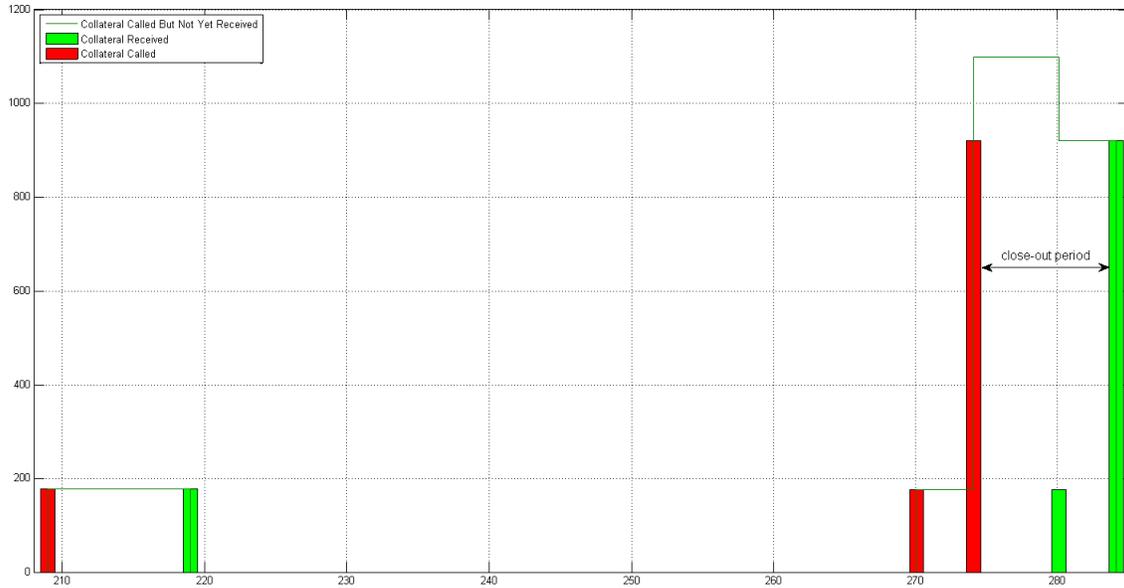


Figure 3.11 - Collateral Agreements: Projected Collateral Calls

It is clear from the discussion above that the close-out period as a parameter of the collateral agreement has a significant impact on the resultant collateralised credit risk exposure as illustrated by the estimated collateralised PFE profile. We will now look at a few examples which illustrate the impact of some of the other parameters of a collateral agreement. Figure 3.11 illustrates the impact that a change in the assumed close-out period has on the resultant collateralised PFE profile. The graph illustrates the collateralised exposure profiles for different values of COP.

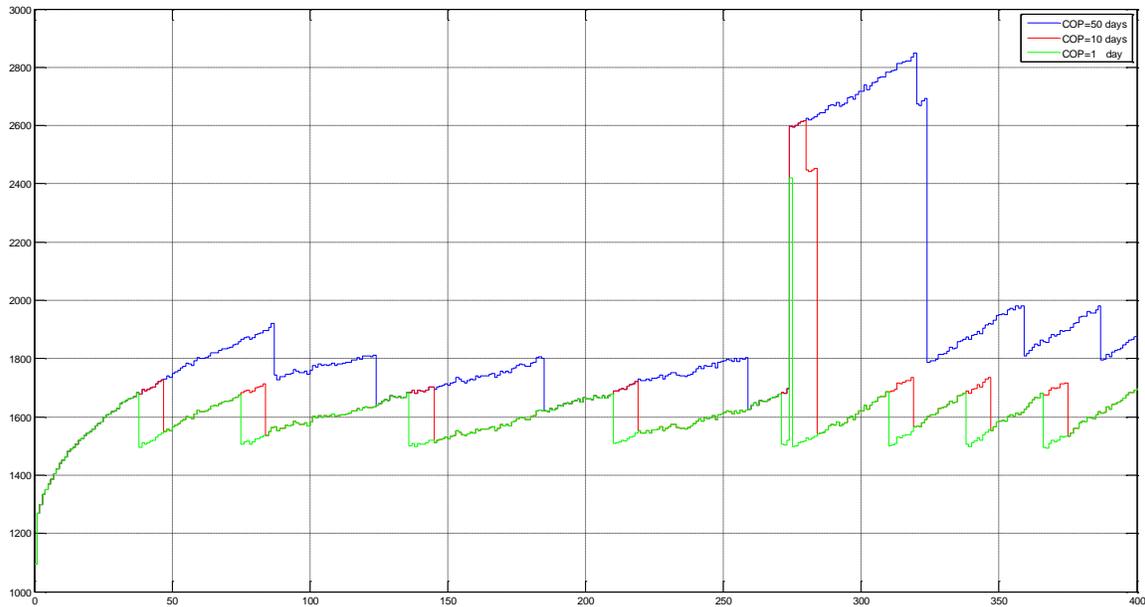


Figure 3.12 - Collateralised PFE for Various Close-Out Periods ($0 < t < 400$)

3.3.1.2 Treating the Value of Collateral as Stochastic

We have thus far assumed that the collateral that was posted remained constant (i.e. we have assumed that the collateral is cash) through time. This is a simplifying assumption and, in practice, it is often not the case. Modelling collateralised PFE profiles assuming risky collateral, and thereby treating the collateral as a stochastic process, is significantly more complex than what has been demonstrated so far. The reader is referred to Cossin and Hricko (2005) who propose a model for stochastic collateral. In the model, the price of the asset given as collateral is assumed to follow a GBM process.

3.3.1.3 Credit State Dependent Collateral

Some CSA agreements define thresholds, upfront collateral amounts and minimum transfer amounts as a function of both the counterparty's and the bank's credit states. If the counterparty is not rated, thresholds may be defined as a function of some financial ratio (e.g. debt to cash-flow) typically used as proxies for its credit state or financial health. The modelling of collateralised PFE profiles with credit-dependent collateral agreement parameters is significantly more complex than the simple case demonstrated in Section 3.3.1.1 above. This typically requires an integrated market and credit risk model

to capture the exposures that are market dependent as well as the collateral balances which are dependent on the current market levels and the credit states of both parties.

Therefore, in addition to the simulated future market values of the risk drivers which determine the potential future exposure of the underlying portfolio, one would also need to simulate the joint credit-state probabilities of both parties at each time-step. A Markov process approach for the case of a one-sided collateral threshold with a publicly-rated counterparty can be used in this case⁵⁵. The Markov process assumes as input a credit migration matrix often published by the credit rating agencies⁵⁶.

3.3.2 Transaction-Specific Documentation

So far we have discussed netting agreements and collateral agreements which both affect the counterparty-level exposure directly and reduce potential future exposure as a result. There are, however, other techniques of credit risk mitigation which impact the contract-level exposure directly.

3.3.2.1 Contract-Level Credit Risk Mitigation

In some derivative contracts, it is possible to make relatively small changes to certain terms and conditions of ‘vanilla’ derivative contracts and significantly reduce the counterparty credit risk associated with these contracts. Although it could be argued that this is merely an application or an extension of the collateral agreement technology. The following is a high-level overview and the purpose is merely to give the reader an overview of the effect of contract-level credit risk mitigation techniques. The absolute values of the exposures are therefore entirely fictitious and are for illustrative purposes only.

Let’s consider an example of a 20 year cross-currency interest rate swap (CC Swap). Consider a USD/ZAR cross-currency interest rate swap based on a notional amount of

⁵⁵ De Prisco and Rosen, (2005).

⁵⁶ E.g. Fitch, Moody’s and Standard and Poor’s (S&P).

1,000,000 USD. A vanilla cross-currency swap involves an exchange of principal at outset with an agreement to swap the principal back at *maturity*. During the life of the contract – i.e. between the original exchange of principal and the final exchange of principal at maturity each party pays the other interest on the principal held, in the appropriate currency (i.e. each party pays the other interest on the notional it holds, in the currency of the notional it holds. The interest rate used to calculate the payments due, is also typically the 3 month inter-bank rate prevalent in that particular currency). Since the payments are in different currencies these amounts are not netted.

Certain banks have altered the terms and conditions of the standard cross currency swap agreement yielding what is sometimes referred to as a mark-to-market reset cross-currency interest rate swap or a pull-to-par cross currency swap. In the case of the MTM reset cross currency swap the two parties exchange principal at outset (as is the case in the vanilla case) but also at each quarterly reset date, thereby reducing the major contributing risk: the foreign exchange risk. Let's now compare the cash flow profiles of a typical vanilla cross currency interest rate swap with that of the mark-to-market reset cross currency interest rate swap to highlight the major differences between the two derivatives:

- At outset, principal is exchanged based on the most current exchange rate in the market at that point in time. This is true in both the vanilla and MTM Reset derivatives.
- During the life of the contract, in the case of the vanilla trade and the MTM Reset derivative, there are quarterly interest payments by each party. In other words, the party receiving the USD principal at outset will pay interest payments on the USD principal at the appropriate USD 3 month inter-bank floating rate. In the case of the MTM reset CC Swap there is also a principal payment at each interest rate reset date (i.e. quarterly). The purpose of this principal payment is to extinguish any mark-to-market value which developed since the last reset as a result of (mainly) exchange rate movements. The result of this exchange in principal is

that, following each quarterly interest and principal payment, the mark-to-market value on the CC Swap will be zero (at least for a short period).

- At maturity, in the case of both the swaps, principal is exchanged based on the most current exchange rates.

Although a small change in the terms and conditions of the CC Swap, it has a significant impact on the counterparty credit risk exposure – especially for swaps with long maturities. The reason for this reduction is that, in the case of the vanilla swap, the potential impact of exchange rate moves during reset dates is much larger than in the case of the MTM reset CC Swap. In essence, both the vanilla cross currency swap and the MTM reset swap exposes both parties to a 20 year exchange rate movement – but the MTM reset cross currency swap does so in quarterly intervals. The following figure illustrates the typical contract-level PFE profiles for the two swaps.

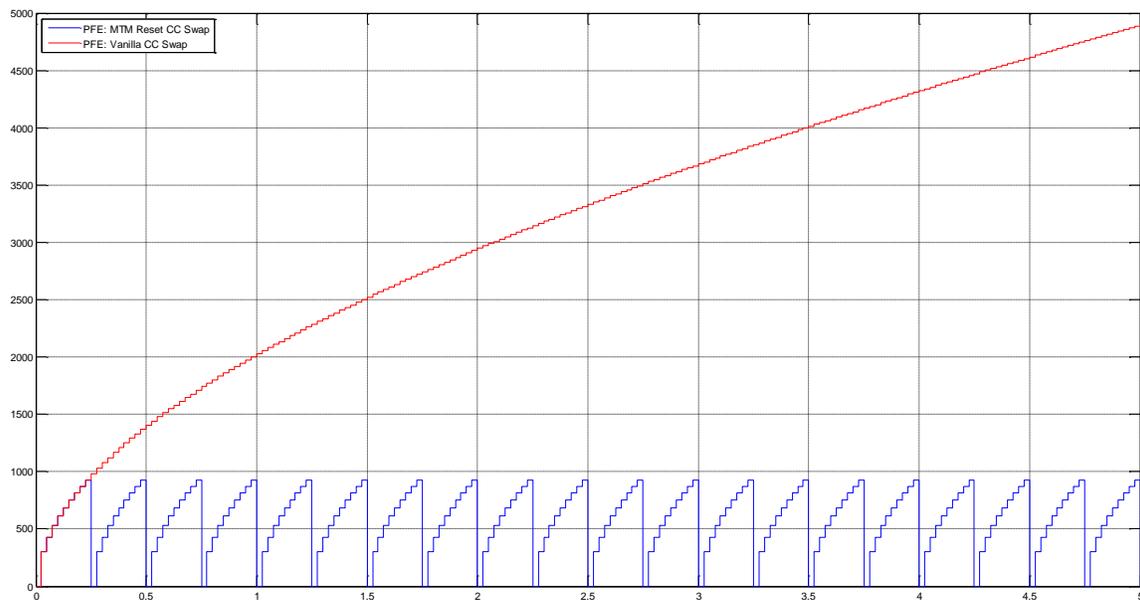


Figure 3.13 - Contract-Level PFE of MTM Reset and Vanilla Cross-Currency Interest Rate Swaps

3.4 Impact of Credit Derivatives on Counterparty Credit Risk

Due to the significant expansion of the OTC Credit Derivatives market over the past decade banks often are faced with the challenge of measuring counterparty credit risk on portfolios which not only include a large number of OTC contracts on equities, interest rates and other underlying assets but also often a significant amount of credit derivatives. Including credit derivatives into a portfolio of non-credit derivatives contracts significantly adds to the complexity of measuring counterparty credit risk.

To illustrate this point, let's consider a bank which buys credit protection, from counterparty ABC, in the form of a credit default swap (CDS) on one of its other counterparty's, DEF. Also, let's assume that the bank already has credit exposure against DEF and ABC in the form of loans (traditional primary risk) and other derivative contracts (counterparty credit risk).

Conceptually, a credit default swap achieves very much the same as a standard insurance contract. In this example, the bank agrees to pay a fixed premium (typically quarterly), to ABC, and in return receives a commitment from ABC to compensate the bank for credit losses in the event of a credit default by DEF. In practice, however, a CDS typically has a specific reference asset (i.e. for example a publicly traded bond issued by DEF) upon which events constituting a credit event (typically restructuring of the debt or defaults on the payments under the debt issued, etc.) are based. CDS contracts are also typically over-the-counter and so can be tailored to the specific needs of the protection buyer. Since the bank already has some credit exposure against DEF, it is reasonable to assume that this credit exposure should, at least to a certain extent, be offset by the amount of credit protection bought. The big question is "by how much?" and also "which credit exposures can one offset?" If we consider the counterparty to the CDS, ABC, it is also important to realise that the bank has a reliance on ABC to honour its obligation under this CDS – in very much the same way as the bank relies on any other counterparty to a non-credit derivative contract. The bank therefore has counterparty credit risk exposure to ABC as a result of the CDS.

The situation above adds a considerable amount of complexity to the modelling of a counterparty-level PFE profile for ABC. The source of the added complexity is the fact that the simulated scenarios would now need to include scenarios of a risk driver which drives the MTM of the CDS. The MTM value of a CDS is influenced by the credit quality of the reference asset – the more likely the reference asset (or name) is to default, the higher the mark-to-market value for the protection buyer. The diagram below illustrates the dynamics of this CDS contract and indicates the resulting credit risk exposure impact.

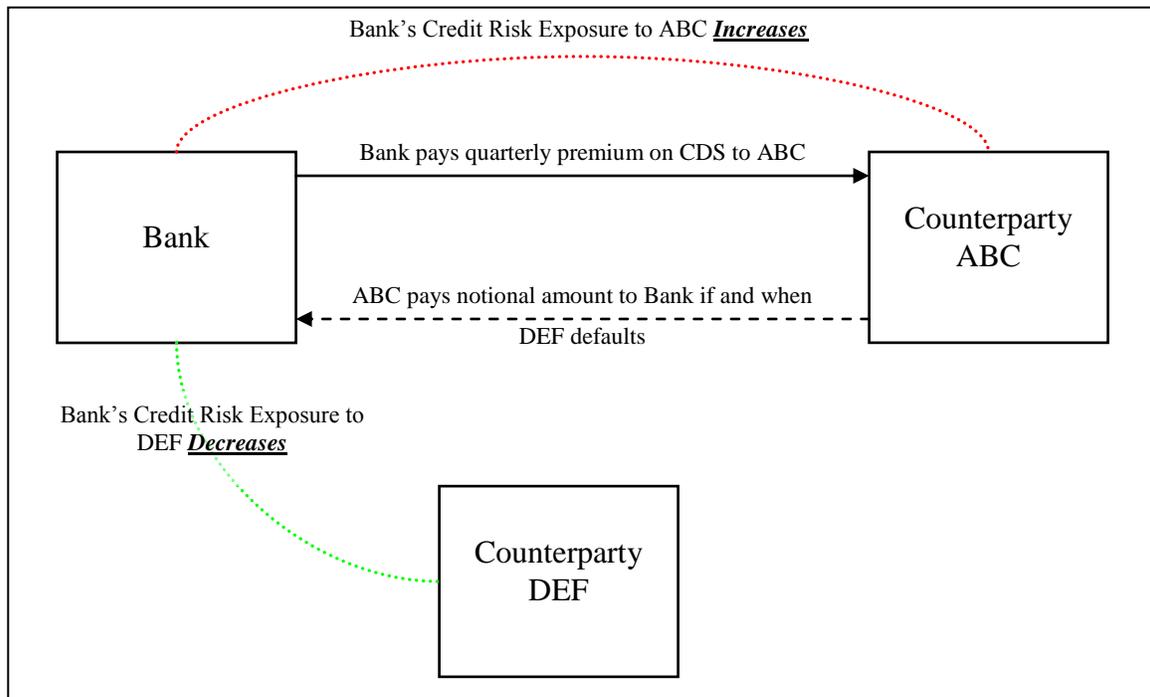


Figure 3.14 - Dynamics of CDS and Impact on Credit Risk Exposure

In our definition of PFE (Section 2.2.2 and Section 3.2) we defined Ω_t^i to contain all prices of market variables required to calculate the value of the derivative(s) under consideration. This can be extended to include credit spreads which are used to value credit default swaps and other credit derivatives. This is necessary for the calculation of counterparty-level PFE profiles on portfolios of derivatives which include credit

derivatives. The difficulty is, however, that one would need to make an assumption regarding the evolution of credit spreads⁵⁷.

Let's firstly consider the problem of modelling the counterparty credit risk associated with the counterparty to the credit derivative and then proceed to discuss the calculation of the impact to the credit exposure to the issuer (or referenced name⁵⁸).

3.4.1 Counterparty Credit Risk Associated with the Credit Derivative Counterparty

In the examples of counterparty credit risk estimation thus far the credit exposure was relatively easy to estimate since the underlying risk drivers are assumed to be independent of the counterparty's financial health (i.e. credit state). For credit derivatives, however, this is not the case since there is more often than not a positive correlation between the credit quality of the referenced credit and the counterparty to the credit derivative. We will look at two models which attempt to address the problem of estimating the counterparty credit risk under a credit derivative.

3.4.1.1 The Hazard Rate Model

The term 'hazard rate', denoted by $h(t)$, originated in survival analysis and can be interpreted as the instantaneous probability of default at t .⁵⁹ Often, practitioners make use of a model for the hazard rate. There are a number of motivations for modelling the hazard rate⁶⁰:

⁵⁷ Assume a stochastic process which can be used to simulate credit spreads in a similar way in which we have applied GBM in modelling the FX spot exchange rate and the CIR model to simulate the short rate of interest.

⁵⁸ It is possible for a CDS contract to merely reference a name and a level of debt seniority. For example, a CDS can be tailored to have as default event any default on debt of the class Senior Unsecured. Clearly this is more general than merely referencing a specific bond and the prices and the risks associated with such contracts is therefore different.

⁵⁹ McNeil and Embrechts (2005)

⁶⁰ Li (2000)

- The hazard rate, $h(t)$, provides information on the immediate default risk of each non-defaulted counterparty, at time t .
- The comparisons of groups of individual counterparties are most incisively made via the hazard rate function,
- A model based on hazard rate can be easily adapted to more complex situations such as multiple-underlying credit derivatives (e.g. basket CDS contracts⁶¹).
- There are a lot of similarities between the hazard rate function and the short rate of interest. Many modelling techniques for short rate processes can, as a result, be readily applied to modelling the hazard rate.

The relationship between the hazard rate, $h(t)$, the forward hazard rate, $h(t,T)$, and the survival probability $\psi(t,T)S(t,T)$ is:

$$\psi(t,T) = \mathbb{E}_t \left[e^{-\int_t^T h(s) ds} \right] = e^{-\int_t^T h(t,u) du}$$

and

$$h(t,t) = h(t).$$

Where \mathbb{E}_t denotes expectation in the risk-neutral measure conditional on the information at time t ⁶².

It is possible to treat the hazard rate similar to an interest rate and use models such as the Hull-White model or the exponential Vasicek model to simulate the hazard rate process $h(t)$ and logarithm of the hazard rate $\ln[h(t)]$ respectively. One very important draw-back of the hazard rate model is the fact that the model is not very applicable to less liquid

⁶¹ Basket CDS contracts are CDS contracts where the underlying is a group ('basket') of names. The pay-out trigger can be a first-to-default or even n^{th} to default event.

⁶² For a detailed derivation of the hazard rate function and some background on the motivation for this approach refer to Li (2000).

credit derivative markets such as in South Africa. When the market is not liquid, it is very difficult to estimate the required model parameters since the observable credit spreads are not always reliable or frequently updated.

3.4.1.2 The Ratings Transition Model

Hille, Ring, and Shimamoto (2005) propose a model for CDS counterparty credit exposure which uses innovative ideas. In summary, the model involves the following steps (for each market scenario and time step):

- Calculate spread curves and risk-neutral default probabilities for each possible credit state (usually credit ratings are used as credit states) of the underlying issuer
- Calculate a set of CDS contract values using the risk-neutral probabilities
- Calculate the probabilities of the issuer being in each credit state (conditional upon the counterparty defaulting at that point in time)
- Enhance the original market scenarios with the new distribution of CDS values and probabilities
- Calculate all exposure measures using this enhanced distribution.

The key model features, which are important to take into account when modelling exposures from credit derivatives, are:

- The underlying risk drivers are not independent of the counterparty's financial health. The joint credit process is modelled and credit quality correlation is an integral part of the model.
- Exposure is conditional on a marginal default window.

- The joint ratings transition is modelled through a combination of Markovian transition matrix evolution and a structural asset-value model. The latter introduces credit quality correlation through correlated asset values, equivalent to using a Gaussian copula credit risk model originally introduced by Li (2000).

This model is semi-analytical, which is an advantage since it enables fast and reliable computation of counterparty credit risk. Another major appeal of this model framework is the use of the credit ratings transition matrices. The short-coming of the hazard rate model in terms of its application to markets which are less developed and less liquid can be overcome to a certain extent in this approach.

Hille, Ring, and Shimamoto (2005) mention the possibility of incorporating a representative or average par spread curve for each rating state (credit rating) from a generic rating-spread-table. This generic rating-spread-table can be constructed by using the average of the credit spreads of credits which are actively traded and publicly rated. This approach is quite a rough approximation since it assumes that changes in credit spreads are only due to changes in credit rating which underestimates credit spread volatility. The one major advantage is that the model can be applied to instances where the reference asset or reference name is not actively traded and does not have an observable and reliable credit spread curve by applying the average par spread curve mentioned above.

3.4.2 The Reduction in Credit Exposure to the Referenced Issuer/Name

As discussed in Section 3.4 above, it is clear that buying protection on DEF from ABC should, in some way, result in a reduction in the credit exposure that the bank has to DEF. The most important motivation for determining such an impact is a practical one. In a bank it is very important to be able to reflect and monitor the credit exposure that the bank has to its various counterparties. It is therefore crucial to the bank to be able to

reflect the impact of the protection bought, through the CDS, on its exposure to DEF. One simple approach to capturing this impact within a limits system is through the creation of an *offsetting instrument* for the CDS. This answers the question of *how* this impact will be reflected. The more important aspect is the problem of quantum. How big should this offset be? It's important to highlight that using CDS contracts to offset existing exposures in general is not entirely clean.

It is very important to determine to what extent the CDS will, in reality, mitigate the current and future credit exposures to the particular counterparty. If the bank, for example, bought the credit protection on a specific bond issued by DEF one would need to determine the seniority of the bond in relation to the other existing exposures and also the impact that this possible mismatch in seniority between the credit protection and credit exposure will be⁶³. If the bank does not hold the exact reference asset but one with the same issuer but different seniority, then the net exposure to the issuer is not completely eliminated and can be calculated using the following⁶⁴:

$$Net\ Exposure = \max \left[0, \frac{Notional \cdot (rr_{CDS} - rr_{Bond})}{1 - rr_{Bond}} \right],$$

where rr_{CDS} is the recovery rate estimated⁶⁵ for the reference asset and rr_{Bond} is the recovery rate estimated for the asset (or exposures) held. The full exposure amount is therefore offset if the CDS is contracted on a reference asset that is of equivalent or more junior quality to the exposure held.

3.5 Other Counterparty-Level Issues to be considered

⁶³ Seniority relates to the 'pecking order' in the event of default. If one bond 1 is senior to bond 2 then, in the event of default, holders of bond 1 have claim on the defaulted counterparty's assets before the holders of bond 2. It is therefore expected that holders of senior bonds will have a higher recovery rate.

⁶⁴ De Prisco and Rosen (2005)

⁶⁵ In practice some CDS contracts have a specified recovery rate, sometimes referred to as a digital or binary CDS. In this case the rr_{CDS} would therefore be known.

3.5.1 Consistency in Modelling across products

It is very important to apply a general and consistent approach to the problem of counterparty credit risk measurement. The interpretation of an exposure estimate should always be consistent across products and should, as far as possible, achieve results which are expected (from an economic and financial perspective). For example, if one measures the contract-level PFE of an FX Forward (Section 2.2.4, pp 17) and, in a separate exercise, measure the counterparty-level PFE with netting applied on a portfolio consisting of two FX Options (using Put-Call Parity) achieving the *exact* same payoff as the FX Forward one should yield the same resultant PFE. This concept is an extension of the no-arbitrage assumption in derivative pricing. Let's consider the following two portfolios⁶⁶ to illustrate this point:

Portfolio 1: Long FX Forward on 1000 USD		Portfolio 2: 2 FX Options on 1000 USD	
<i>Contract Details</i>	<i>Payoff at Maturity</i>	<i>Contract Details</i>	<i>Payoff at Maturity</i>
Long Forward	$S_T - K_{USD/ZAR}$	Long Call Short Put	$\max[S_T - K_{USD/ZAR}, 0]$ $\min[S_T - K_{USD/ZAR}, 0]$
Portfolio 1	$S_T - K_{USD/ZAR}$	Portfolio 2	$S_T - K_{USD/ZAR}$

Table 3.4 - Portfolios Used in Cross-Model Consistency Example

Figure 3.15 illustrates the payoff profiles for the individual derivatives in Portfolio 2, followed by the payoff profile of Portfolio 1. This also shows (visually) that the payoffs under the two portfolios are equal under all scenarios.

⁶⁶ All derivatives in the portfolios have the same, 6 month maturity, and all options are European-style. Also note that $K_{USD/ZAR} = 8.4947$, the 6-month forward exchange rate.

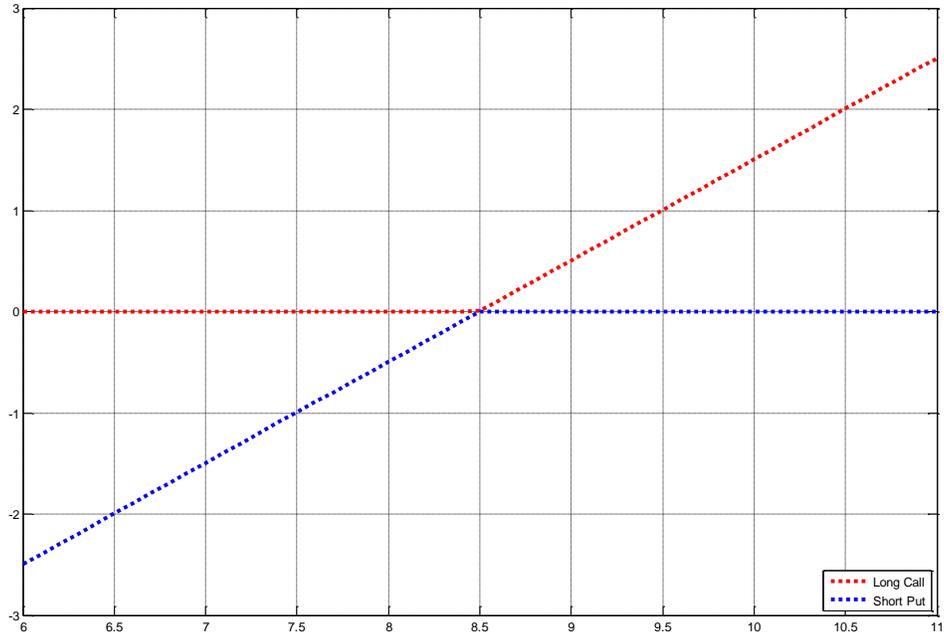


Figure 3.15 - Payoff Profiles of the Derivatives in Portfolio 2

When the payoff of the call and the payoff of the put are added it results in the same payoff under all scenarios as that of the long FX Forward, shown in Figure 3.16 below.

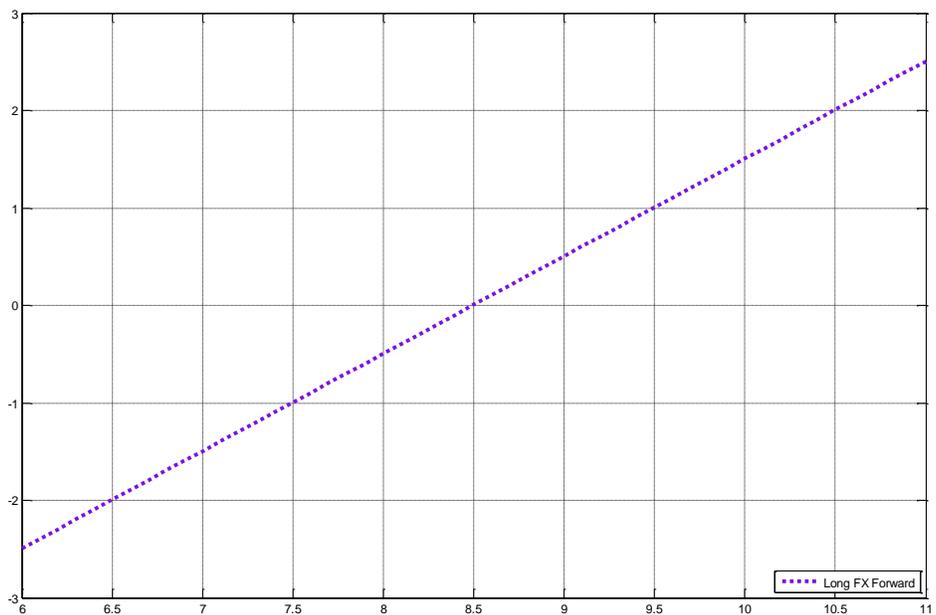


Figure 3.16 - Payoff Profile of the Derivative in Portfolio 1

This is expected since, as shown in Table 3.4 above:

$$\max[S_T - K_{USD/ZAR}, 0] + \min[S_T - K_{USD/ZAR}, 0] = S_T - K_{USD/ZAR}.$$

The approach followed in the generation of future spot exchange-rate scenarios is that derived in section 3.5.3.2 below, which ensures economic consistency. We have also applied the same method of perturbing the forward exchange rates (implicitly assuming that interest rates remain constant) as in section 2.2.4 above. The resulting counterparty-level PFEs are given in Figure 3.17.

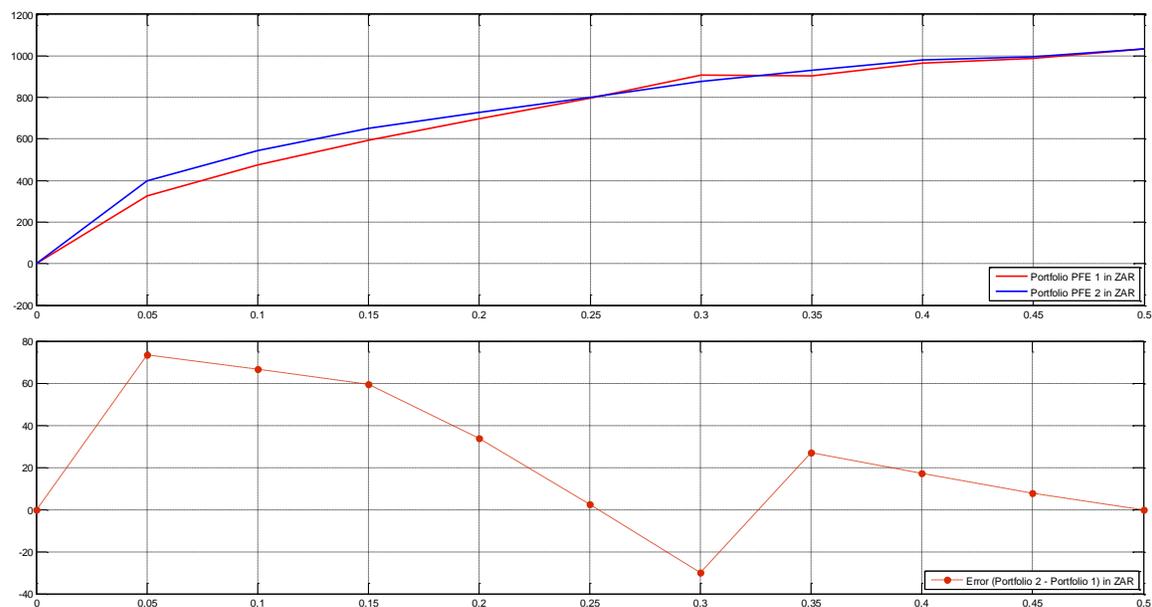


Figure 3.17 - Counterparty-Level PFE Comparison: Synthetic Forward vs. Regular Forward and Associated Differences in Estimates

The results above are satisfactory from a practical point of view since the errors, in relative terms, are not great considering that the contracts are based on a USD 1,000 notional. From a theoretical point of view though, it would be good to understand the reasons for the differences in the estimates. The following points are some of the main reasons for the differences between the counterparty-level PFE estimates in Figure 3.17 above:

- Firstly, we have assumed that there is a flat 12% interest rate for discounting purposes. In addition to that, we have assumed for the sake of pricing the FX

forward through time that the relative shape of the forward FX curve remains constant. This second assumption implies that the interest rate differential for all maturities remains constant. This interest rate differential is however observed from data from the actual market which means that the interest rate curves are certainly not flat. The two assumptions regarding the interest rates are therefore slightly inconsistent and therefore contribute to the disjoint results between the portfolios.

- Secondly, we have assumed that the implied volatility in the Black Scholes pricing formula remains constant. This, again, is unrealistic and should ideally be adjusted according to some volatility smile as the simulated mark-to-market values move in or out of the money over the life of the contract.
- Lastly, the time steps used for valuation were purely for ease of exposition and therefore the forward exchange rates had to be interpolated from the actual market-observed curve. Linear interpolation was used and could therefore also have an impact on the marked differences between the counterparty-level PFE results.

Another elementary method of testing consistency in the modelling of PFEs across products would be the case of the relationship between an Interest Rate Swap (IRS) and a string of Forward Rate Agreements (FRAs). Essentially, a vanilla fixed for float interest rate swap can be broken down into a string of consecutive FRAs. The PFEs of the IRS and the string of FRAs should therefore be equal.

3.5.2 Consistency in Modelling across Risk Drivers

In all the portfolio-level PFE examples we have discussed so far we have only had portfolios which consist of contracts based on very similar risk drivers. For example, we have considered portfolios consisting only of currency derivatives, which make the modelling of the future risk driver scenarios relatively easy. Unfortunately, in practice,

we are less fortunate and the situations in real life can be exponentially more difficult. If one considers, for example, a portfolio which, not only consists of foreign exchange derivatives but also interest rate derivatives and perhaps even a few commodity trades. All of the underlying risk drivers are, to some extent, correlated which would ideally need to be incorporated into the modelling. Moreover, some of the risk drivers in the portfolio are also directly related in practice. An example is the relationship between the exchange rates between two countries and the relative difference between their interest rates (commonly referred to as the interest rate differential). It is therefore of utmost importance that care is taken in the modelling of these various underlying risk drivers and that the impact of one risk driver on another seemingly unrelated risk driver is carefully considered. Failure to do so may result in a gross overstatement, or even worse, understatement of credit exposure.

A good example of a method currently used in practice to overcome related issues is the following: Let's assume a portfolio contains currency derivatives and also some interest rate derivatives. More specifically, let's assume that the portfolio contains a ZAR interest rate swap as well as a few USD/ZAR FX Forward contracts. As we have seen in numerous examples above, the most obvious risk driver to model in the case of the forward contracts is the spot price, but some assumption needs to be made regarding the evolution of the forward exchange rate curve. In contrast to the approach that was taken above (i.e. assuming the forward exchange rate curve moves up and down in parallel shifts) one can use the GBM model to model USD/ZAR spot exchange rates and use a model for the short rate (e.g. the CIR Model used in Section 2.2.7, pp 23) in order to evolve the forward exchange rate curve. In this way there will be a consolidated framework in which the interest rate swaps and the FX forwards can be valued under the same set of simulated scenarios in a consistent fashion.

3.5.3 Consistency in Conversion Rates in Simulated Future Scenarios

In the foreign exchange derivative examples above, we have consistently used exchange rates which are quoted against the Rand, and proceeded to refer to the Rand exposures in

our discussions. This avoided a very important and often underestimated complexity which often exists in converting future simulated exposure numbers between different currencies while maintaining economic and financial consistencies and realities. An example will illustrate this point.

Let's consider the FX forward contract in Section 2.2.4 (pp 17) under which the bank is due to receive USD 1,000 in exchange for paying ZAR 8,170 in 6 months' time. The credit exposure from the bank's perspective will materialise when there is a positive mark-to-market value for the bank. This, in turn, will happen if ZAR weakens against USD during the life of the contract. If, for example, ZAR weakens to a level of USD 1 = ZAR 10 in six months' time then the bank is in a fairly good position since it will pay only ZAR 8,170 for USD 1,000 worth ZAR 10,000 at that point in time. If we take this to the extreme, the best situation that the bank could be in just before the maturity date (purely from a MTM point of view) is if ZAR becomes close to worthless. ZAR being 'worthless' would result in an almost unlimited⁶⁷ MTM (in ZAR terms) of this contract. If, however, the bank monitors this exposure in USD, the amount that can be lost is limited, from an economical point of view, to USD 1,000,000.

We have stated the formula for the MTM (in ZAR) of this type of contract before (Section 2.2.4, pp 17). For the sake of completeness, the formula for the MTM (in ZAR and USD, respectively) from the perspective of the bank (as USD notional receiver/ZAR payer) is given below:

$$MTM_{ZAR} = N_{USD} \cdot PV_{ZAR}(F_{USD/ZAR} - K_{USD/ZAR})$$

$$MTM_{USD} = N_{ZAR} \cdot PV_{USD}(F_{ZAR/USD} - K_{ZAR/USD}).$$

Let's consider the simple PFE calculation similar to that in the example in Section 2.2.4 (pp 17). In section 2.2.4 we measured and illustrated in the PFE in ZAR terms and also concluded that the maximum contract-level PFE value occurs just before maturity (since

⁶⁷ The value of the amount to be received, if measured in ZAR terms could be as large as the exchange rate allows it to be. This is in contrast to the economic reality that this amount, in USD terms, will never exceed the notional amount.

there is no amortisation effect and essentially only the one settlement amount at maturity). What happens if we change the measurement currency to USD?

At first glance one may be tempted to assume that this is trivial and convert every ZAR number in the PFE profile to USD using the current USD/ZAR spot exchange rate. This, as will be shown below is incorrect. Before proving this, it should be highlighted that the modelling of an exchange-rate as risk driver for PFE purposes can be done using both quoting directions. In the example above, this means that we can model the exchange rate as a USD/ZAR rate or as a ZAR/USD rate – the same volatility will apply etc. There is one major requirement though: the two models should yield consistent results, and most importantly, it should not matter which quoting direction we use – we should be able to convert the results presented by the one and arrive at results that would have been produced by the other. More simply, we need to establish which exchange rate we need to convert at throughout the life of the contract into the future in order to achieve economically consistent results (without having to be selective about the quoting convention used in the modelling of the scenarios). It turns out, as will be shown below, that the rate that should be used to convert a time $t > 0$ ZAR exposure produced by a USD/ZAR model in scenario i should be converted to USD using the time t spot rate produced by the USD/ZAR model under that same scenario and not the spot rate at time 0 (today).

This may seem very trivial to some readers but is, nonetheless, a very important result demonstrating the importance of consistency in PFE modelling and financial modelling in general.

3.5.3.1 Example of Consistency in the Conversion of Future Exposures

Let's now apply what we have conceptually deduced, and in the process show more formally the motivation for this result. Let's consider the value of the contract at maturity

(T)⁶⁸. If we choose to calculate the PFE (or exposure) in ZAR using a model for the USD/ZAR exchange rate we will calculate MTM values under each simulated scenario using the following formula (subscripts indicate the currency or quoting convention used):

$$MTM(i, T)_{USD/ZAR} = N_{USD} \cdot (S_{USD/ZAR, T, i} - K_{USD/ZAR}).$$

Also note that

- $S_{USD/ZAR, T, i} = S_0 e^{((\mu - 0.5\sigma^2)t + \sigma\varphi_i\sqrt{t})} = (S_{ZAR/USD, T, i})^{-1}$, with φ_i the random sample from the $N(0, 1)$ distribution in scenario i .
- $K_{USD/ZAR} = (K_{ZAR/USD})^{-1}$
- $N_{USD}K_{USD/ZAR} = N_{ZAR}$.

Now, if we were to simulate the PFE using the model for the ZAR/USD we will use:

$$MTM(i, T)_{ZAR/USD} = N_{ZAR} \cdot (K_{ZAR/USD} - S_{ZAR/USD, T, i}).$$

If we attempt to convert the result from the USD/ZAR model to a USD exposure using the approach motivated above⁶⁹ we get:

⁶⁸ This is purely for ease of exposition and the approach and result can be extended to any hold at any t ($0 \leq t \leq T$).

⁶⁹ Converting using the applicable simulated spot price produced by the model, at that point in time.

$$\begin{aligned}
& MTM(i, T)_{USD/ZAR} \cdot (S_{USD/ZAR, T, i})^{-1} \\
&= (S_{USD/ZAR, T, i})^{-1} \cdot N_{USD} \cdot (S_{USD/ZAR, T, i} - K_{USD/ZAR}) \\
&= K_{USD/ZAR} \cdot N_{USD} \cdot (S_{USD/ZAR, T, i})^{-1} \cdot \left(\frac{S_{USD/ZAR, T, i}}{K_{USD/ZAR}} - 1 \right) \\
&= N_{ZAR} \cdot (K_{ZAR/USD} - S_{ZAR/USD, T, i}) \\
&= MTM(i, T)_{ZAR/USD}.
\end{aligned}$$

This shows that the motivation above is accurate and produces the correct result. We have thus shown that it is possible to apply the model proposed for the estimation of FX Forward PFEs consistently from an economic point of view⁷⁰. We now show that this method also implicitly enforces the reality observed in the real world, also explained above, with regards to the fact that the bank's exposure is capped at the notional amount if the notional amount it receives at maturity is quoted in the currency that it monitors its exposure in.

3.5.3.2 Example of Consistency between the Model Results and Real World Expected Results

Let's look at the specific example in Section 2.2.4 (pp 17) and again consider the exposure at maturity (just before final settlement). In addition we consider this under different assumptions of volatility and maturities in order to illustrate the required result effectively. The following graph shows the 95% contract-level PFE estimates, at maturity (before settlement) for USD/ZAR FX Forwards with various maturities and different assumed USD/ZAR volatilities which are then converted, to USD, using the approach above.

⁷⁰ The model will yield the same results whether the USD/ZAR exchange rate is modelled or whether the ZAR/USD exchange rate is modelled.

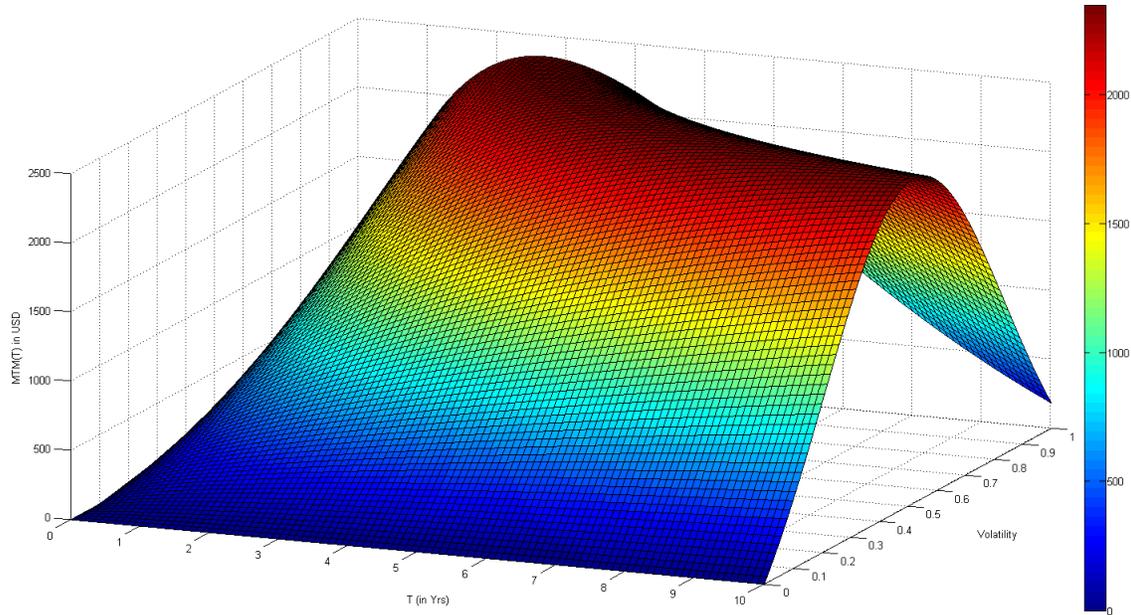


Figure 3.18 - Contract-Level PFE at Maturity for FX Forwards with Different Volatilities and Maturities

Clearly this is not exactly the result we had hoped for since the above graph shows that for certain volatilities and certain maturities the model could produce results which could never materialise in real life. The problem lies in the fact that the function:

$$S_{USD/ZAR,T,i} = S_0 e^{((\mu - 0.5\sigma^2)t + \sigma\varphi_i\sqrt{t})}$$

is not strictly increasing, for a fixed choice of φ (in general). A clever choice for μ , however, does overcome this problem. If we choose $\mu = 0.5\sigma^2$, we get the following:

$$S_{USD/ZAR,T,i} = S_0 e^{(\sigma\varphi_i\sqrt{t})}$$

Using this choice for μ in reproducing Figure 3.18, we get Figure 3.19 which satisfies our criteria of consistency between results produced by our model and the results expected in practice. What's also comforting is the fact that such a result is possible within the model framework without explicit intervention (in the form of explicitly capping the produced exposure amounts) to suit real-world requirements.

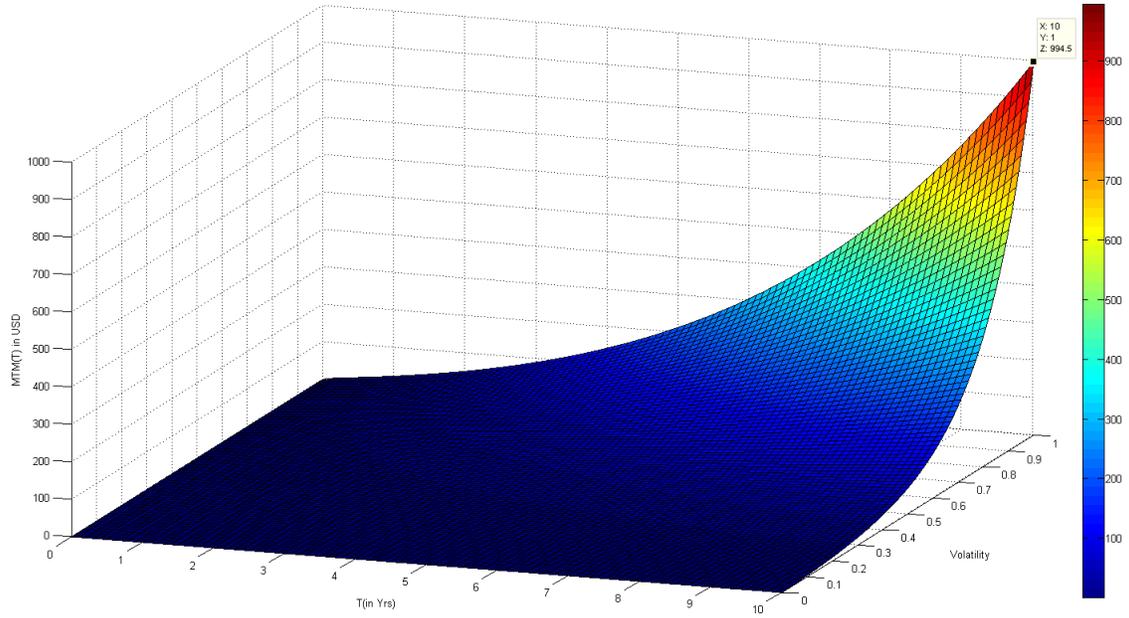


Figure 3.19 - Contract-Level PFE at Maturity for FX Forwards with Different Volatilities and Maturities, Using Drift Adjustment Term

4 Measuring Wrong-Way Risk Exposure

Many financial institutions were caught by surprise during the financial crises in South-East Asia and Russia when corporate and sovereign defaults, as well as downgrades, were accompanied by severe declines in currency values, driving exposures and losses well beyond their expectation. Wrong way exposure (WWE) occurs when counterparty exposure is highly negatively correlated with the counterparty's credit quality, i.e. counterparty exposure increases simultaneously with a weakening of the counterparty's credit quality. For example

- when a bank enters into a cross currency swap agreement with an emerging markets counterparty seeking inexpensive US Dollar funding referencing its domestic currency; or
- When a highly leveraged counterparty seeks to receive fixed payments on an interest rate swap.

Both of these examples are wrong-way in that the situations which would have the derivatives be in the money for the bank, are those situations which coincide with scenarios where the counterparty will have difficulty in fulfilling their obligations under those agreements (rising interest rates or a strengthening dollar).

In the modelling examples discussed so far we have consistently assumed that the exposure is independent of the counterparty's credit quality. More formally, we have defined PFE (in 2.2.2) such that

$$PFE(\alpha, i, t) := \inf\{MTM_i(\omega_t^i) : P[\Lambda_t^i > MTM_i(\omega_t^i)] \leq \alpha\}.$$

Implicitly, we have in fact assumed that

$$\mathbb{P}[\Lambda_t^i > MTM_i(\omega_t^i)] = \mathbb{P}[\Lambda_t^i > MTM_i(\omega_t^i) | CS_{CP} = Def],$$

where CS_{CP} denotes the credit state of the counterparty and Def denotes the default state.

We have therefore assumed the market factor distributions can be used directly to generate or project the set of (unconditional) scenarios which are used to compute the distribution required to estimate the PFE, and finally compute measures of credit exposure. This completely ignored the dependence between the level of the market factors and the impact thereof on the likelihood of the counterparty to default. From a purely conceptual point of view, this problem can also be explained as follows: Instead of saying “at each future scenario of the market price⁷¹, what would the value of the derivative be?”, one should rather be asking: “Under which scenarios⁷² does the counterparty actually default⁷³” and then, focusing on those scenarios, determine the value of the derivative and finally computing the measure of exposure as required. This is, from a more technical perspective, the equivalent to approaching the problem of exposure measurement using a conditional-distribution approach. More specifically, a conditional distribution of exposure would give the exposure profile conditional on default.

It is often suggested that stress tests should be used to handle wrong-way exposures⁷⁴, but stress test measures cannot be used for pricing and limit setting in the same way as the standard measures. It should also be highlighted that in using stress testing to specify the conditional distribution by assuming that counterparties default only when the market significantly moves against them, the cases where the counterparties default for other reasons would be ignored. According to the CRPMG⁷⁵ report, produced by executives

⁷¹ This market price being simulate as we have done in all examples so far – i.e. unconditional of counterparty default.

⁷² These scenarios, in order to be determined, would need to have some relation to the probability of default of the counterparty.

⁷³ Or, equivalently, “...are the counterparty more likely to default”

⁷⁴ Rowe (1999).

⁷⁵ Counterparty Risk Management Policy Group, see <http://www.crpmgroup.org/>

from twelve of the largest commercial banks, the typical assumption⁷⁶ used by banks was heavily criticised. Duffee (1996) investigates the validity of this assumption empirically and concludes that between 1971 and 1992 corporate defaults in the U.S. tended to cluster during periods of falling interest rates. For the ‘receive fix’ side of U.S. interest rate swaps this produced a significant positive correlation between the size of exposure and counterparty default events. Duffee demonstrates that an exposure measure which accounts for wrong-way risk is on average 65% higher than a comparable measure assuming independence.

Typical exposure measures which assume independence between the counterparty default and the value of the contract allow us to determine expected losses quite easily. For example, for a contract with an expected exposure⁷⁷ of R1000,000 and a default probability of 1% we *must* assume that the default event and exposure amount is independent in order to conclude that the expected loss, as a result of default, is $1\% \times R1000,000 = R100,000$. Ideally, we would also want to be able to make such statements when it comes to situations where wrong-way risk is present. In other words, we would like to have an exposure measure incorporating wrong-way risk which, when multiplied by the probability of default of the counterparty, will yield the expected default loss on the transaction. Finger (2000) proposes a framework which achieves this desired property in an exposure measure incorporating the effects of wrong-way credit risk. This approach is summarised below.

4.1 Finger’s Model

4.1.1 Definition of the Framework

⁷⁶ The assumption of the independence of the counterparty exposure and the counterparty’s credit quality – i.e. ignoring wrong-way risk.

⁷⁷ See Table 2.4.

Firstly, we assume that only one risk factor underlies our derivative contract under consideration. Let's define some notation:

- R_t is a stochastic variable and denotes the risk factor at time t . Note that this is a value in Ω_t (as defined in Section 2.2.2 above)⁷⁸.
- f_t denotes the probability density function of R_t . We assume that this is known⁷⁹.
- $MTM(R_t)$ denotes the mark-to-market value of the contract assuming that the price of the underlying risk factor is R_t .
- $\mathbb{E}_t(r) = \max[MTM(r), 0]$ denotes the exposure⁸⁰ at time t , given that $R_t = r$ at t .

Then we have the expected exposure⁸¹ defined as:

$$\mathbb{E}[\mathbb{E}_t[R_t]] = \int_0^\infty \mathbb{E}_t[r] f_t(r) dr.$$

The contract-level PFE⁸² at t using a confidence level of α is the x satisfying:

$$\alpha = \mathbb{P}(\mathbb{E}_t[R_t] < x) = \int_{\{r: \mathbb{E}_t[r] < x\}} f_t(r) dr.$$

We define $p_t(r)$ as the conditional probability of default – i.e. the probability of default given that $R_t = r$. For example, if we assume our contract is an interest rate swap the function p_t would be the relation between the counterparty's default probability and the swap's underlying interest rate – presumably p_t would increase for an increase in the interest rate. In contrast, when default is independent of the underlying factor p_t would be constant. For consistency, the following condition is introduced:

⁷⁸ Note the suppression of the ' i ' in the notation here (compared to Ω_t^i). This is purely for simplicity because we are only considering one contract at this stage.

⁷⁹ For more information on exposure measurement, see Zangari, (1997).

⁸⁰ As mentioned previously, some practitioners define exposure as being strictly positive. This is an example of such a definition.

⁸¹ This is merely the continuous case of the definition in Table 2.4.

⁸² In Finger (2000) this is referred to as the maximum exposure given confidence α .

$$\int_0^{\infty} p_t(r) f_t(r) dr = p_{def},$$

where p_{def} denotes the unconditional probability of default of the counterparty.

The conditional probability that R_t lies below a fixed value y , given that default has occurred, is⁸³:

$$\mathbb{P}[R_t < y | CS_{CP} = Def] = \frac{\mathbb{P}[(R_t < y) \cap (CS_{CP} = Def)]}{\mathbb{P}[CS_{CP} = Def]} = \frac{\int_0^y p_t(r) f_t(r) dr}{p_{def}}.$$

And therefore, the conditional density (f_t^{def}) is easily computed:

$$f_t^{def} = f_t(r) \cdot \left[\frac{p_t(r)}{p_{def}} \right].$$

It's important to highlight here that the conditional density does not depend on the absolute level of the default probability – only on how the conditional default probability is related to the risk factor⁸⁴. This feature makes this framework very appealing from a practical point of view. The reason for this is that two counterparties which have the same dependence on the risk factor (through p_t), but a different p_{def} , would have the same conditional density⁸⁵. It is therefore possible to have certain groups of counterparties with the same p_t 's and it would only be necessary to compute one f_t^{def} irrespective of their different levels of p_{def} . This is very appealing from a systems and computational efficiency point of view.

⁸³ Using Bayes' Rule.

⁸⁴ This point is very important and one which we will build on in the next section.

⁸⁵ More specifically, if the two companies have different unconditional probabilities of default, but react similarly in the relative impact to their conditional probability of default for a given change in r the companies will have the same conditional probability of default.

It is now possible to define the conditional expected exposure in this framework:

$$\begin{aligned}
& \mathbb{E}[\mathbb{E}_t[R_t] | CS_{CP} = Def] \\
&= \int_0^{\infty} \mathbb{E}_t[r] \cdot f_t^{def}(r) dr \\
&= \int_0^{\infty} \mathbb{E}_t[r] \cdot f_t(r) \cdot \left[\frac{p_t(r)}{p_{def}} \right] dr \\
&= \mathbb{E} \left[\frac{p_t(R_t)}{p_{def}} \cdot \mathbb{E}_t[R_t] \right].
\end{aligned}$$

And similarly, we can define the conditional contract-level PFE, using a confidence level of α , as the x satisfying:

$$\alpha = \mathbb{P}[R_t < x | CS_{CP} = Def] = \int_{\{r: \mathbb{E}_t[r] < x\}} f_t^{def}(r) dr = \int_{\{r: \mathbb{E}_t[r] < x\}} f_t(r) \cdot \left[\frac{p_t(r)}{p_{def}} \right] dr.$$

Note that $\mathbb{E} \left[\frac{p_t(R_t)}{p_{def}} \cdot \mathbb{E}_t[R_t] \right]$ and $\int_{\{r: \mathbb{E}_t[r] < x\}} f_t(r) \cdot \left[\frac{p_t(r)}{p_{def}} \right] dr$ can be evaluated using Monte Carlo simulations of R_t . This means that it is not necessary to simulate scenarios from the conditional distribution in order to calculate the conditional expected exposure, which is a very useful result from a practical perspective.

4.1.2 Calibration of the Model: Finger's Approach

Finger proposes a function for $p_{t(r)}$ as follows:

$$p_{t(r)} = p_{max} \cdot g \left(\frac{r - \zeta - \beta_1 \zeta}{\beta_2 \sigma} \right),$$

where

- p_{max} is determined by $\int_0^{\infty} p_{t(r)} f_t(r) dr = p_{def}$, the consistency condition defined above.

- $g(z) = \frac{1 + \tanh(\tanh^{-1}(0.8) \cdot z)}{2}$,
- $\zeta\mu$ is the mean of the risk factor at t ,
- $\zeta\sigma$ is the standard deviation of the risk factor at t ,
- β_1 and β_2 are numbers to be specified through calibration.

The approach followed by Finger in the calibration of β_1 and β_2 is to specify values for the mean (ζ_{def}) and standard deviation (ζ_{def}) of the conditional distribution and find values for β_1 and β_2 such that:

$$\int_0^{\infty} r \cdot f_t^{def}(r) dr = \zeta_{def}$$

and

$$\int_0^{\infty} r^2 \cdot f_t^{def}(r) dr = \zeta_{def}^2 + \zeta_{def}^2.$$

The above set of equations can be solved numerically. The challenge is defining the values for ζ_{def} and ζ_{def} . Finger proceeds to apply a methodology specified in Levy (1999) for obtaining a so-called “residual currency value upon counterparty default” in order to derive values for ζ_{def} and ζ_{def} .⁸⁶

4.2 A Proposed Enhancement to Finger’s Model

In the framework developed by Finger the link between the level of the underlying risk factor which causes the wrong-way risk and the probability of default of the counterparty is crucial. One criticism of this model is the fact that, although very flexible and intuitive, the $p_{t(r)}$ function proposed by Finger almost seems arbitrary and is not transparent. In short, it is not clear what the relationship between the underlying risk driver (causing the wrong way risk) and the level of the probability of default of the counterparty is.

⁸⁶ See Finger (2000) and Levy (1999) for more information.

Moreover, this functional link, $p_{t(r)}$, directly impacts the conditional default distribution from which the conditional PFE or EPE is estimated.

The proposed enhancement to Finger's model is a specification of this $p_{t(r)}$ function through a simulation approach employing a structural model which results in an intuitively appealing framework for the incorporation of wrong-way risk measurement into existing counterparty credit risk simulation models. We will first present an overview of structural models, followed by a detailed specification of the proposed framework for measuring the impact of wrong-way risk exposure on counterparty credit risk measures.

4.2.1 Structural Models of Default Risk

4.2.1.1 Merton's Structural Model

The first version of the structural model appeared in a paper by Merton in 1974. Merton argued that, fundamentally, a firm would default if the value of the firm decreases beyond a certain threshold level. Merton's work relates credit events to economic fundamentals by modelling the dynamics of the assets of the firm.

Merton considers a firm with the following characteristics:

1. The firm has two funding resources:
 - a. Equity
 - b. A single class of debt in the form of a zero-coupon bond with a par value D and maturity T
2. If the firm defaults on the repayment of the debt at time T , the bond holders take control of the firm leaving the equity holders to receive nothing
3. It's assumed that the firm cannot issue any senior claims, pay cash dividends repurchase shares prior to T .

It is further assumed that there are no transaction costs or taxes and that short selling is permitted. Also, there is no problem with the divisibility of assets⁸⁷. Merton's model relates the default risk of a firm to a firm's capital structure. The assumption is that the value of a firm's assets follows lognormal diffusion process with constant volatility, i.e.

$$\frac{dA_t}{A_t} = \mu_A dt + \sigma_A dW_t, \quad (5)$$

where A_t the value of the firm's assets at time t , E_t represent the value of equity at time t and μ_A and σ_A denote the asset drift and volatility respectively. dW_t is a standard Wiener process. Note that (5) has the unique solution:

$$A_t = A_0 e^{(\mu - 0.5\sigma^2)t + \sigma W_t},$$

with A_0 given and

$$W_t \sim N(0, t).$$

As per our argument above, if the firm defaults on its debt at time T the bond holders take control of the firm leaving the equity holders with nothing. Conversely, if at time T the debt is repaid the equity holders will receive the remaining value of the firm. Therefore, under these assumptions, the following relationship (at time T) will hold:

$$E_T = \max[A_T - D, 0]. \quad (6)$$

The relationship above is then equivalent to equating the equity price of a firm to the value of a call option on the firm's assets with strike price equal to its debt repayment at T . If r denotes the risk free interest rate (which, together with σ_A , is assumed to be constant) we have⁸⁸:

⁸⁷ See Merton (1974) for detailed assumptions.

⁸⁸ By application of the famous model by Black and Scholes.

$$E_0 = A_0 N(d_1) - D e^{-rT} N(d_2) \quad (7)$$

where

$$d_1 = \frac{\ln\left(\frac{A_0 e^{rT}}{D}\right)}{\sigma_A \sqrt{T}} + \frac{\sigma_A \sqrt{T}}{2} \text{ and } d_2 = d_1 - \sigma_A \sqrt{T}. \quad (8)$$

The one difficulty with (5) is the fact that the asset volatility is not directly observable in the market. Since the equity value is a function of the assets, it is possible to apply Ito's lemma to determine the instantaneous volatility of the equity from the volatility of the assets⁸⁹:

$$\sigma_E = \frac{A_0}{E_0} \cdot \frac{\partial E}{\partial A} \cdot \sigma_A, \quad (9)$$

with σ_E the instantaneous volatility of the firm's equity at time zero.

It is therefore possible to estimate the volatility of the assets of a firm by using (9) together with the equity price of the firm observable in the market. As mentioned above, if the asset value of the firm falls below the value of the firm's debt payment at time t , the firm is assumed to be in default which implies that the bond holders will take control of the firm, and in practice, make some recovery on the unpaid debt. From (7) it is possible (under the assumptions made) to make statements surrounding the probability of default of the firm. In fact, the probability of default can be seen as the probability of not surviving up to a certain point in time which is equivalent to the probability of the option, with pay-off as in (6), not being exercised.

The motivation for this is that, if the option under consideration is not exercised, then the firm's assets are less than its debt payment due implying that it has defaulted. In short, we can write the survival function⁹⁰ as follows:

⁸⁹ See: Jones, Mason and Rosenfeld (1984), Hull, Nelken and White (2004).

⁹⁰ The survival function, i.e. $S(t)$, denotes the probability that the firm will not default before time t .

$$Y_M(t) = \mathbb{P}[A_0 e^{(\mu - 0.5\sigma^2)t + \sigma W_t} > D] = 1 - \Phi\left(\frac{\ln\left(\frac{D}{A_t}\right) - \left(\mu - \frac{\sigma^2}{2}\right)t}{\sigma_A \sqrt{t}}\right). \quad (10)$$

Therefore, the probability that the firm defaults before t is given by:

$$pd(t) = \Phi\left(\frac{\ln\left(\frac{D}{A_t}\right) - \left(\mu - \frac{\sigma^2}{2}\right)t}{\sigma_A \sqrt{t}}\right), \quad (11)$$

with Φ representing the cumulative standard normal distribution function.

The approach followed above in deriving the probability of default is intuitive and adds a insight into the basic motivations for using structural models of default. The practicality of the approach is however questionable since the variables in (5) are not directly observable in the market and one would therefore need to make certain assumptions in order to apply the approach in practice. To this extent there have been numerous extensions⁹¹ of Merton's model since 1974.

4.2.1.2 The Moody's KMV Model

One of the most widely used extensions of Merton's model is the Moody's KMV model which defines the statistic called 'Distance to Default' which is essentially the number of standard deviations that the asset-value of the firm is away from default and is calculated as:

$$\text{Distance to Default} = \frac{\text{Market Value of Assets} - \text{Default Point}}{\text{Market Value of Assets} \times \text{Asset Volatility}}$$

The Moody's KMV (MKMV) model is essentially a structural Merton model which, using the Distance to Default (DD) measure defined above, has been calibrated to

⁹¹ See Haworth (2004) for a comprehensive overview of the evolution of the structural model.

historical default data from Moody's database. In summary, the MKMV model uses the Moody's database⁹² and, for a given DD value of a firm, looks up the number of companies in the database which have, given that DD, defaulted within the next year. Through this approach the MKMV model is able to provide a framework which is attractive from a theoretical point of view, but also takes account of empirical data addressing some of the known problems associated with using the normal distribution for credit risk modelling. One of the most important shortcomings of the normal distribution in the application of default risk estimation is the fact that empirical data suggests that a fatter tail distribution would be more applicable⁹³. Although a very appealing framework, the MKMV model is proprietary and therefore not free to use because of the Moody's database which is used in the model calibration. The reader is referred to Crosbie and Bohn (2003) for more detail on the Moody's KMV structural model.

4.2.1.3 The CreditGrades Model

Another well known extension of the Merton framework (which is entirely free and transparent) is CreditGrades. The CreditGrades (CG) model was developed by three major international investment banks⁹⁴ in association with RiskMetrics Group. The motivation for the development of the model is to provide a transparent model relating relevant model parameters to market observables and, in the process, attempt to provide transparency in the credit markets. It should be mentioned that, in contrast to the MKMV model, which has been 'trained' using a proprietary default database to model the probability of default of a firm as accurately as possible, the CG model's main goal is "to track credit spreads well and to provide a timely indication of when a firm's credit becomes impaired"(Finger, 2002).

⁹² This database includes more than 250,000 company-years of data and over 4,700 incidents of default or bankruptcy.

⁹³ See Crosbie and Bohn (2003)

⁹⁴ Deutsche Bank, JP Morgan and Goldman Sachs.

Although the CG model's main purpose is not to derive a probability of default for a firm the model does produce satisfactory results in relative credit assessment⁹⁵, i.e. the model is able to distinguish between relative levels of probability of default quite successfully. The basic approach is very similar to the original Merton Model, and the assets of the firm under consideration is assumed to have the dynamics specified in (5) with the only exception that the asset drift is assumed to be zero, i.e. $\mu_A = 0$. The motivation for the asset-drift being set to zero is that it is not the drift of asset itself that is important but rather the drift of the asset value relative to the default barrier⁹⁶.

The CG model defines default as the first time that the assets of the firm cross the default barrier⁹⁷. The default barrier is defined as the amount of the firm's assets that remain after the firm defaults, and is represented by:

$$\text{Default Barrier} = R \cdot D,$$

with

$R = \text{Recovery Rate}$

$D = \text{The firm's debt per share.}$

The main difference in model assumption is as a result of the observation that defaults produced by the traditional Merton model are 'expected' when the model is used in simulating default events using Monte Carlo Simulation. In simple terms, the diffusion model used in the traditional Merton model results in defaults being 'predictable' in the sense that it does not happen 'unexpectedly' because of the default barrier being fixed. This also leads to credit spreads of zero in the short term – which is not observed in practice. As a result, the CG model assumes that the average recovery rate (R) is stochastic. More specifically, it is assumed that the average recovery rate follows a lognormal distribution with mean \bar{R} and volatility of λ , i.e.

⁹⁵ Finger (2002).

⁹⁶ It is assumed that, on average over time, a firm issues more debt in order to maintain a steady level of leverage or else pays dividends such that the debt has the same drift as the stock price.

⁹⁷ The default barrier in this case is defined as the amount of the firm's assets that remain after default. This is merely the recovery value that the debt holders receive, $L \cdot D$.

$$\bar{R} = \mathbb{E}[R],$$

$$\lambda^2 = \text{var}[\ln(R)],$$

and

$$RD = \bar{R}D e^{\lambda Z - 0.5\lambda^2}$$

with

$$Z \sim N(0,1).$$

The random variable Z is independent of the Brownian motion in (5)⁹⁸. From an intuitive point of view, by letting Z be random, the model captures the uncertainty in the actual level of a firm's debt-per-share ratio. There is therefore some true recovery rate which evolves through time which we are unable to observe with certainty (Finger , 2002).

With the uncertain recovery rate, the default barrier can be hit unexpectedly leading to a jump-like process of default. The probability that the firm, given the assumptions above, survives to time t is then given by⁹⁹

$$Y_{CG}(t) = \Phi\left(-\frac{V_t}{2} + \frac{\ln(d)}{V_t}\right) - d \cdot \Phi\left(-\frac{V_t}{2} - \frac{\ln(d)}{V_t}\right), \quad (12)$$

where

$$d = \frac{A_0 e^{\lambda^2}}{RD}$$

and

$$V_t = \sqrt{\sigma_A^2 t + \lambda^2}.$$

The probability of defaulting before time t is therefore trivially given by $1 - Y_{CG}(t)$.

Note that Finger (2002) also suggests a practical method of calculating the parameters used in (11) using observable market variables:

⁹⁸ Also note that Z is unknown at $t=0$ and only revealed at the time of default. More specifically, there is a filtration F to which W is adapted such that F is independent of F_0 but $Z \in F_t \forall t > 0$.

⁹⁹ Finger (2002)

$$d = \frac{E_0 + \bar{R}D}{RD} e^{\lambda^2} \quad (13)$$

$$V_t^2 = \left(\sigma_E^* \frac{E^*}{E^* + RD} \right)^2 t + \lambda^2, \quad (14)$$

where

- E_0 is the initial stock price
- E^* is the reference stock price
- σ_E^* is the reference stock volatility
- D is the debt-per-share
- \bar{R} is the global debt recovery rate
- λ is the percentage standard deviation of the default barrier.

4.2.2 A Structural-Model Approach to the Measurement of Wrong-Way Exposure

We have discussed in some detail the dynamics of the traditional, as well as some extensions of the, Merton structural model. One very useful application of structural models, often used in practice, is that of default simulation using a Monte Carlo method. This technique is often employed, most notably by the rating agencies¹⁰⁰, in estimating a default credit loss distribution on a portfolio level.

The main reason for the popularity of this approach is the ease of implementation through the fact that equity data is readily available in the market, as well as the fact that correlation is easily incorporated into the simulation approach. Another major appeal is the intuitive nature of the approach due to the use of financial fundamentals.

In a similar fashion, the author proposes to apply very similar technology in introducing a relationship between the underlying risk driver in an OTC derivative, which causes

¹⁰⁰ Especially in the rating of Collateralised Debt Obligations (CDOs).

wrong-way risk, and the likelihood of default¹⁰¹ of such a firm. This is achieved through Monte Carlo simulation of the underlying risk driver of the OTC derivative and by calculating expected values of the equity price of the counterparty given the simulated underlying price. These simulated conditional expectation values are then used, through the structural model framework, in estimating conditional probabilities of default. The resultant distribution¹⁰² of values can then be used to establish the link between movements of the underlying risk driver of the OTC derivative and the likelihood of default of the counterparty through the application of the structural model on the simulated equity price(s). Of most interest is the relative change in probability of default given changes in the price of the underlying asset.¹⁰³

For example, let's consider the a typical situation where wrong-way risk exposure arises as mentioned on page 77: *“when a bank enters into a cross currency swap agreement with an emerging markets counterparty seeking inexpensive US Dollar funding referencing its domestic currency”*. In this case it is possible to calculate a measure of dependence (e.g. correlation) between the equity price of the counterparty of the bank and the underlying risk driver of the OTC contract (the counterparty's domestic currency's exchange rate against the dollar). This dependence estimate can be used to simulate future scenarios of the conditional expected equity price and the exchange rate. The simulated conditional expected equity prices, in turn, can be used to derive the implied conditional probability of default of the counterparty under each of these simulated scenarios, yielding the $p_t(r)$ function specified in Section 4.1.1 above. The estimated $p_t(r)$ function can then be applied, using Finger's framework, to yield a relative impact on the probability of default of the counterparty as a result of the changes in the underlying variable. Finally a conditional exposure calculation determined the counterparty exposure incorporating wrong way risk.

¹⁰¹ Also referred to as the probability of default. Typically banks measure the probability of default over a one-year horizon for regulatory capital reasons although some banks have a so-called term structure of PDs.

¹⁰² As we will see, this distribution is a multivariate lognormal distribution.

¹⁰³ More specifically, the ratio between the conditional probability of default and the unconditional probability of default.

This approach is very intuitive and extremely flexible in that it can be applied in almost any market. More specifically, in the South African context where the credit derivative market contains only a few names which are actively traded, this framework can be applied to almost¹⁰⁴ any counterparty as it does not rely on the availability of observable credit spreads.

4.2.2.1 Formal Definition of the Proposed Framework

Consider a single OTC derivative with a single underlying market variable. The counterparty to the OTC derivative has a listed equity. The following stochastic differential equations describe the dynamics of the price processes of the underlying market variable price, θ , and the equity price, E , respectively:

$$\begin{aligned}d\theta_t &= \mu_\theta \theta_t dt + \sigma_\theta \theta_t dW_t^\theta \\dE_t &= \mu_E E_t dt + \sigma_E E_t dW_t^E \\dW_t^\theta dW_t^E &= \rho dt.\end{aligned}$$

In other words, we assume that the underlying price of the OTC contract, as well as the equity price of the counterparty follows Geometric Brownian motion. Moreover, we assume that there is a correlation between these Brownian motions, denoted by $\rho_{\theta,E}$. Note that this is equivalent to saying that the underlying asset price at time t , θ_t , and the equity price at time t , E_t , have a bivariate correlated lognormal distribution. This, in turn, is equivalent to saying that the natural logarithm (\ln) of the prices at time t has a bivariate normal distribution as follows:

¹⁰⁴ The counterparty needs to have an observable equity price in the market – i.e. the counterparty needs to be listed. However, often in practice some proxy can be used in order to get an approximation of the correlation estimate required. Specifically in South Africa there are fortunately some sector-based indices on the JSE that can may be suited to be used as a proxy for unlisted companies in the same industry.

$$\begin{aligned} \begin{pmatrix} \ln\theta_t \\ \ln E_t \end{pmatrix} &\sim N \left(\begin{pmatrix} \ln\theta_0 + (\mu_\theta - 0.5\sigma_\theta^2)t \\ \ln E_0 + (\mu_E - 0.5\sigma_E^2)t \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 t & \sigma_{\theta,E}\sqrt{t} \\ \sigma_{\theta,E}\sqrt{t} & \sigma_E^2 t \end{pmatrix} \right) \\ &= N \left(\begin{pmatrix} \mu'_\theta \\ \mu'_E \end{pmatrix}, \begin{pmatrix} (\sigma'_\theta)^2 & \sigma'_{\theta,E} \\ \sigma'_{\theta,E} & (\sigma'_E)^2 \end{pmatrix} \right). \end{aligned}$$

The structural model used in CreditGrades provides a useful mechanism for deriving a probability of default from an observed equity price. We use the function $g(\cdot)$ to represent the structural model which derives a 1-year probability of default for a given equity price. More specifically, we define

$$g(E) = 1 - \Phi \left(-\frac{V_1}{2} + \frac{\ln(d)}{V_1} \right) + d \cdot \Phi \left(-\frac{V_1}{2} - \frac{\ln(d)}{V_1} \right),$$

where V_t and d as in (13) and (14) above.

In Finger's framework, a seemingly arbitrary function is chosen for $p_t(r)$, the conditional probability of default, given $R_t = r$. In our setup, $R_t = \theta_t$.

From an intuitive point of view there are two major requirements from the function chosen for $p_t(\hat{\theta}_t)$:¹⁰⁵

- Firstly, the function needs to provide a relationship between the level of the underlying asset (θ_t) and the probability of default of the counterparty
- Secondly, when there is no correlation between the underlying asset and the probability of default of the counterparty (i.e. when there is no wrong-way risk) then the function p_t must equal the unconditional probability of default for all values of θ_t .

¹⁰⁵ $\hat{\theta}_t$ denotes a realisation of the random variable θ_t .

We propose the following function for $p_t(\theta_t)$, the conditional probability of default of the counterparty at time t given θ_t , a stochastic variable denoting the value of the underlying asset of the OTC contract:

$$p_t(\hat{\theta}_t) = g(\mathbb{E}[E_t | \Theta_t = \hat{\theta}_t]) = g(\bar{\varepsilon}_t^c).$$

In short, we are defining the conditional probability of default to be the probability of default derived by the structural model using the conditional expected value of the equity price of the counterparty. In addition, we propose the following function for p_t^u , the unconditional probability of default at time t :¹⁰⁶

$$p_t^u = g(\mathbb{E}[E_t]) = g(\bar{\varepsilon}_t).$$

Therefore, we define the unconditional probability of default of the counterparty at time t to be the probability of default derived by the structural model using the expected equity price at time t . It is easily shown that:

$$p_t^u = p_{def}, \forall t$$

if

$$\mu_E = 0.$$

In other words, if $\mu_E = 0$ the unconditional probability of default in the proposed framework yields an unconditional probability of default which is constant over time. The proposed approach is therefore similar to Finger's approach under this condition (i.e. $\mu_E = 0$).

Note that in the absence of wrong-way risk, i.e. $\rho_{\Theta, E} = 0$ we have

¹⁰⁶ Note that the unconditional probability of default in Finger's framework is represented as p_{def} and is assumed to be constant. In our approach we assume a more general case and allow the unconditional probability of default to be time dependent.

$$\mathbb{E}[E_t | \Theta_t = \hat{\theta}_t] = \mathbb{E}[E_t],$$

and therefore

$$p_t(\hat{\theta}_t) = p_t^u, \forall \hat{\theta}_t,$$

which is expected and desirable.

It follows that the conditional distribution of the random variable $\ln E_t | \ln \Theta_t = \ln \hat{\theta}_{i,j}$ has a normal distribution as follows:

$$\ln E_t | \ln \Theta_t \sim N(\mu_E^*, \sigma_E^*),$$

where

$$\mu_E^* = \mu_E' + \rho \frac{\sigma_E' (\ln \hat{\theta}_{i,j} - \mu_\Theta')}{\sigma_\Theta'}$$

and

$$\sigma_E^* = \sqrt{(\sigma_E')^2 (1 - \rho_{\Theta,E}^2)}.$$

Therefore, the conditional expected value of the equity price at time t given the price of the underlying asset is $\hat{\theta}_{i,j}$ can be calculated by:

$$\bar{\varepsilon}_{i,j}^c = \mathbb{E}[E_i | \Theta_i = \hat{\theta}_{i,j}] = e^{\mu_E^* + 0.5(\sigma_E^*)^2}.$$

To summarise, the proposed method for estimating counterparty credit risk exposure, taking into account the effects of wrong-way risk, can be broken down into the following steps:

1. Estimation of the dependence (typically through correlation, ρ) between the underlying asset and the equity price of the counterparty. This will typically done using historical data. In other words, an estimate $\hat{\rho}_{\Theta,E}$ will be calculated from

- historical prices for the equity of the counterparty and prices for the underlying asset.
2. Estimation of the parameters of the structural model. This involves financial data of the counterparty which is observable in the market – for example from a data provider like Bloomberg.
 3. Monte Carlo simulation of future prices for the underlying asset, Θ , of the derivative contract as well as conditional expected values of the equity, E , of the counterparty. These two variables are assumed to have a deterministic correlation of $\hat{\rho}_{\theta,\varepsilon}$. N scenarios will be simulated for M time steps over the interval $[0, T]$. For time step $t_i \in (0, T]$ under each scenario s_j $\{i = 1, 2, \dots, M; j = 1, 2, \dots, N\}$ there will be a simulated value for the underlying asset, $\hat{\theta}_{i,j}$, as well as a conditional expected value of the equity price¹⁰⁷, i.e. $\bar{\varepsilon}_{i,j}^c = \mathbb{E}[E_i | \Theta_i = \hat{\theta}_{i,j}]$. The unconditional expected value of the equity price (i.e. $\bar{\varepsilon}_i = \mathbb{E}[E_i]$) will also be calculated, although this will be time-dependent and not dependent on the scenario under consideration.
 4. At each time step, under each scenario, the probability of default given $\hat{\theta}_{i,j}$ is determined using the structural model with $\bar{\varepsilon}_{i,j}^c$ as the input for the equity price. This step yields the conditional default probability required in Finger's framework. In other words, for each time step t_i under each scenario s_i there will be a estimated conditional probability of default,

$$\hat{p}_t^{i,j}(\hat{\theta}_{i,j}) = \mathbb{P}[CS_{CP} = Def | \theta = \hat{\theta}_{i,j}],$$

- estimated using the structural model. Also, at each time step, the unconditional probability of default is estimated through the structural model using $\bar{\varepsilon}_i$ as the input for the equity price.
5. As in the case of calculating any counterparty credit risk exposure through simulation, this is followed by calculating the value of the derivative contract at

¹⁰⁷ Note the superscript “c” indicates the expected value is conditional.

- each time step t_i under each scenario s_i – i.e. yielding $MTM_{i,j} = MTM(\hat{\theta}_{i,j})$ for each i and each j .
6. Convert the unconditional exposure distribution to a conditional¹⁰⁸ exposure distribution through the application of Finger’s approach – i.e. multiplying each MTM_{ij} with the appropriate ratio $\frac{p_t^{i,j}(r)}{p_t^u}$. Henceforth, we will refer to the term

$$EI^{i,j} = \frac{p_t^{i,j}(r)}{p_t^u}$$

- as the exposure inflation factor. This results in an estimated conditional distribution of values of the contract.
7. Finally, calculate the desired statistic from the conditional exposure distribution. For example, the conditional $\alpha\%$ PFE profile is estimated as the α^{th} percentile of the conditional distribution yielded by the previous step.

4.2.2.2 Practical Application of the proposed Model

We compare the conditional and unconditional counterparty credit risk exposure estimates under a few typical scenarios of wrong-way and right-way risk in order to demonstrate the application of the model and to test, from an intuitive level, the viability of the results. The following table is a summary of the scenarios under consideration. We consider the scenarios from a bank’s perspective – i.e. the bank is considered to be the counterparty to the companies under these hypothetical scenarios in the derivative contracts specified in the table¹⁰⁹.

¹⁰⁸ Conditional on default.

¹⁰⁹ All options have strikes equal to the current spot of the underlying. This is for simplicity sake.

<i>Scenario</i>	<i>Company</i>	<i>OTC Contract Type¹¹⁰</i>	<i>Underlying Asset¹¹¹</i>	<i>Risk Direction</i>
A	Anglo American	Long Put	Gold (USD)	Wrong-Way
B	Sasol	Long Call	Brent Crude (USD)	Right-Way
C	MTN	Long Put	MTN Shares (ZAR)	Wrong-Way

Table 4.1 - Scenarios under consideration for wrong-way risk measurement

Using a 3 year history of daily observed prices the correlation between the underlying asset and the equity price of the applicable counterparty is estimated. These results are summarised below:

<i>Company</i>	<i>Underlying</i>	<i>Correlation Estimate¹¹²</i>
Anglo American	Gold (USD)	86%
Sasol	Oil (USD)	95%
MTN	MTN (ZAR)	100%

Table 4.2 - Correlation estimates for wrong-way exposure measurement

We apply the CreditGrades structural model in these examples. The reason for using this specific structural model is mainly because of the ease of application and because of the fact that the data required in estimating the model parameters are easily obtainable – even for companies listed on the JSE. The expressions¹¹³ in (13) and (14) are used in the estimation of the model parameters.

The structural model uses the cumulative normal distribution in deriving a probability of default from a given equity price (and other inputs). The fact that the normal distribution has very thin tails causes some difficulty in the assessment of relative changes in the probability of default resulting from changes in the equity price – especially for counterparties with extremely low probabilities of default. For example, for a

¹¹⁰ The long or short position refers to the position from the bank's point of view. We assume that the maturity of all contracts is one year.

¹¹¹ The abbreviation in brackets is the currency used to measure the underlying in.

¹¹² The correlation estimate is based on 3 yrs of historical daily observations.

¹¹³ See Finger (2002) for a detailed derivation of these estimates.

counterparty with a probability of default of say 3 basis points one would be considering the very extremes of the normal CDF (cumulative distribution function).

For the purposes of our model, however, this may lead to situations where, for small upward movements in the equity price used to derive the probability of default, one could end up with probabilities of default of zero due to the extremely small area under the normal CDF at this end of the curve. Anglo American has a long term international-scale credit rating from Fitch of A. From historical data this implies a probability of default of approximately 3 basis points. It is possible to calibrate the CreditGrades model to produce a 3 bps probability of default using the debt-per-share input. If we consider the range of possible equity price values over a one year horizon using the distributional assumptions made above it is possible to construct a confidence interval for this purpose. Using this confidence interval, the following associated probabilities of default are returned implied by the CreditGrades model.

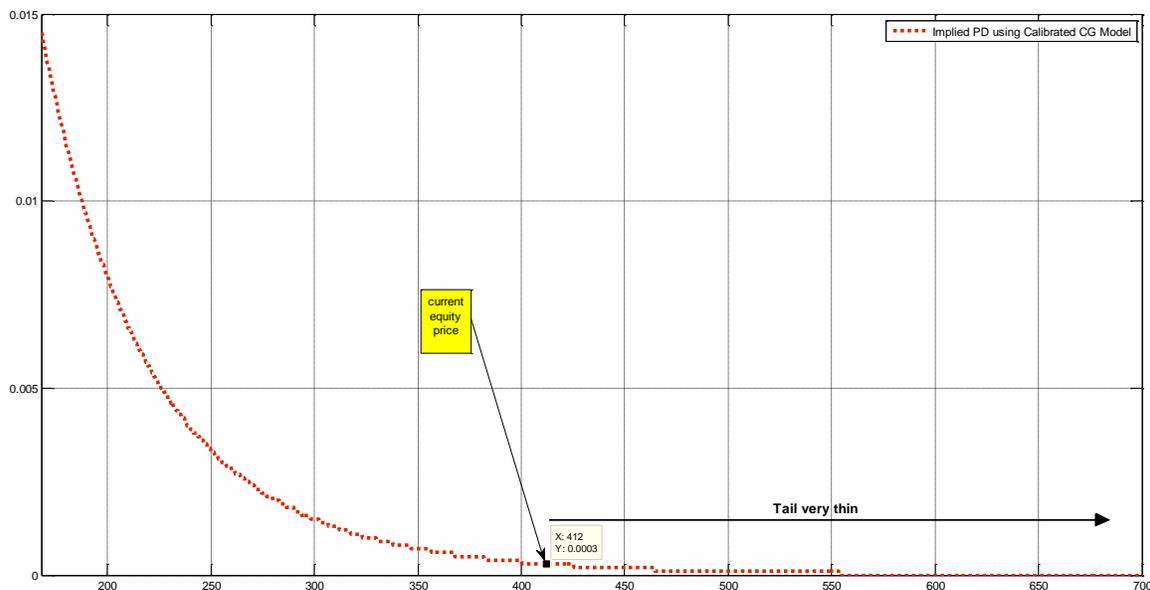


Figure 4.1 - Implied PDs using calibrated CG model

Note that we are considering the implied probabilities of default for equity prices within the 95% confidence interval of possible future values over a one year horizon. More specifically, we are concerned with the implied probabilities of default for equity prices

which are larger than the current equity price leading to lower probabilities of default. Under the normal distribution using the CG model calibrated to the probability of default implied by the rating these lead (very quickly) to implied probabilities of default of zero. This is not only unrealistic but also undesirable from a modelling perspective.

Since we are interested in relative changes in the level of the probabilities of default as opposed to absolute levels of the probability of default it is also possible to use a different area of the normal CDF as a starting point in order to assess changes in probability of default in both directions equally. The only major difference between starting in the middle of the CDF (say) as opposed to the end of the curve is the slope of the CDF curve at these points. We argue that starting at the middle of the curve will lead to smoother credit exposure calculations due to the fact that small changes in the equity price will lead to relatively small changes in the probability of default. In Figure 4.2 we illustrate this point and show the difference in implied probabilities of default using two different methods. The first method uses the probabilities of default implied from equity prices using the CG model calibrated to produce the probability of default implied by the credit rating if the current stock price is used. This method, as discussed above leads to undesirably small PDs for equity prices higher than the current equity price. The second method, i.e. the ‘normalised PD method’ is calibrated to yield a probability of default of 50% when the current equity price is used in deriving the PD. The normalised PD is then derived by the following conversion:

$$\text{Normalised PD} = 2 \times PD_h \times g^*(\varepsilon_t),$$

where

- PD_h denotes the historical PD implied by the rating
- $g^*(.)$ denotes the CG model calibrated to produce a PD of 50% if the function is applied on the current equity price (ε_0).

Note that, if $\varepsilon_t = \varepsilon_0$ in the expression above, then the two methods yield exactly the same result.

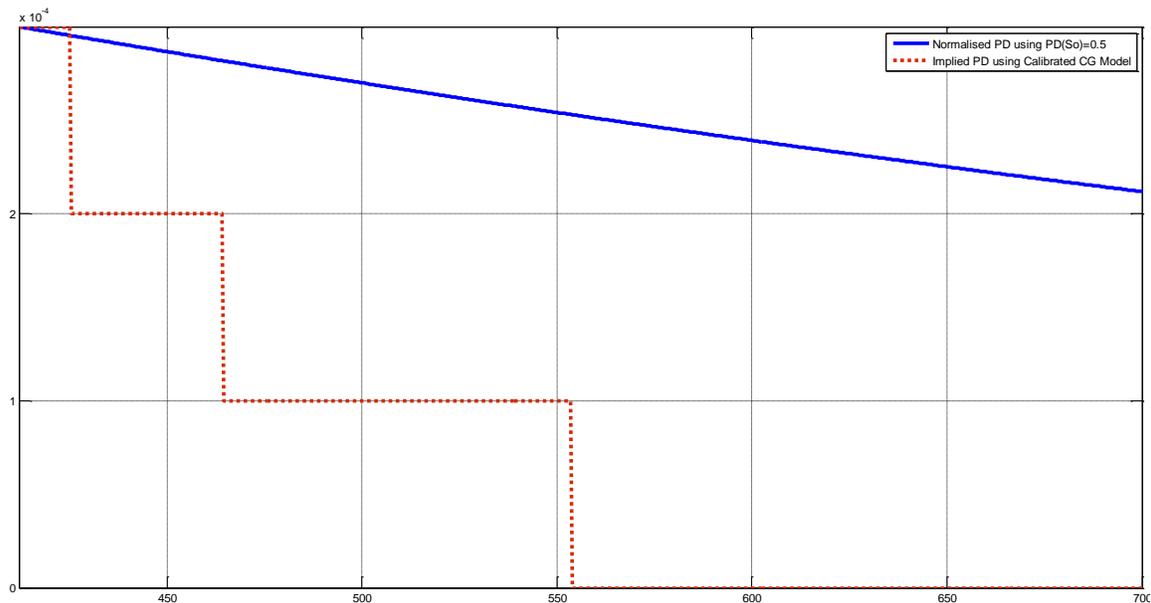


Figure 4.2 - Normalised implied PDs and implied PDs using a calibrated CG model

We will apply the normalised PD method in our example calculations. The debt-per share value is adjusted in order to ensure that the probability of default at $t=0$ is equal to 50% for each counterparty. The following table¹¹⁴ summarises the estimated and calibrated model parameters for each of the three companies under consideration:

<i>Company</i>	S_0	σ_S	D	\bar{R}^{115}	λ^{116}
Anglo American	412.00	36.52%	5,457.86	50%	0.3
Sasol	391.00	35.77%	5,185.16	50%	0.3
MTN	119.30	39.12%	1,574.42	50%	0.3

Table 4.3 - Estimate Model Parameters for the CG model

Next we consider the simulation of future values of the underlying variable of the OTC contract as well as estimates of the conditional expected equity price at each time step and under each scenario taking into account the dependence as measured by the

¹¹⁴ The data was sourced from Bloomberg on 14 August 2008. A 3-year history of daily observed prices was used as historical data in the estimation of the volatilities.

¹¹⁵ We assume a recovery rate of 50%, as also used in the CG Technical Document. See Finger (2002).

¹¹⁶ We use the λ parameter to calibrate the CG model to produce the one-year probability of default implied by the Fitch credit rating assigned to the company.

correlation estimates in Table 4.2. These simulations were performed using $M=10$ time steps and $N=10,000$ simulations. As described above, the model simulates future scenarios for the price of the underlying asset driving the value of the derivative contract. For each simulated value, however, there is an associated conditional expected equity price which, through the use of the structural model, is used in deriving an estimated conditional probability of default for the counterparty, under this scenario. The following figure depicts the relationship between the exposure inflation factor ($EI^{i,j}$) against various simulated values at $t=\{0.2,0.5,0.8\}$ of the underlying asset for each of the three counterparties under consideration.

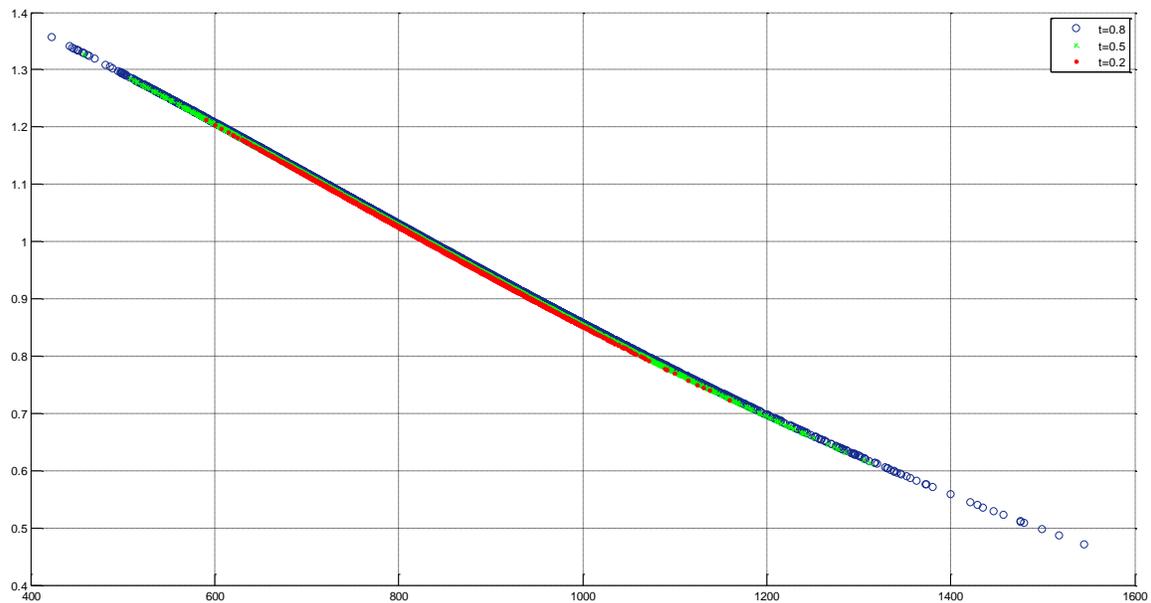


Figure 4.3 - Effect of changes in the gold price on the exposure inflation factor for AngloGold

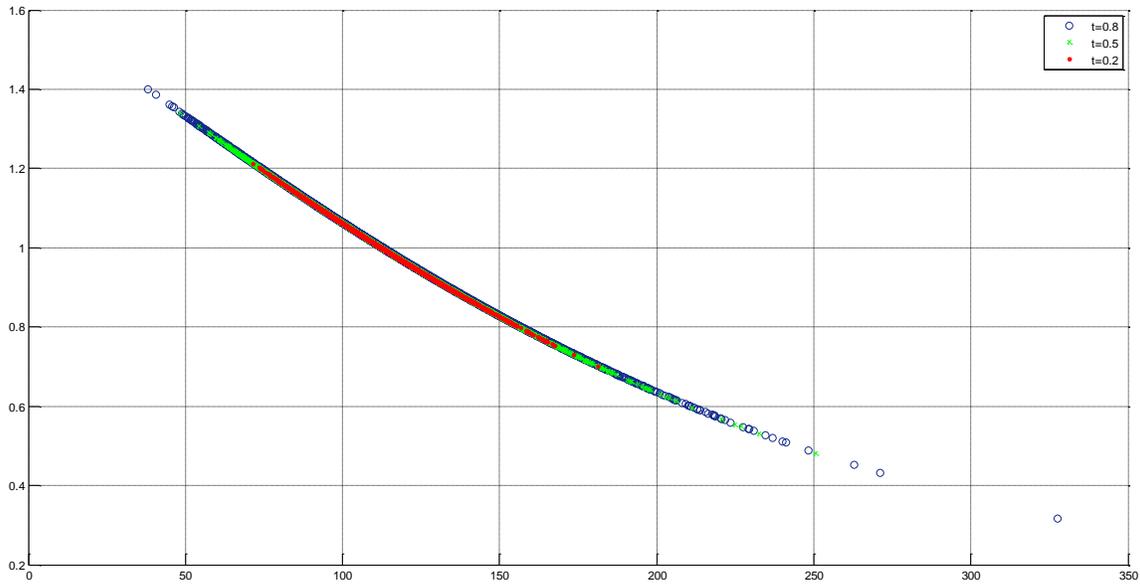


Figure 4.4 - Effect of changes in the oil price on the exposure inflation factor for Sasol

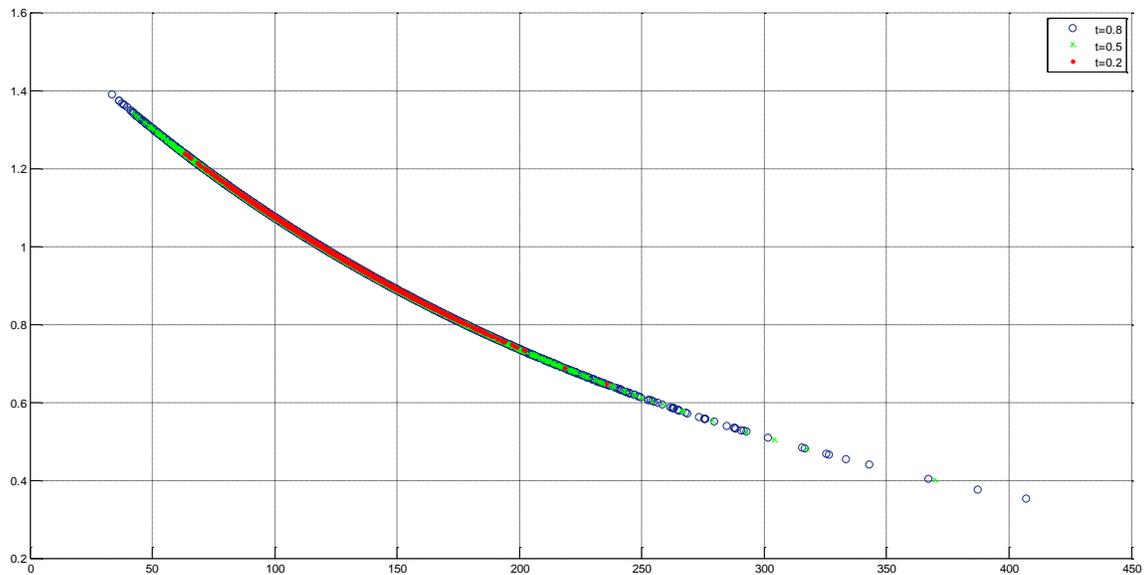


Figure 4.5 - Effect of changes in the MTN equity price on the exposure inflation factor for MTN

The results in Figure 4.3 are intuitive in that the exposure inflation factor decreases for increases in the gold price. This affirms the wrong-way risk exposure that we expect to be present in this scenario in that the exposure inflation factor is higher when the gold price drops since AngloGold is expected to be in a more difficult financial position under that scenario. A similar argument holds in the case of Figure 4.4 with the exposure inflation factor increasing when the oil price falls. Since the contract with Sasol is such that the

bank is exposed to counterparty risk only when the call option that the bank holds is in the money, this situation leads to right-way risk. The call option will be more in the money the higher the oil price – the higher the oil price the better the economic climate for Sasol in which to honour its agreements under the contract with the bank.

The following set of figures show the unconditional and conditional 95% PFE profiles for the three counterparties. As expected, the conditional PFE on AngloGold is higher than the unconditional PFE. This is illustrated in Figure 4.6.

It's also interesting to note in Figure 4.7 that the conditional PFE is lower than the unconditional PFE. This is however an intuitive result since the contract with Sasol is actually a right-way risk scenario – i.e. under scenarios where the contract is in the money (i.e. the oil price goes up) the counterparty is less likely to default. This means that the bank has exposure to the counterparty when the counterparty is in an economic scenario where it is expected that the counterparty will be more likely to be able to honour its obligations.

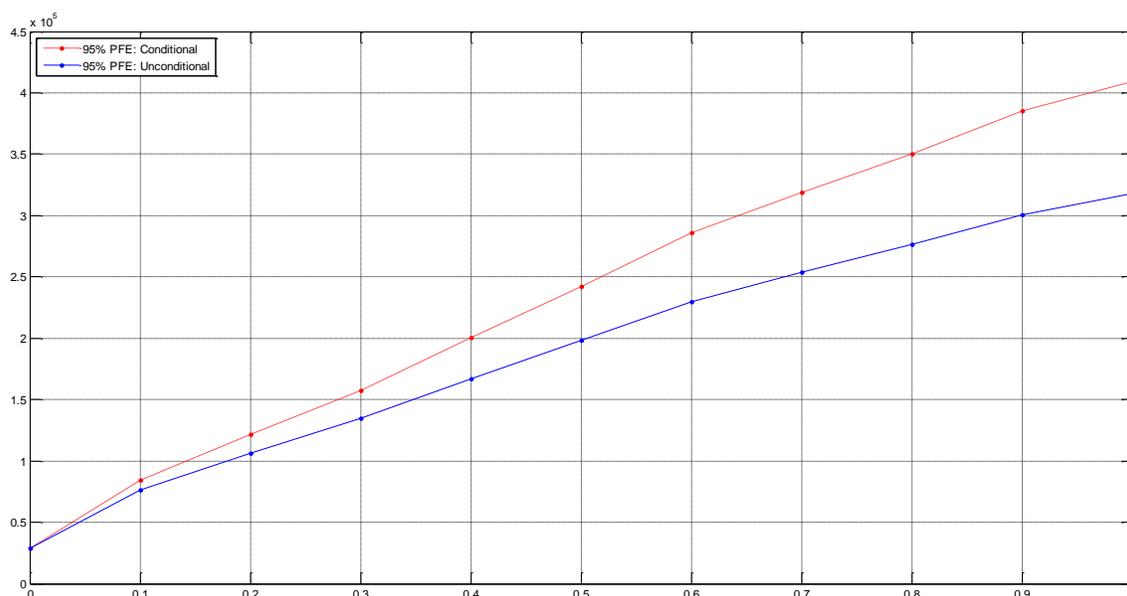


Figure 4.6 – AngloGold Unconditional and Conditional 95% PFE profile in USD

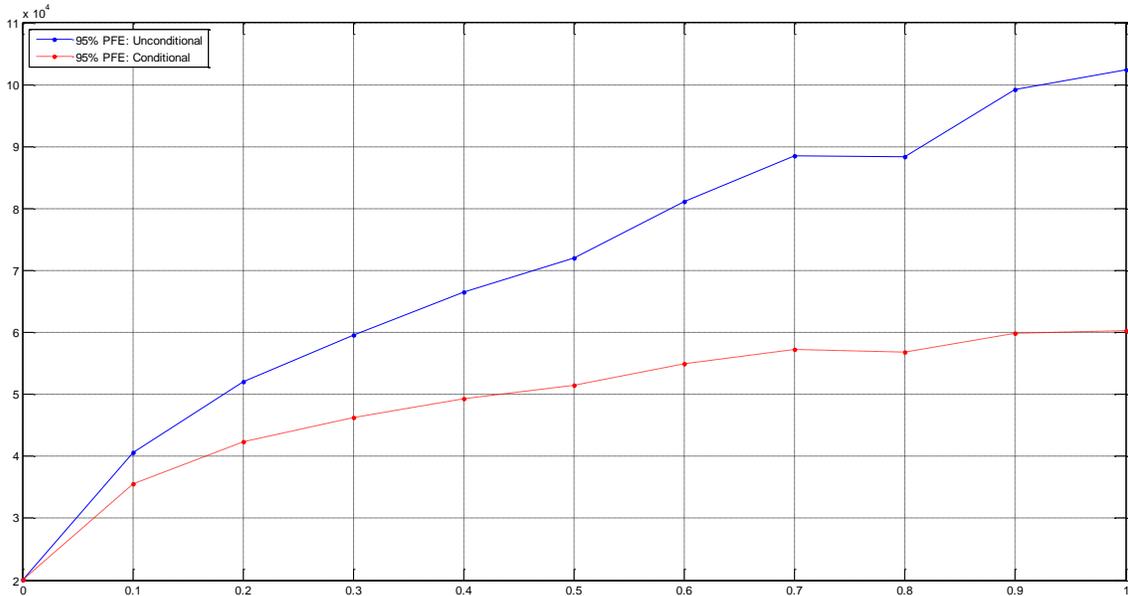


Figure 4.7 - Sasol Unconditional and Conditional 95% PFE profile in USD

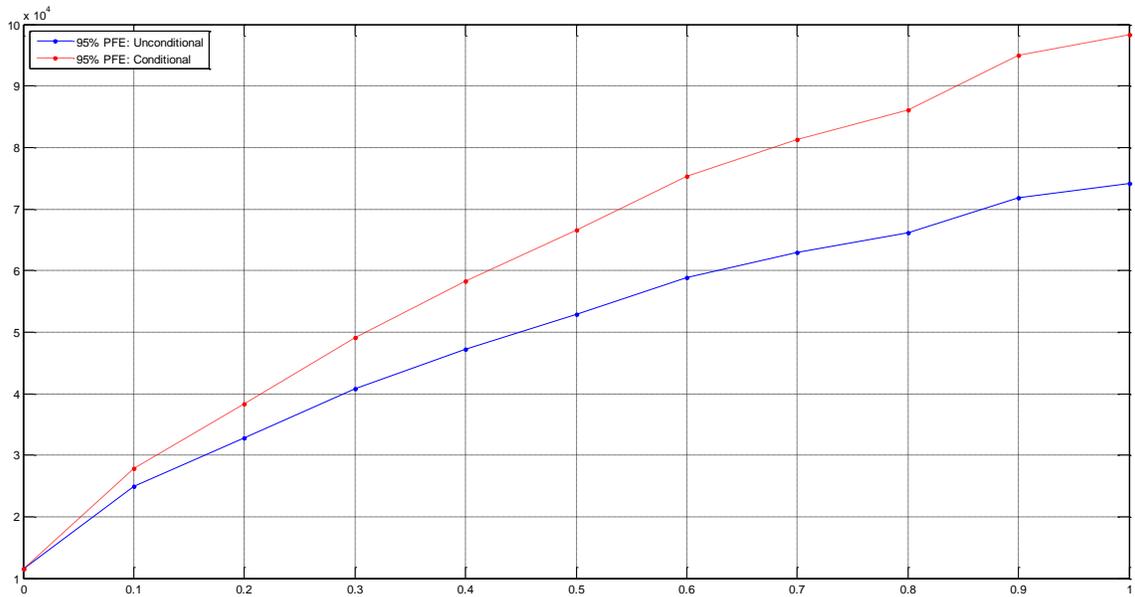


Figure 4.8 – MTN Unconditional and Conditional 95% PFE profile in ZAR

Finally, the following table summarises the relative increase (or decrease) in the maximum potential future exposure (i.e. maximum PFE over the profile) for each of the three scenarios as a result of the incorporation of wrong-way risk into the exposure measure:



<i>Company</i>	<i>Increase</i>
Anglo American	29.79%
Sasol	-41.20%
MTN	32.68%

Table 4.4 – Increase in maximum PFE as a result of wrong-way risk



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UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

5 Regulatory Capital for Counterparty Credit Risk

The models discussed so far for measuring counterparty credit risk was mostly presented in situations applying to internal credit risk management – i.e. measuring the counterparty credit risk exposure against a set limit. This is however only one application of these models and we now proceed to another very important field where the measurement of counterparty credit risk plays a very important role.

5.1 Systemic Risk

Banks (and some other financial institutions) play a very important role of financial intermediation and, most importantly provide liquidity to financial markets. This role is crucial in ensuring the efficient functioning of an economy. For this specific reason, bank failures can be much more disruptive to the economy than the failure of other entities. Systemic risk can be defined as the risk of a sudden shock that would damage the financial system to such an extent that the whole economy would suffer. Kaufman and Scott (2003) refer to systemic risk as: “It matches the fear of a cry of ‘*Fire!*’ in a crowded theatre...” They define systemic risk as “the risk or probability of breakdowns in an entire system as opposed to breakdowns in individual parts or components and is evidenced by co-movements (correlation) amongst most or all of the parts”. In turn, the Bank for International Settlements (BIS) defines systemic risk as “the risk that failure of a participant to meet its contractual obligations may, in turn, cause other participants to default with a chain reaction leading to broader financial difficulties”.

A typical example of a source of systemic risk is the behaviour of panic-stricken depositors or investors. The depositors of a bank become concerned about the stability of the bank and demand immediate return of their funds. This could lead to the failure of a bank. Similarly, a sudden drop in security prices may lead to margin calls forcing leveraged investors to liquidate their positions – furthering the downward pressure on prices. This may lead to a loss of liquidity or even a credit crunch.

The history of systemic risk is profoundly marked with the banking crisis in the United States during the 1930's. The banking system was subject to bank runs, when depositors lost faith in the ability of their deposit bank to make full payment and “ran to the bank” to withdraw their funds. In such situations, typically, the bank may be perfectly solvent – i.e. have assets (in the form of outstanding loans) which exceed the value of its liabilities (in the form of demand deposits). The problem is that these assets are illiquid and so the bank cannot meet its redemptions immediately leading to default. Another more recent example of such an event is that of Northern Rock Bank. Northern Rock Bank had to turn to the Bank of England, as lender of last resort, due to liquidity problems resulting from difficulty in raising funds in the money market. These liquidity problems were as a direct result of the Sub-Prime crisis in the United States during the summer of 2007.¹¹⁷

A prime example of another source of systemic risk, a breakdown in the payment system, is that of the 1974 failure of Bankhaus Herstatt – a small German bank active in the foreign exchange market. The bank was closed down due to insolvency during German banking hours but before the start of the US banking hours. As a result, the bank failed to make payments on the US Dollar (USD) legs of its foreign exchange transactions and cross-currency swap transactions even where it had already received the Deutsche Mark (DM) leg on these transactions. What became known as Herstatt Risk has led to a concerted effort by bank regulators to avoid such situations which ultimately gave birth to the Basel Committee on Banking Supervision (BCBS). The regulation of banks is motivated by two main objectives:

- Minimising Systemic Risk
- Protecting the depositors

¹¹⁷ For more information, see: http://en.wikipedia.org/wiki/Northern_Rock

5.2 The Basel 2 Capital Accord

The Basel Committee¹¹⁸ on Banking Supervision was created by the Central Bank governors of the Group of Ten (G-10) nations in 1974 and meets quarterly each year. The Basel Committee formulates broad supervisory standards and guidelines. It recommends statements of best practice in banking supervision in the expectation that member authorities and other nations' authorities will take steps to implement them through their own national systems (whether in statutory form or otherwise). The main purpose of the committee is to encourage convergence toward common approaches and standards.

Arguably one of the most influential of such standards is the Basel Capital Accord (Basel 1) published in 1988 which specifies a standard capital adequacy framework for banks. The Basel 1 accord represented a landmark financial agreement in terms of the regulation of internationally active commercial banks. The framework specifies a standard that banking regulators can use in passing regulations on the minimum capital that a bank is required to hold to guard against the financial and operational risks and the risks related to financial gearing that banks face in their day-to-day activities. The purpose of this capital is to serve as a buffer against unexpected financial losses, thereby protecting depositors and ensuring the stability of the financial markets. The biggest criticism of the Basel 1 accord was that it was not risk sensitive. More specifically, it required the banks to hold the same amount of capital against two identical loans even in the case of the counterparties (borrowers) being of significantly differing credit quality. This induced banks to shift lending to lower rated borrowers since the return was higher, with the same capital charge.

In simple terms, the new Basel accord (Basel 2) aims to have banks hold more capital the higher their risk exposure. The new accord distinguishes between expected losses (EL) and unexpected losses (UL). Capital is supposed to absorb unexpected losses implying it cannot support expected losses as well. Banks typically have provisions for expected

¹¹⁸ The Basel Committee's members are senior officials from the G-10 (Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Sweden, the United Kingdom and the United States (plus Luxembourg and Switzerland). Its website: <http://www.bis.org>.

losses in the form of funded reserves called general provisions or loan loss reserves. The Basel accord is divided into three sections or pillars:

- Pillar 1

This section deals with the maintenance of regulatory capital calculated for three major risks that a bank faces:

- Credit Risk

This component can be calculated using three different methods varying in degrees of sophistication:

- Standardised Approach
- Foundation Internal Ratings Based Approach (Foundation IRB)
- Advanced Internal Ratings Based Approach (Advanced IRB)

- Operational Risk

This component can be calculated using three different methods varying in degrees of sophistication:

- Basic Indicator Approach
- Standardised Approach
- Advanced Measurement Approach

- Market Risk

For Market Risk the preferred approach is VaR (Value at Risk).

- Pillar 2

This pillar deals with the regulatory response to the first pillar giving regulators much improved ‘tools’ over those available to them under Basel 1.

- Pillar 3

Pillar 3 greatly increases the disclosures that a bank is required to make in order to promote transparency in the banking sector worldwide.

For the discussions following hereafter we will mostly focus on aspects specific to the credit risk section under Pillar 1, and more specifically on the measurement of EAD (Exposure at Default) of trading book products as this is related to what we have discussed so far in terms of the quantification of counterparty credit risk exposure.

Although, strictly speaking, the accord applies only to internationally active banks within the G-10, the capital requirements have been accepted and are being applied in more than 100 countries, including South Africa. Moreover, all four of the major¹¹⁹ banks in South Africa have elected to follow the Advanced Internal Ratings Based (Advanced IRB) approach. These banks, at the time of writing this document, are all implementing the current exposure method for measuring EAD (initially at least). We will therefore focus on both the current exposure method (CEM) and the internal models method (IMM) in order to illustrate the potential benefits to the typical South African bank in using the IMM approach in comparison with the CEM.

5.3 Pillar 1: Credit Risk

Under Pillar 1, there are three main components or inputs to the calculation of the required regulatory capital resulting from credit risk. These parameters are:

- *PD (Probability of Default)*
The probability of default is the likelihood that the counterparty to an obligation with a bank will default over a one year time window and as a result not honour its obligation.
- *EAD (Exposure at Default)*

¹¹⁹ The four major commercial banking groups in South Africa are: ABSA, First National Bank, Standard Bank of South Africa and Nedcor.

An estimate of the extent to which a bank may be exposed to a counterparty in the event of, and at the time of, that counterparty's default. It is a measure of potential exposure, in monetary terms, calculated for a period of one year or until maturity (whichever is sooner).

- *LGD (Loss Given Default)*

LGD is the fraction of the EAD that will not be recovered after default. This is also equal to $1 - RR$ (where RR is the recovery rate). LGD is therefore a number between (and including) 0 and 1.

The estimation of PD and LGD are, however, beyond the scope of this document. The reader is referred to Duffie and Singleton (2003).

In the remainder of this section we will focus on the estimation of EAD under the Advanced IRB approach comparing the Current Exposure Method (CEM) and the Internal Models Method (IMM).

5.3.1 Advanced IRB Approach

Under the Advanced IRB Approach, the minimum regulatory capital requirement, C , is calculated using the following formula:

$$C = EAD \cdot K(PD, LGD) \cdot MA(PD, M),$$

where

- EAD is the exposure at default (defined in Section 5.3 above).
- PD is the obligor's probability of default (defined in Section 5.3 above), floored to 0.03%.
- LGD is the exposure-level loss given default, conditional on economic downturn.

- M is the exposure's effective remaining maturity (subject to a floor of one year and a cap of five years).
- $K(PD, LGD)$ is the default-only capital factor that is calculated from PD and LGD using the following formula:

$$K(PD, LGD) = LGD \cdot \Phi \left(\frac{\Phi^{-1}(PD) + \sqrt{\rho} \cdot \Phi^{-1}(0.999)}{\sqrt{1-\rho}} \right) - LGD \cdot PD,$$

with $\Phi(\cdot)$ the cumulative standard normal distribution and ρ the asset correlation dependent on PD as follows:

$$\rho = 0.12 \cdot \left(\frac{1 - e^{-50 \cdot PD}}{1 - e^{-50}} \right) + 0.24 \cdot \left(1 - \frac{1 - e^{-50 \cdot PD}}{1 - e^{-50}} \right).$$

- $MA(PD, M)$ is the maturity adjustment calculated using the following formula:

$$MA(PD, M) = \frac{1 + (M - 2.5) \cdot b(PD)}{1 - 1.5 \cdot b(PD)},$$

with $b(PD)$ a function of PD defined as:

$$b(PD) = (0.11852 - 0.05478 \cdot \ln(PD))^2.$$

Banks can then further elect to estimate EAD using one of the following three methods, in increasing order of sophistication:

5.3.1.1 The Current Exposure Method (CEM)

Under the CEM approach, EAD is calculated using:

$$EAD = RC + [AddOn],$$

with RC the current replacement cost, typically estimated using the current marked-to-market value.

Add-on is the estimated amount of potential future exposure. For a single transaction, this Add-on is calculated as the product of the notional of the transaction and an Add-on Factor determined from the regulatory tables based on the remaining maturity and the type of underlying asset of the derivative (e.g. interest rates, foreign exchange etc.). For the calculation of EAD on a portfolio of trades within a netting¹²⁰ set (i.e. all transactions in a netting set is covered by the same netting agreement) the current exposure is calculated as the sum of that of the underlying trades and the Add-on ($[AddOn]^P$, with P denoting that the Add-on is calculated on a portfolio level) is calculated using:

$$[AddOn]^P = (0.4 + 0.6 \cdot NGR) \cdot \sum_i [AddOn]_i, \quad (15)$$

where

- $[AddOn]_i$ is the Add-on for transaction i , and
- NGR is the ratio of the current net replacement cost to the current gross replacement cost for all transactions within the netting set.

For a collateralised counterparty, however, the credit exposure for transactions within a netting set is calculated as:

$$EAD = \max[0, MTM^P - C_A] + [AddOn]^P, \quad (16)$$

with MTM^P the mark-to-market value of the portfolio and C_A the volatility-adjusted collateral amount¹²¹.

¹²⁰ Note that only bilateral netting agreements are taken into account. If the netting agreement is not bilateral the effect of netting is not taken into account.

¹²¹ This is the value of the collateral reduced by a volatility-dependent haircut.

5.3.1.2 Standardised Method (SM)

Under the Standardised Method the EAD under a netting set is defined as follows:

$$EAD = \beta \cdot \max[NCV, \sum_j NRP_j \cdot CCF_j],$$

where

- NCV is the current market value of transactions in the netting set reduced by the current market value of the collateral assigned to the netting set.
- NRP_j is the absolute value of the net risk position in the hedging set¹²² j and
- CCF_j is the Credit Conversion Factor (specified in the Basel 2 Accord¹²³) that converts the net risk position in the hedging set j into a PFE measure.¹²⁴

5.3.1.3 Internal Models Method (IMM)

The IMM is the most risk-sensitive and most advanced method for estimating EAD under the Basel 2 accord (Pykhtin and Zhu, 2006). Under this method, both the EAD and the effective maturity (M in the formula for C , the minimum capital requirement, in 5.3.1 above) are estimated from the output of the bank's internal models¹²⁵ of potential future exposure. In strong contrast to the other two methods (CEM and SM), the IMM allows for cross-product netting. EAD calculated under the IMM therefore benefits from full netting. EAD, under the IMM, is calculated using¹²⁶:

¹²² A hedging set is defined as the portfolio risk positions of the same category (in terms of currencies, remaining maturities and market risk factors) that arise from transactions within the same netting set. Within each hedging set, offsets are fully recognised i.e. only the net amount of all risk positions is relevant in the calculation of EAD.

¹²³ Note that these CCFs were derived using a one-year horizon and uses at-the-money volatilities (which lead to conservative estimates since volatility impacts at-the-money contracts more significantly).

¹²⁴ Note that long positions in contracts with a linear risk profile carry positive signs while short positions carry negative signs. Positions with non-linear risk profiles (e.g. options) are represented by their delta-equivalent notional values.

¹²⁵ These models do, however, need to be approved by the bank's regulators before it may be used for this purpose.

¹²⁶ Fleck and Schmidt (2005).

$$EAD = \alpha \cdot [EPE_{eff}],$$

where

- EPE_{eff} is the Effective Expected Positive Exposure calculated for each netting set from the Expected Exposure (EE) profiles (described in Section 5.3.1.5 below).
- α is a multiplier which inflates the EAD number in an attempt to take account of the over-simplifying assumptions in the estimation of Effective EPE (see Section 5.3.1.5 below).

5.3.1.4 Multiplier Alpha (α)

As mentioned above, the reason for holding regulatory capital to ensure that the bank is protected against unexpected losses¹²⁷. From a theoretical mathematical perspective, if one looks at quantifying unexpected losses on a portfolio of exposures one would need (at least an estimate of) the loss distribution of such a portfolio. This, in turn, is quite a non-trivial problem to solve since estimating a loss distribution for a portfolio of counterparty credit exposures involves the modelling of the changes in the exposures together with modelling defaults and timing of defaults of the obligor. From a portfolio perspective (i.e. considering the loss distribution as a result of counterparty credit risk to multiple counterparties) this problem becomes even more complex in that one needs to model dependence among the obligors as well. In other words, the estimation of a counterparty-risk portfolio-level loss distribution involves the modelling of market variables that drive exposure as well as credit risk factors that impact counterparty credit quality.

In an attempt to simplify the Basel 2 framework, the regulators developed the concept of ‘loan equivalent’ which is used as a fixed exposure profile for each counterparty. The

¹²⁷ Theoretically this should be on a portfolio level – i.e. considering all the obligors and the contracts with each.

‘loan equivalent’ is intended to be an amount which can be used as an exposure on a derivative (or portfolio of derivatives) in the same way as a loan balance can be used in the context of a loan exposure (or exposure on a portfolio of loans). This allows the Basel 2 framework to apply the concept of EAD in a relatively consistent manner across the trading and banking book products. The problem is, however, to define the ‘loan equivalent’ amount in such a way as to achieve the desired result.

Canabarro, Picoult and Wilde (2003) showed that, for an infinitely granular portfolio (i.e. a portfolio with infinitely many counterparties with infinitely small exposures) with counterparty-level exposure independent both of themselves and of the counterparty credit quality, EPE becomes the true loan-equivalent exposure. The problem is that real portfolios are not infinitely granular and counterparty exposures are not independent since they are driven by the same market risk factors. In addition, as we have discussed in section 4 above, the existence of wrong-way risk implies that credit exposure may be correlated to the counterparty’s credit quality. EPE is therefore, on its own, not sufficient to be used as a loan equivalent for real portfolios and will lead to understated portfolio capital.

In 2002, Evan Picoult wrote a proposal¹²⁸ to an International Swaps and Derivatives Association (ISDA) working group in which he recommended defining a quantity called “alpha” (α) – a scaling factor to transform the EPE into an effective loan equivalent taking into account of the fact that the characteristics of real portfolios of counterparty risk were quite different from the infinitely granular portfolio assumptions. Picoult defined alpha as the ratio of two quantities (a/b) where:

- a is the portfolio capital estimated using full simulation using uncertain exposures
- b is the portfolio capital estimated using the reduced (simplified) model by replacing the uncertain exposures with EPE.

Picoult suggested that alpha should be measured as a function of:

¹²⁸ Picoult (2003).

- the effective number of independent counterparties
- the effective number of independent market rates which affect the potential exposure
- the PD of each counterparty.

Since capital estimated by the reduced model is a homogenous function of exposures, scaling EPE by the α -factor would match the capital produced by the full model. Therefore, $\alpha.EPE$ is the true loan-equivalent exposure since using $\alpha.EPE$ as the EAD removes the difference in capital treatment between economically equivalent loans and derivatives.

Canabarro, Picoult and Wilde (2003) used a one-factor conditional independence framework to study the sensitivity of alpha under various model inputs and parameters ignoring the effect of wrong-way risk. The results ignoring wrong-way risk resulted in an alpha estimate of $\alpha = 1.09$. After adding the effect of wrong-way risk to the same model, Wilde (2005) estimated the value for alpha at $\alpha = 1.21$. The Basel Accord specifies alpha at a rather conservative level of $\alpha = 1.4$. Banks using the IMM do however have the option of estimating their own value for alpha subject to supervisory approval and floored at 1.2.

In summary the alpha factor can therefore be seen to compensate, albeit in a conservative manner, for the over-simplifying assumptions required in using EPE as a loan-equivalent amount and also, to an extent, of the effect of wrong-way risk.

We will apply the specified $\alpha = 1.4$ in our calculations.

Using Finger's Model presented in Section 4.1 on pp 79, it is therefore possible to deduct an implied level of wrong-way risk in the IMM using an alpha of $\alpha = 1.4$.

5.3.1.5 Expected Positive Exposure (EPE) and Effective EPE

Banks typically have internal models for measuring counterparty credit risk – either using a simple add-on approach or using full simulation as discussed previously. The banks that apply the IMM for estimating EAD typically have internal models which are used to measure counterparty credit risk and, as a result, these model typically estimate (through simulation) portfolio distributions at various points in the future. Calculating an estimate for EPE from these distributions is trivial and summarised below:

For each simulation date, t_k , the bank computes an expectation of exposure EE_k , as a simple average of all the simulated realisations of exposure¹²⁹ at that date. In other words:

$$EE_k = \frac{1}{N} \sum_{i=1}^N \max[0, MTM(\omega_k)]. \quad (17)$$

The various values for EE_k for $0 \leq k \leq 1$ is then referred to the EE profile over the first year. The EPE is then defined to be the average of the EE profile over the first year and is practically computed as the weighted average of the EE as follows¹³⁰:

$$EPE = \sum_{k=1}^{\min[1yr, mat]} (EE_k \cdot (t_k - t_{k-1})). \quad (18)$$

Now, in practice, generally the number of trades and the number of unrealised cash flows decrease over time. Also, trades in the short term mature resulting in a decrease in the EE profile. The reality is that these short-term trades are more often than not, in practice, replaced by new ones and as a result leads to the EPE (as defined above) to risk. To account for this so-called roll-over risk¹³¹, the EPE profile is used in the following recursive formula in defining the Effective EE ¹³²:

¹²⁹ In this instance exposure is meant to only represent positive numbers – i.e. negative exposures are treated as zero.

¹³⁰ Note that *mat* denotes the actual maturity of the transaction.

¹³¹ See Pykhtin and Zhu (2006).

¹³² The initial condition is that EE_0^{eff} should equal the current mark-to-market.

$$EE_k^{eff} = \max[EE_{k-1}^{eff}, EE_k]. \quad (19)$$

The relationship between the Effective EPE and the Effective EE is similar to that between the EPE and EE as follows:

$$EPE_{eff} = \sum_{k=1}^{\min[1yr, mat]} (EE_k^{eff} \cdot (t_k - t_{k-1})). \quad (20)$$

5.3.1.6 Maturity Adjustment

Banks are required, under the Basel 2 accord, to calculate the EE profile out to the expiration of the longest contract in the netting set. For exposures with remaining maturity longer than one year, the effective maturity, M , is given by

$$M = \min[1 + \Delta M, 5yr],$$

with

$$\Delta M = \frac{\sum_{t_k > 1yr}^{mat} (EE_k \cdot (t_k - t_{k-1}) \cdot df_k)}{\sum_{k=1}^{t_k \leq 1yr} (EE_k^{eff} \cdot (t_k - t_{k-1}) \cdot df_k)}$$

5.4 EAD and its Role in the Calculation of Regulatory Capital

5.4.1 Measuring Exposure at Default (EAD)

As discussed in Section 5.3.1 above, the exposure at default estimates used in calculating the minimum capital requirements under Basel 2 differ between the three approaches under the Advanced IRB approach. We now proceed to the calculation of the EAD estimates using the Current Exposure Method (used by the majority of the leading banks in South Africa) and the Internal Models Method (the most advanced method for calculating EAD under Basel 2). It is important to note that the results from both of these

methods will be applied equally in the determination of the minimum capital requirement under Basel 2. Therefore, a higher EAD estimate (in the examples below) will necessarily result in a higher capital requirement.

5.4.2 EAD under the Current Exposure Method (CEM)

As discussed in Section 5.3.1 above, the CEM requires, as inputs into the calculation of EAD, the following parameters (for the contract-level EAD):

- RC – the replacement cost, typically measured using MTM

- Add-On, which in turn depends on the:
 - Type of underlying in the derivative
 - Time to maturity of the contract
 - Notional amount of the contract

Let's consider the portfolio of derivative contracts used in the counterparty-level PFE calculations in Section 3.2.1 (Example Portfolio 1) and Section 3.2.2 above. Refer to Table 3.1 and Table 3.2 for specific details on the portfolios.

Firstly, let us briefly discuss the derivation of and motivation behind our input parameters. Throughout we will use the current mark-to-market value for the replacement cost (RC) of the individual contracts. In the derivation of the Add-On for each contract, we will use the following table¹³³ from the Basel 2 Accord:

¹³³ Source: Basel Committee on Banking Supervision (2005)

<i>Maturity (T)</i>	<i>Interest Rates</i>	<i>FX and Gold</i>	<i>Equities</i>	<i>Precious Metals¹³⁴</i>	<i>Other Commodities</i>
T<1	0.0 %	1.0 %	6.0 %	7.0 %	10.0 %
1≤T<5	0.5 %	5.0 %	8.0 %	7.0 %	12.0 %
5≤T	1.5 %	7.5 %	10.0 %	8.0 %	15.0 %

Table 5.1 - Add-On Factors under the Current Exposure Method

As we are interested in the exposure in Rand (ZAR), all notional amounts will be expressed in ZAR when calculating the Add-On amounts. Since the portfolios under consideration only contain foreign exchange derivatives only column 3 in Table 5.1 will be applicable.

The following is a summary of the input parameters required in the calculation of EAD under the CEM for the example portfolios under consideration:

<i>No.</i>	<i>Contract Type</i>	<i>Underlying</i>	<i>Notional¹³⁵</i>	<i>Maturity (Yrs)</i>	<i>RC¹³⁶</i>	<i>Add-On¹³⁷</i>
1	Forward	USD/ZAR	1,000	0.75	97.77	0.00
2	Forward	USD/ZAR	-850	1.50	-2,194.66	24.78
3	Call Option	USD/ZAR	1,000	2.00	1,871.54	38.85
4	Put Option	USD/ZAR	1,000	2.00	213.63	38.85

Table 5.2 - Example Portfolio 1 with input parameters for the calculation of EAD under the Current Exposure Method

<i>No.</i>	<i>Contract Type</i>	<i>Underlying</i>	<i>Notional¹³⁸</i>	<i>Maturity (Yrs)</i>	<i>RC</i>	<i>Add-On</i>
1	Forward	USD/ZAR	1,000	0.75		0.00
2	Forward	GBP/ZAR	-490	1.50		50.84
3	Call Option ¹³⁹	GBP/ZAR	-500	3.00		38.63
4	Call Option	USD/ZAR	1,000	3.00		37.50

¹³⁴ Excluding Gold.

¹³⁵ The notional is expressed in ZAR. The amount is converted using the strike of the transaction since this produces the ZAR-equivalent of the underlying notional.

¹³⁶ Measured as the marked-to-market value.

¹³⁷ Calculated as the product of the add-on factor and the notional. See the approach described in section 5.3.1.

¹³⁸ See pp 135.

¹³⁹ Note that written (sold) options do not result in increased credit exposure. Written options are excluded from EAD calculations under the CEM.

Table 5.3 - Example Portfolio 2 with input parameters for the calculation of EAD under the Current Exposure Method

The tables above allow us to calculate the EAD on a contract level for each of the contracts in the respective portfolios using the CEM. The contract-level EAD calculations are very simple¹⁴⁰ and the results are summarised below:

<i>Portfolio</i>	<i>Contract</i>	<i>Contract-Level EAD_{CEM}</i>
1	1	179.47
	2	247.78
	3	2,260.04
	4	602.13
2	1	297.64
	2	524.54
	3 ¹⁴¹	-
	4	3,050.53

Table 5.4 - Contract-Level EADs using the CEM

We will compare the above estimates of contract-level EAD with the contract-level EAD estimates using the IMM below.

We now proceed to the calculation of counterparty-level EAD using the portfolios above. We will consider the EAD with a netting agreement in place. Our first portfolio-level calculation is that of the term *NGR*, the ratio of the current net replacement¹⁴² cost to the current gross replacement cost for all transactions within the netting set¹⁴³. This is then followed by the calculation of *Add-On(P)*, the portfolio-level add-on. The results from the calculations of *NGR* and *Add-On(P)* are summarised below, together with the resultant counterparty-level EADs using the CEM.

¹⁴⁰ The contract-level EAD is simply the sum of the current replacement cost (floored at zero) and the add-on (see Table 5.2 and Table 5.3).

¹⁴¹ Excluded: the contract is a written option.

¹⁴² Floored at zero.

¹⁴³ As defined in BCBS (2005).

<i>Portfolio</i>	<i>Gross RC</i>	<i>Net RC</i>	<i>NGR</i>	<i>Add-On(P)</i>	<i>EAD_{CEM}</i>
1	2,182.94	0.00	0.00	442.59	442.59
2	2,907.63	2,907.63	1.00	965.08	3,872.71

Table 5.5 - Counterparty-Level EADs using the CEM

5.4.3 EAD under the Internal Models Method (IMM)

In order to calculate EAD under the IMM, we need to calculate the multiplier alpha (α)¹⁴⁴, Expected Positive Exposure (EPE) and Effective EPE. We proceed with example calculations of EPE, Effective EPE and EAD using the IMM. We will, in contrast to Section 5.4.2 above apply the IMM method to the example portfolios used in Section 3.2.1 above and Section 3.2.2 above.

The first step in the process of estimating EAD under the IMM is the calculation of the Expected Exposure (EE) profile. As mentioned above, this is done using the simulated MTM values of the contract(s) under consideration and simply calculating the average of the positive MTM values at each time step. This step is relatively straight forward since we are merely calculating a portfolio statistic on an already simulated portfolio distribution at each time step. Using the EE profile it is simply a matter of applying (17), (18), (19) and (20) in order to derive the EPE and Effective EPE. The following graphs illustrate the estimated EE, EPE, Effective EE, Effective EPE and the EAD under the IMM for the two portfolios under consideration:

¹⁴⁴ In our examples we will use the prescribed alpha of 1.4.

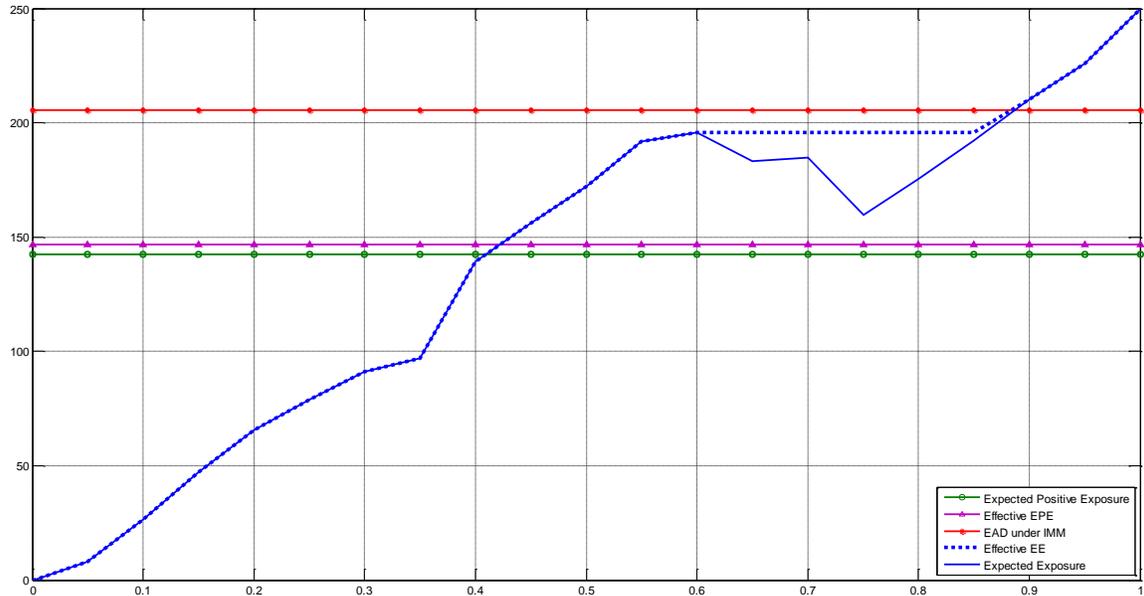


Figure 5.1 - Portfolio 1: EAD estimate and related measures using the IMM

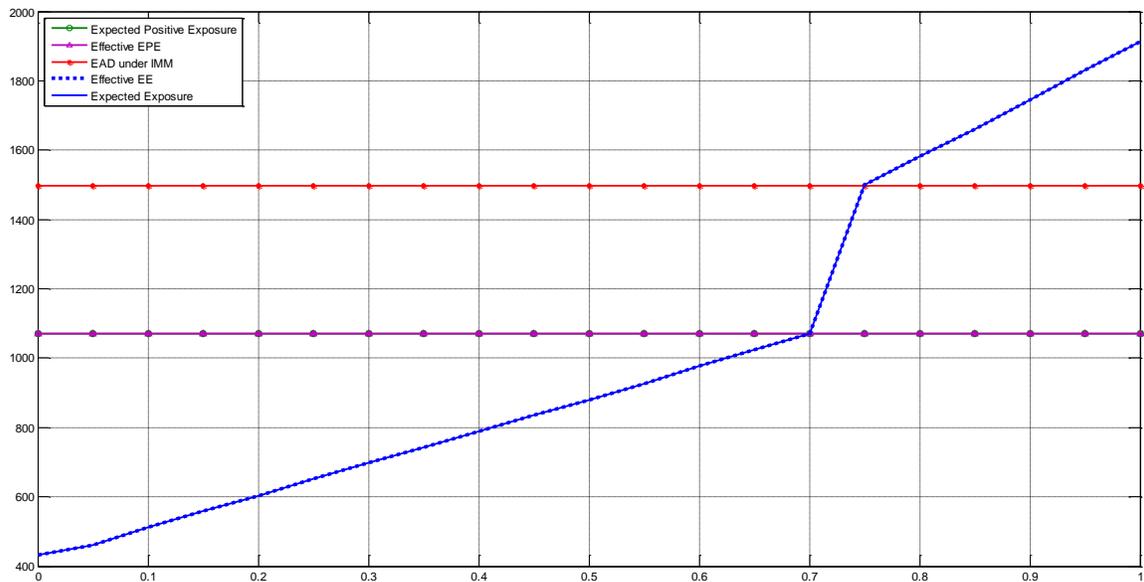


Figure 5.2 - Portfolio 2: EAD estimate and related measures using the IMM¹⁴⁵

The following table summarises (analogous to Table 5.4) the contract-level EADs calculated using the IMM.

¹⁴⁵ Note that in this example the EPE=Effective EPE since the EE profile is strictly increasing.

<i>Portfolio</i>	<i>Contract</i>	<i>Contract-Level EAD_{IMM}</i>
1	1	513.11
	2	7.16
	3	2,751.16
	4	475.77
2	1	422.30
	2	1,314.52
	3	0.00
	4	3,933.02

Table 5.6 - Contract-Level EADs using the IMM

5.4.4 Comparison of EAD under CEM and IMM

We have applied the EAD estimates under both the CEM and the IMM to the same portfolios of derivatives. Firstly, let's compare the counterparty-level EAD estimates and see how these compare to the 95% PFEs estimated in Section 3.2.1 above and Section 3.2.2 above.

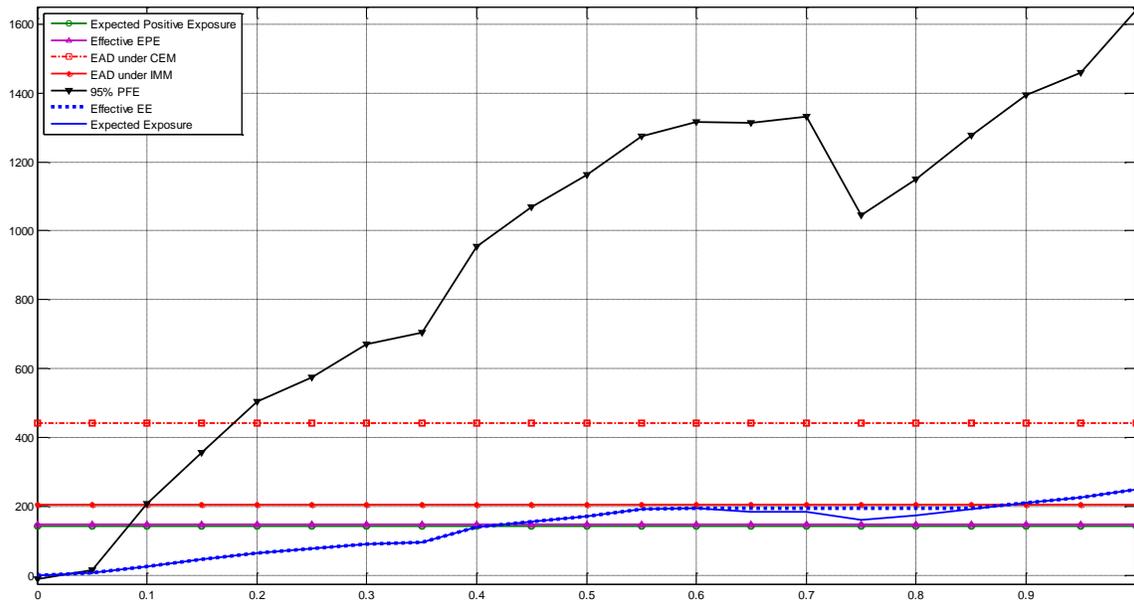


Figure 5.3 - Portfolio 1: EAD estimates using CEM and IMM

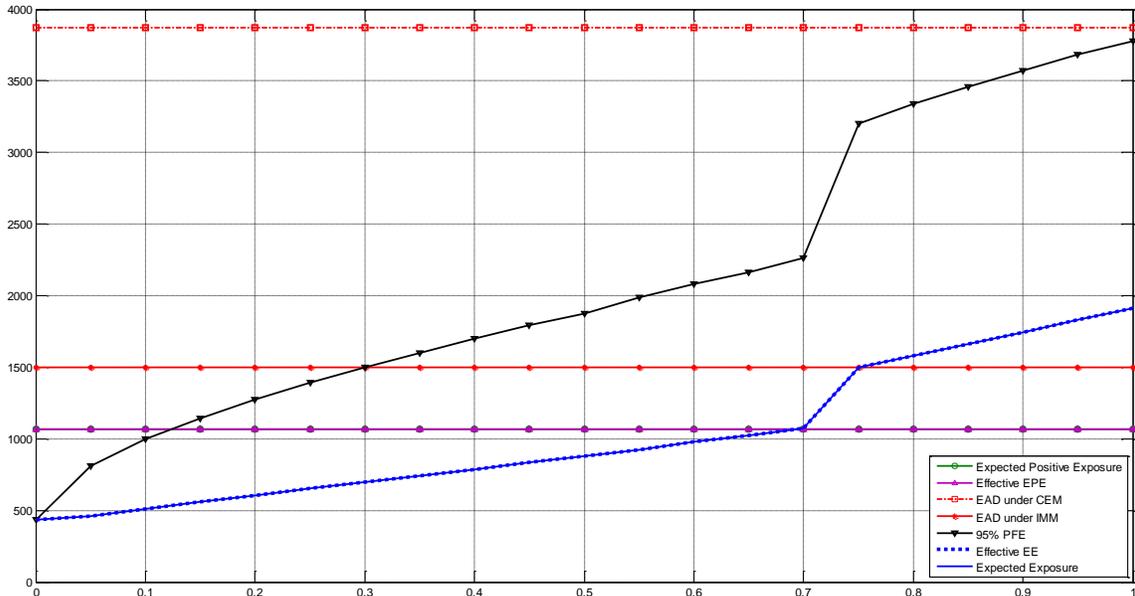


Figure 5.4 - Portfolio 2: EAD estimates using CEM and IMM

The following graph combines the results presented in Table 5.4 and Table 5.6 and illustrates the differences in the results between the two methods.

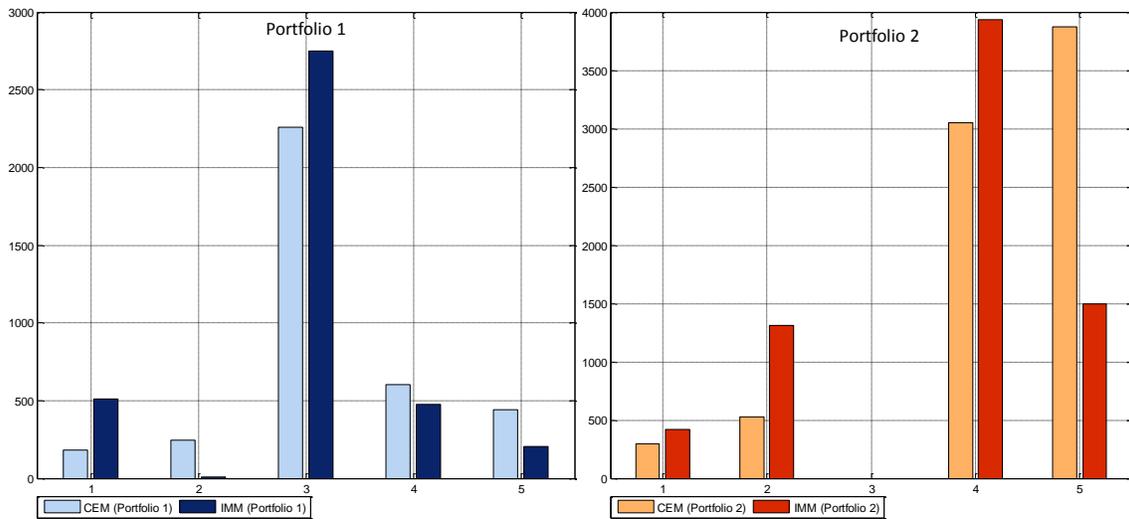


Figure 5.5 - Comparison of IMM and CEM EAD estimates

The figure above compares the EAD estimates for each contract (numbered 1 to 4) as well as the counterparty-level EAD (assigned the number 5) of portfolio 1 and portfolio 2 under the CEM and IMM methods. It is interesting to note that, in the results obtained through our analysis that, on a contract level, the CEM produces lower estimates for EAD

than that produced by the IMM. In contrast it appears that the IMM produces lower estimates for counterparty-level EADs. It is dangerous to make generalisations based on the results from 2 small portfolios, but we can nonetheless make the following valuable observations and comments:

- The fact that the IMM produces lower EAD estimates on a portfolio of contracts is not unexpected and makes intuitive sense. Since the CEM takes no account of correlation it means that it is insensitive to higher or lower correlation assumptions between the underlying market variables. Portfolio 2 does contain very highly correlated underlying market variables and it is therefore expected that the IMM will have significantly lower estimated EAD values.
- In practice some banks incorporate the cost of capital in the pricing of transactions in order to ensure an appropriate level of return. Typically, an assessment is made with regards to the regulatory capital that a proposed (often individual) transaction attracts by calculating a contract-level EAD and determining the resultant regulatory capital charge. The results obtained through our analysis (albeit elementary examples) highlight some potential pitfalls in the practice of calculating capital on a contract-level basis which may lead to undesirably high capital charges (leading to potential loss of business through unnecessarily high pricing). Let's, for example, consider the aggregate EAD (i.e. the sum of the EADs of each of the underlying contracts) for each portfolio under each of the two methods:

	<i>Current Exposure Method</i>			<i>Internal Models Method</i>		
	<i>Aggregate EAD</i>	<i>Portfolio-Level EAD</i>	<i>A-EAD/P-EAD</i>	<i>Aggregate EAD</i>	<i>Portfolio-Level EAD</i>	<i>A-EAD/P-EAD</i>
P/folio 1	3,289.41	442.59	743%	3,747.20	205.43	1,824%
P/folio 2	3,872.70	3,872.70	100%	5,669.84	1,497.84	3,785%

Table 5.7 - Comparison between aggregate and portfolio-level EAD estimates

The above table indicates that if we were to charge capital based on contract-level EAD estimates that we could on a cumulative basis (in our example portfolios) charge up to 2,785% more than what we would need¹⁴⁶ to charge.

- The impact of the *NGR* factor, i.e. the ratio of the current net replacement cost to the current gross replacement cost for all transactions within the netting set, on the CEM EAD estimates is also significant. If the current net replacement value of a portfolio is very low compared to the gross replacement cost the *NGR* factor will significantly reduce the EAD when compared to a similar portfolio with an *NGR* factor close to one. This may seem obvious and trivial but the implications of this need to be fully understood. Let's consider the following two scenarios:
 1. A portfolio of two contracts with a single counterparty. The two contracts have different but very highly-correlated underlying assets. Furthermore, let's assume that the current net replacement cost is zero – resulting in *NGR*=0.
 2. The same situation as above, except that the underlying assets are not correlated.

Firstly, let's consider what effect the fact that *NGR*=0 has on the EAD estimate under the CEM. By inspection it is clear that the condition that *NGR*=0 results in the following:

$$AddOn(P) = 0.4 \cdot \sum_i AddOn_i. \quad (21)$$

The effect of a current net MTM of zero therefore implies a 40% reduction in the gross add-on. The *NGR* factor, it would seem, attempts to use current information (i.e. current replacement costs) to derive information on future scenarios. Let's

¹⁴⁶ Assuming that the internal model EAD is an accurate reflection of the true exposure at default.

consider the effect of changing the *NGR* (which will be achieved by adjusting the strikes of the underlying transactions) and the assumed level of correlation. Obviously the correlation assumption will have no impact on the CEM EAD estimates and therefore we also apply the IMM in comparing the results. The portfolio that we will consider is as follows:

<i>No.</i>	<i>Contract Type</i>	<i>Underlying</i>	<i>Notional</i> ¹⁴⁷	<i>Maturity</i> ¹⁴⁸
1	FX Forward	USD/ZAR	USD 1,000	1.5 Yrs
2	FX Forward	GBP/ZAR	GBP -490	1.5 Yrs

Table 5.8 - Example Portfolio for IMM Correlation Testing

If we assume a high level of correlation between the USD/ZAR and GBP/ZAR exchange rates, it implies that these two variables will move in the same direction most of the time – i.e. if the USD/ZAR exchange rate moves up, we would expect the GBP/ZAR exchange rate to move up as well. Contract 1 in the table above will move more into the money if the USD/ZAR rate moves up, whereas contract 2 will move more into the money if the GBP/ZAR rate moves up. It is therefore fair to assume that these contracts will offset each other to a certain extent¹⁴⁹ as a result of this high level of correlation. In other words, the combined marked-to-market value of the two transactions will be relatively stable. Contract 1, as an example would increase in value when contract 2 decreases in value and vice versa. One would also expect this offsetting effect to diminish when there is no correlation between the two underlying variables and that the offsetting effect will reverse¹⁵⁰ should the correlation be highly negative¹⁵¹. In summary we therefore expect to see the EAD increase as the assumed level of correlation increases¹⁵².

The following chart summarises the resultant EAD estimates using the CEM and

¹⁴⁷ Negative notional amounts reflect a short position. For the first contract the notional is positive which represents a contract to receive 1,000 USD in 1.5Yrs. Similarly contract 2 represents a contract to deliver 490 GBP in 1.5Yrs.

¹⁴⁸ Remaining maturity.

¹⁴⁹ Assuming a high correlation between the USD/ZAR and GBP/ZAR exchange rates.

¹⁵⁰ We expect that the exposure would be amplified. In other words, if contract one is in the money, contract two will also be in the money leading to a higher combined exposure than that under the positive correlation assumption. It is also possible that both contracts could simultaneously move out of the money.

¹⁵¹ Close to -1.

¹⁵² Since the CEM takes no account for correlation we only expect to see these effects in the EAD estimates using the IMM.

the IMM for five¹⁵³ different values of the assumed correlation and three¹⁵⁴ different values of *NGR*:

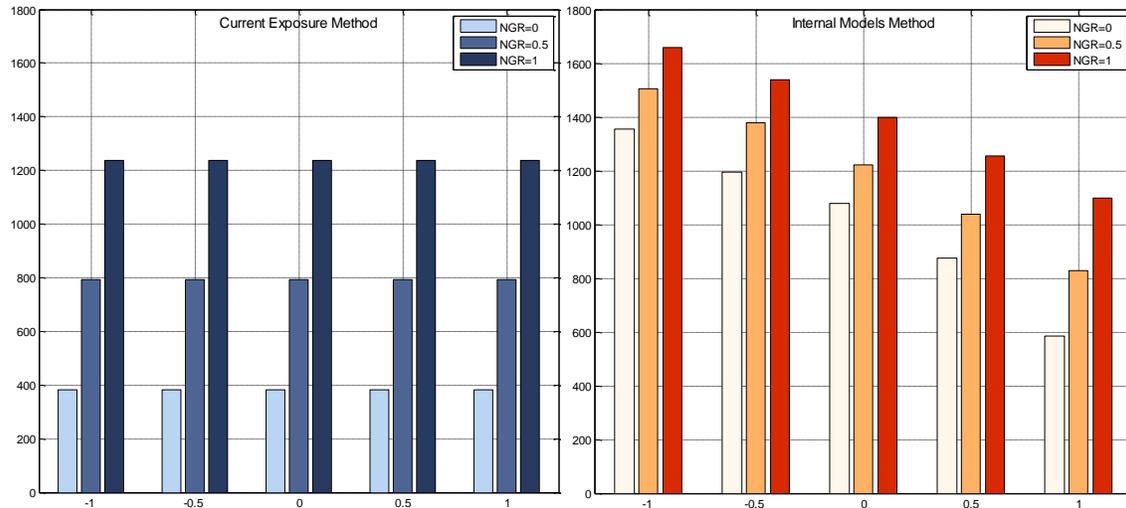


Figure 5.6 - Counterparty-Level EADs for different NGRs and Correlations using CEM and IMM

The results presented in Figure 5.6 allow us to make the following observations:

1. As expected, assumptions relating to the correlation of the underlying market variables have no impact on the exposure estimates produced using the CEM. The IMM on the other hand behaves as expected and the EAD estimates decrease as the correlation increases.
2. It is surprising to note that the CEM EAD estimates are mostly lower than that of the IMM. In fact, for the portfolio under consideration the CEM method grossly understates the EAD when compared to the IMM. The following graph shows the relative difference between the results from the two methods for different correlations and *NGR*:

¹⁵³ $\rho = \{-1, -0.5, 0, 0.5, 1\}$.

¹⁵⁴ $NGR = \{0, 0.5, 1\}$. Note that these values were achieved by adjusting the strike rate(s) of the contracts in the portfolio. More specifically, the strike rate combinations used in achieving the desired *NGR* values are: $X_1 = \{9, 8.65, 8.65\}$ and $X_2 = \{20.65, 20.37, 20.75\}$. The subscripts indicate the contract number.

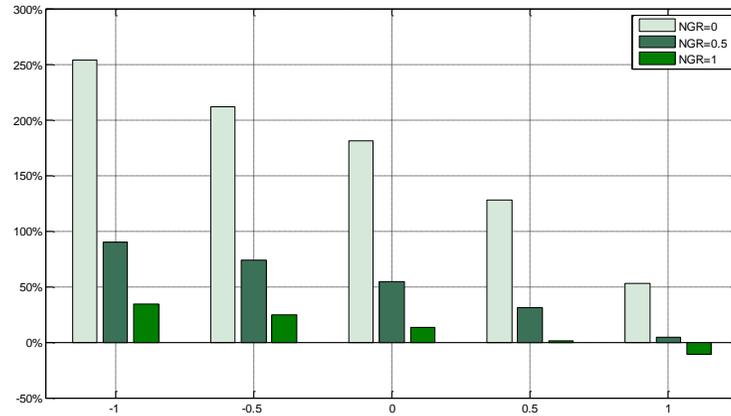


Figure 5.7 - Potential Under-Estimation of EAD using CEM compared to IMM

One possible explanation for this seemingly underestimation of EAD by the CEM is the fact that the NGR impacts the method’s estimation of potential future exposure. It is clear from (15) and (16) that the method implies that the current effect of netting on the portfolio is an indication of what could happen in the future. This example illustrates that this assumption is perhaps not always valid. The fact that the current net replacement cost on a portfolio-level is close to zero does not imply that the underlying contracts, which make up the portfolio, will behave in a similar fashion (i.e. in terms of correlation) in the future. Put simply, current net exposure should not determine the assumed potential future exposure on a portfolio level.

As mentioned above, the analysis on the portfolios conducted is by no means comprehensive and therefore it would not be wise to make general comments based on the results obtained. The results and discussions above do however give some insight into the dynamics of the two methods for calculating EAD under consideration.

6 Pricing and Hedging Counterparty Credit Risk

As with any other risk it would be ideal not only to be able to measure counterparty credit risk accurately but also to price and hedge unwanted risk in a liquid market. Counterparty credit risk is, however, significantly more complex to measure than for example market risk, and also does not yet have (especially in the South African context) a liquid market in which it is possible to transfer counterparty credit risk to. There are however some interesting developments in global developed markets which may, as often is the case, spill over to the local market in some form. Although there are not currently direct ways of trading or transferring counterparty credit risk directly (in South Africa) there does however exist other techniques of essentially achieving a similar result. This chapter gives an overview of some of the current practices of incorporating the effect of counterparty credit risk into the pricing of OTC derivative transaction and also discusses some techniques and motivations for the transfer of such risks.

6.1 Pricing Counterparty Credit Risk

It is clear from what we have discussed so far in this dissertation that the quantification of counterparty credit risk is not trivial. It does however remains a significant risk to an investment bank (for example) which needs to be managed. A bank not only needs to quantify and manage the risks that it takes on – it also needs to be compensated fairly for the risks that it is exposed to. This therefore means that banks need to take into account the counterparty credit risk that they are exposed to in making prices for the derivative products which they offer to their client.

6.1.1 Credit Value Adjustment (CVA)

In a report published by the Counterparty Risk Management Policy Group (CRMPG) mention is made of the emerging practice of so called credit charges or credit transfer

pricing. The counterparty credit risk on a swap or other derivative contract traded by a bank is transferred, within the institution, to a trading desk which manages and hedges this risk out dynamically and actively through the use of credit derivatives technology. In essence this practice is similar to the typical management of liquidity risk within a bank where an asset and liability management area would be responsible for the management of the liquidity risk of the entire bank and also charge a premium or make a price internally for funding the various business units. It makes intuitive sense to consolidate the counterparty credit risk that the bank is exposed to and to have a dedicated area to measure and manage such risks on an active basis. As can be expected, this also has the advantage of benefiting from portfolio-effects such as correlation and diversification.

It is interesting to compare the notion of credit risk transfer pricing to that of liquidity management as explained above. There is, however, one major difference in that the price of money (i.e. what funding costs for the asset and liability management unit) is more transparently observable in the market – whereas the ‘price’ of counterparty credit risk is not. This is especially true in the South African market where the normal credit derivatives market is not very liquid comprehensive in terms of names traded. Regardless of the challenges involved in determining a price for counterparty credit risk, it is more important to price for it than not.

A related concept to that of credit transfer pricing is credit value adjustment (CVA). CVA is an adjustment to the market value of the derivative due to the credit risk of the counterparty. More specifically, CVA is the difference between the the risk-free value of a derivative with a counterparty and the risk-adjusted value after taking the counterparty’s credit risk into account. For many years the derivatives portfolios of commercial banks were marked to market independent of the credit quality or credit rating of the counterparty. In the early 1990s some investment banks introduced the notion of a credit value adjustment into the valuation of their derivatives portfolios (Sorensen and Bollier (1994))¹⁵⁵:

¹⁵⁵ MTM_{CP_k} denotes the MTM of the entire portfolio of trades with counterparty k .

$$MTM_{CP_k} = \sum_j PV_{risk-free,j} - CVA_k.$$

The potential increase in the CVA of a counterparty is an important component of the potential loss of the economic value of the derivative with the counterparty. Hedging the credit value adjustment is the essence of hedging counterparty credit risk (Picoult, 2005).

6.1.2 Definition of CVA

Before we define the CVA formally, let's firstly look at a similar concept defined on a corporate bond. The market value of a single corporate bond can be expressed as follows:

$$\begin{aligned} MV_{bond} &= MV_{bond}^{risk-free} - CVA_{bond} \\ &= \sum_m Bond CF_m \times e^{-(r_{m,risk-free} \times t_m)} - CVA_{bond} \\ &= \sum_m Bond CF_m \times e^{-((r_{m,risk-free} + spread_{bond}) \times t_m)}. \end{aligned}$$

It follows that, using a first order approximation, that the CVA of a corporate bond is:

$$\begin{aligned} CVA_{bond} &= \sum_m Bond CF_m \times e^{-((r_{m,risk-free}))} \times t_m \times spread_{bond} \\ &= MV_{bond}^{risk-free} \times spread_{bond} \times duration_{bond}. \end{aligned}$$

Based on our knowledge of counterparty credit risk, we can make the following statements regarding CVA for counterparty credit risk in relation to the CVA of a risky bond:

- The CVA has to take into account not only the current expected cash flows but also the potential future cash flows that could occur if market rates change. Conceptually we can think of the way that the PFE profile is dependent on current rates and future rate assumptions. A PFE profile is clearly not deterministic over time and so the CVA needs to incorporate this.
- The CVA should be calculated on a counterparty-level across all transactions with the counterparty taking into account the effects of mitigating measures and techniques such as netting and collateral agreements.
- The CVA needs to, ideally, take into account the bilateral nature of counterparty credit risk – i.e. each counterparty to a swap (for example) is exposed to counterparty credit risk on the other party.

An intuitive approach to the definition of CVA in the case of OTC derivatives is to base the CVA on a credit reserve or expected loss-type measure. Moreover, the expected loss (EL) over the life of the portfolio of OTC derivatives can be thought of as the sum, over all forward periods, of the product of the EPE in the period and the historical forward loss norm. The historical loss norm is the product of the probability of default (PD) and the loss given default (LGD) for the forward period. This is consistent with how one might calculate a credit reserve to cover expected losses assuming that the portfolio were to be held to maturity¹⁵⁶. Specifically, we have:

$$EL = \sum_t EPE_t \times LN_t \times df_t.$$

In order to derive the CVA from a credit reserve measure we note that we need to make two adjustments:

¹⁵⁶ Also assuming that historical losses is a good predictor of future losses.

1. Instead of using historical volatilities and correlations in calculating EPE one needs to use, as far possible, market implied information.
2. Instead of using historical loss norms in order to predict losses one uses current credit spread in order to get a market implied view on the expected future states of the world.

We therefore derive the following expression for a market-based measure of unilateral CVA:

$$CVA = \sum_k EPE_k^{(I)} \times spread_k \times \Delta t_k \times df_k.$$

Where $EPE_k^{(I)}$ denotes the expected positive exposure calculated using implied market data – i.e. implied volatilities etc. $spread_k$ is the current credit spread of the counterparty in the forward time interval Δt_k . This form of CVA is typically referred to a unilateral CVA due to the fact that the measure is only based on the counterparty credit risk that the bank takes on against the counterparty and ignores the counterparty credit risk that the counterparty takes on against the bank.

6.2 Hedging Counterparty Credit Risk

As a result of our derivations and discussions above it is possible to view the market value of a portfolio of derivatives as being composed of two main components:

- The risk-free value of the portfolio. In other words ignoring counterparty credit risk
- The credit value adjustment (CVA)

Hedging the total market value of such a portfolio of derivatives can therefore be decomposed into:

- Hedging the risk-free market value of the portfolio
- Hedging the changes in CVA.

From the expression for MTM_{CP_k} above, it is clear that:

$$\Delta MTM_{CP_k} = \Delta \sum_j PV_{risk\ free,j} - \Delta CVA_k.$$

This expression tells us that the change in the mark-to-market value of the portfolio of derivatives with counterparty k is composed of two components:

- The change in the risk-free value of the derivative portfolio
- The change in the credit value adjustment.

The second term is therefore the change in the counterparty credit risk component of the market value of the derivative portfolio. Hedging counterparty credit risk can therefore be considered as the hedging of the CVA.

6.3 Transferring Counterparty Credit Risk

In general it is also possible to transfer credit risk using more traditional means. The impact of counterparty credit risk on a bank's capital requirements from regulatory, as well as an economic point of view, is significant and as a result banks typically try to maximise returns and minimise capital requirements in order to increase the return on equity for its shareholders. Therefore, as far as possible banks try not to warehouse risk on their balance sheets as this is quite expensive and inefficient from a capital point of view. To this extent banks have developed various techniques for reducing and

transferring the risk from their balance sheets to other parties willing to take on and manage the risks.

A very good example of such a technique is securitisation. In securitisations banks sell ‘risks’ from their balance sheets that are typically not traded in the open market (such as home loans) and, by grouping these together and assigning a credit rating to them (through a rating agency), transforms a non-traded asset into a recognisable security which investors are comfortable investing in. Through this process the bank retains some of the margin earned on the home loans (as an example) and passes the rest of the margin through to the investor as compensation for the risk assumed in investing in the securitised asset.

6.3.1 Motivation for Credit Risk Transferral

Banks typically transfer credit risk for the liberation of regulatory and/or economic capital in order to facilitate further loan intermediation. The example of securitisation is a very good one in the sense that it explains a number of the related issues surrounding credit risk transferral. Banks are typically specialists of asset origination in the sense that banks have access to a variety of different markets and market participants who require banking services and, more importantly for our example, funding in the form of loans. Although the banks have access to all of these potential assets for their balance sheets, there at the same time awareness of the scarcity of capital. Banks therefore prefer to focus their ability on the origination of the assets as opposed to the ‘hold to maturity’ or warehousing strategy.

An alternative motivation for the transferral of credit risk is to increase capacity. For example, let’s assume a bank has a client to which it is highly exposed in terms of credit risk. Let’s assume that the client asks the bank for increased credit. The bank may want to assist the client in this respect due to relationship considerations. Perhaps due to capacity or credit limits constraints, the bank may, however be unable to assist. The bank could then consider transferring some of the current exposure to this particular client to a third

party. Duffie (2008) argues that diversification and the reduction in the costs of raising capital for additional loan intermediation are the two major motivations for banks to transfer credit risk.

6.3.1.1 Diversification

In some markets, as in the South African market, there are certain industries which dominate the economy in terms of size and contribution to the country's GDP. The industry which is a good example in South Africa is the mining industry. This often has the result that banks (typically all of the big banks) have very large exposures to the mining industry. If, as an example, the total term-loan book of a bank contains 40% of exposure to mining-related companies then this will be of some concern to the risk managers and specifically the portfolio credit risk management division. In order to alleviate such a concern the bank could typically sell some of the exposure off to potential investors.

One very simple method of doing this is through the use of credit derivatives technology. The investor could enter into a credit default swap in which the investor assumes the default risk for the loan from a mine and the bank, in turn, pays the investor a spread on the nominal amount of the loan. The bank would only do this if the spread that it is required to pay on the CDS is less than the cost of the capital that the loan on the balance sheet consumes. The bank has in this scenario transferred the credit risk from its balance sheet to that of the investor.

It does, however, have an effect on the counterparty credit risk against the investor. The reason for this is that, in the case of the mine defaulting, the bank would call on the investor to compensate it for its loss. If the investor cannot honour its obligation under the CDS the bank would suffer a significant loss. One possible method often used in practice in getting around this issue is to use funded CDSs – also known as Credit Linked Notes (CLNs). A CLN is exactly the same as a CDS except that the potential payment in the event of default is paid upfront instead of upon the default event occurring. This then

removes the credit risk associated with the credit derivative transaction (from the bank's point of view).

6.3.1.2 Reduction in the costs of incremental capital

Under Basel 2 a bank is required to hold regulatory capital for the credit risk that it is exposed to. As mentioned before, banks typically attempt to reduce the amount of regulatory capital that it is required to hold and, at the same time, maximise its returns. If it succeeds it increases its return on equity and return on capital for its shareholders. It often happens that banks have exposures on their balance sheets which consume so much capital¹⁵⁷ that the bank can buy credit protection on the exposure – thereby relieving some regulatory capital due to the fact that credit risk is transferred – at a price which is lower than the cost of the regulatory capital that the exposure consumes. There are however various criteria that such credit protection must meet in order to be eligible under Basel 2. The point is that this situation is essentially giving the bank an opportunity to raise capital (through the reduction in required regulatory capital) at a relatively low price.

6.3.2 Customised Single-Name Credit Default Swaps

It is important to note that credit derivatives such as credit default swaps typically reference an underlying asset observable in the market and specifically a debt instrument. For example, it is possible to buy credit protection in South Africa on Eskom through the use of a CDS. This CDS will then typically reference a bond issued by Eskom and the protection buyer will be compensated for a loss in the event that Eskom defaults on this referenced bond. It is therefore not correct to use a CDS on an Eskom bond in order to offset the counterparty risk that one has against Eskom. One reason for this is that the bond is higher in terms of seniority to that of the derivatives exposure and it could technically happen that Eskom defaults on a derivative transaction and not on the bond

¹⁵⁷ There are a number of reasons why this may be the case. Generally speaking though, the more risky the exposure the more capital it attracts.

which would make this hedge redundant¹⁵⁸. It would therefore be cleaner to have a CDS which would be triggered if there is a default on *any* form of payment due from Eskom as there is no more basis risk. This CDS will then cover derivative transactions as well and is typically used in practice in transferring counterparty credit risk.

6.3.3 Portfolio-Level Risk Transferral

One very popular way of transferring credit risk is on a portfolio basis. More specifically, instead of transferring the credit risk of one name, the credit risk of a list of names is transferred at once. A good example of this is a first to default CDS. Under a first-to-default CDS the protection buyer pays a premium for credit protection on a basket of names and receives a contingent payment of the notional amount in the event of a default by any one of the names in the basket. Once a reference name has defaulted the CDS ceases to exist.

Probably the most widely used method of credit risk transfer is through the use of collateralised debt obligations (CDOs). A specific sub-set of CDOs which typically are used by banks in managing portfolio credit risk is collateralised loan obligations. These structures are complex securitisation structures which transfers the credit risk on a pool of loans from the bank's balance sheet into a special purpose vehicle (SPV). This SPV then, in order to assume this risk, issues notes on the obligations (liabilities) that it has now assumed. These issued notes are then typically invested in by institutional investors who have an appetite for the typical assets that banks redistribute since these assets are normally not available to these investors directly and serve as a source of diversification.

¹⁵⁸ This argument assumes that there are no cross-default clauses which could trigger a default on the bond if there is a default on a derivative payment. Also, the argument is for illustrative purposes only.

7 Practical Considerations and Approximations

Advanced mathematical tools and models are often employed in an attempt to accurately quantify counterparty credit risk exposure requiring significant investments in infrastructure and intellectual capital. These complex systems or models may be advanced from a technological or mathematical point of view but it is important to know and understand the limitations and validity of such models. In this chapter we give some thoughts on methods of calibrating counterparty credit risk models. We also discuss an approach of assessing the validity of such models using historical data analogous to back-testing used in market risk value-at-risk systems. The chapter ends with an overview of a widely used approximation approach for measuring counterparty credit risk: the add-on approach. An example illustrates this method.

7.1 Validation and Calibration of PFE Models

7.1.1 Assessing the validity and accuracy of PFE models

The simulation methods presented and demonstrated in this dissertation has strong grounds from a theoretical point of view. The results from the models however are meaningless if these estimates are not an accurate reflection of what typically realises in actual financial markets. More specifically, the assumptions around the statistical distributions of underlying variables form the foundation of the simulation framework presented. The validity of these distributional assumptions is therefore essential and needs to be tested.

Market risk simulation models for the measurement of value-at-risk (VaR) are validated using historical data in a process known as back testing. Back testing is the process of testing model-projected results against realised results in a retrospective manner in order to assess the model's ability of producing accurate predictions. More specifically, the tested model is typically used over a specific time horizon (for example 1 year) in

predicting VaR estimates for a specific portfolio of trades. At the end of this time horizon the model's predictions are tested against the actual realised results. If for example the estimates are calculated using a 95% confidence level one would expect an accurate model to have been 'wrong' close to 5% of the time. If the model's VaR estimates were breached more than 5% of the time the model is deemed to be underestimating the true VaR and if the model's VaR limits are breached less than 5% of the time the model produces conservative results. VaR estimates are typically over short periods of time – i.e. 5 or 10 days. Counterparty credit risk measurement methodologies typically estimate measures such as PFE over the life of the contract which, in the case of interest rate swap agreements, may be as far out as 30 years or even further. To estimate future market values of underlying market variables over such a long time horizon is significantly more challenging and most likely considerably less accurate. Taking into account the effect of correlation (or dependence) between different variables introduces even additional challenges in terms of complexity.

The models used in the measurement of counterparty credit risk have most application on a counterparty-level since this is the exposure level at which credit decisions are made. Limits set against counterparties are also monitored on a counterparty level and it is therefore important for a bank to be able to accurately measure the counterparty credit risk at this level. If one considers the problem holistically, it makes sense to approach the problem of model validation from the bottom up. If one starts on a counterparty level it may be extremely difficult, or even impossible, to explain differences between the realised and estimated measures of credit exposure. The problem of validation of the models should therefore be approached in the following stages:

- ***Contract-level model validation***

It makes sense to validate the counterparty credit risk models on a contract level first. In this manner the distributional assumptions of individual underlying market variables are tested. Consider, for example, the case of a simple product like an FX Forward contract, as discussed extensively throughout this dissertation. In measuring the contract-level PFE of this contract we have assumed that the

spot exchange rate follows a GBM process, and therefore that the future spot prices follow a lognormal distribution. The Chi-squared test for goodness-of-fit can be used on historical spot exchange rate data in assessing the validity of this assumption. In other words, it is possible to test whether historically a specific exchange rate has exhibited the properties of a lognormal distribution on a given level of statistical significance. There are, however, some interesting challenges in even applying such a simple test in this context. The problem has a lot of dimensions since assuming that future spot exchange rates follow a GBM process implies that there is a different statistical distribution for each time-step over which one simulates these future spot rates. More specifically, we assume that

$$S_t \sim LN(\ln S_0 + (\mu - 0.5\sigma^2)t, \sigma^2 t),$$

and therefore, technically speaking, one needs to test the validity of this assumption over a number of time-horizons. This could potentially be a very laborious task and one would practically only validate the model for a few critical¹⁵⁹ time horizons.

An alternative method of validation, as mentioned above, is through an approach often used in the validation of market risk models called back-testing. Our definition of contract-level potential future exposure attempts to make predictions around the MTM value of the contract under consideration given a certain level of confidence. We are therefore, in modelling PFEs, trying to construct confidence intervals within which the future values (measured by the MTM) of a derivative contract is expected to be, assuming a certain level of statistical significance. It follows that it is also therefore sensible to test the ability of a PFE model to predict future mark-to-market values accurately. Banks generally value every OTC contract on a daily basis for market risk management purposes. These realised MTM values can therefore be recorded and used in testing the actual MTM values against the PFE values estimated during the life of the contract.

¹⁵⁹ Time horizons which are typically more active in terms of volume of active trades.

- ***Counterparty-level model validation***

Counterparty credit risk measurement results are significantly impacted by the assumed dependence between underlying market variables. In section 3.2.2 we demonstrate the potential impact that the assumption of correlation can have on a simple portfolio consisting of derivatives on two correlated underlying market variables. In validating PFE models it is therefore important to realise that the most important factor to consider is the assumption of dependence of the underlying market variables.

Let's consider for example an interest rate swap PFE model which has been satisfactorily validated, using the back-testing approach, on a contract level. This model is then tested on a small portfolio of trades which consists only of interest rate swaps but on a number of different interest rates and the result is significantly different from the observed historical results. Under these conditions it would make sense to firstly consider the correlation assumptions and perhaps the validity of these for the underlying interest rates of this portfolio. Taking this argument into account it is fair to suggest that correlation assumptions should be independently validated even before the realised MTM values of portfolios of derivatives are tested against model-predicted PFE values.

7.1.2 Specification and Calibration of PFE models

The majority of the examples that we have considered have been based on underlying market variables which are typically modelled using a GBM process. Different market variables, however, require different stochastic processes to characterise their evolution through time. In fact, the same market variable can potentially be modelled using different stochastic processes depending on the circumstances. Major foreign exchange rates are usually modelled using a GBM process which is in contrast to the modelling of certain emerging market exchange rates where significant jumps often occur in practice. Jump-diffusion processes are generally employed in these situations to characterise the movements of the prices of emerging markets or pegged currencies. Sometimes the risk

factor to be modelled is not a one-dimensional price but rather a vector such as in the case of interest rate curves or the forward price curve of a commodity. In these cases the simulation model must be sufficiently elaborate to impose the proper arbitrage-free constraints and to take account of possible reshaping and twisting of the curve. Often prices in markets also exhibit some form of mean-reversion which is particularly important in simulating long-term future scenarios in order to prevent unrealistically high exposure estimations.

The calibration of the parameters of the simulation models is an important step in model building (Canabarro and Duffie, 2003). The future values produced by the simulation models are fundamentally determined by the calibration scheme applied in the model. An important and fundamental decision is whether the models need to be calibrated using historical or market-implied parameters. Models calibrated to historical data tend to project future market scenarios based on statistical observations observed in the past whereas models which are calibrated to market prices (such as forward price curves and option-implied volatilities) tend to reflect forward-looking views.

According to Canabarro and Duffie (2003) there are advantages and disadvantages in each approach. Historical calibration implies that the process generating future market price behaviour is the same that was observed in the past. Such a model is often criticised as being slow to react to changes in market conditions and structural changes in financial markets. This problem is however slightly alleviated by the use of time-decay factors to weight more recent observations higher in the calibration process. On the other hand, in the case of models calibrated on market prices and implied volatilities, it can be argued that market prices contain components that are not only determined by the market participants' view about the future. These components include risk premiums, liquidity premiums, carrying costs etc.

The main objective of the simulation model is to project, as realistically as possible, the potential future state of the market being simulated. In that sense the model should operate under the real probability measure. The only justification for using the risk-

neutral measure is that, to some extent, it contains the consensus expectations of market participants on future prices, volatilities etc. When the simulations are used for pricing, as in the case of credit value adjustments (CVA) the risk-neutral measure should be used.

7.2 Add-On Approximation to PFE Estimation

The global market for OTC derivatives is enormous. This market does not only consist of big global financial institutions, in fact there are numerous financial institutions that take part in this market which are relatively small in comparison. As a result, not all market players can afford to spend millions in infrastructure for measuring counterparty credit risk. Moreover, these players typically also do not have the skill set to develop or implement and maintain such complex systems.

There are other banks which are not small in term of balance sheets but just not yet as advanced as other who also typically need to measure counterparty credit risk on a consistent and meaningful way. At the end of the day it is more important to measure the risk than not to measure the risk at all. One would rather be inaccurate than uninformed or ignorant regarding risks that can potentially, when materialised, disrupt global financial markets. As a result, it is often required to make compromises when it comes to accuracy for the sake of practicality. For example, an emerging markets bank which trades in the OTC derivatives markets should preferably measure its counterparty credit risk as accurately as possible. It may, however, not be in the position to implement advanced simulation based systems to measure such risks due to skills or perhaps even financial constraints.

In these and similar cases practitioners often employ simple approximations in an attempt to measure its exposure to counterparty credit risks more easily and less costly. These approximations methods (often referred to as add-on methods) can vary considerably in terms of accuracy and complexity and are significantly less computationally intensive than comparable simulation methods. Some internationally active banks also make use of add-on methods in the calculation of credit exposure for certain product lines where the

volumes of their trades actually make it impractical to estimate the exposures by simulation.

7.2.1 Add-On vs. Simulation

In estimating counterparty credit risk exposure using an approximation method it is expected that some accuracy will be sacrificed. It is important to, where possible, assess such inaccuracies and to understand under which circumstances these inaccuracies may materialise. The add-on approach does however have the benefit of being computationally more efficient and therefore less costly. We will now formally define our version of the add-on approach to PFE estimation and afterwards illustrate how this approach may be applied in estimating PFE profiles.

7.2.1.1 Definition of add-on approach to PFE estimation

Typically, the PFE estimated using Monte Carlo simulation is taken to be the accurate answer and therefore the target to work towards. Banks that typically apply the add-on approximation will therefore firstly quantify the PFE profile of a typical contract of the type under consideration. The PFE profile can be broken up into two main components:

- ***Current Exposure***

The current exposure is typically defined as the current replacement cost of the transaction under consideration. This is typically measured by the MTM value of the contract. This information is also mostly readily available in a bank as a product of the market risk or front-office trading systems.

- ***Potential Future Exposure***

This is the component of the PFE which is estimated under the add-on method. The add-on approach is a discretisation of the potential future exposure

component of the PFE profile. Let's define the potential future exposure at time t as the add-on for time t , i.e.

$$PFE\ Component(t) = AddOn(t)$$

In general terms, the add-on at a specific time point for a specific contract is dependent on the following two components:

- Size of the contract
- Potential value of the underlying asset

If we compare the approach in estimating a PFE profile using the add-on method to the approach using the simulation method some interesting observations can be made. Firstly, in applying the simulation method and under the add-on approach the current exposure is known. The potential future exposure portion of the PFE profile under the simulation approach is estimated using simulations from a known assumed distribution of the underlying asset. More specifically, the simulation approach involves drawing random samples (ω_t^i) from a known distribution (Ω_t^i) of underlying prices at t – i.e. $\omega_t^i \in \Omega_t^i$. The derivative is then valued, using this sample price ω_t^i . This process is then repeated in order to get an accurate estimation of the distribution of possible future contract values at various points in time. The final step is then to calculate a percentile of this distribution at each point in time in order to derive a PFE profile. In contrast, the add-on method is an attempt to bypass the repetitive random sampling algorithm of the simulation method by estimating the percentile of the future distribution of the contract values directly.

More formally, the add-on approach is an attempt to estimate the quantity ξ with

$$\mathbb{P}[MTM(\Omega_t) > \xi] = \alpha,$$

without having to draw samples from the distribution of Ω_t . Ω_t is a random variable with the same distribution as the distribution of the prices of the underlying of the contract at t .

If we assume that the distribution of Ω_t is known, then it is possible, for simple functions $MTM(\cdot)$, to find ξ relatively easily.

In practice the add-on method is often used to construct tables of add-on factors which can be applied to more than one contract. Typically, there are separate add-on tables for different underlying asset and contract type combinations. For example, there would be one table of add-on factors for USD/ZAR FX forwards and another table for GBP/USD FX forward contracts. These tables allow PFE estimates on a contract level using a very simple formula.

The add-on factor is typically a function of the underlying, its volatility and expected value¹⁶⁰, the time step into the future t and the maturity T of the contract under consideration. It is however possible that, for certain products, the add-on factor is quite complex and dependent on a large number of parameters.

7.2.2 Example of Add-On PFE Estimation

We illustrate the add-on method for estimating a contract-level PFE using a simple example of an OTC contract used in Section 2.2.4. We have therefore already calculated the PFE estimate using the simulation method and only need to derive an approximation using the add-on method approach.

7.2.2.1 Model Derivation

As argued above, the PFE profile of this FX forward contract consists of two main components. The first portion, the current exposure, is easy to calculate and is given by the expression:

$$MTM_{ZAR} = N_{USD} \times PV_{ZARi}(F_{USD/ZAR} - K_{USD/ZAR}). \quad (22)$$

¹⁶⁰ It often happens that the forward prices observed in the market is used as an indication of future spot prices. These then serve, from a statistical perspective, as expected future values of the spot price.

The potential future exposure component of the PFE profile is, however, strictly speaking a stochastic variable for each time step in the future at which the PFE is to be calculated. The reason for this is that the spot USD/ZAR exchange rate is assumed to follow the following stochastic differential equation:

$$dS_t = \mu S_t dt + \sigma S_t dW_t.$$

The SDE above implies that

$$S_t \sim LN(\ln S_0 + (\mu - 0.5\sigma^2)t, \sigma^2 t),$$

in other words, the stock price at time t is a random variable with a lognormal distribution with parameters as above.

In the example in Section 2.2.4 we argued that the main risk driver in the PFE of the FX forward is the forward exchange rate at the maturity of the contract. Furthermore, we made a simplifying assumption as to the evolution of the forward exchange rate relative to the simulated spot exchange rate – essentially assuming that the shape of the forward curve remains constant and moves up and down parallel to the spot rate. In order to approximate the results from that simulation process as accurately as possible we need to make the same assumptions in the add-on model.

From a theoretical point of view, the forward exchange rate is determined by interest rate parity. A no arbitrage assumption determines the no-arbitrage forward exchange rate at some future point in time, say t_i , through the interest rate differential in the following expression:

$$F_{0,t_i}^{USD/ZAR} = S_0^{USD/ZAR} e^{(ZARi_{t_i} - USDi_{t_i})t_i},$$

where

- $F_{t_i}^{USD/ZAR}$ is the forward exchange rate for time t_i . In other words this is the point on the forward exchange rate curve corresponding to time point $t_i > t_0 = 0$,
- $S_0^{USD/ZAR}$ is the spot rate observed at time $t_0 = 0$,
- $ZARi_{t_i}$ is the interest rate earned on a ZAR deposit, deposited at $t_0 = 0$ for t_i years. The interest rate in this instance is a continuously compounded rate.

In assuming that the shape of the forward exchange rate curve remains constant we are therefore implicitly assuming that the interest rate differential remains constant. In determining simulated forward exchange rates at some future point in time, t , we have replaced the S_0 with an S_t where

$$S_t \sim LN(\ln S_0 + (\mu - 0.5\sigma^2)t, \sigma^2 t).$$

We can therefore derive an expression (under our assumptions) for the simulated forward exchange rate at time t ($F_{t_i,t}^{USD/ZAR}$, a random variable as a result of $S_t^{USD/ZAR}$ being a random variable) as follows:

$$F_{t_i,t}^{USD/ZAR} = S_t^{USD/ZAR} e^{(ZARi_{t_i} - USDi_{t_i})t_i}.$$

We have expanded our notation to indicate that $F_{t_i,t}^{USD/ZAR}$ is the t_i year forward exchange rate observable at time $t > 0$. The factor $e^{(ZARi_{t_i} - USDi_{t_i})t_i}$ has therefore remained unchanged.

Under the simulation method the value of the FX Forward contract at a future point in time is calculated using random samples from the distribution of future forward exchange rates, of the form $F_{t_i,t}^{USD/ZAR}$. More specifically, the *MTM* formula in (22) is applied to a simulated future forward exchange rate in order to derive a sample from the distribution of future contract values ($MTM_{t,ZAR}$) using the following expression:

$$MTM_{t,ZAR} = N_{USD} \times PV_{ZARi} (F_{t_i,t}^{USD/ZAR} - K_{USD/ZAR}). \quad (23)$$

Note that $MTM_{t,ZAR}$ is therefore a random variable with a distribution which is a linear transformation of the distribution of $F_{t_i,t}^{USD/ZAR}$. Also, $F_{t_i,t}^{USD/ZAR}$ in turn has a distribution which is a linear transformation of that of $S_t^{USD/ZAR}$ which is known.

We define the function $p^\alpha(X)$ to be a function of a random variable X such that

$$\mathbb{P}(X < p^\alpha(X)) = \alpha.$$

In other words, $p^\alpha(X)$ is the α^{th} percentile of the distribution of X . Using this notation, it is possible to write the PFE of the FX Forward using the following notation:

$$\begin{aligned} PFE(\alpha, i, t) &= p^\alpha(MTM_{t,ZAR}) \\ &= p^\alpha(N_{USD} \times PV_{ZARi} (F_{t_i,t}^{USD/ZAR} - K_{USD/ZAR})) \\ &= N_{USD} \times PV_{ZARi} (p^\alpha(F_{t_i,t}^{USD/ZAR}) - K_{USD/ZAR}) \\ &= N_{USD} \times PV_{ZARi} (p^\alpha(S_t^{USD/ZAR} e^{(ZARi_{t_i} - USDi_{t_i})t_i}) - K_{USD/ZAR}) \\ &= N_{USD} \times PV_{ZARi} (p^\alpha(S_t^{USD/ZAR}) e^{(ZARi_{t_i} - USDi_{t_i})t_i} - K_{USD/ZAR}). \end{aligned}$$

This result is extremely helpful in deriving an expression for an add-on for an FX forward contract. It shows that the $\alpha\%$ PFE of an FX Forward at some future point in time, $t > 0$, can be calculated by applying the MTM formula in (23) using the $(100 \times \alpha)^{th}$ percentile of the distribution of the future spot exchange rates $S_t^{USD/ZAR}$.

Since the distribution of $S_t^{USD/ZAR}$ is lognormal it is therefore possible to determine $p^\alpha(S_t^{USD/ZAR})$ using the properties of the standard normal distribution. It follows that the

$\alpha\%$ contract-level PFE of an FX Forward at some future point in time, $t > 0$, can be calculated directly (without the need to simulate) using the following expression:

$$PFE(\alpha, i, t) = MTM_0 + N_{USD} \times PV_{ZARi} \left(\overset{\alpha USD/ZAR}{\tilde{F}}_{t_i, t} - K_{USD/ZAR} \right),$$

with

$$\overset{\alpha USD/ZAR}{\tilde{F}}_{t_i, t} = \mathcal{P}^\alpha(F_{t_i, t}^{USD/ZAR}) = S_0 e^{(\mu - 0.5\sigma^2)t + \sigma\sqrt{t}\varepsilon_\alpha} e^{(ZARi_{t_i} - USDi_{t_i})t_i},$$

and ε_α such that

$$\mathcal{P}^\alpha(Z) = \varepsilon_\alpha,$$

and

$$Z \sim N(0, 1).$$

In other words, ε_α is the $(100 \times \alpha)^{th}$ percentile of the standard normal distribution. For example, for $\alpha = 0.95$ we have $\varepsilon_\alpha \approx 1.645$.

We now turn to the formal derivation of the add-on factor for the FX Forward contract. Recall that we are looking for an expression for the potential future exposure portion of the PFE profile in the form:

$$AddOn_t = Notional \cdot AddOn Factor(.).$$

Therefore, we need to consider and solve the following:

$$AddOn_t = Notional \cdot AddOn Factor(.) = N_{USD} \times PV_{ZARi} \left(\overset{\alpha}{\tilde{F}}_{T,t}^{USD/ZAR} - K_{USD/ZAR} \right).$$

We therefore have¹⁶¹:

$$AddOn Factor(\alpha, t, T, K) = PV \left(\overset{\alpha}{\tilde{F}}_{T,t} - K \right).$$

Note that in the expression above the add-on factor depends mainly on the time point, t , in the PFE profile under consideration, the maturity of the contract (T) and the strike of the specific contract under consideration. As mentioned before, practitioners mostly employ the add-on method through so-called add-on tables which are typically generic tables which can be used for any contract of a specific type.

In our example, therefore, we are looking for add-on factors for USD/ZAR FX Forwards which are as generic as possible¹⁶². The strike of the forward contract is, however, a very contract-specific variable and the add-on factor should therefore ideally be independent of the strike of the contract. Moreover, the add-on tables are typically not updated daily or often even weekly. It is therefore ideal to have the add-on factors as insensitive to short-term market fluctuations as possible. To this extent the following adjustment is made to the add-on factor:

$$AddOn Factor(\alpha, t, T, K) = K \times PV \left(\frac{\overset{\alpha}{\tilde{F}}_{T,t}}{K} - 1 \right).$$

Although this add-on factor is strictly speaking not independent of the strike of the forward contract it is in a more elegant form and easily applied to generic USD/ZAR FX Forwards.

¹⁶¹ Note that for ease of exposition we have suppressed the notation indicating the quoting convention with respect to the two currencies.

¹⁶² Although we are only considering FX Forward contracts on USD/ZAR.

If one considers the term

$$\begin{aligned}
 & \frac{\overset{\alpha}{F}_{T,t}}{K} \\
 &= \frac{S_0 e^{(\mu - 0.5\sigma^2)t + \sigma\sqrt{t}\varepsilon_\alpha} e^{(ZARi_{t_i} - USDi_{t_i})t_i}}{K} \\
 &= \frac{S_0 e^{(ZARi_{t_i} - USDi_{t_i})t_i}}{K} e^{(\mu - 0.5\sigma^2)t + \sigma\sqrt{t}\varepsilon_\alpha} \\
 &= \frac{F}{K} e^{(\mu - 0.5\sigma^2)t + \sigma\sqrt{t}\varepsilon_\alpha},
 \end{aligned}$$

the add-on factor may be considered as

$$AddOn\ Factor(\alpha, t, T, K) = K \times PV(MF \times SF - 1),$$

where

- $MF = \frac{F}{K}$ is a ‘moneyness’ factor
- $SF = e^{(\mu - 0.5\sigma^2)t + \sigma\sqrt{t}\varepsilon_\alpha}$ is a ‘stress factor’.

Using this expression it is possible to create add-on factors, and more importantly add-on factor tables which are very generic in the sense that all parameters in the add-on may be thought of as a generic parameter. It also transforms the add-on factor, conceptually at least, into a relative number instead of an absolute number since it only depends on a relative quantity – the moneyness factor.

A specific organisation will typically use only one value of α for all PFE calculations and therefore the stress factor can be treated as deterministic for a specific value of t . Also, recall from section 3.5.3.1 that $N_{USD}K_{USD/ZAR} = N_{ZAR}$. It is therefore possible to simplify the expression for the PFE using the add-on method developed above to:

$$PFE(\alpha, t, T) = N_{ZAR} \times AF_{t,T}.$$

This expression enables us to construct a 3-dimensional table of add-on factors using the following three factors:

- t the time step into the future at which the PFE is to be calculated
- T the maturity of the contract under consideration
- MF the moneyness factor of the particular contract under consideration.

Using this add-on factor table it is then possible to calculate the entire¹⁶³ PFE profile for any USD/ZAR FX Forward contract. Note that it is, in this specific add-on approach, not necessary to know the MTM_0 value (current exposure) since this is given by $PFE(\alpha, 0, T)$.

7.2.2.2 Model Application

Let's construct a typical add-on factor table using the method described above. We will focus our efforts on the specific example USD/ZAR FX forward discussed in Section 2.2.4 (pp17). The details of this particular contract is summarised below:

<i>Underlying</i>	<i>Notional (USD)</i>	<i>Maturity (yrs)</i>	<i>Strike</i> ¹⁶⁴
USD/ZAR	1,000	0.5	8.17

Table 7.1 - Repetition of Table 2.1

Using the add-on approach described above we construct a table of add-on factors for the following ranges of the variables on which the add-on factors depend:

- $t = \{0.05, 0.1, \dots, 0.5\}$

¹⁶³ Only at certain predetermined points t for which add-on factors exist in the add-on table.

¹⁶⁴ Quote in number of ZAR per USD.

- $T = 0.5$
- $MF = \{0.2, 0.4, \dots, 2.0\}$

The resultant table of add-on factors (in %) is given in Table 7.2 below¹⁶⁵. These add-on factors allow us to calculate the PFE profile of a specific USD/ZAR FX Forward using the add-on approximation method. The simulated PFE profile was calculated using 500,000 Monte Carlo simulations.

Figure 7.1 illustrates the PFE profiles produced using the Monte Carlo and add-on methods. We also demonstrate, in Figure 7.2 an example of a deep out of the money FX forward with the same contract details as in Table 7.1 except that the strike is 20.43. This Strike value coincides with a moneyness factor of approximately 0.4. Linear interpolation is typically used in practice in deriving add-ons for between tabulated values.

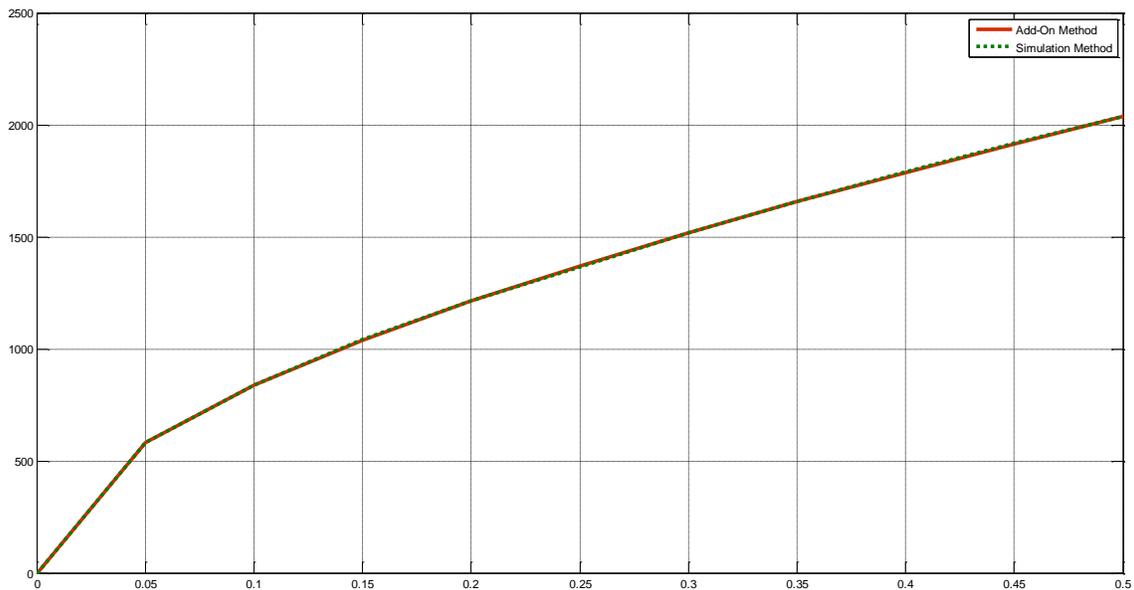


Figure 7.1 – ATM Contract-level PFE: add-on approach vs. simulation approach

¹⁶⁵ Since we have used only one value for T we have essentially reduced the number of dimensions of the add-on factor table and it is therefore possible to represent the add-on table as a two dimensional table.

		<i>Time Step into the future</i>									
		0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
<i>Moneyness Factor</i>	0.20	-74.37	-74.20	-74.17	-74.20	-74.28	-74.39	-74.52	-74.67	-74.83	-75.01
	0.40	-53.99	-53.09	-52.45	-51.94	-51.51	-51.15	-50.82	-50.53	-50.27	-50.03
	0.60	-33.62	-31.98	-30.73	-29.68	-28.75	-27.91	-27.12	-26.39	-25.70	-25.04
	0.80	-13.24	-10.87	-9.01	-7.42	-5.99	-4.66	-3.43	-2.25	-1.13	-0.05
	1.00	7.13	10.24	12.70	14.84	16.78	18.58	20.27	21.88	23.43	24.93
	1.20	27.50	31.35	34.42	37.10	39.54	41.82	43.97	46.02	48.00	49.92
	1.40	47.88	52.46	56.14	59.37	62.31	65.06	67.66	70.16	72.57	74.91
	1.60	68.25	73.57	77.86	81.63	85.07	88.30	91.36	94.30	97.14	99.89
	1.80	88.63	94.68	99.58	103.89	107.84	111.54	115.06	118.44	121.70	124.88
	2.00	109.00	115.79	121.29	126.15	130.60	134.78	138.76	142.57	146.27	149.87

Table 7.2 - Example of a table of add-on factors (in percent)

The difference between the PFE profiles computed using the add-on and simulation approaches are due to two main factors: sample errors due from the simulations and errors from interpolation (where applicable) in the add-on approach.

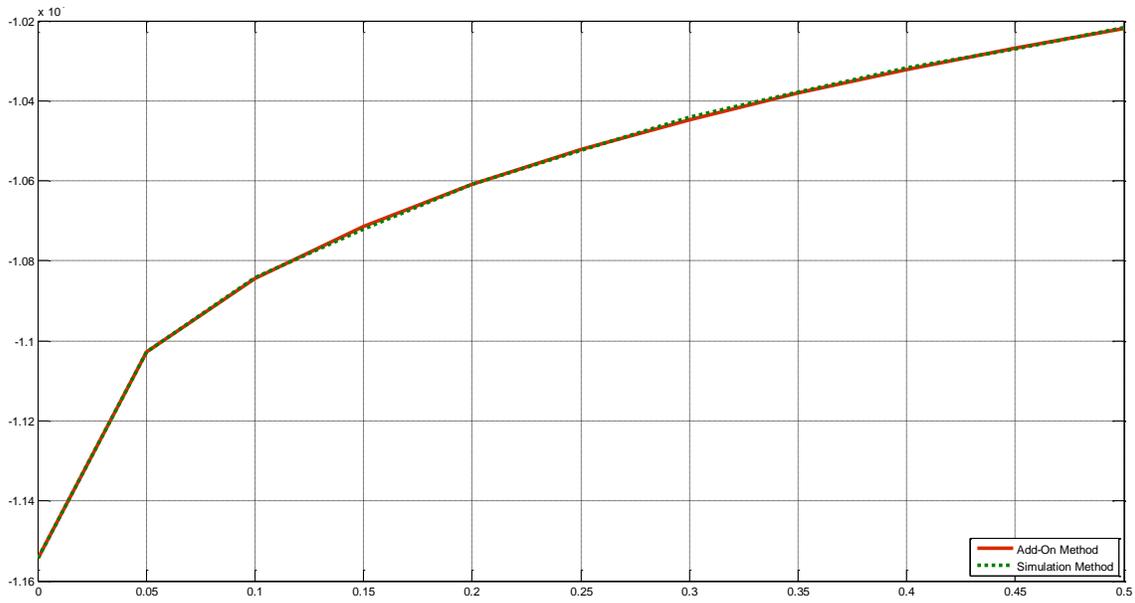


Figure 7.2 - OTM Contract-level PFE: add-on approach vs. simulation approach

7.2.2.3 Some thoughts on approximation methods

In the examples above the add-on approach is clearly a good method for estimating contract-level PFE profiles for FX Forwards when compared to the simulation results. The examples were, however, specifically chosen to demonstrate the ability of approximation methods for simple products. Also note that we have only considered contract-level PFEs and in extending this framework beyond one underlying results in significant losses in accuracy.

Approximation methods of the type discussed above are very accurate on a contract level but fail to incorporate dependence among correlated underlying assets and therefore result in significantly over-stated exposure numbers on a counterparty-level. The extent of these errors is the similar to those demonstrated in Section 3.2.2. The reason for this is that one typically needs to implicitly assume a worst-case correlation between the underlying variables in order to ensure that the estimated exposure result does not understate the actual exposure number.

This worst-case correlation assumption does ensure that exposure estimates are conservative but at a significant cost: a potentially large level of inaccuracy.

7.2.3 Further Considerations

As illustrated above, add-on methods can (under certain conditions) provide extremely accurate estimates to contract-level PFE profiles. Although these approximation methods fail to incorporate dependence among different underlying assets it still serves as a useful tool for institutions that may not have the capacity to support a complex simulation system for counterparty credit risk measurement. In short, a conservative measure of risk is better than no measure of risk at all.

There are products for which it is not possible to create generic add-on tables which accurately estimate, even on a contract level, the counterparty credit risk exposure. A typical example of such products include exotic path-dependent options. In practice these products are handled using a simulation approach and then manually added onto existing counterparty-level exposure profiles calculated using approximation methods. Although not ideal, this approach does at least give the risk manager a consolidated, albeit conservative, view of the risk position.

Another important aspect of add-on methods from a practical point of view is the frequency of add-on factor updates. If the add-on factors are not updated frequently enough inaccuracies in estimates become more and more likely and what may have been perceived to be conservative measures of risk may become less so. It is therefore of crucial importance that the add-on factor tables are updated as frequently as possible as this will also ensure that exposure numbers remain ‘smooth’ and do not significantly increase (or decrease) when add-on factors are updated.

8 Conclusion

The importance of counterparty credit risk measurement is apparent from the sheer size of the global OTC derivatives markets. Large financial institutions are increasingly linked at a global level as is evident from the credit crises which started in 2007. This dependency between market participants introduces a significant amount of systemic risk into the global financial industry. This highlights the importance of efforts by banking regulators globally to minimise this risk through the enforcement of best practices and capital adequacy requirements in order to guard against unexpected losses.

This dissertation discussed in detail the current methods used in practice for measuring counterparty credit risk. The concept of potential future exposure was introduced – firstly on a contract-level and then on a portfolio-level. In addition, we have also discussed other issues relating to the measurement of counterparty credit risk exposure which typically require attention in practice. These issues include risk mitigation techniques such as netting agreements and collateral agreements which are used extensively in practice for mitigating the potential impact of counterparty default events.

One important aspect of counterparty-level PFE calculations which have been simplified in this dissertation is the effect of dependency among underlying market variables on the resultant counterparty-level exposure estimates. We have used the measure of correlation to measure dependency and, in addition, assumed that this correlation is constant through time. This assumption is not realistic and often breaks down in practice. It is therefore suggested that this issue is addressed in further research.

The model presented in Chapter 4 for measuring the impact of wrong-way risk provides a very practical framework for measuring counterparty credit risk in the presence of wrong-way risk. It should however be noted that the structural model approach to measuring probability of default is by no means perfect. The structural model merely provides an intuitive link between the market moves in the underlying asset of a derivative contract and the potential impact thereof on the probability of default of the counterparty through

the correlation between its equity price and the underlying market variable. The model has also only been demonstrated for instances where there is a single underlying asset. Furthermore, the model has also only been demonstrated in situations where the underlying market variable is assumed to follow a GBM process. This scenario leads to a simple closed-form bivariate lognormal distribution for the joint process of the underlying asset and the equity price of the counterparty. In the case of the underlying variable not following a GBM process it may be more complex and may require an undesirable and unpractical amount computational time.

The validation of counterparty credit exposure models is an area which is not widely discussed in literature. The task of estimating potential future market scenarios over significant time horizons is complex from a modelling point of view. The potential impact of errors in these models may be significant. It is therefore important to validate and test these models frequently in order to ensure that they produce meaningful results.

9 Appendix: Matlab Code and Data Files

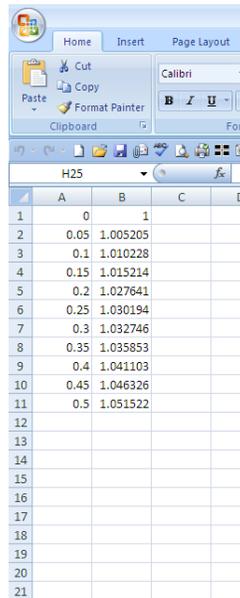
This section contains the majority of the Matlab code used in the calculations and simulations and calculations performed as part of this dissertation.

9.1 FX Forward Contract-Level PFE Calculation

```
%This Matlab code calculates the PFE, using Monte Carlo Simulation, of a single
FX Forward.'

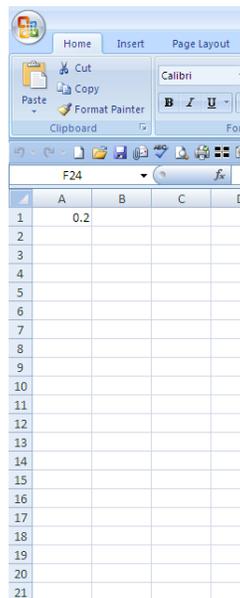
Notional=1000;
NoSimulations=250000;
TimeSteps=11;
T=0.5;
Mu=0.0;
InterestRate=0.12;
Strike=20.4258290193026;%8.17;
Sigma=csvread('c:\Data Files\Vol.csv');
RelFwdCurve=csvread('c:\Data Files\RelativeForwardCurve.csv');
RelFwdCurve=RelFwdCurve' ;
S0=7.77;
CurrentMTM=Notional*exp(-InterestRate*T)*...
(RelFwdCurve(2,TimeSteps)*S0-Strike);
NormRandVars=randn(NoSimulations,TimeSteps);
TimeStepVector=0:0.05:0.5;
Simulations=ones(size(NormRandVars));
StressedSpots=ones(size(NormRandVars));
StressFactors=ones(size(NormRandVars));
StressFwds=ones(size(NormRandVars));
StressMTM=ones(size(NormRandVars))*CurrentMTM;
for I=1:1:NoSimulations
    StressedSpots(I,1)=S0;
    for J=1:1:TimeSteps -1
        StressFactors(I,J+1)=exp((Mu-(Sigma.^2)/2)*...
            (TimeStepVector(1,J+1)-TimeStepVector(1,J))+...
            Sigma*NormRandVars(I,J)*sqrt((TimeStepVector(1,J+1)-...
            TimeStepVector(1,J))));
        StressedSpots(I,J+1)=StressedSpots(I,J)*StressFactors(I,J+1);
        StressFwds(I,J)=RelFwdCurve(2,TimeSteps)*StressedSpots(I,J+1);
        StressMTM(I,J+1)=Notional*exp(-InterestRate*...
            (TimeStepVector(1,TimeSteps)-TimeStepVector(1,J+1)))...
            *(StressFwds(I,J)-Strike);
    end
    StressFwds(I,TimeSteps)=StressedSpots(I,TimeSteps);
end
FinalPFE=prctile(StressMTM,95);
t=0:T/(TimeSteps-1):T;
%This plots all scenarios at once as well as a line indicating the 95% PFE'
Line1=plot(t,StressMTM,'LineWidth',1);
grid on;
Line2=line(t,FinalPFE,'LineWidth',1.5,'Color',[1 1 1],'LineStyle',':');
```

9.1.1 Data files used in the calculation of the FX Forward Contract-Level PFE Calculations



	A	B	C	D
1	0	1		
2	0.05	1.005205		
3	0.1	1.010228		
4	0.15	1.015214		
5	0.2	1.027641		
6	0.25	1.030194		
7	0.3	1.032746		
8	0.35	1.035853		
9	0.4	1.041103		
10	0.45	1.046326		
11	0.5	1.051522		
12				
13				
14				
15				
16				
17				
18				
19				
20				
21				

9.1 - RelativeForwardCurve.csv



	A	B	C	D
1	0.2			
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
21				

9.2 - Vol.csv

9.2 Interest Rate Swap Contract-Level PFE Calculation

```
%This Procedure Generates the short rate paths for the Interest Rate Swap
%PFE Calculation using the Cox, Ingersoll Ross model.
I=0;
J=0;
```



```
Rates=csvread('c:\Data Files\ZeroCurve.csv');
NoSimulations=1000;
CIRPaths=ones(NoSimulations,21);
CIRZCYCs=ones(NoSimulations,21,21);
CIRSimulatedMTMs=zeros(NoSimulations,21);
PFEPProfile=zeros(21);
CIRa=0.2417;
CIRb=0.0809;
CIRvol=0.0212;
CIRr0=0.1186;

%Initialise the interest rate environment
RateSpec=intenvset('Rates',Rates,'StartDates',StartDates,...
    'EndDates',EndDates,'Basis',5,'Compounding',-1);
LegRate=[NaN 0];
LegType=[1 0];
LegReset=[4 4];
Maturity=1801;
Settle=91;
[Price SwapRate]=swapbyzero(RateSpec,LegRate,Settle,Maturity,...
    LegReset,'Basis',5,LegType);
LegRate=[SwapRate 0];

for I=1:NoSimulations
    CIRPaths(I,:)=cirpath(t,CIRa,CIRb,CIRvol,CIRr0);
    for J=1:1:21
        CIRZCYCs(I,J,:)=CIRPaths(I,J).*RelativeZCYC';
        %Change the maturity and ZCYC used in the calculation
        Rates=squeeze(CIRZCYCs(I,J,:));
        RateSpec=intenvset('Rates',Rates,'StartDates',StartDates,...
            'EndDates',EndDates,'Basis',5,'Compounding',-1);
        if J~=21
            CIRSimulatedMTMs(I,J)=Principal*swapbyzero(RateSpec,...
                LegRate,Settle,Maturity,LegReset,'Basis',5,LegType);
        end
        Maturity=Maturity-90;
    end
    Maturity=1801;
end
for I=1:1:21
    PFEPProfile(I)=prctile(CIRSimulatedMTMs(:,I),95);
end
End
```

9.2.1 Data files used in the calculation of the Interest Rate Swap Contract-Level PFE Calculations



	A	B	C	D
1	0.11862			
2	0.12624			
3	0.12582			
4	0.12347			
5	0.12158			
6	0.12028			
7	0.11928			
8	0.11832			
9	0.11736			
10	0.11626			
11	0.11515			
12	0.11394			
13	0.11282			
14	0.11173			
15	0.11056			
16	0.10948			
17	0.10846			
18	0.10754			
19	0.10665			
20	0.10587			
21	0.10515			

9.3 - ZeroCurve.csv



9.3 FX Forward Counterparty-Level PFE Calculation with Netting Functionality

```
%This m-file calculates the Counterparty-Level PFE, using Monte Carlo
%Simulation, of a portfolio of derivatives with the same counterparty
Notional1=1000;
Notional2=-490;
Notional3=-500;
Notional4=1000;

%Define the number of simulations and time steps
NoSimulations=15000;
TimeSteps=61;

%Specify the parameters of the underlying trades and market information
T1=0.75;
T2=1.5;
T3=3;
T4=T3;
T=3;
Mu=0.0;
InterestRate=0.12;
Strike1=8.17;
Strike2=20.75;
Strike3=15.45;
Strike4=7.5;
SigmaUSDZAR=0.1548;
SigmaGBPZAR=0.1475;
Correlation=0.9289;
RelFwdCurveUSDZAR=csvread...
    ('c:\Data Files\RelativeForwardCurve_PortfolioPFE2_USDZAR.csv');
RelFwdCurveGBPZAR=csvread...
    ('c:\Data Files\RelativeForwardCurve_PortfolioPFE2_GBPZAR.csv');
RelFwdCurveUSDZAR=RelFwdCurveUSDZAR';
RelFwdCurveGBPZAR=RelFwdCurveGBPZAR';
USDZARS0=7.86;
GBPZARS0=15.62;

%Initialise the calculations - time zero exposures (current mark to market
%values
CurrentMTM1=Notional1*exp(-InterestRate*T1)*...
    (RelFwdCurveUSDZAR(2,T1/0.05+1)*USDZARS0-Strike1);
CurrentMTM2=Notional2*exp(-InterestRate*T2)*...
    (RelFwdCurveGBPZAR(2,T2/0.05+1)*GBPZARS0-Strike2);
[CurrentMTM3,Dummy1]=blsprice(GBPZARS0, Strike3,...
    InterestRate, T3, SigmaGBPZAR);
[CurrentMTM4,Dummy1]=blsprice(USDZARS0, Strike4,...
    InterestRate, T4, SigmaUSDZAR);
CurrentMTM3=CurrentMTM3*Notional3;
CurrentMTM4=CurrentMTM4*Notional4;

%Generate the random variables required for the simulations
NormRandVarsUSDZAR=randn(NoSimulations,TimeSteps);
NormRandVarsGBPZAR=randn(NoSimulations,TimeSteps);

%Apply transformation for correlated random vars
NormRandVarsGBPZAR=Correlation*NormRandVarsUSDZAR+...
    sqrt(1-(Correlation).^2)*NormRandVarsGBPZAR;

TimeStepVector=0:0.05:T;
```



```
StressedSpots_USDZAR=ones(size(NormRandVarsUSDZAR));
StressedSpots_GBPZAR=ones(size(NormRandVarsUSDZAR));
StressFactors_USDZAR=ones(size(NormRandVarsUSDZAR));
StressFactors_GBPZAR=ones(size(NormRandVarsUSDZAR));
StressFwds_USDZAR=ones(size(NormRandVarsUSDZAR));
StressFwds_GBPZAR=ones(size(NormRandVarsUSDZAR));
StressMTM1=ones(size(NormRandVarsUSDZAR))*CurrentMTM1;
StressMTM2=ones(size(NormRandVarsUSDZAR))*CurrentMTM2;
StressMTM3=ones(size(NormRandVarsUSDZAR))*CurrentMTM3;
StressMTM4=ones(size(NormRandVarsUSDZAR))*CurrentMTM4;
NettedPFEArray=ones(size(NormRandVarsUSDZAR));
NonNettedPFEArray=ones(size(NormRandVarsUSDZAR));
for I=1:1:NoSimulations
    StressedSpots_USDZAR(I,1)=USDZARS0;
    StressedSpots_GBPZAR(I,1)=GBPZARS0;
    NettedPFEArray(I,1)=CurrentMTM1+CurrentMTM2+CurrentMTM3+CurrentMTM4;
    NonNettedPFEArray(I,1)=max(CurrentMTM1,0)+max(CurrentMTM2,0)+...
        max(CurrentMTM3,0)+max(CurrentMTM4,0);
    for J=1:1:TimeSteps -1
        StressFactors_USDZAR(I,J+1)=exp((Mu-(SigmaUSDZAR.^2)/2)*...
            (TimeStepVector(1,J+1)-TimeStepVector(1,J))+SigmaUSDZAR*...
            *NormRandVarsUSDZAR(I,J)*sqrt((TimeStepVector(1,J+1)-...
            TimeStepVector(1,J))));
        StressFactors_GBPZAR(I,J+1)=exp((Mu-(SigmaGBPZAR.^2)/2)*...
            (TimeStepVector(1,J+1)-TimeStepVector(1,J))+SigmaGBPZAR*...
            NormRandVarsGBPZAR(I,J)*sqrt((TimeStepVector(1,J+1)-...
            TimeStepVector(1,J))));
        StressedSpots_USDZAR(I,J+1)=StressedSpots_USDZAR(I,J)*...
            StressFactors_USDZAR(I,J+1);
        StressedSpots_GBPZAR(I,J+1)=StressedSpots_GBPZAR(I,J)*...
            StressFactors_GBPZAR(I,J+1);
        if TimeStepVector(1,J+1)<=T1
            StressFwds_USDZAR(I,J)=RelFwdCurveUSDZAR(2,T1/0.05+1-J+1)*...
                StressedSpots_USDZAR(I,J+1);
        end
        if TimeStepVector(1,J+1)<=T2
            StressFwds_GBPZAR(I,J)=RelFwdCurveGBPZAR(2,T2/0.05+1-J+1)*...
                StressedSpots_GBPZAR(I,J+1);
        end
        StressMTM1(I,J+1)=(TimeStepVector(1,J+1)<T1)*Notional1*...
            exp(-InterestRate*(TimeStepVector(1,J+1)-...
            TimeStepVector(1,J)))*(StressFwds_USDZAR(I,J)-Strike1);
        StressMTM2(I,J+1)=(TimeStepVector(1,J+1)<T2)*Notional2*...
            exp(-InterestRate*(TimeStepVector(1,J+1)-...
            TimeStepVector(1,J)))*(StressFwds_GBPZAR(I,J)-Strike2);
        [StressMTM3(I,J+1),Dummy1]=blsprice(StressedSpots_GBPZAR(I,J+1)...
            , Strike3, InterestRate, T3-TimeStepVector(1,J+1),SigmaGBPZAR);
        [StressMTM4(I,J+1),Dummy1]=blsprice(StressedSpots_USDZAR(I,J+1),...
            Strike4, InterestRate, T4-TimeStepVector(1,J+1),SigmaUSDZAR);
        StressMTM3(I,J+1)=(TimeStepVector(1,J+1)<T3)*StressMTM3(I,J+1)...
            *Notional3;
        StressMTM4(I,J+1)=(TimeStepVector(1,J+1)<T4)*StressMTM4(I,J+1)...
            *Notional4;
        NettedPFEArray(I,J+1)=StressMTM1(I,J+1)+StressMTM2(I,J+1)+...
            StressMTM3(I,J+1)+StressMTM4(I,J+1);
        NonNettedPFEArray(I,J+1)=max(StressMTM1(I,J+1),0)+...
            max(StressMTM2(I,J+1),0)+max(StressMTM3(I,J+1),0)+...
            max(StressMTM4(I,J+1),0);
    end
    StressFwds_USDZAR(I,TimeSteps)=StressedSpots_USDZAR(I,TimeSteps);
    StressFwds_GBPZAR(I,TimeSteps)=StressedSpots_GBPZAR(I,TimeSteps);
end
FinalPFE1=prctile(StressMTM1,95);
```



```
FinalPFE2=prctile(StressMTM2,95);
FinalPFE3=prctile(StressMTM3,95);
FinalPFE4=prctile(StressMTM4,95);
AggregatePFEGross=max(FinalPFE1,0)+max(FinalPFE2,0)+...
    max(FinalPFE3,0)+max(FinalPFE4,0);
AggregatePFENet=FinalPFE1+FinalPFE2+FinalPFE3+FinalPFE4;
NettedPFE=prctile(NettedPFEArray,95);
NonNettedPFE=prctile(NonNettedPFEArray,95);
t=0:0.05:T;
%This plots the Contract-Level PFEs of the 4 Derivatives
Line1=stairs(t,FinalPFE1,'LineWidth',1.5,'Color',[0 0 1]);
grid on;
hold;
Line2=stairs(t,FinalPFE2,'LineWidth',1.5,'Color',[1 0 0],'LineStyle','-');
Line3=stairs(t,FinalPFE3,'LineWidth',1.5,'Color',[0 1 0],'LineStyle','-');
Line4=stairs(t,FinalPFE4,'LineWidth',1.5,'Color',[1 0.69 0.39]...
    ,'LineStyle','-');
%This plots the Portfolio-Level PFEs(Netted and Non-Netted) of the 4 Derivs
figure;
Line1=stairs(t,NettedPFE,'LineWidth',1.5,'Color',[0 0 1]);
grid on;
hold;
Line2=stairs(t,NonNettedPFE,'LineWidth',1.5,'Color',[1 0 0],...
    'LineStyle','-');
Line3=stairs(t,AggregatePFEGross,'LineWidth',1.5,'Color',...
    [0 1 0],'LineStyle','-');
Line4=stairs(t,AggregatePFENet,'LineWidth',1.5,'Color',...
    [1 0.69 0.39],'LineStyle','-');
```

9.4 Counterparty-Level PFE taking into account CSA Agreement

```

%This M-File applies the effects of a Collateral Agreement to a
%predetermined counterparty-level PFE profile (NettedPFE) and produces the
%collateralised PFE. The inputs to the collateral agreement which impact
%the collateralised PFE profile are:
ThresholdAmount=1500;
MinimumTransferAmount=175;
CloseOutPeriod=10;
BarrierAmount=2250;

%Define a few variables
CollateralReceived=zeros(size(NettedPFE)); %represents the collateral
%received at that point in time
CollateralHeld=zeros(size(NettedPFE)); %represents the collateral
%received up to that point in time
CollateralCalled=zeros(size(NettedPFE)); %represents the collateral
%called at that point in time
CollateralCalledNotReceived=zeros(size(NettedPFE));
%We now proceed to the calculation of the collateralised PFE:
CollateralReceived(1)=0;
NumberOfSteps=size(NettedPFE);
NumberOfSteps=NumberOfSteps(2);
for time=2:1:NumberOfSteps
    %Adjust the balances of the variables to keep track of collateral on
    %call and in possession
    if time>CloseOutPeriod
        CollateralReceived(time)=CollateralCalled(time-CloseOutPeriod);
        CollateralHeld(time)=CollateralHeld(time-1)+CollateralReceived(time);
    end
    CollateralCalledNotReceived(time)=CollateralCalledNotReceived...
        (time-1)-CollateralReceived(time);
    %Determine whether a Collateral Call or Collateral Refund should be made:
    if NettedPFE(time)-(CollateralHeld(time)+...
        CollateralCalledNotReceived(time))-ThresholdAmount...
        >MinimumTransferAmount
        if CollateralHeld(time)+CollateralCalledNotReceived(time)...
            <BarrierAmount
            CollateralCalled(time)= NettedPFE(time)-...
                (CollateralHeld(time)+CollateralCalledNotReceived(time))...
                -ThresholdAmount;
        end
        elseif ThresholdAmount-(NettedPFE(time)-(CollateralHeld(time)+...
            CollateralCalledNotReceived(time)))>MinimumTransferAmount
            %We have now established that we have to make a collateral refund
            %and so we need to firstly establish what the amount is that we
            %have to refund (i.e. what the excess collateral is)
            CollateralReceived(time)=min(-(NettedPFE(time)-...
                (CollateralHeld(time)+CollateralCalledNotReceived(time))...
                -ThresholdAmount)), -CollateralHeld(time));
            CollateralHeld(time)=CollateralHeld(time)+min(-(NettedPFE(time)...
                -(CollateralHeld(time)+CollateralCalledNotReceived(time))...
                -ThresholdAmount)), -CollateralHeld(time));
        end
        CollateralCalledNotReceived(time)=CollateralCalledNotReceived(time)...
            +CollateralCalled(time);
    end
end
figure;
t=0:1:NumberOfSteps-1;
ThresholdLine=ones(size(CollateralCalled))*ThresholdAmount;
BarrierLine=ones(size(CollateralCalled))*BarrierAmount;

```



```
Line1=stairs(t,CollateralReceived,'LineWidth',1,'Color',[0 1 0],...  
'LineStyle','-');  
grid on;  
hold;  
Line2=stairs(t,CollateralCalled,'LineWidth',1,'Color',[1 0 0],...  
'LineStyle','-');  
Line3=stairs(t,CollateralCalledNotReceived,'LineWidth',1,'Color',...  
[0 0.5 0],'LineStyle','-');  
  
figure;  
Line4=stairs(t,NettedPFE,'LineWidth',1.5,'Color',[0 0 1]);  
hold;  
Line5=stairs(t,CollateralHeld,'LineWidth',1.5,'Color',[0 0 0],...  
'LineStyle','-');  
Line6=stairs(t,ThresholdLine,'LineWidth',1.5,'Color',[1 0 0],...  
'LineStyle','-');  
Line7=stairs(t,BarrierLine,'LineWidth',1.5,'Color',[1 0 0],...  
'LineStyle',':');  
Line8=stairs(t,ThresholdLine+MinimumTransferAmount,'LineWidth',...  
1.5,'Color',[0 0 1],'LineStyle',':');  
Line8=stairs(t,NettedPFE-CollateralHeld,'LineWidth',1.5,'Color',...  
[0 1 1],'LineStyle','-');  
grid on;
```

9.5 Wrong-Way Risk Model Implementation

```

%The purpose of this m-file is to demonstrate the wrong-way risk model
%proposed by le Roux (2008). The model uses a structural approach
%(CreditGrades Model) to simulate conditional and unconditional PDs.
%These equity prices are correlated with another underlying variable and in
%the process this achieves a models which links the probability of default
%with the changes in the underlying market variable. In so doing the effect
%of wrong way risk is encapsulated in the measures for counterparty credit
%risk.
%Notional of the underlying asset
Notional_Underlying=1000;

%Define the number of simulations and time steps
NoSimulations=2001;
TimeSteps=11;

%Specify the parameters of the underlying trades and market information
T1=1;
T=T1;
Mu=0;
Mu_Underlying=Mu;
Mu_Stock=0;
InterestRate=0.12;
Strike1=826.8;
Vol_Stock=0.3652;
Vol_Underlying=0.2103;
Vol_Barrier=0.3;
Correlation=0.8614;
S0_Stock=412;
S0_Underlying=826.8;
S0_Barrier=0.4;
ContractIsCall=false;
%Determine the MTM at t=0;
if ContractIsCall
    [CurrentMTM1,Dummy1]=blsprice(S0_Underlying, Strike1, InterestRate,...
        T1, Vol_Underlying);
    CurrentMTM1=CurrentMTM1*Notional_Underlying;
else
    [Dummy1,CurrentMTM1]=blsprice(S0_Underlying, Strike1, InterestRate,...
        T1, Vol_Underlying);
    CurrentMTM1=CurrentMTM1*Notional_Underlying;
end
%Generate the random variables required in the simulations
NormRandVars_Underlying=randn(NoSimulations,TimeSteps);
NormRandVars_Barrier=randn(NoSimulations,TimeSteps);

%Determine the time-grid
TimeStepVector=0:0.1:T;
%Initiate the simulates spot price variables
SimulatedSpots_Underlying=ones(size(NormRandVars_Underlying));
SimulatedBarrier=ones(size(NormRandVars_Underlying));
%Initiate the simulates GBM variables
StressFactors_Underlying=ones(size(NormRandVars_Underlying));
%Initiate the simulated MTM variable
SimulatedMTM1=ones(size(NormRandVars_Underlying))*CurrentMTM1;
SimulatedConditionalMTM1=ones(size(NormRandVars_Underlying))*CurrentMTM1;
%Determine the Current Undconditional PD
DebtPerShare=243.27
R_Bar=0.5;
d=((S0_Stock+R_Bar*DebtPerShare)/(R_Bar*DebtPerShare))*exp(Vol_Barrier^2);

```



```
At=sqrt(((Vol_Stock*S0_Stock)/(S0_Stock+R_Bar*DebtPerShare)).^2)...
+Vol_Barrier^2);
CurrentUnconditionalPD=1-(normcdf(-0.5*At+log(d)/At,0,1)-d*...
normcdf(-0.5*At-log(d)/At,0,1));
%Initiate the simulated Conditional PD variable and associated variables
%
ExposureInflationFactor=ones(size(NormRandVars_Underlying));

%Define a variable for storing the expected value and conditional expected
%value of the stock prices for various values of t
ExpectedSpot_Stock=ones(size(TimeStepVector))*S0_Stock;
UnconditionalPD=ones(size(NormRandVars_Underlying))*CurrentUnconditionalPD;
ConditionalExpectedSpot_Stock=ones(size(NormRandVars_Underlying))*S0_Stock;
ConditionalPD=ones(size(NormRandVars_Underlying))*CurrentUnconditionalPD;
MuPrime_Stock=ones(size(NormRandVars_Underlying))*log(S0_Stock);
SigmaPrime_Stock=zeros(size(NormRandVars_Underlying));
MuPrime_Underlying=ones(size(NormRandVars_Underlying))*log(S0_Underlying);
SigmaPrime_Underlying=zeros(size(NormRandVars_Underlying));
MuStar_Stock=ones(size(NormRandVars_Underlying))*log(S0_Stock);
SigmaSqStar_Stock=zeros(size(NormRandVars_Underlying));

for I=1:1:NoSimulations
    SimulatedSpots_Underlying(I,1)=S0_Underlying;
    SimulatedBarrier(I,1)=S0_Barrier*DebtPerShare;
    ExposureInflationFactor(I,1)=1;
    for J=1:1:TimeSteps -1
        ExpectedSpot_Stock(J+1)=S0_Stock*exp(((Mu_Stock-0.5*(Vol_Stock^2))...
            *TimeStepVector(J+1))+0.5*(Vol_Stock^2)*TimeStepVector(J+1));
        MuPrime_Stock(I,J+1)=log(S0_Stock)+(Mu_Stock-0.5*(Vol_Stock^2))...
            *TimeStepVector(J+1);
        SigmaPrime_Stock(I,J+1)=Vol_Stock*sqrt(TimeStepVector(J+1));
        MuPrime_Underlying(I,J+1)=log(S0_Underlying)+(Mu_Underlying...
            -0.5*(Vol_Underlying^2))*TimeStepVector(J+1);
        SigmaPrime_Underlying(I,J+1)=Vol_Underlying*sqrt(TimeStepVector(J+1));

        StressFactors_Underlying(I,J+1)=exp((Mu-(Vol_Underlying.^2)/2)...
            *(TimeStepVector(1,J+1)-TimeStepVector(1,J))+Vol_Underlying...
            *NormRandVars_Underlying(I,J)*sqrt((TimeStepVector(1,J+1)...
            -TimeStepVector(1,J))));

        SimulatedSpots_Underlying(I,J+1)=SimulatedSpots_Underlying(I,J)...
            *StressFactors_Underlying(I,J+1);

        MuStar_Stock(I,J+1)=MuPrime_Stock(I,J+1)...
            +(Correlation/SigmaPrime_Underlying(I,J+1))...
            *SigmaPrime_Stock(I,J+1)*(log(SimulatedSpots_Underlying(I,J+1))...
            -MuPrime_Underlying(I,J+1));
        SigmaSqStar_Stock(I,J+1)=(SigmaPrime_Stock(I,J+1)).^2)...
            *(1-Correlation^2);
        ConditionalExpectedSpot_Stock(I,J+1)=exp(MuStar_Stock(I,J+1)+0.5...
            *SigmaSqStar_Stock(I,J+1));

        SimulatedBarrier(I,J+1)=S0_Barrier*DebtPerShare;
        if ContractIsCall %Call
            [SimulatedMTM1(I,J+1),Dummy1]=blsprice(...
                SimulatedSpots_Underlying(I,J+1), Strike1, InterestRate,...
                T1-TimeStepVector(1,J+1),Vol_Underlying);
            SimulatedMTM1(I,J+1)=SimulatedMTM1(I,J+1)*Notional_Underlying;
        else %Put
            [Dummy1,SimulatedMTM1(I,J+1)]=blsprice...
                (SimulatedSpots_Underlying(I,J+1), Strike1, InterestRate,...
                T1-TimeStepVector(1,J+1),Vol_Underlying);
```



```
        SimulatedMTM1(I,J+1)=SimulatedMTM1(I,J+1)*Notional_Underlying;
    end
    %Calculate the conditional default probability using simulated stock
    At_Unconditional=sqrt(((Vol_Stock*ExpectedSpot_Stock(J+1))/...
        (ExpectedSpot_Stock(J+1)+R_Bar*DebtPerShare))^2)+Vol_Barrier^2);
    d_Unconditional=((ExpectedSpot_Stock(J+1)+R_Bar*DebtPerShare)/...
        (R_Bar*DebtPerShare))*exp(Vol_Barrier^2);
    UnconditionalPD(I,J+1)=1-(normcdf(-0.5*At_Unconditional...
        +log(d_Unconditional)/At_Unconditional,0,1)-d_Unconditional...
        *normcdf(-0.5*At_Unconditional-log(d_Unconditional)...
        /At_Unconditional,0,1));

    At_Conditional=sqrt(((Vol_Stock...
        *ConditionalExpectedSpot_Stock(I,J+1))...
        /(ConditionalExpectedSpot_Stock(I,J+1)...
        +R_Bar*DebtPerShare))^2)+Vol_Barrier^2);
    d_Conditional=((ConditionalExpectedSpot_Stock(I,J+1)...
        +R_Bar*DebtPerShare)/(R_Bar*DebtPerShare))*exp(Vol_Barrier^2);

    ConditionalPD(I,J+1)=1-(normcdf(-0.5*At_Conditional...
        +log(d_Conditional)/At_Conditional,0,1)-d_Conditional...
        *normcdf(-0.5*At_Conditional-log(d_Conditional)...
        /At_Conditional,0,1));
    %Determine the conditional MTM
    ExposureInflationFactor(I,J+1)=(ConditionalPD(I,J+1)...
        /UnconditionalPD(I,J+1));

    SimulatedConditionalMTM1(I,J+1)=SimulatedMTM1(I,J+1)...
        *ExposureInflationFactor(I,J+1);
end
    StockPath=1;
end
FinalPFE1_Unconditional=prctile(SimulatedMTM1,99);
FinalPFE1_Conditional=prctile(SimulatedConditionalMTM1,99);

t=0:0.1:T;
plot(t, FinalPFE1_Unconditional, 'DisplayName', 'Unconditional',...
    'XDataSource', 't', 'YDataSource', 'Unconditional'); figure(gcf)
grid on;
hold on;
line1=plot(t, FinalPFE1_Conditional, 'DisplayName', 'Conditional',...
    'XDataSource', 't', 'YDataSource', 'Conditional','Color', 'red');...
    figure(gcf)
figure;
hold on;
scatter (SimulatedSpots_Underlying(:,9),ExposureInflationFactor(:,9),'o');...
    figure(gcf)
grid on;
scatter (SimulatedSpots_Underlying(:,6),ExposureInflationFactor(:,6),'x');...
    figure(gcf)
hold on;
scatter (SimulatedSpots_Underlying(:,3),ExposureInflationFactor(:,3),'.');...
    figure(gcf)
hold on;
maxincrease=max(FinalPFE1_Conditional)/max(FinalPFE1_Unconditional)-1;
```

9.6 EAD Calculations under the Internal Models and Current Exposure Methods

```

%The following Parameters are used for the calculation of the EAD under the
%CEM approach in Basel2:

AddOnFactor1=0.01;
AddOnFactor2=0.05;
AddOnFactor3=0.05;
AddOnFactor4=0.05;
ZARNotional1=abs(Notional1)*Strike1;
ZARNotional2=abs(Notional2)*Strike2;
ZARNotional3=abs(Notional3)*Strike3;
ZARNotional4=abs(Notional4)*Strike4;%Note that in portfolio 1 Strike3=Strike4
EAD1_CEM=max(CurrentMTM1,0)+ZARNotional1*AddOnFactor1;
EAD2_CEM=max(CurrentMTM2,0)+ZARNotional2*AddOnFactor2;
EAD3_CEM=max(CurrentMTM3,0)+ZARNotional3*AddOnFactor3; %Note that contract
% 3 of portfolio two is a written option and is therefore left out of the
% EAD calculations under the CEM
EAD4_CEM=max(CurrentMTM4,0)+ZARNotional4*AddOnFactor4;
GrossRC=max(0,CurrentMTM1)+max(0,CurrentMTM2)+max(0,CurrentMTM3)+...
max(0,CurrentMTM4);
NetRC=max(CurrentMTM1+CurrentMTM2+CurrentMTM4,0);%CurrentMTM3+CurrentMTM4,0);
NGR=NetRC/GrossRC;
AOP=(0.4+0.6*NGR)*(ZARNotional1*AddOnFactor1+ZARNotional2*...
AddOnFactor2+ZARNotional4*AddOnFactor4);
EADCounterparty_CEM=NetRC+AOP;

%The following loop calculates the EAD using the IMM under Basel2
%Note: TS1Yr are the number of time steps up to one year
TS1Yr=21;
Deltat=0.05; %the size of each time step
ExpectedExposure1=zeros(TS1Yr,1);
ExpectedExposure2=zeros(TS1Yr,1);
ExpectedExposure3=zeros(TS1Yr,1);
ExpectedExposure4=zeros(TS1Yr,1);
ExpectedExposureCounterparty=zeros(TS1Yr,1);
EPE1=0;
EPE2=0;
EPE3=0;
EPE4=0;
EPECounterparty=0;
PFE1=zeros(TS1Yr,1);
PFE2=zeros(TS1Yr,1);
PFE3=zeros(TS1Yr,1);
PFE4=zeros(TS1Yr,1);
PFECounterparty=zeros(TS1Yr,1);

for I=1:1:TS1Yr %We use the first year only as this is the tenor over which
%EE, EPE and Effective EPE is defined
for J=1:1:NoSimulations
ExpectedExposure1(I)=ExpectedExposure1(I)+max(StressMTM1(J,I),0);
ExpectedExposure2(I)=ExpectedExposure2(I)+max(StressMTM2(J,I),0);
ExpectedExposure3(I)=ExpectedExposure3(I)+max(StressMTM3(J,I),0);
ExpectedExposure4(I)=ExpectedExposure4(I)+max(StressMTM4(J,I),0);
ExpectedExposureCounterparty(I)=ExpectedExposureCounterparty(I)+...
max(NettedPFEArray(J,I),0);
end
ExpectedExposure1(I)=ExpectedExposure1(I)/NoSimulations;

```



```
ExpectedExposure2(I)=ExpectedExposure2(I)/NoSimulations;  
ExpectedExposure3(I)=ExpectedExposure3(I)/NoSimulations;  
ExpectedExposure4(I)=ExpectedExposure4(I)/NoSimulations;  
ExpectedExposureCounterparty(I)=ExpectedExposureCounterparty(I)/...  
NoSimulations;  
EPE1=EPE1+ExpectedExposure1(I)*Deltat; %since our time steps are equal  
EPE2=EPE2+ExpectedExposure2(I)*Deltat;  
EPE3=EPE3+ExpectedExposure3(I)*Deltat;  
EPE4=EPE4+ExpectedExposure4(I)*Deltat;  
EPECounterparty=EPECounterparty+ExpectedExposureCounterparty(I)*Deltat;  
PFE1(I)=FinalPFE1(I);  
PFE2(I)=FinalPFE2(I);  
PFE3(I)=FinalPFE3(I);  
PFE4(I)=FinalPFE4(I);  
PFECounterparty(I)=NettedPFE(I);  
end  
EffectiveEE1=ones(TS1Yr,1)*ExpectedExposure1(1);  
EffectiveEE2=ones(TS1Yr,1)*ExpectedExposure2(1);  
EffectiveEE3=ones(TS1Yr,1)*ExpectedExposure3(1);  
EffectiveEE4=ones(TS1Yr,1)*ExpectedExposure4(1);  
EffectiveEECounterparty=ones(TS1Yr,1)*ExpectedExposureCounterparty(1);  
  
EffectiveEPE1=Deltat*ExpectedExposure1(1);  
EffectiveEPE2=Deltat*ExpectedExposure2(1);  
EffectiveEPE3=Deltat*ExpectedExposure3(1);  
EffectiveEPE4=Deltat*ExpectedExposure4(1);  
EffectiveEPECounterparty=Deltat*ExpectedExposureCounterparty(1);  
for I=2:1:TS1Yr  
    EffectiveEE1(I)=max(EffectiveEE1(I-1),ExpectedExposure1(I));  
    EffectiveEE2(I)=max(EffectiveEE2(I-1),ExpectedExposure2(I));  
    EffectiveEE3(I)=max(EffectiveEE3(I-1),ExpectedExposure3(I));  
    EffectiveEE4(I)=max(EffectiveEE4(I-1),ExpectedExposure4(I));  
    EffectiveEECounterparty(I)=max(EffectiveEECounterparty(I-1),...  
        ExpectedExposureCounterparty(I));  
  
    EffectiveEPE1=EffectiveEPE1+Deltat*EffectiveEE1(I);  
    EffectiveEPE2=EffectiveEPE2+Deltat*EffectiveEE2(I);  
    EffectiveEPE3=EffectiveEPE3+Deltat*EffectiveEE3(I);  
    EffectiveEPE4=EffectiveEPE4+Deltat*EffectiveEE4(I);  
    EffectiveEPECounterparty=EffectiveEPECounterparty+Deltat*...  
        EffectiveEECounterparty(I);  
end  
EAD1_IMM=EffectiveEPE1*1.4;  
EAD2_IMM=EffectiveEPE2*1.4;  
EAD3_IMM=EffectiveEPE3*1.4;  
EAD4_IMM=EffectiveEPE4*1.4;  
EADCounterparty_IMM=EffectiveEPECounterparty*1.4;  
  
x=0:Deltat:1;  
%This plots the Contract-Level PFEs of the 4 Derivatives  
%Contract1  
Line1=stairs(x,ExpectedExposure1,'DisplayName','Expected Exposure',...  
    'LineWidth',1.5,'Color','b');  
grid on;  
hold;  
Line2=stairs(x,ones(TS1Yr,1)*EPE1,'DisplayName',...  
    'Expected Positive Exposure','LineWidth',2,'MarkerSize',5,...  
    'Marker','o','Color',[0 0.5 0]);  
Line3=stairs(x,EffectiveEE1,'DisplayName','Effective EE','LineWidth',...  
    3,'Color','b','LineStyle',':');  
Line4=stairs(x,ones(TS1Yr,1)*EffectiveEPE1,'DisplayName',...
```



```
'Effective EPE', 'LineWidth', 2, 'MarkerSize', 5, 'Marker', '^', 'Color', ...
[0.75 0 0.75]);
Line5=stairs(x, ones(TS1Yr, 1)*EAD1_CEM, 'DisplayName', 'EAD under CEM', ...
'LineWidth', 2, 'MarkerSize', 5, 'Marker', 's', 'Color', 'r', 'LineStyle', '-. ');
Line6=stairs(x, ones(TS1Yr, 1)*EAD1_IMM, 'DisplayName', 'EAD under IMM', ...
'LineWidth', 2, 'MarkerSize', 5, 'Marker', '*', 'Color', 'r');
Line7=stairs(x, PFE1, 'DisplayName', '95% PFE', 'LineWidth', 2, 'MarkerSize', ...
5, 'Marker', 'v', 'Color', 'k');
legend show;
figure;

%Contract2
Line1=stairs(x, ExpectedExposure2, 'DisplayName', 'Expected Exposure', ...
'LineWidth', 1.5, 'Color', 'b');
grid on;
hold;
Line2=stairs(x, ones(TS1Yr, 1)*EPE2, 'DisplayName', ...
'Expected Positive Exposure', 'LineWidth', 2, 'MarkerSize', 5, ...
'Marker', 'o', 'Color', [0 0.5 0]);
Line3=stairs(x, EffectiveEE2, 'DisplayName', 'Effective EE', 'LineWidth', ...
3, 'Color', 'b', 'LineStyle', ':');
Line4=stairs(x, ones(TS1Yr, 1)*EffectiveEPE2, 'DisplayName', ...
'Effective EPE', 'LineWidth', 2, 'MarkerSize', 5, 'Marker', '^', 'Color', ...
[0.75 0 0.75]);
Line5=stairs(x, ones(TS1Yr, 1)*EAD2_CEM, 'DisplayName', 'EAD under CEM', ...
'LineWidth', 2, 'MarkerSize', 5, 'Marker', 's', 'Color', 'r', 'LineStyle', '-. ');
Line6=stairs(x, ones(TS1Yr, 1)*EAD2_IMM, 'DisplayName', 'EAD under IMM', ...
'LineWidth', 2, 'MarkerSize', 5, 'Marker', '*', 'Color', 'r');
Line7=stairs(x, PFE2, 'DisplayName', '95% PFE', 'LineWidth', 2, 'MarkerSize', ...
5, 'Marker', 'v', 'Color', 'k');
legend show;
figure;

%Contract3
Line1=stairs(x, ExpectedExposure3, 'DisplayName', 'Expected Exposure', ...
'LineWidth', 1.5, 'Color', 'b');
grid on;
hold;
Line2=stairs(x, ones(TS1Yr, 1)*EPE3, 'DisplayName', ...
'Expected Positive Exposure', 'LineWidth', 2, 'MarkerSize', 5, ...
'Marker', 'o', 'Color', [0 0.5 0]);
Line3=stairs(x, EffectiveEE3, 'DisplayName', 'Effective EE', 'LineWidth', ...
3, 'Color', 'b', 'LineStyle', ':');
Line4=stairs(x, ones(TS1Yr, 1)*EffectiveEPE3, 'DisplayName', ...
'Effective EPE', 'LineWidth', 2, 'MarkerSize', 5, 'Marker', '^', 'Color', ...
[0.75 0 0.75]);
Line5=stairs(x, ones(TS1Yr, 1)*EAD3_CEM, 'DisplayName', 'EAD under CEM', ...
'LineWidth', 2, 'MarkerSize', 5, 'Marker', 's', 'Color', 'r', 'LineStyle', '-. ');
Line6=stairs(x, ones(TS1Yr, 1)*EAD3_IMM, 'DisplayName', 'EAD under IMM', ...
'LineWidth', 2, 'MarkerSize', 5, 'Marker', '*', 'Color', 'r');
Line7=stairs(x, PFE3, 'DisplayName', '95% PFE', 'LineWidth', 2, 'MarkerSize', ...
5, 'Marker', 'v', 'Color', 'k');
legend show;
figure;

%Contract4
Line1=stairs(x, ExpectedExposure4, 'DisplayName', 'Expected Exposure', ...
'LineWidth', 1.5, 'Color', 'b');
grid on;
hold;
Line2=stairs(x, ones(TS1Yr, 1)*EPE4, 'DisplayName', ...
'Expected Positive Exposure', 'LineWidth', 2, 'MarkerSize', 5, 'Marker', ...
'o', 'Color', [0 0.5 0]);
```



```
Line3=stairs(x,EffectiveEE4,'DisplayName','Effective EE','LineWidth',3,...
'Color','b','LineStyle',':');
Line4=stairs(x,ones(TS1Yr,1)*EffectiveEPE4,'DisplayName',...
'Effective EPE','LineWidth',2,'MarkerSize',5,'Marker','^','Color',...
[0.75 0 0.75]);
Line5=stairs(x,ones(TS1Yr,1)*EAD4_CEM,'DisplayName','EAD under CEM',...
'LineWidth',2,'MarkerSize',5,'Marker','s','Color','r','LineStyle','-');
Line6=stairs(x,ones(TS1Yr,1)*EAD4_IMM,'DisplayName','EAD under IMM',...
'LineWidth',2,'MarkerSize',5,'Marker','*','Color','r');
Line7=stairs(x,PFE4,'DisplayName','95% PFE','LineWidth',2,'MarkerSize',...
5,'Marker','v','Color','k');
legend show;
figure;

%Counterparty
Line1=stairs(x,ExpectedExposureCounterparty,'DisplayName',...
'Expected Exposure','LineWidth',1.5,'Color','b');
grid on;
hold;
Line2=stairs(x,ones(TS1Yr,1)*EPECcounterparty,'DisplayName', ...
'Expected Positive Exposure','LineWidth',2,'MarkerSize',5,'Marker',...
'o','Color',[0 0.5 0]);
Line3=stairs(x,EffectiveEECounterparty,'DisplayName','Effective EE',...
'LineWidth',3,'Color','b','LineStyle',':');
Line4=stairs(x,ones(TS1Yr,1)*EffectiveEPECcounterparty,'DisplayName',...
'Effective EPE','LineWidth',2,'MarkerSize',5,'Marker','^','Color',...
[0.75 0 0.75]);
Line5=stairs(x,ones(TS1Yr,1)*EADCounterparty_CEM,'DisplayName',...
'EAD under CEM','LineWidth',2,'MarkerSize',5,'Marker','s','Color',...
'r','LineStyle','-');
Line6=stairs(x,ones(TS1Yr,1)*EADCounterparty_IMM,'DisplayName',...
'EAD under IMM','LineWidth',2,'MarkerSize',5,'Marker','*','Color','r');
Line7=stairs(x,PFECounterparty,'DisplayName','95% PFE','LineWidth',2,...
'MarkerSize',5,'Marker','v','Color','k');
legend show;
```

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