

**OPTIMAL TUNDISH DESIGN METHODOLOGY**  
**IN A**  
**CONTINUOUS CASTING PROCESS**

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## ABSTRACT

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**Title:** Optimal Tundish Design Methodology in a Continuous Casting Process  
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*The demand for higher quality steel and higher production rates in the production of steel slabs is ever increasing. These slabs are produced using a continuous casting process. The molten steel flow patterns inside the components of the caster play an important role in the quality of these products. A simple yet effective design method that yields optimum designs is required to design the systems influencing the flow patterns in the caster. The tundish is one of these systems. Traditionally, experimental methods were used in the design of these tundishes, making use of plant trials or water modelling. These methods are both costly and time consuming. More recently, Computational Fluid Dynamics (CFD) has established itself as a viable alternative to reduce the number of experimentation required, resulting in a reduction in the time scales and cost of the design process. Furthermore, CFD provides more insight into the flow process that is not available through experimentation only. The CFD process is usually based on a trial-and-error basis and relies heavily on the insight and experience of the designer to improve designs. Even an experienced designer will only be able to improve the design and does not necessarily guarantee optimum results. In this thesis, a more efficient design methodology is proposed. This methodology involves the combination of a mathematical optimiser with CFD to automate the design process. The methodology is tested on a four different industrial test cases. The first case involves the optimisation of a simple dam-weir configuration of a single strand caster. The position of the dam and weir relative to inlet region is optimised to reduce the dead volume and increase the inclusion removal. The second*

*case involves the optimisation of a pouring box and baffle of a two-strand caster. In this case, the pouring box and baffle geometry is optimised to maximise the minimum residence time at operating level and a typical transition level. The third case deals with the geometry optimisation of an impact pad to reduce the surface turbulence that should result in a reduction in the particle entrainment from the slag layer. The last case continues from the third case where a dam position and height is optimised in conjunction with the optimised impact pad to maximise the inclusion removal on the slag layer. The cases studies show that a mathematical optimiser combined with CFD is a superior alternative compared to traditional design methods, in that it yields optimum designs for a tundish in a continuous casting system.*

**Keywords:** *tundish, continuous casting, computational fluid dynamics, mathematical optimisation, computational flow optimisation, design methodology, steelmaking, inclusion removal, tundish design*

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## SAMEVATTING

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**Titel:** Optimale Gietbak Ontwerpsmetodiek van 'n Kontinue-gietproses  
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*Daar bestaan 'n alomtoenemende aanvraag vir hoër kwaliteit staalplate wat teen 'n hoër tempo geproduseer word. Die plate word geproduseer deur 'n kontinue gietproses. Die vloeistaal patrone in die proses speel 'n belangrike rol in die kwaliteit van die produkte. 'n Eenvoudige maar tog effektiewe ontwerpsmetode word verlang wat optimum ontwerp lewer van die sisteme wat die vloeipatrone beïnvloed. Die gietbak is een van die sisteme. Histories is eksperimentele metodes gebruik in die ontwerp van die gietbakke, met gebruikmaking van aanlegtoetse of water modellering. Meer onlangs is Berekenings Vloei Dinamika (BVD) gebruik as 'n alternatief om die aantal eksperimentele toetse te verminder en om sodoende die tydskaal en kostes van die ontwerpsproses te verminder. Verder verskaf BVD baie meer insig in die vloei proses wat nie beskikbaar is met tradisionele eksperimentele metodes nie. Die BVD proses is gewoonlik gebaseer op 'n tref-of-fouteer basis en steun sterk op die ontwerper se insig en ondervinding om die ontwerp te verbeter. Selfs 'n ontwerper met baie ondervinding sal slegs die ontwerp kan verbeter en sal nie noodwendig 'n optimum ontwerp kan gee nie. In die tesis word 'n meer effektiewe ontwerpsmetodiek voorgestel. Die metodiek behels die kombinerings van 'n wiskundige optimerings program met BVD om die ontwerpsproses te outomatiseer. Die metodiek word getoets op vier verskillende toetsgevalle. Die eerste geval is die optimering van 'n eenvoudige dam-keermuur konfigurasie vir 'n enkelstring gietproses. Die posisie van die dam en keermuur relatief tot die inlaat area is geoptimeer om die dooie volume verminder en die inklusie verwydering te vermeerder. Die tweede geval beskryf die optimering van die gietboks en skermplaat van 'n tweestring gietbak. In die geval*

*word die gietboks en skermplaat geometrie geoptimeer om die minimum residensie tyd te maksimeer by die bedryfsvlak en by 'n tipiese transisievlak. Die derde geval beskou die geometrie optimering van 'n impakstuk om die oppervlak turbulensie te verlaag, wat veroorsaak dat minder partikels in die vloeï ingetrek sal word vanaf die slaklaag. Die laaste geval volg op geval drie waar 'n dam se posisie en hoogte geoptimeer word saam met die geoptimeerde impakstuk om die inklusie verwydering op die slaklaag te maksimeer. Die toetsgevalle wys dat 'n wiskundige optimeerder gekombineerd met BVD 'n beter alternatief is as tradisionele ontwerpmetodes, om sodoende optimum ontwerpe van 'n gietbak van 'n kontinue stringgiet proses te lewer.*

**Sleutelwoorde:** *gietbak, stringgiet, berekenings vloeï dinamika, wiskundige optimering, berekenings vloeï optimering, ontwerpmetodiek, staalmaak, inklusieverwydering, gietbak ontwerp*

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## NOMENCLATURE

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**Symbols:**

| <b>Variable</b> | <b>Description</b>  | <b>Unit</b>       |
|-----------------|---|-------------------|
| $a_1, a_2, a_3$ | Constants for particle drag law                             | -                 |
| $A$             | Approximated Hessian matrix                                 | -                 |
| $a$             | Approximated curvature                                      | -                 |
| $C_D$           | Particle drag coefficient                                   | -                 |
| $C_L$           | Constant for eddy lifetime                                  | -                 |
| $C_\mu$         | Empirical constant for $k$ - $\varepsilon$ turbulence model | -                 |
| $C_1$           | Empirical constant for $k$ - $\varepsilon$ turbulence model | -                 |
| $C_2$           | Empirical constant for $k$ - $\varepsilon$ turbulence model | -                 |
| $c_p$           | Specific heat at constant pressure                          | J/kg·K            |
| $D_{eff}$       | Local effective diffusion coefficient                       | m <sup>2</sup> /s |
| $D_0$           | Laminar diffusion coefficient                               | m <sup>2</sup> /s |
| $D/Dt$          | Substantial derivative                                      | -                 |
| $d_p$           | Particle diameter   | M                 |
| $E$             | Empirical constant for the wall function in turbulent flow  | -                 |
| $f$             | Objective function  | ♦                 |
| Fr              | Froude number   | -                 |
| $g$             | Gravitational acceleration                                  | m/s <sup>2</sup>  |
| $g_j$           | $j$ -th inequality constraint                               | ♦                 |
| $H$             | Fluid enthalpy  | J/kg              |
| H               | Hessian matrix  | -                 |
| $h_k$           | $k$ -th inequality constraint                               | ♦                 |
| $I$             | Identity matrix   | -                 |
| $K$             | Thermal conductivity  | W/m·K             |

---

♦ Problem dependant

| Variable           | Description   | Unit                                  |
|--------------------|---|---------------------------------------|
| $k$                | Turbulent kinetic energy                              | $\text{m}^2/\text{s}^2$               |
| $k_P$              | Turbulent kinetic energy at point $P$                 | $\text{m}^2/\text{s}^2$               |
| $L$                | Reference length                                      | m                                     |
|                    | Turbulent length scale                                | m                                     |
| $L_e$              | Eddy length scale                                     | m                                     |
| $m$                | Number of inequality constraints                      | -                                     |
| $n$                | Number of design variables                            | -                                     |
| $\bar{p}$          | Time-averaged pressure equations                      | Pa                                    |
| $p$                | Pressure  | Pa                                    |
|                    | Penalty function                                      | -                                     |
|                    | Number of equality constraints                        | -                                     |
| Re                 | Reynolds Number                                       | -                                     |
| $Sc_t$             | Turbulent Schmitt number                              | -                                     |
| $S_m$              | Mass source   | $\text{kg}/\text{m}^3 \cdot \text{s}$ |
| $t$                | Time  | s                                     |
| $t_{cross}$        | Particle eddy crossing time                           | s                                     |
| $T$                | Temperature   | K                                     |
| $T_0$              | Reference temperature                                 | K                                     |
| $U$                | Reference velocity                                    | m/s                                   |
| $u$                | Velocity in $x$ -direction                            | m/s                                   |
| $U^*$              | Dimensionless velocity                                | -                                     |
| $u_i$              | Instantaneous velocity in the $i$ -th direction       | m/s                                   |
| $U_i$              | Mean velocity in the $i$ -th direction                | m/s                                   |
| $u_i'$             | Fluctuating part of velocity in the $i$ -th direction | m/s                                   |
| $U_P$              | Mean velocity of the fluid at point $P$               | m/s                                   |
| $u_p$              | Particle velocity                                     | m/s                                   |
| $\bar{\mathbf{V}}$ | Time-averaged velocity vector                         | m/s                                   |
| $\mathbf{V}$       | Velocity vector                                       | m/s                                   |
| $v$                | Velocity in $y$ -direction                            | m/s                                   |
| $V_{dpv}$          | Dispersed plug volume                                 | -                                     |
| $V_{dv}$           | Dead volume   | -                                     |
| $V_{mv}$           | Well-mixed volume                                     | -                                     |



| Variable        | Description  | Unit                    |
|-----------------|--|-------------------------|
| $V_{pv}$        | Plug flow volume                                     | -                       |
| $\nu_t$         | Eddy viscosity                                       | $\text{m}^2/\text{s}$   |
| $w$             | Velocity in $z$ -direction                           | $\text{m}/\text{s}$     |
| $x$             | Distance in $x$ -direction                           | $\text{m}$              |
| $\mathbf{x}$    | Design variable vector                               | ♦                       |
| $\mathbf{x}^*$  | Optimum design variable vector                       | ♦                       |
| $x_i$           | Spatial co-ordinate in the $i$ -direction            | $\text{m}$              |
|                 | $i$ -th design variable                              | ♦                       |
| $y$             | Distance in $y$ -direction                           | $\text{m}$              |
| $Y$             | Inclusion mass fraction                              | -                       |
| $y^*$           | Dimensionless distance fro the wall                  | -                       |
| $y_P$           | Distance from point $P$ to the wall                  | $\text{m}$              |
| $z$             | Distance in $z$ -direction                           | $\text{m}$              |
| $\alpha$        | Penalty function parameter for inequality constraint | -                       |
| $\beta$         | Thermal expansion coefficient                        | $1/\text{K}$            |
|                 | Penalty function parameter for equality constraint   | -                       |
| $\gamma$        | Penalty function parameter for objective function    | -                       |
| $\delta_i$      | Move limit on $i$ -th design variable                | ♦                       |
| $\delta_{ij}$   | Kronecker delta function                             | -                       |
| $\varepsilon$   | Turbulent dissipation rate                           | $\text{m}^2/\text{s}^3$ |
| $\zeta$         | Normally distributed random number                   | -                       |
| $\theta_{av}$   | Average residence time                               | -                       |
| $\theta_{min}$  | Minimum break-through time                           | -                       |
| $\theta_{peak}$ | Time to reach peak concentration                     | -                       |
| $\kappa$        | Von Kármán constant                                  | -                       |
| $\lambda$       | Bulk viscosity                                       | $\text{m}/\text{s}^2$   |
| $\mu$           | Kinematic viscosity                                  | $\text{m}^2/\text{s}$   |
|                 | Large positive number                                | -                       |
| $\rho$          | Density  | $\text{kg}/\text{m}^3$  |
|                 | Penalty function parameter                           | -                       |

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♦ Problem dependant

| Variable                 | Description   | Unit              |
|--------------------------|---|-------------------|
| $\rho_0$                 | Reference density   | kg/m <sup>3</sup> |
| $\rho_p$                 | Particle density  | kg/m <sup>3</sup> |
| $\sigma_\varepsilon$     | Empirical constant for $k$ - $\varepsilon$ turbulence model | -                 |
| $\sigma_k$               | Empirical constant for $k$ - $\varepsilon$ turbulence model | -                 |
| $\tau$                   | Theoretical residence time                                  | -                 |
|                          | Particle relaxation time                                    | -                 |
| $\tau_e$                 | Characteristic lifetime of an eddy                          | s                 |
| $\omega$                 | Turbulent vorticity   | 1/s               |
| $\Delta x_{\text{norm}}$ | Normalized step size  | -                 |
| $\Delta f_{\text{norm}}$ | normalized change in function value                         | -                 |
| $\Delta x_i$             | Step size for $i$ -th design variable                       | ♦                 |
| $\Phi$                   | Viscous dissipation   | W/m <sup>3</sup>  |

**Abbreviations:**

|      |  |
|------|--|
| CFD  | Computational Fluid Dynamics             |
| RANS | Reynolds Averaged Navier-Stokes equation |
| RTD  | Residence time distribution              |
| MRT  | Minimum residence time                   |
| RSM  | Reynolds Stress Models                   |
| ASM  | Algebraic Stress Models                  |
| LES  | Large-Eddy Simulation                    |
| DNS  | Direct Numerical Simulation              |
| SEN  | Submerged Entry Nozzle                   |

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♦ Problem dependant