

# Adaptive Multi-Population Differential Evolution for Dynamic Environments

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# Abstract

Dynamic optimisation problems are problems where the search space does not remain constant over time. Evolutionary algorithms aimed at static optimisation problems often fail to effectively optimise dynamic problems. The main reason for this is that the algorithms converge to a single optimum in the search space, and then lack the necessary diversity to locate new optima once the environment changes.

Many approaches to adapting traditional evolutionary algorithms to dynamic environments are available in the literature, but differential evolution (DE) has been investigated as a base algorithm by only a few researchers. This thesis reports on adaptations of existing DE-based optimisation algorithms for dynamic environments.

A novel approach, which evolves DE sub-populations based on performance in order to discover optima in an dynamic environment earlier, is proposed. It is shown that this approach reduces the average error in a wide range of benchmark instances. A second approach, which is shown to improve the location of individual optima in the search space, is combined with the first approach to form a new DE-based algorithm for dynamic optimisation problems.

The algorithm is further adapted to dynamically spawn and remove sub-populations, which is shown to be an effective strategy on benchmark problems where the number of optima is unknown or fluctuates over time.

Finally, approaches to self-adapting DE control parameters are incorporated into the newly created algorithms. Experimental evidence is presented to show that, apart from reducing the number of parameters to fine-tune, a benefit in terms of lower error values is found when employing self-adaptive control parameters.

**Keywords:** Differential evolution, dynamic environments, competing populations, self-adaptive control parameters, dynamic number of populations, moving peaks.

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