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A DISAGGREGATED MARSHALLIAN MACROECONOMETRIC MODEL OF SOUTH AFRICA

by

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SUMMARY

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Abstract

The thesis enticingly describes a synergetic mix of productivity related topics at macroeconomic level. It aims at whetting potential readers to understand in more insightful ways topics such as: (1) the use of human capital in sectoral growth; (2) the role played by rising public expenditures (health and education) in strengthening production activities; (3) the role played by disaggregation in improving models' forecasting ability and policy guidance; etc. The current research constitutes a valuable tool for understanding and predicting a country's overall economic behavior and the behavior of important industrial sectors.

In the present study, lack of data on important variables at sectoral level led to the use of advanced econometric estimation methods such as the implied transfer function equations system. As cited in the thesis, the literature reports a set of interesting economic investigations in this field that have been successful in describing some of the features included in this study. However, this research not only enhances the theoretical discussion on the issue but also provides empirical evidence using South African data. It is anticipated that further use and development of the outcomes of this thesis will yield additional explanatory, predictive and policy-making results that will be useful to many. In addition to the usefulness of this thesis' contribution to the body of knowledge, several suggestions for further improvement are considered.

Most predominantly, the work presented in this thesis has been reported in two interrelated papers (chapters). In the first paper, a methodical discussion is provided on the use and the size of social ingredients estimated as the level of normalized human capital per capita together with the conditional convergence process applied to South African sectoral growth. In the second paper, the parameters obtained are embodied into a full-fledged Macroeconometric (Marshallian) Model employing South African economic sectors. In fact, the second paper goes beyond the simple discussion of a Disaggregated Macroeconometric Model. It provides a comprehensive analysis of the effects that freedom (Thatcher-like) reforms may induce to the South African economy.

ACKNOWLEDGMENTS

The completion of the present thesis was made possible by an everlasting ‘divine providence’ that was materialised all through the support (academic and psychological) of several individuals to whom I remain mostly grateful. I extend a deep sense of gratitude toward Prof. Charlotte du Toit as well as Prof. Reneé van Eyden for their supervision that was conducted with utmost dexterity and high professionalism. I also extend words of gratitude to Professor Arnold Zellner who hosted me at the University of Chicago where most of this thesis was completed and presented.

Additionally my gratitude is extended toward my near and dear ones. Let my late parents in law find respect and honour throughout this work that received tremendous moral support from my wife, Marie. Let this thesis be considered as the crowning of my parents’ (including my uncle Benjamin) support throughout my entire life. My elder son JOVIC and his brother Daniel with the entire circle of cousins (including Hellene) and friends will not be ignored as their presence always made my leisure time more inspiring.

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LIST OF ABBREVIATIONS

AIDS	Acquired immune deficiency syndrome
ARIMA	Autoregressive Integrated Moving Average
ARLI	Autoregressive leading indicator
ARV	Antiretroviral
CGE	Computable General Equilibrium
DMC	Direct Monte Carlo Simulations
DSGE	Dynamic Stochastic General Equilibrium
EBMF	Excel-based model for forecasting
ECA	Economic Commission for Africa
EL	Effective labour per sector
GDP	Gross Domestic Product
GLS	Generalised Least Square
HIV	Human immunodeficiency virus
IID	Identically Independently Distributed
IMF	International Monetary Fund
ISUR	Iterative Seemingly Unrelated Regressions
MAE	Mean Absolute Error
MCMC	Markov Chain Monte Carlo Simulations
MDG	Millennium Development Goal
MLE	Maximum Likelihood Estimator
MMM	Marshallian Macroeconometric Model
MMM-DA	Marshallian Macroeconometric Model (Disaggregated)
RFE-DA	Reduced-Form Equations disaggregated by sector
RMMM-DA	Restricted Marshallian Macroeconometric Model (Disaggregated)
RMSE	Root Mean Squared Error
RMSM	Revised Minimum Standard Models
SADC	South African Development Community
SARB	South African Reserve Bank
SSA	Sub-Saharan Africa
SSA	Statistics South Africa
SUR	Seemingly Unrelated Regression

TFP	Total Factor Productivity
UN	United Nations
VAR	Vector Autoregression

CHAPTER 1: GENERAL INTRODUCTION

1.1 INTRODUCTION

In many ways, macroeconometric modelling has provided crucial support to decision-making units at both national and international level. The prediction ability of reliable macroeconomic models has helped to improve the performance of world economies over the years. The building of advanced structural dynamic equation models tested on past data for forecasting experiments has improved the reliability of macroeconomic modelling. More policy makers are making use of such models while most donors rely on the output of these models when it comes to making decisions on fund allocation. In addition, credit allocated to progressive macroeconomic modelling has largely been enhanced by the introduction of social ingredients such as health and education.

The understanding of the macroeconomic returns on human capital has stimulated larger interest from both academics and donors. On many occasions, the two groups have been called to work on common grounds to bring helpful solutions to the issue. However, several aspects of the dilemma remain unattended. Regions of the world that portray the poorest macroeconomic progress are the most vulnerable to human calamities. This constitutes a heavier burden placed on national budgets. The South-to-North dependency relationship remains pertinent as far as education and/or health is concerned. The inadequate health and education systems found in the poor regions of the world (mainly Sub-Saharan Africa) have largely contributed to the widening of the technological gap among developed countries and many of the less fortunate underdeveloped economies. This issue has been identified as part of the world's developmental priorities on several occasions (see Millennium Development Goals, 2000)[1]. Nevertheless, for several economies where lack of good governance has eroded the efficiency of donors' actions, more drastic solutions are needed.

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[1] The **Millennium Development Goals** comprise of eight goals that were established by 189 UN (United Nations) member states during the Millennium Summit held in 2000. The aim is to achieve the goals by the year 2015.

On many occasions, budget restrictions have forced policy makers to prioritise funding requirements using the criteria of return on investment. Development opportunities (sectors or areas) that have been identified as more productive (higher and faster return) have received larger consideration than others.

1.2 PROBLEM STATEMENT AND OBJECTIVES OF THE STUDY

This thesis investigates the prediction ability of a Marshallian Macroeconometric Model Disaggregated (MMM-DA) by economic sectors, accounting for health and education through a coefficient of 'labour effectiveness'. This coefficient of effectiveness[2] as well as the country's technological diffusion process are thoroughly analysed in the second chapter of this thesis using complementary econometric techniques such as: (1) Iterative Seemingly Unrelated Regressions (ISUR); (2) the Fixed Effects Model; and (3) the Holt Winter Exponential Filter. As part of its findings, the second chapter suggests that the use of effective labour does not reduce the model's prediction ability. It rather renders the model more flexible, making it more accommodating for social impediments.

The core model presented in this work constitutes a major contribution to both the South African modelling literature and the existing literature on MMM-DA. At present, a MMM-DA has never been built for the South African economy. Also, the original version of the model did not include: (1) social ingredients; (2) an entry cost; and (3) the foreign sector. However, all these components have been successfully incorporated in the MMM-DA in the third chapter of this thesis. Additionally, this chapter suggests that, when carefully implemented, freedom (Thatcher-like) reforms might uplift overall South African economic growth to as high as 5.1 percent (for a 1 percent shock) and 8.1 percent (for a 10 percent shock). These figures are obtained within a reasonable margin of error (1.28). The Thatcher-like policy experiments conducted mainly focus on: (1) the entry cost (to promote a free market); (2) tax cuts (to raise consumers' power parity); and (3) improving the level of human capital. The modelling approach developed in chapter 3 is supported by a comparative analysis

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[2] The level of human capital per capita modelled with health and education as explanatory variables.

between: (1) the Autoregressive Leading Indicators Model of order three (benchmark model); (2) the MMM-DA using ISUR without shrinkage; (3) the MMM-DA using ISUR with shrinkage; and (4) the MMM-DA run using Markov Chain Monte Carlo Simulations (MCMC) and Direct Monte Carlo Simulations. Enough evidence could be garnered to advocate the use of the MMM-DA as a tool in improving forecasting models.

1.3 SIGNIFICANCE OF THE STUDY

As it was highlighted earlier, the flexibility and the comprehensiveness of the present thesis in terms of: (1) macroeconomic returns on human capital; and (2) improved forecasting ability of the model through sectoral disaggregation and entry/exit of firms; contribute in providing accurate guidance to policy makers (South African in particular). The modelling exercise described in this thesis has been used to assess several types of reforms (including reforms on human capital) and their applicability to the South African economy.

As it has been shown, and emphasised strongly for many years by Richard Stone, Guy Orcutt, Milton Friedman, Franco Modigliani, and many others, disaggregating wisely and using reliable disaggregated data can lead to significantly improved explanations and predictions. For some recent empirical examples and references, see Zellner and Israilevich (2005). Note that with disaggregation, one not only avoids the negative effects of "aggregation biases" of the sort that is emphasised in the literature, but one also gains extra precision in estimation and prediction. Added precision is gained not only in predicting outcomes for individual sectors, which is impossible with aggregate data, but also extra precision in predicting "totals" or macroeconomic variables as shown theoretically and empirically in some more recent papers, with or without shrinkage (Stein, 1962). These views are supported by De Alba (1991) and others. The rigor undertaken in assessing the forecasting ability of the models used in this thesis will reassure the reader concerning their reliability.

1.4 OUTLINE OF THE REST OF THE STUDY

An outline of the study is as follows. In chapter 2, the functioning and the impact size of social ingredients, essentially health and education, at sectoral level is investigated using South African data. Additionally, the chapter analyses the country's conditional convergence process for the selected sectors. Afterwards, the MMM-DA model and its variants are developed in chapter 3. This chapter presents (1) all the estimation techniques used, (2) the fit and the predictive performance of the MMM-DA as compared to its benchmark autoregressive leading indicator (ARLI) model, and (3) an evaluation of the (Thatcher-like) Freedom Reforms' effects on the growth rate of the South African economy using our MMM-DA. Finally, in the concluding chapter (chapter 4), the results are summarized and direction for future work is indicated. The more technical algebra related to the models together with additional figures and tables of estimates can be found in the different appendices. The last two sections of the appendix section briefly describe (1) the market dynamics for medical products, and (2) the life cycle utility maximization under the probability of death between periods.

CHAPTER 2: SOCIAL INGREDIENTS AND CONDITIONAL CONVERGENCE IN THE STUDY OF SECTORAL GROWTH

2.1 INTRODUCTION

The importance of health as a component of human capital has captivated the attention of several researchers in macroeconomics as well as policy makers. Developments in the world economy are closely linked to health related predicaments. The labour force through its productivity sees its contribution to economic growth enhanced by human factors such as: the workers' endurance and capacities (mental or physical); the workers' aptitude to make use of their reasoning ability; the workers devotion to delivering efficiently on time; etc. (Canning & Bloom, 2005). The design of any valid macroeconomic policy cannot be performed without inclusion of a health component. Health and education too, might be considered as human capital determinants. Although health on its own constitutes an important ingredient in any growth or development study, both require a particular consideration, especially with regard to their effects on effective labour.

The disaggregating approach used in this study helps with comparison of the effects of increased investment in health or schooling at the sectoral and national levels. One cannot disregard the fact that a healthier worker with higher educational background and more experience is usually more productive. Therefore, the use of physical labour force features while ignoring the effectiveness aspect is no longer sufficient in explaining the production setting. This study acknowledges the fact that technological components also have a labour-related contribution. The coefficient of effectiveness used in this study implicitly includes the level of health and schooling investment per worker as well as the level of experience. It remains plausible that other labour augmentation factors have been omitted in this analysis. Nevertheless, useful outcomes can be extracted.

The importance of health in macroeconomic models is much more perceptible in the developing world where the majority of economies are labour intensive. A stronger level of labour effectiveness will tend to give rise to higher economic growth and *vice* *versa*. Further evidence of these effects has been garnered using microeconomic approaches (Strauss & Thomas, 1998).

The objective of this chapter is to show that the development of a full-fledged macroeconomic model employing effective labour is a valid exercise and that it is much more informative than traditional macroeconomic models. Secondly, the chapter aims to present evidence that the outcomes of investment in health and/or schooling differ according to the sector targeted. Parameter estimates for South African economic sectors are discussed and, under very specific assumptions, the model can be regarded as representative of other African economies. Accordingly, this chapter incorporates an analysis of the technological diffusion process in South African growth sectors.

Health has often been measured in terms of life expectancy. From an expenditure perspective, per capita (or per worker) health expenditure can also be used as an indicator of health when data on life expectancy are unavailable. The same measure, *i.e.* using expenditures as a proxy, is applicable for schooling. The pathways investigated in this chapter are plausible in explaining the macroeconomic effect of health and schooling in the South African economy, although, data restrictions impose limitations on the study. Consequently, analysis could only be performed on five sectors, namely: (1) Agriculture; (2) Mining; (3) Construction; (4) Transport and Communications; and (5) Manufacturing; spanning the period from 1995 to 2006.

2.2 BACKGROUND

As mentioned earlier, social ingredients, which appear in various forms in the growth literature, have a relatively rich history. They underscore most of economic thinking on the issue. Health and education are among the most important social ingredients referred to in macroeconomic studies. As mentioned earlier, in most of the references studied, health is presented in the form of life expectancy while a weighted average of total years of schooling is used as a proxy for education.

The use of effective labour, defined in terms of social variables, has produced interesting outcomes in terms of policy analysis. A study conducted by Fogel (1994) provides evidence that a large part of British economic growth in the 1970s was the result of a larger volume of effective labour inputs. Effective labour input was associated with workers with improved health and sufficient nutrition. Very similar results were obtained for the Korean economy where improved nutrition caused available labour input to rise by one percent for the period 1962 to 1995 (Sohn, 2000).

The effects of health improvement on economic growth follow different channels that converge towards income growth (Bloom *et al.* 2000, 2003, 2004). Investment in human capital associated with labour market participation and worker productivity have influenced the path of economic growth.

An interesting debate raised around the macroeconomic effects of health is that many regressions run in past studies were unable to indicate whether the coefficients obtained were the true reflection of the direct benefits of health on growth or whether they were just a proxy for other mismeasured variables (Bloom *et al*. 2003). In order to assuage this criticism, Bloom *et al.* included health in a full-fledged production function and conducted several tests to determine the direct effect of health on labour productivity. Their model encompasses multiple dimensions of human capital in an aggregate growth function. The combination of life expectancy and years of schooling used by Bloom *et al.* (2005) in their modelling of a coefficient of effectiveness using a panel of countries (Penn World Tables version 6.0) for the time period 1960 to 1995 remains a major contribution to the macroeconomics of health. A few questions could be raised with regard to the assumption that the coefficient of effectiveness equals one whenever life expectancy and years of schooling simultaneously equal zero. In this regard it is important to highlight the fact that the two parameters are specified as summing to unity. When health (life expectancy) equals zero the coefficient of effectiveness will automatically equal one no matter what value the parameter of schooling takes. In fact, it is hardly conceivable that a workforce unit can increase its effectiveness just by using schooling. In this study, a third factor has been introduced, namely a constant that capture any omitted variable. When a worker has no life expectancy, none of the other factors can improve his effectiveness. However, when a worker has some life expectancy with a certain level of education, his effectiveness will be increased by a higher level of experience. The present research does not involve enough tests which indicate whether or not the impact of experience in the coefficient of effectiveness is mixed up with other mismeasured factors.

It is a difficult exercise to establish whether quantitative evidence of the relationships between education (schooling) and economic growth exists. The majority of studies previously conducted on schooling as a social ingredient to economic growth have made use of variables such as: school enrolment; literacy rates; years of schooling; etc. Schooling implies better skills and higher productivity and a higher level of education in the workforce increases the absorption rate of technology (Barro *et al*., 2000). The interesting question raised in Barro's study relates to the adequacy of these variables in the measurement of the stock of available human capital. The matter is addressed by measuring education levels for a panel of countries conducted on intervals of five years. The research provides relevant findings in terms of advice on how to measure the macroeconomic impact of schooling. The authors make adjustments to cover missing observations using gross school enrolment features capturing the movement from students repeating years. Additionally, the average years of schooling used in the research account for amendments in the total number of years of schooling in the panel. This chapter addresses the issue slightly differently, by taking the public expenditure side into consideration. Both health and schooling are defined in terms of per capita expenditure[3]. It might be unwise to argue that more money spent by the government on schooling or health will directly translate into a larger contribution of these two factors to economic growth. However, once it is assumed that government expenditure is efficient, higher per capita expenditures on health or schooling translates into a greater investment in human capital which can then be expected to generate higher worker productivity. By expenditures on schooling we mean real expenditures per member of the school age population. We could not use a direct measure of average years of schooling of the labour force since this type of data is currently unavailable or hard to access at sectoral level. Addressing the issue from an expenditure point of view eliminates some of the criticisms made towards earlier studies concerning potential bias that could occur in estimating the macroeconomic effects of health (through life expectancy) in countries or sectors with

[3] The use of public expenditures figures on health and education may present some drawbacks. During major health outbreaks, public expenditures on health may be subject to unusual increases. In this study, the use of seasonally adjusted data helps assuage this weakness.

high life expectancy. These countries or sectors tend to have older workforces (the ageing phenomenon) meaning that expenditure on health and schooling do not always translate into greater labour productivity. Nevertheless, older workforces with higher experience are meant to be more productive as long as they remain within the working age. Data constraints have forced us to use expenditure measures in lieu of superior direct measures for education and health.

2.3 THE THEORETICAL MODEL

Production functions

In this chapter an economy with *n* sectors operating at time *t*, each with a Cobb-Douglas production specification (Zellner, 2003) is assumed:

$$Q_{it} = A_{Ni}(z_{it}L_{it})^{\alpha} K_{it}^{\beta} \qquad (1)$$

with: - A_{Ni} :Neutral technological change factor in sector i;

- z_{it} :Labour augmentation factor reflecting changes in labour quality (level of human capital in per capita terms).

From the existing literature (see Bloom *et al.*), the coefficient of effectiveness is developed through the following equations:

Effective wage: $w_e = zw$ (2)

Labour effectiveness[4]: $z = e^{\gamma s + \delta h}$ (3)

Aggregate level of human capital: $Z = \sum_i e^{\gamma s_i + \delta h_i}$ (4)

Log of aggregate level of human capital: $\ln Z = \sum_i (\gamma s_i + \delta h_i)/L + \sigma^2/2$ (5)

New logged aggregate production: $\ln Q = a + \alpha(\ln gL + \gamma s + \delta h) + \beta \ln K$ (6)

By logging the production function while including the z function, two equations are obtained:

[4] This normalisation of the effective labour unit to one with zero *s* (per capita expenditure on schooling) and zero *h* (per capita expenditure on health) has been borrowed from Bloom *et al.* (2005) However, the use of a constant or a time trend improves the definition of estimation or calibration of *z*.

$$\ln Q_{it} = \ln A_{N\,i} + \alpha \ln z_{it} + \alpha \ln L_{it} + \beta \ln K_{it} \tag{7}$$

$$\ln Q_{it} = \ln A_{N\,i} + \alpha\gamma s_t + \alpha\delta h_t + \alpha c_i + \alpha \ln L_{it} + \beta \ln K_{it} \,. \tag{8}$$

In (5), σ represents the standard deviation of the log of wages (lnw). The growth accounting equation is obtained by differentiating both sides of equation (8) with respect to time. In this regard, this section provides a theoretical discussion of two variants of the problem (see sub-sections 2.3.1 and 2.3.2). Another plausible option (see sub-section 2.3.3), relies on the fact that workforces go through a process of recruitment before they form part of a specific industrial sector. Should the given sector decide to recruit a worker, a minimum level of investment on health and education should be observed in the individual. In other words, at recruitment, the worker is expected to have a certain level of education while being in good health. Therefore the model specification can be written as follows:

$$z_{it} = e^{\gamma(s_t - s_o) + \delta(h_t - h_o) + c_t}, \tag{9}$$

where s_o and h_o are the minimum levels of money invested in schooling (education) and health per unit of workforce, respectively. The more money is invested in the worker in terms of health and education, the more productive the worker will be. It is a delicate exercise to find the threshold in terms of basic requirements per industrial sectors (s_o and h_o).

2.3.1 A general approach without specific disentangling of A_{iN} (assumed to be constant over time)

$$\dot{Q}_{it}/Q_{it} = \alpha\gamma \dot{s}_t + \alpha\delta \dot{h}_t + \alpha \dot{L}_{it}/L_{it} + \beta \dot{K}_{it}/K_{it}\,; \tag{10}$$

$G_Q = \dot{Q}/Q$;

$G_L = \dot{L}/L$; and

$G_K = \dot{K}/K$.

The growth accounting equation can be written as follows:

$$G_Q = \alpha\gamma \dot{s}_t + \alpha\delta \dot{h}_t + \alpha G_L + \beta G_K \text{, and} \tag{11}$$

$$G_Q = \alpha(\gamma \dot{s}_t + \delta \dot{h}_t + G_L) + \beta G_K \,. \tag{12}$$

Assuming an annual increase (*dt* = *1*), increasing investment in human capital, for example health per capita by one monetary unit, will lead to a '$\alpha\delta$' increase in the growth rate of output. Additionally, an increase by one monetary unit of schooling expenditure per capita will cause the growth rate of output to increase by '$\alpha\gamma$'. Using these outcomes, a comparison between the effects of more investment in human capital on the growth rate can be validly made and some policy recommendations in terms of a sectoral scheme of expenditures in both health and schooling can be suggested.

2.3.2 A more specific approach that includes HIV factors which affect z_{it} , assuming that technological factors vary across time

The two HIV-related factors included in this scenario are the death rate and the absenteeism rate due to an advanced stage of the infection. These factors are considered among variables affecting the labour augmentation factor.

$$Q_{it} = A_{Nit}\,[z_{it}(a_{it}; d_{it}; o_{it})L_{it}]^{\alpha}\, K_{it}^{\beta} \tag{13}$$

with:

- a_{it}: Work absenteeism observable in HIV patients;
- d_{it}: Death rate associated to HIV pandemic;
- o_{it}: Other omitted factors linked to labour productivity.

Once again, by logging both sides of equation 13 and deriving it with respect to time, the following growth accounting equation is obtained:

$$\ln Q_{it} = \ln A_{Nit} + \alpha \ln z_{it}(a_{it}; d_{it}; o_{it}) + \alpha \ln L_{it} + \beta \ln K_{it} \,. \tag{14}$$

Thereafter, by deriving the total equation with respect to time, the following is obtained:

$$\frac{\dot{Q}_{it}}{Q_{it}} = \frac{\dot{A}_{Nit}}{A_{Nit}} + \alpha\left[\frac{\partial \ln z_{it}}{\partial a_{it}}.\frac{da_{it}}{dt} + \frac{\partial \ln z_{it}}{\partial d_{it}}.\frac{dd_{it}}{dt} + \frac{\partial \ln z_{it}}{\partial o_{it}}.\frac{do_{it}}{dt}\right] + \alpha\frac{\dot{L}_{it}}{L_{it}} + \beta\frac{\dot{K}_{it}}{K_{it}} \text{ and} \tag{15}$$

$$G_Q = G_{AN} + \alpha\left[\frac{\partial \ln z_{itL}}{\partial a_{it}}.\frac{da_{it}}{dt} + \frac{\partial \ln z_{itL}}{\partial d_{it}}.\frac{dd_{it}}{dt} + \frac{\partial \ln z_{itL}}{\partial o_{it}}.\frac{do_{it}}{dt}\right] + \alpha G_L + \beta G_K \,. \qquad (16)$$

Assuming that ARV (Antiretroviral) policies are well implemented and that both absenteeism and death rates decrease over time, the result will be that the output growth rate in the economy will be strengthened. Alternatively, assuming an increase in *a* and *d* over time, the overall output growth rate will be reduced accordingly. Good health policies in terms of HIV should cause a reduction in both absenteeism and death rates in all economic sectors in turn supporting more sustainable economic growth. Both *a* (absenteeism rate) and *d* (death rate) are assumed to be diminishing over time assuming that ARV policies reduce the magnitude of both *a* and *d*. We assume that the HIV prevalence rate itself follows a sigmoid pattern (see figure 2.1).

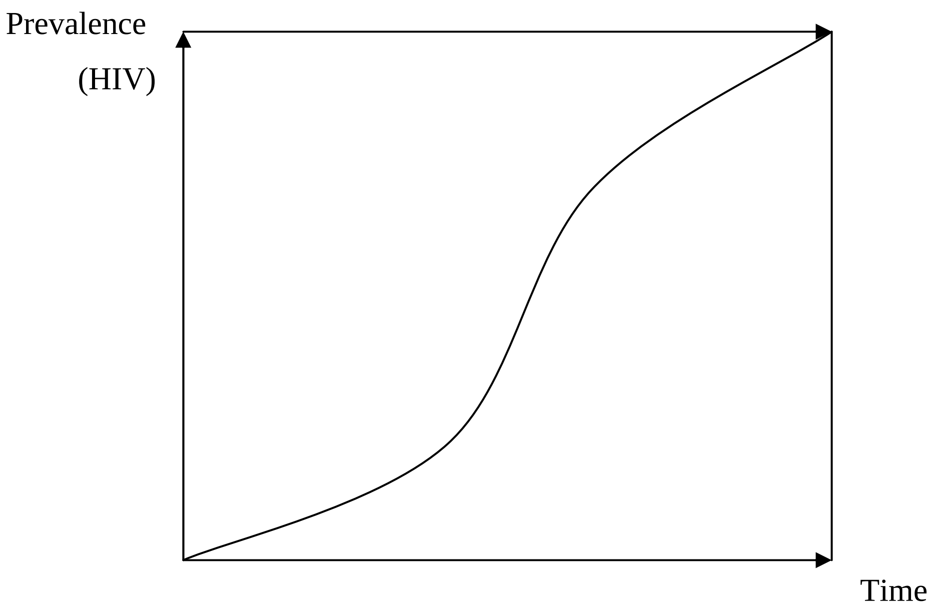

Figure 2.1: HIV Prevalence over time

Considering the sigmoid approach, this issue can be addressed in a slightly different manner. In the African context, policy measures have very little effect on controlling the dynamics of HIV/AIDS. For this reason, referring to related literature, one can depict the production implications of HIV/AIDS through a non-linear function assumed to be logistic. The first stage of HIV prevalence is expected to be exponential. However, as antiretroviral treatment is supplied together with other preventive and counter-cyclical actions, the prevalence decreases and is expected to become completely preventable.

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Herewith the parameters introduced in the labour augmentation factor concerning HIV:

- *h(t):* HIV prevalence rate;
- *a(t):* work absenteeism observable in HIV/AIDS patients;
- *d(t):* death rate associated with HIV/AIDS pandemic.

The following is assumed:

$$h(t) = \varphi_i \frac{1}{1+e^{-t}} = \varphi_i \frac{e^t}{1+e^t}; \tag{17}$$

$$a(t) = \varphi_i \frac{e^{t-\approx{a}}}{1+e^{t-\approx{a}}} \text{ ; and} \tag{18}$$

$$d(t) = \varphi_i \frac{e^{t-\approx{d}}}{1+e^{t-\approx{d}}}; \tag{19}$$

with:

- φ_i : parameter, assumed to be constant[5] over time in the model that captures the link of HIV prevalence with sectoral production;
- $\overset{\approx}{a}$: average period[6] observed for a tested HIV positive individual to develop AIDS symptoms;
- $\overset{\approx}{d}$: average period observed for a tested HIV positive individual to die of AIDS: $\overset{\approx}{d} > \overset{\approx}{a}$.

Absenteeism occurs with a time lag of $\overset{\approx}{a}$ periods relative to the infection stage. In other words, the longer $\overset{\approx}{a}$ is, the larger the gap becomes, and the smaller the negative effects of the pandemic on economic growth. The same applies to the death rate. Death occurs with a time lag of $\overset{\approx}{d}$ periods relative to the infection stage.

Applying the concept of derivatives to this second variant, the relevant information can be derived in a similar way to the first variant discussed earlier.

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[5] This assumption can validly be removed since this parameter is supposed to change over time.
[6] This period could also be assumed as the average across SSA (Sub-Saharan African) countries (3 years) with *t* (time of reference based on the HIV prevalence).

$$
\begin{aligned}
a(t) &= \varphi_i \frac{e^{t-\tilde{\tilde{a}}}}{1+e^{t-\tilde{\tilde{a}}}} \\
a(t) &= \varphi_i \frac{1}{1+e^{-(t-\tilde{\tilde{a}})}}
\end{aligned} \tag{20}
$$

$$\lim_{\tilde{\tilde{a}} \to \infty} a(t) = \varphi_i .$$

The larger $\tilde{\tilde{a}}$ is, the smaller a becomes (work absenteeism as a function of time observable in an infected patient). An adequate ARV treatment supply or a complete eradication of the infection leads $\tilde{\tilde{a}}$ to tend toward infinity. In fact, infinity in this case only means that the worker will actually never be absent from work because of an HIV infection. Infinity therefore refers to the time of the worker's normal resignation. In other words, an infected patient who receives adequate ARV supply will probably never be absent from work due to HIV infection[7]. Linking this to the growth accounting equation, the negative effect of *a(t)* in A_{iN} will disappear or rather, the derivative of a(t) goes to zero at t equals infinity because absenteeism levels off once it reaches a ceiling. In other words, the effect stops getting worse.

$$\frac{\partial a(t)}{\partial t} = \varphi_i \frac{e^{-(t-\tilde{\tilde{a}})}}{\left[1+e^{-(t-\tilde{\tilde{a}})}\right]^2} = \varphi_i \frac{1}{\left[1+e^{-(t-\tilde{\tilde{a}})}\right] . \left[1+e^{(t-\tilde{\tilde{a}})}\right]} \text{ and} \tag{21}$$

$$\frac{\partial d(t)}{\partial t} = \varphi_i \frac{e^{-(t-\tilde{\tilde{d}})}}{\left[1+e^{-(t-\tilde{\tilde{d}})}\right]^2} = \varphi_i \frac{1}{\left[1+e^{-(t-\tilde{\tilde{d}})}\right] . \left[1+e^{(t-\tilde{\tilde{d}})}\right]} \text{, with} \tag{22}$$

$$\lim_{\tilde{\tilde{a}} \to \infty} \frac{\partial a(t)}{\partial t} = 0 \text{ and } \lim_{\tilde{\tilde{d}} \to \infty} \frac{\partial d(t)}{\partial t} = 0 .$$

As $\tilde{\tilde{a}}$ increase, *a(t)* across time will be reduced until it reaches 0.

[7] It is important to note that several forms of drug-resistant HIV virus with more rapid mutation exist and therefore traditional ARV treatment can simply not produce the expected results. However, in this study the concern is much more on the commonly known form of the virus.

2.3.3 Each of the two social ingredients includes a minimum level required at recruitment

In this sub-section, we introduce a minimum level required for each worker in terms of health (h_0) and schooling (s_0) prior to enrolment in an industry. This expansion in our reasoning is much more in compliance with job market realities. Each industry has particular set of requirements that need to be met by workers prior to their employment.

$$z_{it} = e^{\gamma(s_t - s_o) + \delta(h_t - h_o) + c_t}, \text{ and} \quad (23)$$

$$\ln Q_{it} = \ln A_{Ni} + \alpha\left[\gamma(s_t - s_o) + \delta(h_t - h_o) + c_t\right] + \alpha \ln L_{it} + \beta \ln K_{it}. \quad (24)$$

The growth accounting equation is redefined as follows:

$$\dot{Q}_{it}/Q_{it} = \dot{A}_{Ni}/A_{Ni} + \alpha\gamma\dot{\bar{s}}_t + \alpha\delta\dot{\bar{h}}_t + \alpha\dot{L}_{it}/L_{it} + \beta\dot{K}_{it}/K_{it}, \quad (25)$$

with: $\bar{s}_t = s_t - s_o$;

$\bar{h}_t = h_t - h_o$; and

$$G_Q = G_A + \alpha\gamma\dot{\bar{s}}_t + \alpha\delta\dot{\bar{h}}_t + \alpha G_L + \beta G_K. \quad (26)$$

Using this form of the growth accounting equation it is understandable that, whenever recruitment criteria are tightened, the per capita expenditures need to be increased as well, otherwise a negative effect on growth will be observed.

2.3.4 Considering the diffusion process

Using the diffusion[8] process (Bloom, Canning & Sevilla, 2002b) across sectors, the following equation is introduced:

$$\Delta A_{it} = \lambda(\bar{A}_{it} - A_{i,t-1}) + \varepsilon_{it}. \quad (27)$$

Each sector has a 'ceiling' level given by $\bar{A}_{it}$. Recall that A_{it} represents the level of technological factor productivity of country *i* in period *t*. The sector adjusts toward

[8] From a broad perspective, the diffusion process can be referred to as the outcome (solution) of a system of stochastic differential equations. Several processes such as 'Brownian Motion' can be referred to as being part of a diffusion process.

this level at a rate λ. λ depends on the sector characteristics and the country's level of technology.

$$\overline{A}_{it} = \delta X_{it} + b_t \text{, with} \tag{28}$$

with b_t : time dummy representing the current level of national TFP (Total Factor Productivity).

By including this specific dummy variable, it is assumed that the convergence of sectoral TFPs is analysed in accordance with a national TFP. For the latter reason, this study makes use of fixed effects models. Note that in this subsection, A_{it}, which accounts for all the components of TFP including labour effectiveness, is used instead of A_{Nit}, which only includes the neutral part.

Lagged technology can be measured by substituting equation 28 into equation 27 so that the following is obtained:

$$\Delta A_{it} = \lambda_i (\delta X_{it} + b_t - A_{i,t-1}) + \varepsilon_{it} \,. \tag{29}$$

with X_{it} : the complete set of sector-specific variables that has an impact on *A*.

The higher λ_i is, the faster the movement towards a complete diffusion process and the lower λ_i, the slower the diffusion process. Complete diffusion is achieved when the difference $\overline{A}_{it} - A_{i,t-1} = 0$. Therefore, if $\Delta A_{it} = \varepsilon_{it}$, technological change will only depend on random shocks. This chapter presents estimated figures for the diffusion factor given the five sectors considered. When the diffusion process is complete the growth equation is presented as follows:

$$\frac{\dot{Q}_{it}}{Q_{it}} = \left(\frac{\varepsilon_{it}}{A_{it}}\right) + \alpha\gamma \dot{s}_t + \alpha\delta \dot{h}_t + \alpha \frac{\dot{L}_{it}}{L_{it}} + \beta \frac{\dot{K}_{it}}{K_{it}} \,. \tag{30}$$

Including equation 29 into the generic growth accounting equation the following is obtained:

$$\frac{\dot{Q}_{it}}{Q_{it}} = \lambda_i \delta\left(\frac{X_{it}}{A_{it}}\right) + \lambda_i\left(\frac{b_t}{A_{it}}\right) - \lambda_i\left(\frac{A_{i,t-1}}{A_{it}}\right) + \left(\frac{\varepsilon_{it}}{A_{it}}\right) + \alpha\gamma \dot{s}_t + \alpha\delta \dot{h}_t + \alpha \frac{\dot{L}_{it}}{L_{it}} + \beta \frac{\dot{K}_{it}}{K_{it}} \,. \tag{31}$$

For a diffusion coefficient λ that tends to zero, the growth equation can be reformulated as follows:

$$\frac{\dot{Q}_{it}}{Q_{it}} = \left(\frac{\varepsilon_{it}}{A_{it}}\right) + \alpha\gamma \dot{s}_t + \alpha\delta \dot{h}_t + \alpha \frac{\dot{L}_{it}}{L_{it}} + \beta \frac{\dot{K}_{it}}{K_{it}} \cdot \tag{32}$$

The two extreme cases seem to present similar evidence when $\Delta A_{it} = \varepsilon_{it}$. This equality holds when either the speed of adjustment toward the ceiling rate is zero, or when the speed of adjustment is very high and $\overline{A}_{it} - A_{i,t-1} = 0$. The difference in the two cases is therefore that when there is no movement towards the ceiling rate, growth will be hindered by slower shares from both growth in labour and growth in capital. Investment in schooling and education together with a higher level of experience will be even more essential to assist the slow speed of adjustment. However, less additional investment in schooling and education as well as experience will be required when the ceiling rate is achieved, assuming a growth rate that is acceptable and sustainable (such as in the case of developed countries). If there is no technological diffusion among sectors, it is observed that TFP differentials persist among sectors. The latter case can be measured using a fixed effect panel data model. Additionally, in the case where TFPs narrow over time because of high technological diffusion, TFP differentials decrease over time.

2.4 THE DATA

In order to investigate the role played by social ingredients in South African sectoral growth and to study the diffusion process in the country's economic sectors, this chapter has made use of secondary (official) data, from the following sources: (1) the South African Reserve Bank (SARB); (2) Statistics South Africa (SSA); and (3) Quantec Research.

The 'output labour ratio' extracted from the SARB series, is the proxy used for 'labour effectiveness' (z). Mainly due to limitations on sectoral employment data, the sample size considered ranges from 1995 to 2006. No reliable data on employment could be located for earlier periods. Other sectoral data such as: capital (with fixed capital stock as the proxy); and output (with gross value-added at basic prices as the proxy); are readily available from official sources. However, data on social ingredients such as s (public expenditures on schooling) and h (public expenditure on health) could not be located at the sectoral level. To overcome this challenge, national estimates have been used here.

Using Cobb-Douglas specifications for the sectoral growth equations, it is simple to compute a TFP (Total Factor Productivity) series from the ISUR regressions estimations.

2.5 EMPIRICAL RESULTS

2.5.1 Sectoral output equations: Testing variant 1 in 2.3.1

2.5.1.1 Isolated regressions per sectors

Before proceeding to an ISUR specification, it is important to present the long-run estimates of labour effectiveness (equation 3). In this regression, labour effectiveness (z) is the dependent variable while h as well as a dummy for s (DUMEDU) are the explanatory variables.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
s*DUMEDU[9]	0.000230	5.14E-05	4.483760	0.0001
hL	0.000392	3.86E-05	10.14534	0.0000
C	-0.620592	0.020738	-29.92478	0.0000

R-squared	0.974292	Mean dependent var	-0.309726
Adjusted R-squared	0.972634	S.D. dependent var	0.126168
S.E. of regression	0.020872	Akaike info criterion	-4.816746
Sum squared resid	0.013504	Schwarz criterion	-4.682067
Log likelihood	84.88468	F-statistic	587.4282
Durbin-Watson stat	0.644542	Prob(F-statistic)	0.000000

Table 2.1a: The long-run estimate with z as dependent variable

The long-run estimate presents coefficients with theoretically acceptable signs and magnitudes.

[9] s*DUMEDU is a variable used to remove the regression time periods that are affected by unexplained structural breaks.

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.221096	0.0279
Test critical values:	1% level	-3.653730	
	5% level	-2.957110	
	10% level	-2.617434	

*MacKinnon (1996) one-sided p-values.

Table 2.1b: Cointegration test of the long-run estimate

From table 2.1b cointegration (at least at the 5 percent level) from the long-run regression can be observed.

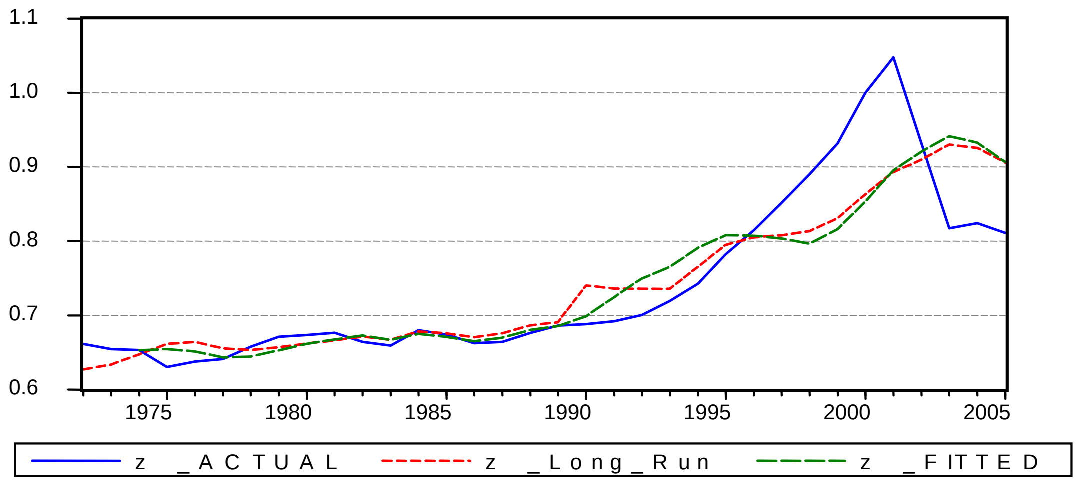

Figure 2.2: Actual and fitted values for Effective Labour, z

From this graph, it is observable that the actual series, as presented by the SARB, contains several irregularities and therefore misrepresents the country's labour productivity. The problem seems to be aggravated from the year 1996 onward. This is mainly due to miscalculation errors caused by the South African Reserve Bank that does not account for workers from the illegal sector. This tends to erroneously inflate productivity figures especially after the end of apartheid.

Using the fitted values for labour effectiveness z , as described in table 2.1 and in figure 2.2 [10], a new series of effective labour per sector (EL or zL) has been computed and the computed series are included in sectoral growth equations. These estimation results are included in the Appendix to Chapter 2 (from table 2.7 to table 2.21). With the exception of 'agriculture', all other sectors portray highly reliable estimation results. The fact that accurate results for the agricultural sector cannot be obtained might be caused by poor data and/or other factors in this sector. For this reason, the final parameters obtained for this sector are not included in the summary table.

For each sector, a long-run cointegration equation is obtained. By making a graphical comparison between the fitted series and the actual series (from figure 2.9 to figure 2.18), it can be seen how interesting and reliable these estimations are. In fact, this exercise has produced insightful outcomes in terms of the impact of changes in expenditures on health or education on sectoral growth. The modelling process used does not enforce any specific input-output scaling. Regressions are conducted on the basis of varying returns to scale and the input shares estimated here are supported by the underlying theories. There is a major weakness in conducting sectoral regressions individually and in isolation from others. That approach ignores the 'cross-sectoral' effects that exist in every economy. For this reason the ISUR approach is conducted in order to obtain parameters.

2.5.1.2 A cross-section ISUR model

This sub-section contains our estimation results using the 'Iterative Seemingly Unrelated Regressions' (ISUR) model. This modelling exercise presents the advantage of providing GLS (Generalised Least Squares) estimates through a correction of contemporaneous correlation and any type of heteroskedasticity related to the cross-sections. Iterative SUR can either be utilised under the form of purely 'cross-section SUR' or 'period SUR'. The 'period SUR' contains the major advantage of correcting for heteroskedasticity related to the period and it also corrects for correlation within cross sections. In this research, a 'period SUR' could not be performed because the number of pool cross-sections (5) does not exceed the number of periods (12). In fact, by using ISUR, a set of sectoral growth equations, allowing

[10] Since the series used as explanatory variables of z (*s* and *h*) are only available at national level, the coefficient of labour effectiveness (z) constitutes a national estimate. However, including it into sectoral growth regressions has helped to produce the disaggregated impacts of schooling and health for each sector considered.

for different coefficient vectors, have been estimated. As mentioned earlier, ISURs have the advantage of capturing efficiency observed due to the correlation of cross-section disturbances.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
_AGRIC--LNK_AGRIC	-2.641124	0.994132	-2.656714	0.0106
_MAN--LNK_MAN	0.653484	0.051183	12.76773	0.0000
_MIN--LNK_MIN	0.839629	0.035988	23.33086	0.0000
_COMTRS--LNK_COMTRS	1.391167	0.201518	6.903428	0.0000
_CONSTR--LNK_CONSTR	0.683151	0.034240	19.95200	0.0000
_AGRIC--LNEL_AGRIC	2.957131	0.829575	3.564634	0.0008
_MAN--LNEL_MAN	0.332578	0.056233	5.914308	0.0000
_MIN--LNEL_MIN	0.079076	0.033303	2.374452	0.0215
_COMTRS--LNEL_COMTRS	-0.524691	0.210324	-2.494678	0.0160
_CONSTR--LNEL_CONSTR	0.292972	0.025089	11.67714	0.0000
Weighted Statistics				
R-squared	0.999995	Mean dependent var		128.4121
Adjusted R-squared	0.999994	S.D. dependent var		437.1014
S.E. of regression	1.067537	Sum squared resid		56.98173
F-statistic	1099021.	Durbin-Watson stat		1.260427

Table 2.2a: Estimates of sectors' production functions using cross-section SUR (no constant)

Tables 2.2 (a, b and c) present the results of a cross-section ISUR with output (gross value added) per sector as dependent variable. In both regressions (table 2.2a and 2.2b) EL represents the effective labour series obtained by multiplying our estimated z (per capita level of human capital) by L (number of workers). With the exception of agriculture, all sectoral growth regressions are well behaved and the levels of significance of the obtained coefficients do not differ much from expectations. In table 2.2a, the ISUR is run without a constant while table 2.2b includes a constant. The use of a constant term for each cross section (table 2.2b) produces an improvement to the estimations mainly by correcting for the negative sign obtained for 'Agric-lnK' (the natural logarithm of capital in the agricultural sector). Table 2.2a does not account for heterogeneity amongst the sectors, i.e. individual effects caused by variables not included as explanatory variables will not be captured.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
_AGRIC--LNK_AGRIC	3.085694	2.109151	1.463003	0.1504
_MAN--LNK_MAN	0.813206	0.420156	1.935487	0.0592
_MIN--LNK_MIN	0.858043	0.153644	5.584618	0.0000
_COMTRS--LNK_COMTRS	2.911776	0.138921	20.95986	0.0000
_CONSTR--LNK_CONSTR	0.689436	0.039532	17.43978	0.0000
_AGRIC--LNEL_AGRIC	22.48967	8.031474	2.800192	0.0075
_MAN--LNEL_MAN	0.346627	0.101458	3.416439	0.0014
_MIN--LNEL_MIN	0.091185	0.036649	2.488031	0.0166
_COMTRS--LNEL_COMTRS	-0.039257	0.053427	-0.734781	0.4663
_CONSTR--LNEL_CONSTR	0.343484	0.079474	4.321955	0.0001
_AGRIC--C	-328.4704	129.8093	-2.530408	0.0150
_MAN--C	-2.162191	4.338682	-0.498352	0.6207
_MIN--C	-0.377566	1.932149	-0.195412	0.8459
_COMTRS--C	-25.31996	1.812443	-13.97007	0.0000
_CONSTR--C	-0.699835	1.040588	-0.672538	0.5047
	Weighted Statistics			
R-squared	0.999997	Mean dependent var		265.2649
Adjusted R-squared	0.999996	S.D. dependent var		573.8779
S.E. of regression	1.090217	Sum squared resid		53.48580
F-statistic	1167712.	Durbin-Watson stat		1.549311
Prob(F-statistic)	0.000000			

Table 2.2b: Estimates of sectors' production functions using cross-section SUR (with constant)

As mentioned earlier, estimates from 'agriculture' do not always meet theoretical expectations. For this reason another set of ISURs which exclude agriculture have been run (table 2.2c). The results show great improvement with the exception of effective labour (EL) for 'communication and transport' (COMTRS).

Variable	Coefficient	Std. Error	t-Statistic	Prob.
_MAN--LNK_MAN	0.758977	0.443535	1.711201	0.0956
_MIN--LNK_MIN	0.899211	0.157137	5.722449	0.0000
_COMTRS--LNK_COMTRS	2.918442	0.136757	21.34036	0.0000
_CONSTR--LNK_CONSTR	0.685548	0.041674	16.45008	0.0000
_MAN--LNEL_MAN	0.359985	0.107256	3.356303	0.0019
_MIN--LNEL_MIN	0.112361	0.038110	2.948304	0.0056
_COMTRS--LNEL_COMTRS	-0.060607	0.053886	-1.124713	0.2682
_CONSTR--LNEL_CONSTR	0.283971	0.088989	3.191078	0.0029
_MAN--C	-1.634881	4.554426	-0.358965	0.7217
_MIN--C	-1.145281	1.947750	-0.588002	0.5602
_COMTRS--C	-25.14415	1.764504	-14.24999	0.0000
_CONSTR--C	0.091927	1.181993	0.077773	0.9384

Weighted Statistics

R-squared	0.999996	Mean dependent var	323.3420
Adjusted R-squared	0.999994	S.D. dependent var	455.5017

Table 2.2c: Estimates of sectors' production functions using cross-section SUR (with constant excluding Agricultural sector)

2.5.2 The effects of an increase in *h* and *s* on the sectoral growth rates

This sub-section provides a thorough discussion of the size and economic meaning of the parameters $\alpha\gamma$ and $\alpha\delta$. These parameters represent the effects of increasing *h* or *s* on sectoral output growth rates. The calculation of these parameters is based on the following regression:

$$\dot{Q}_{it}/Q_{it} = \dot{A}_{itN}/A_{itN} + \left[\frac{\partial \ln z_{it}}{\partial T_{it}}.\frac{dT_{it}}{dt} + \frac{\partial \ln z_{it}}{\partial a_{it}}.\frac{da_{it}}{dt} + \frac{\partial \ln z_{it}}{\partial d_{it}}.\frac{dd_{it}}{dt} + \frac{\partial \ln z_{it}}{\partial o_{it}}.\frac{do_{it}}{dt}\right] +$$

$$\alpha\gamma\,\dot{s}_t + \alpha\delta\,\dot{h}_t + \alpha\,\dot{L}_{it}/L_{it} + \beta\,\dot{K}_{it}/K_{it}\,,$$

With: $\dot{h}_t = \frac{dh(t)}{dt} = \frac{h_{t+1} - h_t}{dt}$; and $\frac{\dot{Q}_{it}}{Q_{it}} = \frac{(Q_{it+1} - Q_{it})/dt}{Q_{it}}$.

Considering a one-year ahead forecast, an increase in h_{t+1} by one Rand will lead to a '$\alpha\delta$' increase of $\frac{\dot{Q}_{it}}{Q_{it}}$. Similarly a one Rand increase in s_{t+1} leads to a $\alpha\gamma$ increase in

$\frac{\dot{Q}_{it}}{Q_{it}}$. Using estimates from this regression analysis the size of $\alpha\gamma$ and $\alpha\delta$ in percentage for each of the 4 sectors has been calculated (see table 2.3).

Variable	Mining	Construction	Transport & Communication	Manufacturing
Health	0.023 %	0.08566 %	0.00155 %	0.08657 %
Schooling	0.00422 %	0.01549 %	0.00028 %	0.015657 %

Table 2.3: The size of the calculated parameters of health ($\alpha\delta$) and schooling ($\alpha\gamma$) on sectoral output growth

From table 2.3, several observations may be drawn. A one-Rand increase on health expenditures or schooling expenditures (per capita) in both sectors leads to higher output growth. The size of this rise differs from one sector to another. The manufacturing sector together with the construction sector has the highest parameters followed by mining and finally transport and communication. The size of these parameters depends on the role played by effective labour variables in the sectoral growth equations. The contribution that a one-Rand increase in per capita expenditures, at sectoral level, has on aggregate growth depends on the size of the sector's contribution to national economic growth. In the fourth quarter of 2006, South Africa reached an economic growth rate of 5.6 percent with several major contributors: the manufacturing industry which contributed 1.4 percent; the finance, real estate and business services industry which contributed 1.0 percent; the wholesale trade, hotels and restaurant industry which contributed 0.8 percent; the storage and communication industry which contributed 0.5 percent (SARB Quarterly Bulletin). When expenditures on health are categorised according to productive sectors, the return on national growth is much higher than an aggregate increase which disregards sectoral differences.

Additionally, lagging h and s will most likely improve these results assuming that both health and schooling policies take years before their effects become noticeable in the economy.

2.5.3 Analysis of the technological diffusion process using a fixed effects model

A 'fixed effects model' (table 2.4) is used to determine the size of TFP across sectors over time and to assess the speed of convergence[11]. This information is needed to comment on the diffusion process.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.937235	1.805991	1.072671	0.2895
LNK?	0.494892	0.109752	4.509205	0.0001
LNL?	0.257374	0.107738	2.388886	0.0215
Fixed Effects (Cross)				
_AGRIC--C	-0.851961			
_MAN--C	0.903629			
_MIN--C	-0.157423			
_COMTRS--C	-0.114270			
_CONSTR--C	0.220025			
Fixed Effects (Period)				
1995--C	-0.156320			
1996--C	-0.101010			
1997--C	-0.069780			
1998--C	-0.065816			
1999--C	-0.042143			
2000--C	0.001738			
2001--C	0.013244			
2002--C	0.048587			
2003--C	0.062101			
2004--C	0.093687			
2005--C	0.113865			
2006--C	0.101848			

R-squared	0.992211	Mean dependent var	10.92981
Adjusted R-squared	0.989058	S.D. dependent var	0.740116
Sum squared resid	0.251744	Schwarz criterion	-1.407508
Durbin-Watson stat	0.472393	Prob(F-statistic)	0.000000

Table 2.4: Fixed effects model

[11] Bloom *et al.* (2005) have used a fixed effects model to assess convergence of TFP across sectors.

The values of TFPs are obtained by taking the exponential value of TFP (cross-sectional fixed effects) coupled with TFP (period fixed effects). In fact, a fixed effects specification is found very appropriate in this analysis as it implies the use of orthogonal projections involving the removal of cross-sectional or period-specific means from the dependent variable and exogenous regressors (Baltagi, 2001). This approach indicates that 'demeans' are used in the specific set of regressions performed. The results from the fixed effects model are presented as 'multiple-graph' (see figure 2.3) and are used to assess the overall convergence tendency of the TFP series in the selected South African industrial sectors.

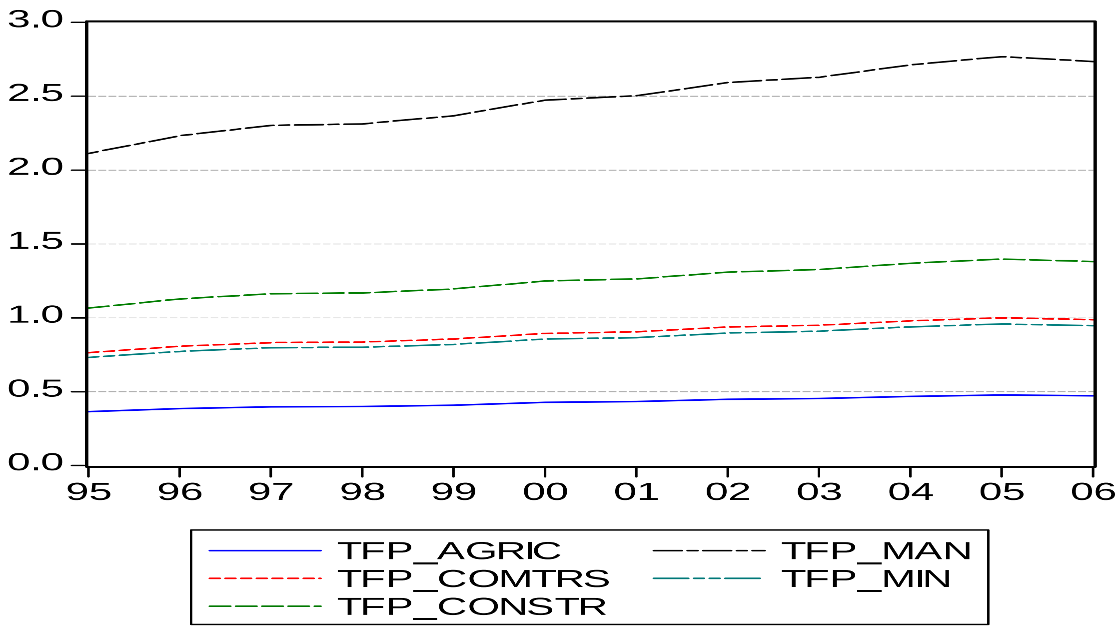

Figure 2.3: TFP across sectors over time

From this graph, very close trends between the Communication & Transport sector and the Mining sector are observed. However, the general view suggests that the speed of adjustment remains very low. It is a fact that sectoral TFPs converge toward a sectoral steady state, although sector differentials remain considerable.

2.6 CONCLUSION

Five sectors were used for the purpose of generating effective labour variables using a coefficient of effectiveness for each sector. Sectoral production functions were estimated using the obtained effective labour series. The data was difficult to obtain due to the lack of a well disaggregated data warehousing system. Nevertheless, the broadest conclusion from the analysis at this stage can only be that it pays to allocate social expenditures according to sectoral productivity and it also pays to include a coefficient of effectiveness in production functions. In many cases it is evident that the use of an effective labour variable does not reduce the predictive ability of the model and that the introduction of this variable opens new channels for "shocking" the model by means of social variables. Additionally, outcomes from the theoretical models can be used with validity to advise policy makers on the harmful effects that the HIV pandemic has on the economic growth. Simply by controlling absenteeism rates and death rates related to the pandemic, the negative impact of the disease can easily be assuaged. However, this chapter does not include other channels through which HIV/AIDS might affect economic growth nor does it consider other types of direct or indirect costs at both the private and national level.

CHAPTER 3: SOME POLICY EXPERIMENTS USING A MARSHALLIAN MACROECONOMETRIC MODEL: THE CASE OF SOUTH AFRICA

3.1 INTRODUCTION

The progress and reliability of the use of forecasting macro models for world economies has been examined in several forums internationally. The scarcity of sound forecasting frameworks weakens budgeting and planning processes in the developing regions of the world when it comes to public investment in human capital. A massive shortfall in the required expertise, combined with the fact that there is no continuously-maintained and centralised data warehousing system, both of which are associated with massive financial requirements, constitute an obstacle to the development of forecasting frameworks. In one of its recent reports, the Economic Commission for Africa (ECA, 2005) depicted the challenges faced by African governments in their modelling exercises.

Forecasting can be perceived as a tool for guiding policy-making units towards achieving long-term goals. South Africa in particular has set very specific developmental goals which are aligned with the Millennium Development Goals (MDGs). These goals are mainly: (1) long-term economic growth (6 – 7%); (2) poverty eradication; and (3) improved health and education for the population; etc. Good projections are linked to the sustainability of socio-economic policies. In line with the MDGs and facing continuous pressure from the International Monetary Fund (IMF), several attempts have been made to build forecasting models and some models have been successfully run. The traditional types of economy-wide models often employed in policy making processes include: (1) the IMF financial programming framework; (2) the World Bank Revised Minimum Standard Models (RMSM); (3) ‘three-gap’ models or ‘two-gap’ models for Africa; (4) Computable General Equilibrium (CGE) Models; (5) dynamic, large-scale models; (5) Project Link; (6) generalised Neo-Keynesian macro models; and (7) Dynamic Stochastic General Equilibrium models (DSGE) (more recently).

There is an extensive use of the two-gap model in African forecasting with regard to the attraction of foreign direct investment needed for economic growth. In fact, the two-gap model constitutes a variant (extension) of the original Harrod-Domar model.

It introduces a second gap (foreign trade gap) in addition to the traditional savings gap and that renders the two-gap model much more relevant in dealing with the countries' import dynamics.

Producing reliable macro models able to describe a country's economy with the aim of evaluating alternative policies has been the major concern of several researchers for many years. Different schools of thought have and are still competing with one another concerning this issue. Some place more emphasis on the key role played by money in the economy (monetarist and neo-monetarist), while others prefer to place more emphasis on the cyclicality of economic systems (real business cycle models and generalised business cycle models). For several years, macro modellers have also made use of the Keynesian principles of economic systems for building models (Keynesian models and Neo-Keynesian models). More recent literature has been enriched by several improvements obtained with regard to the empirical performance of the three-equation New-Keynesian macro model (the Dynamic Stochastic General Equilibrium Model) applied using European panel data. The improvement obtained was generated by the use of real-time information under the Taylor rule[12].

The use of a benchmark to evaluate the performance of any new model is highly advisable. The benchmark models that are most often utilised are the following: (1) ARLI(3) (Autoregressive Leading Indicators of order 3); (2) univariate ARIMA (Autoregressive Integrated Moving Average, Box-Jenkins); and (3) VAR (Vector Autoregressive, Bayesian or Non-Bayesian).

In order to highlight the use of a Marshallian Macroeconometric Model (MMM) as an efficient tool for policy analysis, this chapter makes use of this type of model to predict the impact that Thatcher-like reforms would have on the South African economy, if implemented. However, it is important to dissociate the role played by Margaret Thatcher during her tenure as British Prime Minister, in support of the apartheid regime. In fact, public opinion will recall that, as far back as 1986, Mrs Thatcher together with the Reagan administration engaged in several agreements with the apartheid government while the United Nations tried to impose sanctions against

[12] Paloviita M: 'Estimating a Small DSGE Model under Rational and Measured Expectations: Some Comparisons'. Bank of Finland. Session Paper no. 14/2007.

it. Both Mrs Thatcher and President Reagan acted as supporters of free market policies and therefore vetoed UN sanctions against South Africa. Three years later (1990), she was forced to resign as British Prime Minister and not longer than four years later (1994), South Africa saw its first democratic election. As time went on, Mrs Thatcher somehow revised her radical position toward the South African political crisis and she began to support the idea of negotiations between different parties.

The main focus in this study is the economic reforms that Mrs Thatcher introduced that led to a considerable economic upturn in the United Kingdom. These reforms were initiated by other countries such as Georgia with successful results. There is strong evidence that some of Thatcher's reforms, mainly those concerning labour unions, were inspired by Hutt[13] who is among the pioneers who understood how labour unions can easily turn into major obstacles to workers' prosperity, highlighting clearly the diverging interests of both parties. He did, however, support some form of labour unionisation compatible with the principle of classical liberalism[14].

3.2 BACKGROUND

On the basis of the two-gap model, the 'Bretton Woods' Institutions have developed the 'Revised Minimum Standard Model'. Fund-related models are meant to address balance of payment deficit problems in member states. Several attempts have been made to forecast African growth using the regression approach. However, this approach requires the future values of the exogenous variables, which is a significant weakness. Forecast values of exogenous series must be obtained from a univariate framework or its multivariate counterpart VAR (Vector Autoregression). Countries like Kenya have made use of a VAR model to obtain a period forecast for their exchange rate, while the policy impact of key variables was captured through impulse-response functions. VAR models have been extensively used in many African countries although their outcomes are mostly used for policy evaluation.

[13] WH Hutt: "The Theory of Collective Bargaining", New York, March 1954.
[14] See Baird CW: 'Labour Relations in the 21st Century: Lessons from WH Hutt'. *South African Journal of Economics*, 1998.

AR(3) models including lagged leading indicators have been successfully used in point forecasting frameworks. However, empirical results (Zellner & Tobias, 2000) have shown significant improvement effects of disaggregation in both ARLI (Autoregressive Leading Indicators) as well as Marshallian Models.

The model developed in this chapter is based on sectoral disaggregation including: demand; supply; and entry/exit relations, with sound consideration of labour productivity, as affected by social ingredients such as health and education. A small number of pilot studies on comparative analysis between 'Aggregation and Disaggregation in term of forecasting performance' were located. Zellner and Tobias (1999) published a paper that focused on a comparative analysis between an Aggregated Forecasting Model and Disaggregated Forecasting Model of median growth for eighteen industrialised countries. They made use of MAE and RMSE results to support the hypothesis that 'disaggregation' produces better forecasting outcomes. The aggregated approach used in their paper included median rate variables obtained from all eighteen countries[15]. When employing a disaggregated approach, Zellner and Tobias referred to the same 'Autoregressive Leading Indicators of order 3', ARLI(3) process, while each of the estimates carried two subscripts. One subscript for the country and the other one for the year considered. The disaggregated model allows for all the sectors' equations to be estimated and this provides[16]. Outcomes of Zellner's and Tobias' research paper suggest that disaggregation is more likely to produce better forecasts than 'aggregation', although their disaggregated equations included one aggregated variable: the annual median growth of real GDP. Other evidence of improved forecasting results could be drawn from such comparative studies especially when considering the fact that disaggregation provides more observations to estimates, leading to marginally-better model specification.

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[15] Zellner and Tobias modelled the median growth rate of GDP (aggregated) using an ARLI(3) process that includes three lagged variables of the median growth of GDP together with two other median growth variables: the median growth rate of Real Money; and the median growth rate of Real Stock Prices.

[16] Alternatively, in their disaggregated model, Zellner and Tobias made use of ARLI relationships using the same variables as the ones used in the aggregated model. Firstly, they allowed all coefficients to vary across countries; secondly, they set all coefficients across countries to be equal; and thirdly, they set leading indicators' coefficients equal to one another across countries.

Multiple equation forecasting models involve the use of: (1) the single information estimation technique; (2) limited information system methods (Two Stage Least Squares, Instrumental Variable Estimation, Limited Information Maximum Likelihood); and (3) the full information system technique (Three Stage Least Squares and Full Information Maximum Likelihood); in forecasting frameworks (Challen and Hagger, 1983) using the VAR forecasting approach. The MMM belongs to the group of chaotic models that generate 'booms' and 'busts' as compared to the sine-waves generated by linear models. Its origins lie in Newton's theory of motion, and scientists have in recent times elaborated on their understanding of chaotic models. In fact, in 1975, researchers in the field became aware of this third kind of motion which they called 'chaos' with 'chaotic' being the description for the erratic and quasi-periodic models that are found in several systems.

When discussing the general forecasting literature, as suggested for developing economies, it is important to mention the "Excel-based model for forecasting (EBMF)" developed in 2004 by Huizinga and Alemayehu. The EBMF is based on an AD-AS framework. The model includes sectoral differentials using the CES production function, although the closing of the systems differs from the Marshallian approach and output growth is obtained by aggregating investment consumption, exports and government expenditures, etc.

3.3 OVERVIEW OF INDUSTRIAL SECTORS IN SOUTH AFRICA

This study makes use of a disaggregated model of the South African economy for the purposes of policy analysis. The model is disaggregated into the sectors as outlined in figure 3.1. In this section, a brief overview of South Africa's industrialised sectors is given. South Africa is an economic frontrunner in the African region. Several factor-endowments have helped the nation to be positioned among middle-income countries. However, South Africa remains challenged by a clear division which exists between a world-class economy and a lesser developed economy and which affects the majority of the population through considerable unemployment and high poverty, etc. The developed part of the country's economy benefits from a highly sophisticated financial system (world-renowned stock exchange), a very competitive communication system, and a viable infrastructure among many other attributes.

South Africa has the world's largest reserve of different types of minerals such as: (1) platinum; (2) vanadium; (3) chromium; (4) gold; (5) manganese ores; (6) alumino-silicates; and (7) uranium among others (SADC Review, 2006). The mining sector is not the largest contributor to the country's GDP. It is, however, the largest generator of foreign exchange since mining constitutes an important exporting sector. Sustained production of diamonds, gold, platinum, and coal has significantly enhanced the sector's performance. The rising world demand for mineral products supported by high international prices is also a factor that has contributed to increased growth in recent years.

During years with good weather conditions, South Africa's agriculture has been able to meet local demand with surplus output available for export. Nevertheless, agricultural export is fragile. It is often subject to weather fluctuations as well as international market requirements. The government subsidises the sector by means of a 'profitable' pricing system and technical assistance. Agriculture is backed by a relatively strong research network in the form of the Agricultural Research Council. The country has recorded a sustainable increase in production of different commodities such as: (1) maize; (2) horticulture products; and (3) livestock farming outputs. The country has a vast cultivable surface area helping to promote the sector's growth.

The South African manufacturing sector is a highly promising sector which exhibits a continuous increase in its contribution to the country's growth. It has seen progressive diversification making it more competitive in the international arena. Manufacturing in South Africa makes use of numerous opportunities offered by both the country's climate and a reasonably low level of production cost. The sector comprises different industries that have all faced continuous development such as: metallurgy; chemical industry; agro industry; and electronics; etc. The textile industry is currently facing severe competition and many items that were produced locally are now made in China. The stimulating effect that manufacturing has on other developing sectors is as observable as it is in the textile industry, especially looking at the issue of job creation. The automotive industry, which is one of the fastest growing industries in

South Africa, saw a tremendous rise in car sales for the year 2006 (± 710 000 cars sold locally) with Toyota achieving the highest number of cars ever sold in its history.

Tourism in South Africa constitutes a flourishing industry that generates considerable revenue for the country. The industry has been on the rise for the past years and more than 7 million tourists visited the country in 2006. Tourism generates job opportunities on a large scale and a much larger return from the industry is expected in coming years with the country hosting the 2010 Football World Cup. The organisation of such an event is already producing fruits through impressive investment projects that are currently in operation in the country. South Africa possesses many natural wonders attracting millions of tourists every year.

Expectations on the future might differ for some of the industries considering the current rising trend of the local interest rate which leads to sensibly higher investment costs. Most of these industries benefit from the new openness resulting from the end of the embargo (1991)[17]. Both information and communications technology have seen considerable improvement with many multinational companies manufacturing providing adequate equipment.

The Government is concerned with securing a viable investment environment which promotes competition among business actors. The country has instituted a commission in charge of promoting competition among business agents. Several milestones have been achieved when it comes to moving towards openness. Exchange controls are limited, there are now fewer regulations on foreign investments and easier recruitment of non-national job seekers, etc. The government has implemented many programs to support Research and Development in order to provide a better business environment. It has also become easier to procure the services of foreign firms for provision of technological assistance.

[17] This refers to the embargo imposed on South Africa by the international community (mainly the United Nations) during the apartheid era.

3.4 THE USE OF DISAGGREGATION

The use of a disaggregating process in the MMM developed and used in this study is supported by the sectoral differentials that prevail in the South African economy. The output growth per sector presents disparate behaviour to such an extent that using aggregate data entails the loss of useful information (see figure 3.1). Moreover, aggregate models are unable to analyse detailed policy shocks such as Thatcher-like reforms. Aggregate frameworks suffer from loss of crucial information which leads to inaccurate policy recommendations. A major concern is then raised regarding the veracity, or rather accuracy, of existing forecasts used for policy analysis. If sectoral differentials are not considered, forecasting frameworks remain questionable. Previous studies have improved on the effects of disaggregation as indicated by reduced 'Mean Absolute Errors' (MAEs) and 'Root Mean Squared Errors' (RMSEs). While using disaggregated frameworks, MAEs and RMSEs displayed smaller error figures compared to aggregate models which constitute a noticeable improvement in forecasting performance.

Here it was decided to disaggregate by economic sector as each sector portrays specific characteristics. Although, labour force is most often perceived from an aggregate point of view, labour, capital, and technology function differently from one market to another. In addition, both labour and capital evolve in markets that differ according to sector. Although growth rates may be similar for different sectors, it is important to predict the behaviour of these sectors individually. Marshall emphasised that the process of entry and exit of firms is instrumental in producing long-run equilibrium. Assuming that sectoral error structures are correlated across sectors (see correlation test of the panel), joint estimations with Stein-like shrinkage techniques can be combined in order to improve the predictive accuracy of estimates at both the disaggregate and aggregate levels. Stein-like shrinkages work reasonably well using time varying parameters to allow for possible 'structural breaks'. Theses shrinkages also have the advantage of dealing with parameters that can vary with time. Synchronisation in sectoral rates of growth is very weak (see figure 3.1).

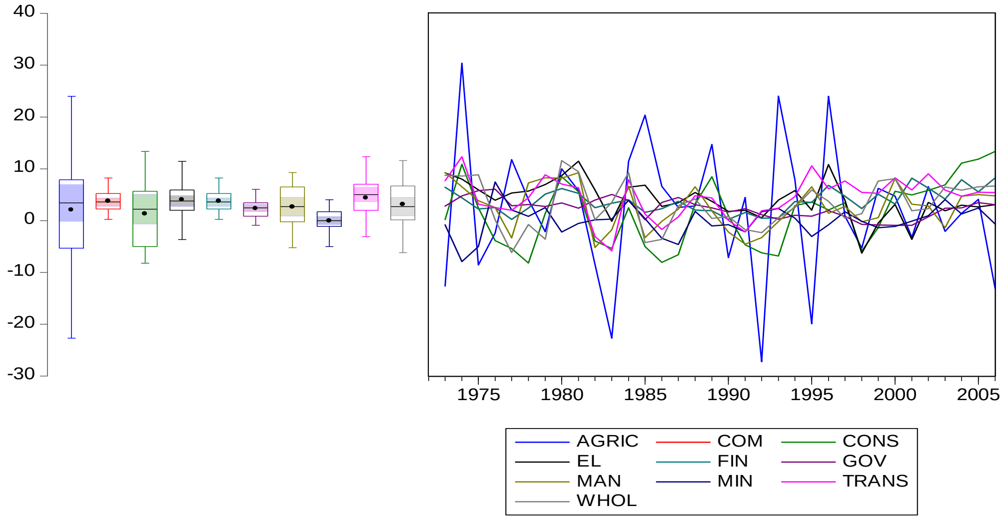

Figure 3.1: South African annual real output growth rates per industrial sector

With:

- AGRIC: Agriculture, Fishing and Forestry;
- EL: Electricity, Gas and Water;
- FIN: Financial Intermediates and Real Estate;
- WHOL: Wholesale, Retail trade, Catering and Accommodation;
- MAN: Manufacturing;
- COM: Community, Social, and Personal Services;
- MIN: Mining and Quarrying;
- CONS: Construction;
- GOV: Government; and
- TRANS: Transport, Storage, and Communication.

3.5. MODEL SPECIFICATION

3.5.1 General characteristics of a macroeconometric model

In the present chapter, the use of a disaggregated Marshallian macroeconometric model (MMM-DA) is justifiable considering its ability to produce better forecasts and reliable policy guidance. Several criteria (statistical or theoretical) can be used to assess the model's prediction[18] (forecasting) ability. However, in the present chapter,

[18] Most often the concepts 'prediction' and 'forecasting' are considered interchangeable. However, in this study, 'prediction' is the most appropriate concept since a structural model is used.

for the most part, only two statistical criteria are referred to: the RMSE and the MAE. A thorough discussion of the estimates and the prediction results obtained is provided as a theoretical assessment. Predictions are analysed in order to establish whether they are supportive of underpinning economic theory or not. The use of a MMM-DA provides greater ability to apply policy simulations and therefore generate the effects at both sectoral and national levels. The greater the degree of disaggregation of a model, the better the results obtained when applying policy shocks.

The literature suggests some general characteristics that can be used to assess the validity of macroeconometric models. In various cases, the following properties need to be observed: (1) simplicity and consistency of the model; (2) relevance and support of economic theory; (3) reliability and possibility of estimating the functional form; (4) solid stochastic specification of the model; (5) consistency and significance of explanatory variables; (6) appropriate number of equations as compared to the number of endogenous variables; (7) adequate identification of parameters; and (8) appropriate use of mathematical theory in obtaining the results. The use of simulation experiments is a very intuitive way of acquiring information about the dynamic properties of the model. The scrupulous consideration of the above-mentioned criteria for the properties of the model does not exclude the fact that all related statistical tests must be conducted cautiously. These range from basic data evaluation to more advanced diagnostic and forecasting tests. The larger a model becomes in terms of the number of non-linear equations used, the more difficult it is to determine whether the model carries a unique solution or not.

Concerning the choice of asymptotically justified estimates (Zellner, 1984), the size of the finite-sample matters. Asymptotically justified estimates may induce different values in relation to the type of specification errors used. The present study does not provide any sensitivity analysis which can be used in order to assess the choice of asymptotically justified estimates, as it is assumed that specification errors do not affect alternative estimates differently.

3.5.2 Deriving the Marshallian Model (Disaggregated Model)

3.5.2.1 Firms' optimisation process

Assuming a Cobb-Douglas production function as follows:

$$Q = A_N (z L)^{\alpha} K^{\beta}, \tag{2.1.1}$$

with: - A_N : Neutral technological change per sector;

- z : Level of human capital in per capita terms[19];

- zL : Effective labour input (Z); and

$$Z = zL. \tag{2.1.2}$$

Effective labour is modelled using the above expression where *h* represents health (per capita expenditure on health) and *s* represents schooling (per capita expenditure on education). Assuming that firms in the sector operate under a competitive market, the profit function may be defined as follows:

$$\pi = TR - TC, \tag{2.1.3}$$

$$TC = wzL + rK + \Gamma, \tag{2.1.4}$$

with: - w : wage rate;

- r : user cost of capital (the proxy often used is the interest rate);

- Γ : entry cost.

Two output prices are assumed: the expected price (P_Q^e); and the current price (P_Q).

At the beginning of period *t*, firms base all their production decisions on the expected price. However, should the actual price be set, firms follow an adjustment process. Since the producer moves according to expectations, mainly considering price, the producer's problem can be stated as follows:

Max: $$\pi = P_Q^e Q - w L - r K - \Gamma, \tag{2.1.5}$$

Constraint: $$Q = A_N (z L)^{\alpha} K^{\beta}. \tag{2.1.6}$$

Using first order conditions, the following optimal solutions are obtained, before price adjustment:

$$K^* = \left[\frac{\beta A P_Q^e}{r} \left(\frac{\alpha r}{\beta w} \right)^{\alpha} \right]^{\frac{1}{1-\alpha-\beta}}; \tag{2.1.7}$$

$$z L^* = \frac{\alpha}{\beta} . \frac{r}{w} \left[\frac{\beta A . P_Q^e}{r} \left(\frac{\alpha r}{\beta w} \right)^{\alpha} \right]^{\frac{1}{1-\alpha-\beta}}; \tag{2.1.8}$$

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[19] This specification follows Weil (2001) and Bloom (2005) and the series was estimated in chapter 2.

$$L^{*} = \frac{\alpha}{\beta}.\frac{r}{w}\left[\frac{\beta A.P_Q^e}{r}\left(\frac{\alpha r}{\beta w}\right)^{\alpha}\right]^{\frac{1}{1-\alpha-\beta}}.z^{-1}. \qquad (2.1.9)$$

After the price adjustment mechanism, the optimal solutions will be the following:

$$K^{**} = \left[\frac{\beta A P_Q^e}{r}\left(\frac{\alpha r}{\beta w}\right)^{\alpha}\right]^{\frac{1}{1-\alpha-\beta}}.\left[\frac{P_Q}{P_Q^e}\right]^{\phi_K}; \qquad (2.1.10)$$

$$L^{**} = \frac{\alpha}{\beta}.\frac{r}{w}\left[\frac{\beta.A.P_Q^e}{r}\left(\frac{\alpha r}{\beta w}\right)^{\alpha}\right]^{\frac{1}{1-\alpha-\beta}}\left[\frac{P_Q}{P_Q^e}\right]^{\phi_L}.z^{-1}; \qquad (2.1.11)$$

$$Q = A^{\frac{1}{1-\alpha-\beta}}.\alpha^{\frac{\alpha}{1-\alpha-\beta}}.\beta^{\frac{\beta}{1-\alpha-\beta}}.(P_{Q_{it}}^e)^{\frac{\alpha+\beta}{1-\alpha-\beta}}.w^{\frac{-\alpha}{1-\alpha-\beta}}.r^{\frac{-\beta}{1-\alpha-\beta}}.\left(\frac{P_Q}{P_Q^e}\right)^{\alpha\phi_L+\beta\phi_K}.z^{-\alpha}.$$

(2.1.12)

When they start producing, firms have expected prices in mind and they make the necessary adjustments when they face the actual prices set within the market. The entry cost (much more of an administrative cost) is assumed to be independent of both inputs and output. In the optimisation process, the variable is treated as a constant variable and therefore does not appear in the optimised K and L. However it directly affects the number of firms operating in the sector.

3.5.2.2 The Sales Supply equation

$$S_S = A^{\frac{1}{1-\alpha-\beta}}.\alpha^{\frac{\alpha}{1-\alpha-\beta}}.\beta^{\frac{\beta}{1-\alpha-\beta}}.\Phi N(\Gamma).w^{\frac{-\alpha}{1-\alpha-\beta}}.r^{\frac{-\beta}{1-\alpha-\beta}}.P^{1+\alpha\phi_L+\beta}.\left(P_Q^e\right)^{-\alpha\phi_L-\beta\phi_K+\frac{\alpha+\beta}{1-\alpha-\beta}}.z^{-\alpha},$$

(2.2.1)

with:

$$\frac{\partial N}{\partial \Gamma} \prec 0$$

The sales supply equation was developed from the basic definition of sales:

$$S_S = (\Phi N)\ .P_Q\ .q; \qquad (2.2.2)$$

where $\Phi = \sum_{j=1}^{N} \hbar_j$;

with N being the total number of firms operating in the sector.

It is important to note that barriers to entry and a low $N(\Gamma)$ imply oligopoly power. With oligopoly power firms might consider the effect of their output level on the price and hence choose a *lower* level of production than the one given by (2.1.2). This can be modelled a Nash equilibrium with output quantities as the choice variable. However if the implied mark-up as a proportion of marginal cost is constant over time, the growth rate equations will still be valid.

Also, $\hbar_j$ is the j^{th} firm's share in sector sales activities (size characteristics) and q is the individual firm's production. Whenever $\hbar_1 = \hbar_2 = ... = \hbar_N$, it simply means that all firms are identical and have the exact same share of the sector's sales activities. In fact, the sum of all $\hbar_j$ will always be equal to one ($\Phi = \sum_{j=1}^{N} \hbar_j = 1$). That is the reason why $\hbar_j$ does not appear in the sector's sales supply equation. However, it is important to highlight the fact that it is most likely that firms in the same sector are not identical. Considering Q as the total sector's production then the sales supply equation will be written as follows: $S_S = P_Q.Q$. Importantly, it has to be highlighted that the expected price is nonlinear and has the following specification:

$$P_Q^e = \prod_{l=1}^{T} P_l^{\sigma_k}\,; \tag{2.2.3}$$

with T being the total number of variables. Reducing the number of price variables to one, P_l can be interchanged with P_Q or simply P.

The sales equation can be expressed in growth terms by logging both sides of equation 2.2.1 and differentiating it with respect to time:

$$\frac{\dot{S}_S}{S_S} = \theta_1 \frac{\dot{A}}{A} + \left(\frac{\dot{\Phi}}{\Phi} + \frac{\dot{N}(\Gamma)}{N(\Gamma)} \right) + \theta_2 \frac{\dot{P}}{P} + \theta_3 \frac{\dot{w}}{w} + \theta_4 \frac{\dot{r}}{r} + \sum_{l=1}^{T} \sigma_l \frac{\dot{P}_l}{P_l} + \theta_5 \frac{\dot{z}}{z}, \tag{2.2.4}$$

where: - $\theta_1 = \dfrac{1}{1-\alpha-\beta}$; $\theta_2 = 1 + \alpha\phi_L + \beta$; $\theta_3 = \dfrac{-\alpha}{1-\alpha-\beta}$; $\theta_4 = \dfrac{-\beta}{1-\alpha-\beta}$;

- $\theta_5 = -\alpha$; and $\theta_6 = -\nu$.

3.5.2.3 The Sales Demand equation

The sales demand equation can be formulated as follows:

$$S_D = (\mathrm{B})P_Q.q \ , \qquad (2.3.1)$$

where $\mathrm{B} = \sum_{k=1}^{D} \upsilon_k$,

with D being the total number of demanders of the sector's products and where υ_k represents the k^{th} demander's size (share) of the sector's product demand. The demanders include: (1) firms; (2) private households; (3) government; as well as (4) foreign entities. The sum of all υ_k will always be equal to one: $\mathrm{B} = \sum_{k=1}^{D} \upsilon_k = 1$. Whenever all demanders are assumed to be identical, all υ_k will be equal. The likelihood of the latter happening is very low. Otherwise, with Q being the total demand, the sales equation can be written as follows:

$$S_D = P_Q.Q \ . \qquad (2.3.2)$$

Another way to present the sales demand function providing more detail on explanatory variables is to make use of the following equation:

$$S_D = P.\left[C_S (P_Q^e)^{\lambda_1}.(Y_d)^{\lambda_2}.(\mathrm{B}D)^{\lambda_3} \prod_{j=1}^{m} X_j^{\chi_j}.\left(\frac{P_Q^e}{P_Q}\right)^{\Delta} \right]. \qquad (2.3.3)$$

It is assumed that the future expected price drives today's demand – the reason why the sales demand function also accounts for price adjustment. Logging both sides and applying derivatives with respect to time the following growth equation is obtained:

$$\frac{\dot{S}_D}{S_D} = (1-\Delta)\frac{\dot{P}_Q}{P_Q} + (\lambda_1 + \Delta)\frac{\dot{P}_Q^e}{P_Q} + \lambda_2 \frac{(\dot{Y}_d)}{(Y_d)} + \lambda_3 \frac{\mathrm{B}\dot{D}}{\mathrm{B}D} + \chi_{j1}\frac{\dot{WY}}{WY} \, , \qquad (2.3.4)$$

where:

- Y_d : Gross National Disposable Income;
- S_D : Sales Demand;
- X : Other variables affecting sales demand; and
- WY : World Income.

3.5.2.4 Factor market

i) Labour

Labour Supply Equation

Under the assumption of perfect competition with *m* maximising firms in the sector, a Cobb-Douglas function for labour is used. The aggregate labour supply is therefore presented as follows:

$$L = C_L \left(\frac{w}{P_Q}\right)^{\psi_1} \left(\frac{S_S}{P_Q}\right)^{\psi_2} \left(\frac{P_Q}{P_Q^e}\right)^{\psi_3} \left(\vartheta(\rho D)\right)^{\psi_4}, \qquad (2.4.1)$$

where: $\vartheta = \sum_{k=1}^{H} \vartheta_k$.

It was specified earlier that *D* represents the total number of demanders of market products. These include: (1) households; (2) firms; (3) government; and (4) the rest of the world. When it comes to the economic units that supply labour, it is correct to think of households (local or even international). Households constitute only a portion ρ of *D* ($H = \rho D$). ϑ_k is an index capturing the share of household *k* in supplying effective labour (*zL*) for the given sector. ϑ_k constitutes the link between the HPM (Household Production Model) and the factor market. When household production activities function efficiently, households can supply a highly efficient and productive labour force. Household Production Equations also appear on the demand side of the model through sales demand. Households with higher production activities will demand more sales.

$$\frac{\dot{(zL)}}{(zL)} = \psi_1\left(\frac{\dot{w}}{w} - \frac{\dot{P}}{P}\right) + \psi_2\left(\frac{\dot{S}_S}{S_S} - \frac{\dot{P}}{P}\right) + \psi_3\left(\frac{\dot{P}_Q}{P_Q} - \frac{\dot{P}_Q^e}{P_Q^e}\right) + \psi_4\left(\frac{\dot{\vartheta}}{\vartheta} + \frac{\dot{\rho D}}{\rho D}\right). \qquad (2.4.2)$$

Labour Demand Equation (Efficient Labour)

The demand for efficient labour is determined as follows:

$$zL = \alpha.\frac{S_S}{w}.\left(\frac{P_Q^e}{P_Q}\right)^{1+\beta\phi_K+(\alpha-1)\phi_L} ; \qquad (2.4.3)$$

$$\frac{\dot{(zL)}}{(zL)} = \frac{\dot{S}_S}{S_S} - \frac{\dot{w}}{w} - (1 + \beta\phi_K + (\alpha - 1)\phi_L)\frac{\dot{P}_Q}{P_Q} + (1 + \beta\phi_K + (\alpha - 1)\phi_L)\frac{\dot{P}_Q^e}{P_Q^e}; \tag{2.4.4}$$

$$\frac{\dot{(zL)}}{(zL)} = \frac{\dot{S}_S}{S_S} - \frac{\dot{w}}{w} + (1 + \beta\phi_K + (\alpha - 1)\phi_L)\left[\frac{\dot{P}_Q^e}{P_Q^e} - \frac{\dot{P}_Q}{P_Q}\right]. \tag{2.4.5}$$

The model implicitly includes the level of unionisation that plays a major role in the labour market through wage determination. This provides more flexibility to the model and allows for the application of more sensitive policy simulation. A simulation based on Thatcher's reforms conducted on the labour market would lessen the impact of unions on wage determination. This would theoretically raise the level of labour demand.

ii) Capital

Concerning the market for capital, this study maintains the use of a Cobb-Douglas function for maximising firms.

Capital Supply Equation:

$$K = C_K\left(\frac{r}{P_Q}\right)^{\gamma_1}\left(\frac{S_S}{P_Q}\right)^{\gamma_2}\left(\frac{P_Q}{P_Q^e}\right)^{\gamma_3}(\delta D)^{\gamma_4} \text{ and} \tag{2.4.6}$$

$$\delta = \sum_{k=1}^{D}\delta_k ,$$

where δ_k is the k^{th} demander's share in capital supply.

$$\frac{\dot{K}}{K} = \gamma_1\left(\frac{\dot{r}}{r} - \frac{\dot{P}_Q}{\dot{P}_Q}\right) + \gamma_2\left(\frac{\dot{S}_S}{\dot{S}_S} - \frac{\dot{P}_Q}{\dot{P}_Q}\right) + \gamma_3\left(\frac{\dot{P}_Q}{P_Q} - \frac{\dot{P}_Q^e}{P_Q^e}\right) + \gamma_4\left(\frac{\dot{\delta}}{\delta} + \frac{\dot{D}}{D}\right). \tag{2.4.7}$$

Capital Demand Equation:

$$K = \beta\frac{S_S}{r}\left(\frac{P_Q^e}{P_Q}\right)^{1+\alpha\phi_L+(\beta-1)\phi_K} , \tag{2.4.8}$$

$$\frac{\dot{K}}{K}=\frac{\dot{S}_S}{S_S}-\frac{\dot{r}}{r}-[1+\alpha\phi_L+(\beta-1)\phi_K]\left(\frac{\dot{P}_Q}{P_Q}\right)+[1+\alpha\phi_L+(\beta-1)\phi_K]\left(\frac{\dot{P}_Q^e}{P_Q^e}\right),\ \text{and} \tag{2.4.9}$$

$$\frac{\dot{K}}{K}=\frac{\dot{S}_S}{S_S}-\frac{\dot{r}}{r}+[1+\alpha\phi_L+(\beta-1)\phi_K]\left(\frac{\dot{P}_Q^e}{P_Q^e}-\frac{\dot{P}_Q}{P_Q}\right). \tag{2.4.10}$$

3.5.2.5 The Money Market

i) Money Supply Equation

The money market presumably follows the traditional route including a constant at which money supply is fixed and two shifters: consumer price index (*P*); and interest rate (*r*).

$$M_S=C_{M_S}.P^{\pi_1}.r^{\pi_2}\ \text{and} \tag{2.5.1}$$

$$\frac{\dot{M}_S}{M_S}=\pi_1\left(\frac{\dot{P}}{P}\right)+\pi_2\left(\frac{\dot{r}}{r}\right). \tag{2.5.2}$$

ii) Money Demand Equation

The demand for money is also a twice-differentiable function of several variables, including household and firm variables together with other traditional series such as interest rate, etc.

$$M^d=C_{M^d}.(\Re D)^{\nabla_1}.(\Im N)^{\nabla_2}.\left(\frac{r}{P_Q^e}\right)^{\nabla_3}.\left(\frac{S_S}{P_Q^e}\right)^{\nabla_4}.\left(\frac{P_Q}{P_Q^e}\right)^{\nabla_5},\ \text{and} \tag{2.5.3}$$

$$\frac{\dot{M}^d}{M^d}=\nabla_1\left(\frac{\dot{\Re}}{\Re}+\frac{\dot{D}}{D}\right)+\nabla_2\left(\frac{\dot{\Im}}{\Im}+\frac{\dot{N}}{N}\right)+\nabla_3\left(\frac{\dot{r}}{r}-\frac{\dot{P}_Q^e}{P_Q^e}\right)+\nabla_4\left(\frac{\dot{S}_S}{S_S}-\frac{\dot{P}_Q^e}{P_Q^e}\right)+\nabla_5\left(\frac{\dot{P}_Q}{P_Q}-\frac{\dot{P}_Q^e}{P_Q^e}\right). \tag{2.5.4}$$

3.5.2.6 Entry/Exit

The present model specification includes an entry/exit set of equations that constitutes the cartilage (point of junction) between supply, demand, and factor markets. In fact, firms do enter and exit the sector quite often and this should be reflected in the modelling exercise to make it more realistic. Firms are mainly attracted by any excess profit obtained through higher sales as compared to equilibrium profit. Firms will be likely to leave once sales go below equilibrium profit.

$$\frac{\dot{N}}{N} = C_E (S_S - \pi^e). \tag{2.6.1}$$

N is the total number of firms operating in the sector while π^e is the equilibrium profit level of individual firms in the sector. A systematic modelling of π^e is required in order to obtain a perfect understanding of firms' movements in the sector and this will be investigated in further studies. In fact, several factors are involved in the determination of π^e. Equation 2.6.1 is a transformation of $\frac{\dot{N}_i}{N_i} = C'_{Ei} (\pi_i^a - \overline{\pi}_i)$ where the market equilibrium profit is represented by $\overline{\pi}_i$ assuming that the firm's actual profit π_i^a constitutes a proportion ℓ of its sales supply S_{Si}: $\pi_i^a = \ell S_{Si}$. It appears that $\pi_i^e = \frac{\overline{\pi}_i}{\ell}$ which is the equilibrium profit at a given time considering $C_{Ei} = a\Gamma_i^{\kappa} = C'_{Ei}\ell$ with Γ being the firms' entry cost per sector that exert a negative impact on firms' entry.

Also, equation 2.6.1 is not estimated, but rather described as an identity that is solved out through the total system of reduced-form equations.

3.5.3 A note on 'expected price, P_Q^e'

Referring to the general literature, this section broadly discusses some features of the theories on price expectation. However, in this modelling exercise, it is not necessary to deeply explore any of these features since reduced form equations are used to help address data constraints.

It is plausible and rather realistic to make the assumption that consumers as well as producers take into consideration expected prices and adjust their consumption or production accordingly. P_Q^e may be assumed to be a nonlinear function. Its formulation depends on many variables. The literature argues that previous and actual prices are the determinants of people expectations[20]. Expected price can therefore be defined as the (geometric) mean of $P_{Q(t-1)}$(lagged actual price) and $P^e_{Q(t-1)}$(lagged expected price)[21]. It can be formulated as follows:

$$\ln P^e_{Qt} = \varpi \ln P_{Q(t-1)} + (1-\varpi)\ln P^e_{Q(t-1)} \quad \varpi \in (0,1)\,. \qquad (3.1)$$

Furthermore, this equation could be developed such that:

$$\ln P^e_{Qt} = \varpi \ln P_{Q(t-1)} + \varpi(1-\varpi)\ln P_{Q(t-2)} + \varpi(1-\varpi)^2 \ln P_{Q(t-3)}\text{, and} \qquad (3.2)$$

$$\ln P^e_{Q_t} = \ln\left[P^{\varpi}_{Q_{(t-1)}}.P^{\varpi(1-\varpi)}_{Q_{(t-2)}}.P^{\varpi(1-\varpi)^2}_{Q_{(t-3)}}\right]. \qquad (3.3)$$

The expected price is thus a nonlinear function of actual prices and can be written in lag form:

$$P^e_{Q_t} = \prod_{j=1}^{n} P^{\sigma_j}_{Q_{(t-j)}}\,, \qquad (3.4)$$

where: $\sigma_j = \varpi(1-\varpi)^{j-1}$.

The general assumption that supports the theory of rational expectation stipulates that no information is wasted in the economic system. The use of an average weight for expectations, although it is not a completely truthful representation of reality, has helped improve the accuracy of models.

Considering equation 3.1, in the event that ϖ equals one, means that the product's expected price in period *t* ($P^e_{Q_t}$) is exactly equal to the product's price in period *t-1* ($P_{Q_{(t-1)}}$). In other words, people expect that the price level remains unchanged. The smaller the value of ϖ, the larger the gap between $P^e_{Q_t}$ and $P_{Q_{(t-1)}}$.

Somehow, people refer to domestic inflation rate while setting price expectations. Therefore the value of ϖ is linked to the CPI (Consumer Price Index). However, to

[20] Muth, JF "Rational Expectations and the Theory of Price Movements", *Econometrica*, 29, 1961.
[21] Kim, KH "Empirical Evidence of Forecasting Improvements from Disaggregating the US Economy", Working Paper, University of Chicago, October 2006.

some extent, consumers might rather consider the trend of international indicators such as the oil price while fixing their price expectations.

In this analysis, since all sectors of the South African economy are considered, it is unrealistic to assume that market agents, and specifically consumers, have a perfect understanding of the dynamics of international markets. Therefore, expected price can be estimated as a function of the equilibrium price plus some error term[22]. Expectations vary according to different variables such as: (1) the type of economic system; (2) the quantity of information available; (3) the volatility of leading indicators; and (4) the level of market openness; etc.

A basic and simple approach used in order to determine price expectations consists of conducting a survey and directly asking respondents regarding their expectations on market prices. The University of Michigan has made use of this approach for several years. Other alternative approaches of determining price expectations include: (1) extrapolative expectation (Goodwin, 1947); (2) adaptive expectation (Nerlove, 1958); (3) classical theory of expectation (Schultz *et al*, 1958); (4) the conditional (weighted) expectation theory (Muth, 1961); (5) the C-P method (Carlson and Parkin, 1975); (6) rational expectation (Lucas, 1981); (7) inflation indexed bonds (Kitamura, 1997); and (8) the expectation-augmented Philips curve (Hori *et al*, 2003); etc. Since theories of expectations have not yet been able to predict human behaviour, the way forward is to establish the links between predictions and market equilibrium dynamics. The real measure of human behaviour in the determination of expectations remains unclear. The choice of an appropriate series for expected price is an important one, and making the wrong choice increases the probability of obtaining biased estimates.

In reference to the general understanding of price expectations, equation 3.1 can be reformulated by deriving it as follows with respect to time:

$$\frac{\dot{P}^{e}_{Q_t}}{P^{e}_{Q_t}} = \varpi \frac{\dot{P}_{Q_{(t-1)}}}{P_{Q_{(t-1)}}} + \varpi (1 - \varpi) \frac{\dot{P}^{e}_{Q_{(t-1)}}}{P^{e}_{Q_{(t-1)}}} + \varepsilon_t \, . \tag{3.5}$$

[22] The error term here represents all unobserved elements that affect the determination of price expectations.

3.5.4 Including leading indicators

Following the supporting arguments that were raised earlier for reduced-form equations (RFEs), the use of leading indicators such as Money (M2) and Stock Prices (SP) enhances the model's forecasting ability. It is interesting to compare the forecasting ability of the ARLI(3) approach and RFEs. Once the two leading indicators are included, and allowing for three lag terms of the price variable, the reduced-form equations can be written as follows:

$$\ln\left(\frac{S_{St}}{S_{S(t-1)}}\right) \approx \theta_0^{''} + \theta_1^{''} S_{S(t-1)} + \theta_2^{''} \ln\left(\frac{P_{Q_{(t-1)}}}{P_{Q_{(t-2)}}}\right) + \theta_3^{''} \ln\left(\frac{P_{Q_{(t-2)}}}{P_{Q_{(t-3)}}}\right) + \theta_4^{''} \ln\left(\frac{P_{Q_{(t-3)}}}{P_{Q_{(t-4)}}}\right) + \theta_5^{''} \ln\left(\frac{M_{(t-1)}}{M_{(t-2)}}\right)$$
$$+ \theta_6^{''} \ln\left(\frac{SP_{Q_{(t-1)}}}{SP_{Q_{(t-2)}}}\right) + \theta_7^{''} \ln\left(\frac{Y_t}{Y_{(t-1)}}\right) + \theta_8^{''} \ln\left(\frac{\dot{A}_t}{A_{(t-1)}}\right) + \varepsilon_{St}\,, \qquad (4.1)$$

$$\ln\left(\frac{P_{Q_t}}{P_{Q_{(t-1)}}}\right) \approx \sigma_0^{''} + \sigma_1^{''} S_{S(t-1)} + \sigma_2^{''} \ln\left(\frac{P_{Q_{(t-1)}}}{P_{Q_{(t-2)}}}\right) + \sigma_3^{''} \ln\left(\frac{P_{Q_{(t-2)}}}{P_{Q_{(t-3)}}}\right) + \sigma_4^{''} \ln\left(\frac{P_{Q_{(t-3)}}}{P_{Q_{(t-4)}}}\right) + \sigma_5^{''} \ln\left(\frac{M_{(t-1)}}{M_{(t-2)}}\right)$$
$$+ \sigma_6^{''} \ln\left(\frac{SP_{Q_{(t-1)}}}{SP_{Q_{(t-2)}}}\right) + \sigma_7^{''} \ln\left(\frac{Y_t}{Y_{(t-1)}}\right) + \sigma_8^{''} \ln\left(\frac{A_t}{A_{(t-1)}}\right) + \varepsilon_{Pt}\,. \qquad (4.2)$$

The RFEs include both leading indicators and lag terms. This type of specification exhibits some similarities to the ARLI (3) models, however, the literature provides enough evidence that RFEs forecast much better (see Zellner *et al*, 2005).

A simple ARLI (3) model is formulated as follows:

$$\ln\left(\frac{S_{St}}{S_{S(t-1)}}\right) \approx \theta_0^{'''} + \theta_1^{'''} s_{S(t-1)} + \theta_2^{'''} s_{S(t-2)} + \theta_3^{''} s_{S(t-3)} + \theta_4^{''} \ln\left(\frac{SP_{Q_{(t-3)}}}{SP_{Q_{(t-4)}}}\right) + \theta_5^{''} \ln\left(\frac{M_{(t-1)}}{M_{(t-2)}}\right) + \varepsilon_{St}^{'}$$
$$(4.3)$$

3.5.5. Reduced-Form Equations disaggregated by sector (RFE-DA)

As mentioned earlier, the set of disaggregated reduced-form equations (RFE-DA) is an expansion of the aggregate reduced form equations (RFEs) including all industrial sectors[23] (see appendix A, chapter 3). The RFE-DA can be formulated as set out below.

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[23] The subscript *i* represents the economic sector. The model includes 10 sectors of the South African economy.

For sector's output (sales supply),

$$\ln\left(\frac{S_{it}}{S_{i(t-1)}}\right) \approx \lambda_0 + \lambda_1 S_{S,i(t-1)} + \lambda_2 \ln\left(\frac{P_{Q_{i(t-1)}}}{P_{Q_{i(t-2)}}}\right) + \lambda_3 \ln\left(\frac{P_{Q_{i(t-2)}}}{P_{Q_{i(t-3)}}}\right) + \lambda_4 \ln\left(\frac{P_{Q_{i(t-3)}}}{P_{Q_{i(t-4)}}}\right) + \lambda_5 \ln\left(\frac{M_{(t-1)}}{M_{(t-2)}}\right)$$
$$+ \lambda_6 \ln\left(\frac{SP_{Q_{(t-1)}}}{SP_{Q_{(t-2)}}}\right) + \lambda_7 \ln\left(\frac{Y_{d,t}}{Y_{d,(t-1)}}\right) + \lambda_8 \ln\left(\frac{r_t}{r_{(t-1)}}\right) + \lambda_9 \ln\left(\frac{w_{it}}{w_{i(t-1)}}\right) + \lambda_{10} \ln\left(\frac{H_t}{H_{(t-1)}}\right)$$
$$+ \lambda_{11} \ln\left(\frac{z_{it}}{z_{i(t-1)}}\right) + \lambda_{10} \ln\left(\frac{\vartheta_t}{\vartheta_{(t-1)}}\right) + \lambda_{12} \ln\left(\frac{\Gamma_{it}}{\Gamma_{i(t-1)}}\right) + \lambda_{13} \ln\left(\frac{WY_t}{WY_{(t-1)}}\right) + \lambda_{14} \ln\left(\frac{A_{it}}{A_{i(t-1)}}\right) + \upsilon_{St} \tag{5.1}$$

and, for sector's output price,

$$\ln\left(\frac{P_{Q_{it}}}{P_{Q_{i(t-1)}}}\right) \approx \iota_0 + \iota_1 S_{S,i(t-1)} + \iota_2 \ln\left(\frac{P_{Q_{i(t-1)}}}{P_{Q_{i(t-2)}}}\right) + \iota_3 \ln\left(\frac{P_{Q_{i(t-2)}}}{P_{Q_{i(t-3)}}}\right) + \iota_4 \ln\left(\frac{P_{Q_{i(t-3)}}}{P_{Q_{i(t-4)}}}\right) + \iota_5 \ln\left(\frac{M_{(t-1)}}{M_{(t-2)}}\right)$$
$$+ \iota_6 \ln\left(\frac{SP_{Q_{(t-1)}}}{SP_{Q_{(t-2)}}}\right) + \iota_7 \ln\left(\frac{Y_{d,t}}{Y_{d,(t-1)}}\right) + \iota_8 \ln\left(\frac{r_t}{r_{(t-1)}}\right) + \iota_9 \ln\left(\frac{w_{it}}{w_{i(t-1)}}\right) + \iota_{10} \ln\left(\frac{H_t}{H_{(t-1)}}\right)$$
$$+ \iota_{10} \ln\left(\frac{\vartheta_t}{\vartheta_{(t-1)}}\right) + \iota_{11} \ln\left(\frac{Op_t}{Op_{(t-1)}}\right) + \iota_{12} \ln\left(\frac{\Gamma_{it}}{\Gamma_{i(t-1)}}\right) + \iota_{13} \left(\frac{WY_t}{WY_{(t-1)}}\right) + \iota_{14} \ln\left(\frac{A_{it}}{A_{i(t-1)}}\right) + \upsilon_{PS} \tag{5.2}$$

This model specification can be used to forecast Sales and Inflation rates at sectoral level. The RFE-DA follows a continuous time specification. Applying log approximation, these can be transformed into a discrete model specification where:

$$\frac{\dot{S}_{S,it}}{S_{S,it}} = \frac{d \ln S_{S,it}}{dt} \approx \ln\left(\frac{S_{S,it}}{S_{S,i(t-1)}}\right). \tag{5.3}$$

3.5.6. Transfer function

Transfer function equations are obtained by using a matrix of lag operators from the derivations of selected transfer functions (see Kim, 2006). Referring to equation 5.1, the following equations can be obtained for: sales supply; sales demand; the entry/exit equations:

$$\ln\left(\frac{S_{Si,t}}{S_{Si,t-1}}\right) = \ln\left(\frac{N_{i,t}}{N_{i,t-1}}\right) + \kappa_{1,i} \ln\left(\frac{w_{i,t}}{w_{i,t-1}}\right) + \kappa_{2,i} \ln\left(\frac{r_t}{r_{t-1}}\right) + \kappa_{3,i} \ln\left(\frac{A_{i,t}}{A_{i,t-1}}\right) + \kappa_{4,i} \ln\left(\frac{z_{i,t}}{z_{i,t-1}}\right)$$
$$+ \kappa_{5,i} \ln\left(\frac{\Gamma_{i,t}}{\Gamma_{i,t-1}}\right) + \kappa_{6,i} \ln\left(\frac{x_{it}}{x_{i,t-1}}\right) + \lambda(L) \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) + \varepsilon_{Ti,t}; \tag{6.1}$$

$$\ln\left(\frac{S_{Di,t}}{S_{Di,t-1}}\right) = \Delta_{1,i}\ln\left(\frac{Y_{d,t}}{Y_{d,t-1}}\right) + \Delta_{2,i}\ln\left(\frac{WY_t}{WY_{t-1}}\right) + \Delta_{3,i}\ln\left(\frac{D_{i,t}}{D_{i,t-1}}\right) + \gamma(L)\ln\left(\frac{P_{Qi,t}}{P_{Qi,t-1}}\right) + \mu_{Ti,t} \tag{6.2}$$

$$\ln\left(\frac{N_{i,t}}{N_{i,t-1}}\right) = \delta_{0,i} + \delta_{1,i}S_{i,t-1} + v_{Ti,t}; \tag{6.3}$$

where:

- $\lambda(L)$ and $\gamma(L)$: are lag operators;
- X: is the set of other exogenous variables obtained from the ARLI (3) model: *SP* (Stock Prices) and M (Money Supply: M2).

The structural equations can be represented in matrix form as follows:

$$\begin{bmatrix} 1 & -\lambda(L) & -1 \\ 1 & -\gamma(L) & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} s_{i,t} \\ p_{i,t} \\ n_{i,t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \delta_{0,i} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \delta_{1,i} \end{bmatrix} S_{i,t-1} + \begin{bmatrix} \kappa_{1,i} \\ 0 \\ 0 \end{bmatrix} w_{i,t} + \begin{bmatrix} \kappa_{2,i} \\ 0 \\ 0 \end{bmatrix} r_t + \begin{bmatrix} \kappa_{3,i} \\ 0 \\ 0 \end{bmatrix} a_{i,t} + \begin{bmatrix} \kappa_{4,i} \\ 0 \\ 0 \end{bmatrix} z_{i,t}$$
$$+ \begin{bmatrix} \kappa_{5,i} \\ 0 \\ 0 \end{bmatrix} \Gamma_{i,t} + \begin{bmatrix} \kappa_{6,i} \\ 0 \\ 0 \end{bmatrix} X_t + \begin{bmatrix} 0 \\ \Delta_{1,i} \\ 0 \end{bmatrix} y_t + \begin{bmatrix} 0 \\ \Delta_{2,i} \\ 0 \end{bmatrix} wy_t + \begin{bmatrix} 0 \\ \Delta_{3,i} \\ 0 \end{bmatrix} d_t + \begin{bmatrix} \varepsilon_{Ti,t} \\ \mu_{Ti,t} \\ v_{Ti,t} \end{bmatrix}. \tag{6.4}$$

The following definitions are used in equation 6.4:

- $\ln\left(\frac{S_{i,t}}{S_{i,t-1}}\right) = s_{i,t}$; $\ln\left(\frac{N_{i,t}}{N_{i,t-1}}\right) = n_{i,t}$; $\ln\left(\frac{w_{i,t}}{w_{i,t-1}}\right) = w_{i,t}$; $\ln\left(\frac{r_{i,t}}{r_{i,t-1}}\right) = r_{i,t}$;
- $\ln\left(\frac{A_{i,t}}{A_{i,t-1}}\right) = a_{i,t}$; $\ln\left(\frac{z_{i,t}}{z_{i,t-1}}\right) = z_{i,t}$; and $\ln\left(\frac{\Gamma_{i,t}}{\Gamma_{i,t-1}}\right) = \Gamma_{i,t}$.

In order to obtain the final equations (MMM-DAs), it is important to multiply both sides of equation 6.4 by the matrix A^* ($A^* = \det A.A^{-1}$), with:

$$A = \begin{bmatrix} 1 & -\lambda(L) & -1 \\ 1 & -\gamma(L) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad A^{-1} = \frac{1}{\det A}\begin{bmatrix} -\gamma(L) & \lambda(L) & -\gamma(L) \\ -1 & 1 & -1 \\ 0 & 0 & \lambda(L)-\gamma(L) \end{bmatrix}.$$

Therefore: $A^* = \begin{bmatrix} -\gamma(L) & \lambda(L) & -\gamma(L) \\ -1 & 1 & -1 \\ 0 & 0 & \lambda(L)-\gamma(L) \end{bmatrix}$.

After multiplying both sides of equation 6.4 by A^*:

$$[\lambda(L)-\gamma(L)]\begin{bmatrix} s_{i,t} \\ p_{i,t} \\ n_{i,t} \end{bmatrix} = \begin{bmatrix} -\gamma(L)\delta_{0,i} \\ -\delta_{0,i} \\ \delta_{0,i}[\lambda(L)-\gamma(L)] \end{bmatrix} + \begin{bmatrix} -\gamma(L)\delta_{1,i} \\ -\delta_{1,i} \\ \delta_{1,i}[\lambda(L)-\gamma(L)] \end{bmatrix} S_{i,t-1} + \begin{bmatrix} -\gamma(L)\kappa_{1,i} \\ -\kappa_{1,i} \\ 0 \end{bmatrix} w_{i,t}$$
$$+ \begin{bmatrix} -\gamma(L)\kappa_{2,i} \\ -\kappa_{2,i} \\ 0 \end{bmatrix} r_t + \begin{bmatrix} -\gamma(L)\kappa_{3,i} \\ -\kappa_{3,i} \\ 0 \end{bmatrix} a_{i,t} + \begin{bmatrix} -\gamma(L)\kappa_{4,i} \\ -\kappa_{4,i} \\ 0 \end{bmatrix} z_{i,t} + \begin{bmatrix} -\gamma(L)\kappa_{5,i} \\ -\kappa_{5,i} \\ 0 \end{bmatrix} \Gamma_{i,t}$$
$$+ \begin{bmatrix} -\gamma(L)\kappa_{6,i} \\ -\kappa_{6,i} \\ 0 \end{bmatrix} X_t + \begin{bmatrix} \lambda(L)\Delta_{1,i} \\ \Delta_{1,i} \\ 0 \end{bmatrix} y_t + \begin{bmatrix} \lambda(L)\Delta_{2,i} \\ \Delta_{2,i} \\ 0 \end{bmatrix} wy_t + \begin{bmatrix} \lambda(L)\Delta_{3,i} \\ \Delta_{3,i} \\ 0 \end{bmatrix} d_t +$$
$$+ \begin{bmatrix} -\gamma(L)\varepsilon_{Ti,t} + \lambda(L)\mu_{Ti,t} - \gamma(L)v_{Ti,t} \\ -\varepsilon_{Ti,t} + \mu_{Ti,t} - v_{Ti,t} \\ [\lambda(L)-\gamma(L)]v_{Ti,t} \end{bmatrix}. \tag{6.5}$$

Equation 6.5 can be transformed into a system of linear equations for both price and sales supply:

$$[\lambda(L)-\gamma(L)]s_{i,t} = -\gamma(L)\delta_{0,i} - \gamma(L)\delta_{1,i}S_{i,t-1} - \gamma(L)\kappa_{1,i}w_{i,t} - \gamma(L)\kappa_{2,i}r_t - \gamma(L)\kappa_{3,i}a_{i,t} - \gamma(L)\kappa_{4,i}z_{i,t}$$
$$-\gamma(L)\kappa_{5,i}\Gamma_{i,t} - \gamma(L)\kappa_{6,i}X_t + \lambda(L)\Delta_{1,i}y_t + \lambda(L)\Delta_{2,i}wy_t + \lambda(L)\Delta_{3,i}d_t - \gamma(L)\varepsilon_{Ti,t} + \lambda(L)\mu_{Ti,t} - \gamma(L)v_{Ti,t} \text{ ; and} \tag{6.6}$$

$$[\lambda(L)-\gamma(L)]P_{i,t} = -\delta_{0,i} - \delta_{1,i}S_{i,t-1} - \kappa_{1,i}w_{i,t} - \kappa_{2,i}r_t - \kappa_{3,i}a_{i,t} - \kappa_{4,i}z_{i,t} - \kappa_{5,i}\Gamma_{i,t} - \kappa_{6,i}X_t + \Delta_{1,i}y_t + \Delta_{2,i}wy_t + \Delta_{3,i}d_t - \varepsilon_{Ti,t} + \mu_{Ti,t} - v_{Ti,t}. \tag{6.7}$$

3.5.7 The MMM-DA (Transfer equations)

3.5.7.1 Sector's Sales Supply equation

$$\ln\left(\frac{S_{S,it}}{S_{S,i(t-1)}}\right) \approx \partial_{0i} + \partial_{1i}S_{S,i(t-1)} + \partial_{2i}S_{S,i(t-2)} + \partial_{3i}S_{S,i(t-3)} + \partial_{4i}\ln\left(\frac{A_{it}}{A_{i(t-1)}}\right) + \partial_{5i}\ln\left(\frac{M_{(t-1)}}{M_{(t-2)}}\right)$$
$$+ \partial_{6i}\ln\left(\frac{SP_{(t-1)}}{SP_{(t-2)}}\right) + \partial_{7i}\ln\left(\frac{Y_{dt}}{Y_{d(t-1)}}\right) + \partial_{8i}\ln\left(\frac{r_t}{r_{(t-1)}}\right) + \partial_{9i}\ln\left(\frac{w_{it}}{w_{i(t-1)}}\right) + \partial_{10i}\left(\frac{\dot{\Re}}{\Re} + \frac{\dot{D}_t}{D_{(t-1)}}\right)$$
$$+ \partial_{11i}\ln\left(\frac{z_{it}}{z_{i(t-1)}}\right) + \partial_{12i}\ln\left(\frac{\Gamma_{it}}{\Gamma_{i(t-1)}}\right) + \partial_{13i}\ln\left(\frac{WY_t}{WY_{(t-1)}}\right) + \varepsilon_{Sit}. \tag{7.1}$$

3.5.7.2 Sector's price equation

$$\ln\left(\frac{P_{Q,it}}{P_{Q,i(t-1)}}\right) \approx \kappa_{0i} + \kappa_{1i} S_{S,i(t-1)} + \kappa_{2i} S_{S,i(t-2)} + \kappa_{3i} S_{S,i(t-3)} + \kappa_{4i} \ln\left(\frac{\dot{A}_{it}}{A_{i(t-1)}}\right) + \kappa_{5i} \ln\left(\frac{M_{(t-1)}}{M_{(t-2)}}\right)$$
$$+ \kappa_{6i} \ln\left(\frac{SP_{(t-1)}}{SP_{(t-2)}}\right) + \kappa_{7i} \ln\left(\frac{Y_{dt}}{Y_{d(t-1)}}\right) + \kappa_{8i} \ln\left(\frac{r_t}{r_{(t-1)}}\right) + \kappa_{9i} \ln\left(\frac{w_{it}}{w_{i(t-1)}}\right) + \kappa_{10i} \left(\frac{\dot{\Re}}{\Re} + \frac{\dot{D}_t}{D_{(t-1)}}\right).$$
$$+ \kappa_{11i} \ln\left(\frac{z_{it}}{z_{i(t-1)}}\right) + \kappa_{12i} \ln\left(\frac{\Gamma_{it}}{\Gamma_{i(t-1)}}\right) + \kappa_{13i} \ln\left(\frac{WY_t}{WY_{(t-1)}}\right) + \varepsilon_{Pit}.$$

(7.2)

3.6 ESTIMATION TECHNIQUES

3.6.1 The Iterative SUR (Seemingly Unrelated Regressions Model): Bayesian versus Non-Bayesian Perspective

In order to estimate the MMM-DA model, the ISUR specification has been utilised in this chapter. The ISUR specification provides estimates using the GLS (Generalised Least Squares) method. Among several other advantages, the use of ISUR allows for correction of contemporaneous correlation and heteroskedasticity biases related to the different cross-sections. Also, the iterative SUR approach permits estimation transfer equations with different coefficient vectors[24]. The correlation of cross-section disturbances increases efficiency, which is one of the most striking aspects of the ISUR approach.

Alternatively, a Bayesian approach using techniques such as MCMC (Markov Chain Monte Carlo Simulations) or DMC (Direct Monte Carlo Simulations) can be utilised as opposed to the GLS. This section compares the different approaches considering criteria of good forecast (ie RMSE and MAE). The use of MCMC requires a good specification of the likelihood function while the iterative SUR method provides the MLE (Maximum Likelihood Estimators) using an algorithm. Although, MCMC is based on the use of an algorithm for computing the posterior distribution, it requires a

[24] When a restriction is imposed to have the same coefficient vectors across sectors, it is called complete shrinkage. The use of complete shrinkage will be discussed later on in order to provide improvement to the estimates presented in this study.

prior and a likelihood function. While iterating the SUR with a normal likelihood function with a flat prior, the modal value of the distribution is obtained. The modal value will be optimal to a zero-one loss function. In a large sample, the posterior distribution is normal and the posterior-mean equals the maximum likelihood estimator (MLE). The posterior shapes up to be normal and the '*t-statistic*' is approximated based on the normal distribution. The iterative SUR procedure iterates in the MLE under a broad range of conditions. In the case of large sample situations, the MLE will be equal to the mean of the posterior and to the modal value. This is what is meant by a Bayesian estimate.

3.6.2 MCMC in estimating the SUR model

As a highly appropriate method in micro-econometric applied research, MCMC is a method that has helped improve the exploration of the posterior density using algorithms. Referring to the fact that the posterior distribution $\pi^*(\theta)$ is not normal, building simulations-based estimates in the given distribution is the core focus of the MCMC procedure[25]. The use of MCMC is more relevant in cases where θ is very large. This study includes 10 economic sectors of the South African economy with as set of equations (at least 7) solved as reduced forms, containing 12 to 13 variables per sector. Therefore, the space dimension to be explored rises to a certain maximum of more or less 840. That is a relatively large sample and therefore the use of MCMC is justified.

The MCMC method consists of using the underpinning theory of 'Markov Chains' as often referred to by the parameter space using simulation methods developed under the theory of Monte Carlo. In other, word, the problem is specified as a Markov chain with $\pi(\theta)$ being the equilibrium distribution of the chain. The estimates of the integrals can be obtained using Monte Carlo Simulations (see Rossi *et al*), because the simulation process consists of several draws (5000 iterations in our case) used to produce estimates of θ or any function of θ. Random variables of θ are generated using a sequential process until the iteration process is stopped at time T. Therefore, the move from θ_t to θ_{t+1} is operated using the principle of a transition matrix,

[25] Rossi PE; Allenby GM; and McCulloch R: 'Bayesian Statistics and Marketing', Wiley Series in Probability and Statistics, 2005.

inducing that θ_{t+1} is conditioned by θ_t. Assuming D_C, a conditional distribution, the use of Markov chains allows the creation of a joint distribution of the process. MCMC requires adamant restrictions to be specified, so that the posterior distribution converges toward a unique distribution. Once the convergence condition is met by the posterior distribution, which is already stationary, the Markov chain process used under Monte Carlo simulations can produce reliable estimates (Rossi *et al*). After a number of draws T, MCMC can be used to construct the posterior expectation of θ or a function of θ $f(\theta)$. Referring to its assumption on convergence, MCMC can validly be said to be asymptotic.

Here is an ergodic chain:

$$\lim_{T\to\infty}\frac{1}{T}\sum f(\theta_t) = E_\pi[f(\theta)],$$

with $E_\pi[f(\theta)]$ being the expected value of the posterior distribution.

Most often, the use of Markov chains entails that the first set of draws be disregarded since it constitutes the initial observations. In fact, the draws start to be considered from the distribution once the initial period is passed, and from that point the chain starts equilibrating.

$$E_\pi[f(\theta)] = \frac{1}{T-S}\sum_{t=S+1}^{T} f(\theta_t),$$

with S being the period of initial observations excluded from the draws (the 'burn in' period). The various draws of θ might be dependant on one another, which renders the process to be a non IID process. However, in most cases, in the literature on large number simulation experiments, the draws are assumed to be independent. MCMC requires very specific types of algorithms and the Gibbs Sampler is one of the most popular applications of MCMC.

In this regard, the Metropolis-Hastings algorithm is used to update the parameters. Assuming the set of parameters α (dimensional coefficient vector) and Ω (squared matrix), sized up based on the number of cross-sections, with diagonal elements being $\{\omega_1^2,\ldots,\omega_m^2\}$ and off-diagonal elements being ω_{ij}, the process can be summarised as follows (Zellner *et al*, 2008): (1) initialise the parameters α and Ω as the maximum likelihood estimates; (2) sample both coefficient parameters; and (3) repeat the sampling process throughout several iterations. The end result of such a process is to obtain a sample from the posterior density function after the 'burn-in' period.

From the parameter α, a candidate parameter $\alpha^{(t)}$ at the tth iteration is generated, with a maximum likelihood estimate Ω_{MLE}. The probability of accepting the candidate draw is ρ_α, and $1-\rho_\alpha$ is the probability of rejecting the draw.

$$\rho_\alpha = \min\left\{1, \frac{g_1(\alpha^{(t)}, \Omega^{(t-1)}}{g_2(\alpha^{(t-1)}, \Omega^{(t-1)})}\right\},$$

where: $g(\alpha, \Omega) = |\Omega|^{-(n+m+1)/2} \exp\left[-\frac{1}{2}\mathrm{A}\{M(\alpha)\Omega^{-1}\}\right]$,

with: - A : the trace of a matrix;

- $M(\alpha)$ is a squared matrix (sized up based on the number of cross-sections) with the ijth elements being: $(y_i - X_i\alpha_i)'(y_j - X_j\alpha_j)$.

Concerning Ω, a candidate $\Omega^{(t)}$ is generated from $f(\Omega|\alpha_{MLE}, Data)$, which is Wishart proposal density. The probability of accepting the candidate draw is therefore:

$$\rho_\Omega = \min\left\{1, \frac{g_3(\alpha^{(t)}, \Omega^{(t)})/h_1(\Omega^{(t)}|\alpha_{MLE}, Data)}{g_1(\alpha^{(t)}, \Omega^{(t-1)})/h_2(\Omega^{(t-1)}|\alpha_{MLE}, Data)}\right\},$$

while $1-\rho_\Omega$ is the probability of rejecting the candidate draw.

3.6.3 Estimating the ISUR model using DMC (Direct Monte Carlo)

The DMC approach is a computational technique that shows some similarities to the MCMC and it is also used to compute Bayesian quantities using data generated from known models[26]. The literature provides enough evidence that the DMC approach is a valid method for computing Bayesian quantities (posterior (marginal) as well as predictive density functions). The DMC approach has the advantage of being easily applicable and useful in solving several problems with fewer concerns per comparison than the MCMC approach. In comparison to the MCMC approach, in this sub-section, some technicalities of the DMC algorithm applicable to a ISUR specification are provided.

[26] Zellner A and Tomohiro A: 'A Direct Monte Carlo Approach for Bayesian Analysis of the Seemingly Unrelated Regression Model', Working paper, 2008.

Considering the parameters α and Ω, the DMC algorithm process can be summarised as follows (Zellner *et al,* 2008): (1) fix the set of equations and samples to be generated; (2) sample a (parameter obtained from a transformation applied on X, the matrix of the unknown) and Ψ (the covariance matrix) using conditional posterior densities, $\{a^{(t)}, \Psi^{(t)}; t = 1,\ldots, S\}$ with S being the samples to be generated; (3) transform $\Psi^{(t)}$ into $\Omega^{(t)}$; and (4) sample the coefficient vector $\alpha^{(t)}$, using a normal density with:

- $\hat{\alpha}^{(t)} = \left\{X'(\Omega^{(t)^{-1}} \otimes S)X\right\}^{-1} X'(\Omega^{(t)^{-1}} \otimes S)y$ being the mean;

- and $\hat{\Omega}_{\alpha}^{(t)} = (X'(\Omega^{(t)^{-1}} \otimes S)X)^{-1}$.

Considering the conditional posterior densities, the DMC approach consists of several draws from the conditional inverse gamma density, with sequential repetition of the process.

3.7 LOSS FUNCTIONS FOR ESTIMATING THE MMM-DA

The concept ‘loss’ is associated with the ‘regret’ which is felt when a target is not reached during an event. It captures the error orchestrated when an estimate is far away from the true value. Therefore, the key issue is to determine a consistent estimator that represents the loss experienced when the target is not reached. As soon as the estimator is well identified, the type of loss function to be used aims at minimising the expected loss observed when the target is not met. Even though both loss functions are subject to a similar objective (minimisation of the expected loss), it is imperative that the choice of the loss function be based on the kind of loss that will be faced in the case of a specific problem. Factors such as the socio-political environment matter most in this type of choice. Therefore, whenever a model is designed for policy guidance, with the knowledge that a specific target has been set, the loss functions to be used should be different. A major advantage linked to Bayesian loss functions is that the experimental data used in the model is not the only factor that orients the final decision. The prior probability plays a larger role. A combination of the following factors plays a role: (1) the prior probability; (2) the experimental data; and (3) the selected loss function; leading to the final decision obtained through the maximisation of the subjective expected loss function (Savage, 1954). Referring to Savage's theory[27] on 'subjective expected utility', the objective function (the subjective expected loss function SE) can be presented as follows:

$$SE = \int_{-\infty}^{\infty} L(y)P(y)dy\,,$$

where $L(y)$ is the loss function and y is the continuous random variable defined under a probability density function. As y can take different values, different subjective expected loss functions can therefore be set:

$$S_i = \int_{-\infty}^{\infty} L_i(y_i)P_i(y_i)dy_i \text{ , and}$$

$$S_j = \int_{-\infty}^{\infty} L_j(y_j)P_j(y_j)dy_j \,.$$

The preferred decision between the two is driven by the lowest subjective expected loss. Taking convex combinations of the different decisions might constitute a better option (Savage 1954).

Quadratic loss functions, which are in popular usage, are not always deemed appropriate when specific policy targets are pursued. Although quadratic loss functions are consistent with mathematical principles based on their frequent reference to variances, they might undermine the definition of loss in certain cases. Whether the error is above or below the target, it generates the same loss provided that it has the same magnitude. This concept makes little sense whenever the target is 'inflation' or any similar macroeconomic indicator (such as GDP growth; poverty line; etc).

3.8 IMPROVING PREDICTIONS USING SHRINKAGE TECHNIQUES

Stein shrinkage techniques constitute an appropriate way to improve predictions of the MMM-DA. The use of shrinkage techniques has produced better outcomes in country as well as regional forecasting. The Stein's Mean approach consists of estimating the vector mean using a quadratic loss function including the goodness of fit. The model specifications of the balanced loss function as well as the squared-error loss function (Dey *et al*, 1994) are as follows:

- Balanced loss function:

[27] Savage LJ (1954): 'Foundations of Statistics', New York, Wiley.

$$L(\hat{\theta},\theta) = w(y-\hat{\theta})'(y-\hat{\theta}) + (1-w)(\hat{\theta}-\theta)'(\hat{\theta}-\theta)\text{ ; and} \tag{8.1}$$

- Squared-error loss function:

$$L_b = w(\iota' y - \hat{T})^2 + (1-w)(\hat{T}-T)^2, \tag{8.2}$$

with: - θ: the vector of means;

- $\hat{\theta}$: estimate from outside information;
- *y*: the series considered;
- *w*: a weight imposed; and
- *T*: total of the mean observation vectors.

The shrinkage techniques make use of future vectors of observations presented in quadratic loss functions (squared errors) to predict future totals using the predictive means. For each sector, specific mean vectors need to be estimated and their totals need to be computed. Considering the sectors' disparities, it is important to generate observation vectors using SUR (Seemingly Unrelated Regression) models:

$$y_{ni} = x_{ni}\gamma_{ni} + \mu_{ni}, \tag{8.3}$$

with $n = 1,\ldots,10$; and

$i = 1, \ldots, 25$.

The shrinkage estimation of the γ_{ni} will affect the prediction of a future observation vector and the total. For the n_i mean, an informative prior needs to be added (Leonard *et al.*, 2001), when considering both the prior and the normal likelihood. The shrinkage estimators are accepted under lemma conditions for quadratic loss functions. For disaggregated models it is important to consider sectoral shrinkage assumptions regarding the estimates of the vectors of means. The evaluation of the predictive performance of the shrinkage estimates requires running comparative analysis with existing forecasting models looking at criteria like the MAEs, the RMSEs and the Akaike or Schwarz criteria.

The shrinkage can be standard, as it has been described above, or complete. Complete shrinkage, which is much easier to perform, assumes common coefficient vectors for each cross-section. Similarly to standard shrinkage, it allows for the reduction of the variance and therefore also for reduction of the RMSE (and MAE). Complete shrinkage entails very strong assumptions on the functioning of sectors and will most likely not reflect economic reality.

3.9 IMPOSING RESTRICTIONS

Given the relatively small sample size in practical applications, it is often recommended that equations with fewer parameters be estimated. In this study, an attempt is made to reduce the number of parameters by using the Wald test on parameter restrictions. In order to reduce the number of parameters in the model, a careful process (see Zellner and Chen, 2000) can be followed.

Considering the Marshallian model with both endogenous and predetermined variables, with L being the lag operator, the variables can be written in vector form as:

Set of endogenous variables and exogenous variables per sector:

$$\underbrace{\begin{pmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} \\ y_{5t} \\ y_{6t} \\ y_{7t} \end{pmatrix}}_{Y} = \begin{pmatrix} (1-L)\ln S_{St} \\ (1-L)\ln WY_t \\ (1-L)\ln w_t \\ (1-L)\ln r_t \\ (1-L)\ln y_{dt} \\ (1-L)\ln \Gamma_t \\ (1-L)\ln (D)_t \end{pmatrix} \quad \text{with} \quad \underbrace{\begin{pmatrix} - \\ \alpha_{21} \\ \alpha_{31} \\ \alpha_{41} \\ \alpha_{51} \\ \alpha_{61} \\ \alpha_{71} \end{pmatrix}}_{\alpha}$$

α (matrix of coefficient); and

Vector x of lagged variables in reference to the ARLI(3) model:

$$x_t^{'} = \left[1; S_{S,t-1}; S_{S,t-2}; S_{S,t-3}; (1-L)\ln M2_{t-1}; (1-L)\ln SP_{t-1}\right]. \quad (9.1)$$

The matrix equalities can be written as follows:

$$y_{1t} = \alpha_{21}y_{2t} + \alpha_{31}y_{3t} + \alpha_{41}y_{4t} + \alpha_{51}y_{5t} + \alpha_{61}y_{6t} + \alpha_{71}y_{7t} + \beta_1 x_{1t}^{'} + \mu_{1t}\text{, and} \quad (9.2)$$

$$y_1 = \alpha_1 Y_1 + \beta_1 X_1 + \mu_1 . \quad (9.3)$$

Assuming that π is the coefficient matrix of X, the unrestricted MMM-DA equations can be written as follows:

$$Y_1 = \prod\nolimits_1 X + \varepsilon_1 . \quad (9.4)$$

The Restricted MMM-DA (RMMM-DA) is obtained by substituting equation 9.4 into equation 9.3:

$$y_1 = \alpha_1 \prod\nolimits_1 X + \beta_1 X_1 + \varepsilon_1^{*} + \mu_1 ; \quad (9.5)$$

and

$$y_1 = \theta_1 \overline{M} + (\varepsilon_1 + \mu_1) ; \quad (9.6)$$

with: $\overline{M}$ being a full column rank such as:

$\overline{M} = (\prod_1 X; X_1)$ and $\varepsilon_1^* + \mu_1 = \nu_1$.

3.10 DATA

The data used in this chapter was collected from 1972 on a yearly basis. Ten economic sectors were considered and regarded to constitute the contributors to the overall national sales output[28]. The main sources of data collection are all official in addition to the links-related forecasting models established. It is usually expected that link-forecasting units are most likely to provide clean and appropriate information. Aggregate output figures are the easiest to locate. Data regarding input components such as: Investment; Employment; Wages; etc; were much more difficult to locate, especially in sectors without proper data warehousing systems. Generic sources include: (1) the SARB (South African Reserve Bank); (2) Statistics South Africa[29]; (3) the IMF (International Monetary Fund); (4) the World Bank; and (5) the African Development Bank. With the exception of employment data, other sectoral data was obtained from the SARB. Data on local leading indicators (Stock (share) prices and Money Supply (M2)) were obtained from the IMF while data on world leading indicators originated from the World Bank Statistics. Other types of national data such as: (1) disposable income; (2) interest rates; (3) wage rates; (4) the number of households; (5) labour productivity; and (6) firms' entry cost; were collected from the SARB database.

Generally, though, the above-mentioned sources provide data with some missing sectoral information, which makes the model synchronisation process more difficult to achieve. To address the missing data problem, the analysis considers the option of solving for reduced form equations for output and uses the outcome for forecasts (Zellner and Israilevich, 2003). As it could be garnered from the number of participating sectors or forecasting links, the interest in this modelling exercise is very high, although the disaggregated or sectoral data problem constitutes one of the major obstacles faced by the present research.

[28] The sectors considered are the following: (1) Manufacturing; (2) Agriculture, Fishing and Forestry; (3) Construction and Buildings; (4) Mining; (5) Government; (6) Community services; (7) Transport and Telecommunication; (8) Financial services; (9) Wholesales, Retail, Catering and Accommodation; and (10) Electricity, Gas and Water.

[29] Data on employment per sector was obtained from Quantec. It constitutes a private source that provides the data collected by Statistics South Africa.

3.11 RESULTS DISCUSSION

3.11.1 General discussion: Thatcher-like reforms

Several economies in the world such as Georgia and Estonia have implemented Thatcher-like reforms and have experienced tremendous results. The reforms, as initiated by Margaret Thatcher during her tenure as British Prime Minister (1979 – 1990), have encountered several controversies, although Positive outcomes could be observed in the British economy. She applied her reforms in several areas of the economy and most of her ideas were seen as highly revolutionary. Her emphasis on free enterprises and perfect competition produced remarkable economic growth in the British economy. Promoting free enterprises entails the removal of any sort of barriers, including entry costs, which firms had to face while deciding to enter an industrial sector. In order to assess the effect of any drastic reform meant to promote free enterprises in South Africa at both sectoral and national levels, the present Marshallian model has been fitted by introducing an entry cost (time variant). In fact, the entry price (Γ) is a non-linear combination of all financial requirements that a firm needs to undergo prior to entering the sector. The latter excludes the normal firms operating and or production costs. The model makes use of the National Accounting aggregate ‘other taxes on production’ as a proxy for this entry cost since no other data series is available at this stage. The results obtained from the ISUR estimations show a significant and negative relationship between Γ and sales growth. A higher Γ is also likely to raise inflation, while any policy to lower the entry cost of new firms in the sector increases growth.

Thatcher’s revolution consisted of a series of socio-economic reforms that reshaped the British economy. Margaret Thatcher initiated major changes in the quality of education as well. British parents were given the ability to choose schools for their children using some criteria of quality. That contributed to an increase in the quality of education in Britain since schools needed to compete in order to be selected by parents. At this point, Margaret Thatcher seems to have been a faithful follower of Friedman’s theory on educational voucher in the United States. In this model, any improvement in the quality of education is captured through the rise of z (with better quality of schooling) and ϑ (through higher I) leading to higher sales growth. As mentioned earlier, a negative sign does not directly entail a negative relation, since variables are expressed in growth terms. Linked to the improvement in the quality of education, Thatcher's revolution also included privatisation of the National Health Service increasing the value of z (through higher quality health services) and of ϑ (through higher H). Once again this results into higher sectoral growth.

Additionally, Thatcher's reforms covered a much broader area. They included other components such as: (1) the reduction of trade union influence, rendering the labour market much less rigid; (2) the control of money supply; and (3) a tax-cut for high income groups. The reform initiated concerning tax-cuts was carefully implemented as, later on, a 'poll tax' was introduced in order to ascertain government's ability to act at the local level. The poll tax was directly paid to the local government as a way to improve local service delivery. 'Poll tax' is a tax of fixed and uniform amount to be paid by all individuals. This is a rather unusual form of tax as it is completely different to a percentage income tax (e.g. personal income tax). The word 'poll' refers to head - a per-person tax. In the United Kingdom, the poll tax was first levied in the year 1275 as a 'lay subsidy' in order to collect war funds. Later on, in 1989, under Thatcher's reforms, the poll tax was reinstated to support local government actions in the United Kingdom. The poll tax came up as replacement of the 'rates tax' previously levied based on housing rental value. The implementation of the poll tax underwent drastic resistance from the public since it was perceived as a transfer of the tax burden from the wealthy to the poor. It was paid per head, meaning that poor families who were more likely to have bigger sizes, ended up paying more than wealthier families that have fewer members. However, the abusive usage of fiscal requirements by local government led to stronger resistance against the poll tax. This in turn generated massive and generalised riots in England. In retaliation, Thatcher's government instated draconian measures to prosecute tax evaders. The overall controversy around the payment of poll tax constituted a major determinant in Thatcher's toppling. There is evidence that Thatcher's reforms aimed at making use of the poll tax to maintain adequate resources for the government, since the tax-cut instated earlier had reduced tax revenue significantly. Any policy that considers tax-cuts needs to find accompanying mechanisms to maintain sufficient revenue for the government. In the present model, when it is assumed that disposable income increases due to an income-tax cut, the government's ability to provide public service is presumably not eroded. The government is therefore allowed to make use of alternative ways to finance its budget. With regards to how all these elements of Thatcher's reforms would have successful results in the South African economy, in this model, it is observed that the control of M2 (Money Supply) helps to curb inflation. Any reduction in the growth of personal income tax leads to higher growth in disposable income and a noticeable increase in sectoral sales. This model has been used to assess the reaction of the different sectors toward such reforms.

In addition to all previously mentioned elements of her reforms, Margaret Thatcher also initialised the privatisation of national industries. She clearly expressed her refusal to support the European currency. There is sufficient evidence in these results that Thatcher-like reforms would produce plausible improvements in the South African economy.

As we mentioned earlier, this section focuses on three types of freedom reforms: (1) freeing firms' entry by lowering their cost of entry into the specific sector; (2) a tax-cut on working groups captured by an increase in the national disposable income; and (3) an improvement of labour effectiveness throughout better public investment in education and health. Due to data constraints, our analysis refers to 'other cost on production' as a proxy for entry cost. The tax-cut is simply captured through an overall increase in the national disposable income. This leaves more room of action to policy makers. In fact, due to the complexity of the analysis, we do not suggest a very specific type of tax-cut. We rather allow the policy makers to choose any combination that help raising national disposable income. Also, when we suggest a rise in z, it should be the consequence of an improvement in public policies on health and education (using the appropriate parameters) that is translated into a more effective labour force. Most obviously, any reform that will result in higher labour effectiveness requires a thorough design and implementation process. In this study, we simply point out the outcome of such a reform without necessarily providing the design technicalities behind it.

From our understanding, freeing up the market undeniably induces more competitiveness in different sectors and therefore we may observe an increase in the number of firms seeking to make their employees more productive. The increase in the number of firms in the market as well as an increase in the number of workers will help raise tax revenue that will compensate for the loss incurred by the initial tax cuts. With more money available, the government can invest much more in good health and education programs. The three sets of freedom reforms that we have implemented using this model are interlinked.

3.11.2 Specific focus on the Labour market

The different effects of any Thatcher-like reform that deals with the labour market are now argued. Going back to equation 3.1, the relation that exists between the growth of current prices (inflation) and the growth of expected price plays a crucial role in explaining unions' impact on labour demand. Assuming that both, the current price and the expected price grow at the same rate, the growth in the demand for labour will be positive provided that the growth of sales supply exceeds the growth of wages[30]. Therefore, any action conducted by unions in order to raise wages, should be conducted in accordance with an increase in the sector's sales supply, in order to avoid negative effects on the labour market. Whenever unions require wage hikes without serious consideration of the growth of sales supply, the demand for labour will be hindered. This will most likely result in a slow down of unemployment reduction. Another more plausible assumption would be that expected price grows faster than current price ($\frac{\dot{P}_Q^e}{P_Q^e} > \frac{\dot{P}_Q}{P_Q}$). In this case, the imbalances between the growth in sales supply and the growth in wage rate could be balanced by the difference between the rate of growth of expected price and the rate of growth of current price. Whenever $\frac{\dot{P}_Q^e}{P_Q^e} \approx \frac{\dot{P}_Q}{P_Q}$, the increase in the growth rate of demand for labour is determined by the difference between growth of sales supply and growth of wage rate ($\frac{\dot{S}_S}{S_S} - \frac{\dot{w}}{w}$). Therefore, any Thatcher-like reform restricting unions' demands for wage increases might be seen as a reasonable option to curb unemployment in South Africa.

Additionally, if the elasticity of demand for labour is less than 1 in absolute value, a rise in wages imposed by the unions could raise aggregate income to labour even as unemployment rises. The rise in pay for those who still have jobs might therefore exceed the lost wages of

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[30] This assumption should not be misunderstood. Assuming that both prices grow at the same rate simply means that consumers refer to the current path of inflation to determine the growth in expected price. It does not mean that both prices are equal.

those who lose their job. Also, over the long run the divergence between expected price change and actual price change will be asymptotically unimportant.

Due to lack of reliable data on price expectations, this subsection is limited to describing the potential impact of Thatcher-like reforms on the South African labour market from a theoretical point of view. Further research might involve empirical verification of the theory. The role played by labour unions and other forces in the wage determination process in South Africa has been subject to thorough investigation recorded in the literature (see Du Toit and Koekemoer, 2003). It is important to highlight that several forms of imperfections characterise the South African labour market. Higher wage rates inevitably have some negative impact on the demand for labour. However, higher wage rates could also translate into more disposable income available to raise the level of consumption generating higher/increased sales demand.

South Africa is known for its aggressive union's actions that have often hindered labour market dynamics. Besides overemphasised unions' activities, we can also name other factors that contribute to the slow adjustment of the country's labour market such as: (1) insufficiency of required skilled labour force; (2) disproportionate social benefits for employees and; (3) slow technological growth process. Heavier criticisms have been put forward by several labour economists (Blair, 1998) about the South African Labour Relations Act. The foundations of Hutt's classical liberal version of labour unions are grounded on several principles such as: (1) parties are all legally equal and free to enroll or quit without any pressure (voluntarism); and (2) all actions are conducted with total respect for human rights. In many occasions, strikes organised by unions in South Africa have overlooked these basic principles. In most cases, workers are not given any option but to follow exactly what unions decide. Unions in South Africa have established themselves as the unique providers of a better life for workers while displaying very little consideration for classical liberal principles. There is very little evidence to suggest that unions in South Africa are subject to antitrust laws. In several instances, workers have not been given a real option in the choice of the unions they need to belong to. Workers are often threatened with loss of job whenever they decided not to take part in mass action.

The principle of 'majority rule' drives the labour union system in South Africa. In most cases, workers are forced to belong to a 'majority union' because the employer has set up a 'closed shop' (Blair 1998) and no freedom of association is left for employees.

The South African Labour Relations Act entails that workers are required to use a centralised type of collective bargaining.

3.11.3 Model estimations and predictions (see Appendix B)[31]

Considering the criteria set earlier, such as RMSE and MAE, it can be concluded that the MMM-DA constitutes a valid model for estimating and predicting the South African economy. MMM-DA predicts better than the benchmark (see appendix B.2). Despite the low quality of data used, the model provides plausible outcomes. Some sectors such as: (1) Electricity, Gas and Water; (2) Government; (3) Agriculture; and (4) Financial Services; have dynamics more complex than the simple profit maximising rules, and as a result of this, the results obtained are relatively difficult to understand.

Considering that all these reforms[32] are implemented simultaneously, the South African economy is expected to gain 0.44 percent on the 2006 aggregate sales supply growth (of 4.66 percent). That gives a prediction of 5.1 percent. This constitutes a remarkable improvement if sustained over a long period of time. Supposing that reforms are much stronger and lead to a 10 percent increase in the growth rate of the same variables, a gain of 4.4 percent is expected. That would lift the sales supply growth rate to 8.06 percent.

In fact, after obtaining the individual sectors' growth predictions in reaction to Thatcher-like reforms for the South African economy, it is important to produce some features (see Appendix C) of the national growth of sales supply (GVA). In order to establish accurate aggregate figures using sectoral forecasts, it is necessary to determine the weights carried by each sector on the national level. These weights can be measured using several techniques. In this study, an exponential filter (Holt-

[31] The predictions presented in this sub-section have been obtained with a standard error of **1.28**. The prediction of 8.1 percent therefore ranges from 6.1 percent to 10.1 percent.
[32] A 1 percent shock on: disposable income; labour effectiveness; and entry cost.

Winter, Multiplicative) is used and applied to the series reflecting the sector's sales within the aggregate economy.

In order to assess the prediction ability of the model, the forecasted series (sector's sales supply growth) is compared to actual sales supply growth. That is the underlying concept driving the calculation of either the MAE or the RMSE. The MAE enforces the use of absolute values without any allusion to square losses, while the RMSE considers the square or the error loss function. Different values of either MAE or RMSE can be obtained depending on the sample size or the sector being considered. Sectors with large sales growth rates tend to have larger MAE or RMSE as compared to sectors with smaller sales growth rates. However, the average error obtained in this model accounts for less than 50 percent of the individual sector's sales growth rate. That is a clear indication of the model's ability to make predictions within a reasonable range. Related studies conducted on the US economy (Kim, 2006) have produced very similar percentage errors (average 50 percent). Regarding the early 2000s, the model presents much less forecasting ability, probably due to several unpredicted policy changes that occurred in the South African economy.

	ARLI (3)	MMM-DA (no shrinkage)	MMM-DA (shrinkage)	MCMC (Bayesian)	DMC (Bayesian)
RMSE	2.75	1.61	1.72	1.51	1.39
MAE	2.17	1.28	1.31	1.26	1.18

Table 3.1 Aggregate RMSEs and MAEs obtained from sectors' RMSEs and MAEs[33]

Results presented in table 3.1 represent forecasted RMSEs and MAEs of the total economy obtained by the weighted sum of sectors' RMSEs and MAEs. Forecasts have been obtained over a period of 11 years (1996 – 2006). Also, referring to table

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[33] Formulas:

- $MAE = \frac{1}{T}\sum_{t}^{T}\left|\hat{y}_t - y_t\right|$

- $RMSE = \sqrt{\frac{1}{T}\sum_{t}^{T}(\hat{y}_t - y_t)^2}$

Simulations using MCMC (1500 iterations) and DMC have been performed but the MMM-DA still forecasts substantially better than the ARLI (3).

3.1, it is noticeable that the MMM-DA predicts much better than the benchmark (ARLI (3)) and the use of shrinkage does not seem to lead to much improvement. However, some sectors have presented major forecasting improvements while using shrinkage techniques as opposed to other techniques. The use of shrinkage requires a supportive understanding of the interrelationship that exists among sectors. When diversity is very large, shrinkage may not be the best option to take.

Considering the graphical representation of forecasted series, it is also noticeable that the MMM-DA performs significantly well in forecasting turning points (see figures in appendix B.2).

Besides predictive ability, the usefulness of an approach also depends on its ability to assess effects of policy changes or external shocks (such as to world income.) The MMM approach is superior to ARLI(3) in this regard. On the other hand, for real time forecasting purposes the MMM model seems to be less useful than ARLI(3), as the future values of some of the explanatory variables in the MMM would be unknown.

A set of restrictions has also been tested on the model in order to reduce the number of parameters. The test used (Wald test, see appendix K of chapter 3) suggests that the first two lagged variables of the sectoral sales supply ($S_{i,t-1}$ and $S_{i,t-2}$)) can be assigned a zero coefficient. A similar finding was obtained in an earlier study (Zellner, 2003).

3.12 CONCLUSION, STUDY LIMITATION AND FUTURE RESEARCH

The present chapter has mainly focused on the improvement of the forecasting process through disaggregation and the use of shrinkage estimations. In this chapter, some policy experiments, mainly Thatcher-like reforms, have been successfully applied to a disaggregated Marshallian Model of the South African economy. The main conclusions drawn from this exercise suggest that such reforms are most likely to produce remarkable improvements in the country’s growth pattern. If carefully implemented, reforms in the impact that unions have on the economy, coupled with freer sectors production activities (less entry requirements) and higher labour productivity are expected to raise the South African growth rate as high as 8.1 percent. This chapter provides clear evidence that a Marshallian Model constitutes a useful tool for understanding and predicting a country’s growth through sectoral production activities. Despite the fact that the quality of some of the sectors' series remains questionable, first approximations can be taken from a full-fledged Marshallian Model for the South African economy. The empirical approach used in this chapter in terms of Marshallian macroeconomic techniques and the shrinkage estimations is not without flaws. In some cases, as this chapter suggests, the use of shrinkage estimations does not always produce improvement in the model's prediction ability considering the fact that sectors often have disparate dynamics. However, the MMA-DA when carefully used, can be valuably introduced to guide output growth forecasting in multiple output sectoral units, as long as the data used in the analysis and the vector means is representative of the production process and can be compared to appropriate peer production units. In addition, restrictions can validly be imposed on the first two lags of the MMM-DA without affecting the results. This conclusion has been supported in previous studies (Zellner *et al*, 2002). In addition to the existing literature, the present chapter has provided some extensions to the Marshallian modelling process by introducing: (1) an entry cost; (2) the human capital aspect of production (labour effectiveness); (3) a broader aspect of the sales demand function (including more household dynamics); and (4) the foreign sector (introducing international shifters such as world income).

This portion of the thesis opens new horizons for further research in forecasting models including more detailed leading indicators and probably deeper disaggregation (i.e. regional disaggregation).

Possible further expansions could also include features such as: (1) the dynamic of inventories; (2) the capital market; (3) the entry of new sectors (Schumpeterian innovations); (4) the distinction between skilled and unskilled labour force; (5) the use of a generalised production function instead of restricting the process to the use of a Cobb-Douglas production function; etc. The latter expansions might increase the prediction ability of the model further. In connection with earlier discussion (see section 3.6.1) on the optimality of a maximum likelihood estimator iterated while using ISUR, when the loss function is defined as a zero-one loss function, the modal value obtained using ISUR is optimal. A zero-one loss function seems very appropriate and appealing in the policy making process since policymakers act according to specific targets. When the target is missed, there is a big loss, and vice versa. However, in some further studies this issue will be addressed more thoroughly.

CHAPTER 4: GENERAL CONCLUSION

4.1 RESTATING THE OBJECTIVE OF THE STUDY

Consistent and comprehensive macroeconometric modelling is required for the improvement of several facets of modern society in both developing and developed nations. Most obvious are the direct effects of socio-economic indicators in a country's economic growth pattern. Also clear is the impact of specific reforms on economic growth enhancement. Further, there are heavy technical requirements that need to be accurately met before a macroeconometric model receives appropriate credentials from policy makers. Among many others details, academics usually assess the performance of a macroeconometric model looking at: (1) its level of comprehensiveness for the different facets of the studied economy; (2) the appropriateness of its specification; and most importantly, (3) its prediction ability. A model that predicts well is more trustworthy and therefore more recommendable. The generic criteria used for prediction ability are the forecasted RMSE as well as the forecasted MAE.

Simply stated, the main objective of the present study was to provide a better understanding of the working of: (1) social interventions (ingredients) in macroeconomics; (2) conditional convergence across economic sectors; and (3) impact of firms' dynamics (entry/exit) as well as disaggregation in improving macroeconometric models' prediction ability; using South African data. It is our goal to have this new approach considered as a piece of scholarship by macroeconomists while it captivates the attention of policy makers.

4.2 DETAILED OUTLINE OF THE STUDY

This study was developed through two main interlinked papers or chapters. The first paper (chapter 2) discussed the use and the size of social predicaments – health and education – and, mainly due to constraints related to employment data, the results are reported for only four South African economic sectors. Also, in this chapter the process of conditional convergence across the ten sectors of the South African economy is examined.

The second paper (chapter 3) constituted the core of the study since it includes the complete development and policy usage of the main model, the MMM-DA. This model made use of features on human capital – labour effectiveness – that have been discussed in the previous chapter and help understanding the role played by firms' entry/exit into our modelling exercise. Transfer equations, disaggregated by economic sector, were used to assess the model's prediction ability by comparing it to its benchmark using different estimation techniques. Concomitantly, a description of the economic effects of a set of freedom reforms on the economic growth pattern of the South African economy was provided.

4.3 BASIC MODEL OUTLINE AND METHODOLOGY

The main objective of this study was to develop a core macroeconometric model, the MMM-DA, with a set of interrelated models, that: (1) includes human capital components and therefore describes the size of human ingredients in the South African economy; (2) is properly disaggregated across 10 South African economic sectors; (3) encompasses firms dynamics – entry and exit; (4) has a strong prediction ability at both sectoral and national level; and (5) is usable in assessing the impact of policy reforms at both sectoral and national level.

In chapter 2, we made use of the 'Iterative Seemingly Unrelated' equations systems with other smoothing techniques such as the Holt Winters Exponential filters in order to estimate labour effectiveness through sectoral production functions. Additionally, different features of the conditional convergence process of South African economic sectors were explained using fixed effects model.

However, the core model was developed in the third chapter where a Marshallian Macroeconometric model disaggregated by economic sectors was developed. The use of disaggregating process in this MMM is justifiable when we consider sectors' differentials prevailing in the South African economy. Also, as it was emphasized throughout this thesis, aggregate models are unable to analyze detailed policy shocks and entail major losses of relevant information. This leads to inaccuracy of policy recommendations made. Improvement effects of disaggregation have been captured in previous studies as measured by reduced MAEs and RMSEs.

In addition to its improved prediction ability due to the use of disaggregation, the MMM-DA captures firms' entry/exit movements and it includes human capital components. Besides its strong macroeconomic specification based on Marshall's theory of firms' entry and exit, the MMM-DA is statistically built on an AR(3) process with leading indicators: the ARLI(3). The estimations of the model were run using Bayesian and Non-Bayesian techniques. As part of the Bayesian techniques, this thesis provided a comparative analysis between the MCMC technique and the DMC technique using RMSEs and MAEs even though both MCMC and DMC are computational techniques which should produce similar results, *ceteris paribus.*

4.4 SUMMARY OF KEY FINDINGS

In chapter 2, a set of sectoral growth equations were estimated using ISUR. With the exception of Agriculture, all sectoral growth regressions were well behaved and the levels of significance of the obtained coefficients did not differ much from *a priori* expectations.

Using a fixed effects model, the size of the parameters measuring the impact of public expenditures on health and schooling in the sectoral growth functions was computed. As expected, it has been observed that any increase in public expenditures for health and education induces higher output growth. However the magnitude of these effects differs from one sector to another. For example, Manufacturing and Construction has recorded the largest impacts. They were followed by Mining and Transport as well as Communication.

In chapter 3, results obtained from the forecasted RMSEs and MAEs provided evidence that the MMM-DA forecasts much better than its benchmark, the ARLI(3). Also, as part of the Bayesian techniques considered, the DMC is more recommendable as it is more direct while producing relevant estimates.

Considering the use of the MMM-DA to assess a set of freedom reforms, the results indicated that institution of these policy reforms would result in a real GVA growth rate of 8.1% with a standard error of 1.28 percentage points. The freedom reforms considered included: (1) freeing up barriers to firms' and workers' abilities to start up new firms and to obtain new employment; (2) tax-cut for the middle class; and (3) health and educational programs that free individuals from poor health and ignorance, thereby enhancing their productivity. The use of disaggregation in our modelling exercise has enabled us to see how these set of reforms affect sectors individually and therefore more accurate policy recommendations can be extracted from this thesis.

4.5 AREAS OF FUTURE RESEARCH

This study opens new horizons for further research in forecasting models including more detailed leading indicators and probably deeper disaggregation (i.e. regional disaggregation). One can validly consider expanding this modelling exercise further by including more features to the research. This include additions such as: (1) the dynamics of inventories; (2) the market for capital with different levels of quality; (3) the entry of new sectors (Schumpeterian innovations); (4) the distinction between skilled and unskilled labour force; (5) the use of a generalised production function instead of restricting the process to the use of a Cobb-Douglas production function; etc; which may increase the prediction ability of the model.

As it is largely emphasized throughout this research, using further disaggregation, one not only avoids the negative effects of ‘aggregation biases’ of the sort that Theil and many others emphasised, but also gets the gain of extra precision in estimation and prediction. Added precision is gained not only in predicting outcomes for individual sectors, that is impossible with aggregate data, but also extra precision in predicting ‘totals’ or macroeconomic variables as shown theoretically and empirically in some more recent papers, with or without shrinkage.

Also, an interesting way of implementing reforms will be to consider different sets of allocations and observe how well the final result could be improved due to a more optimal decision. For example, instead of reducing entry cost for sectors that are naturally regulated monopolies, this money can be used to further reduce entry cost of more open sectors. As it was realised earlier, sectors’ differentials include different types of reactions that sectors might have when it comes to improving labour effectiveness. Capital intensive sectors will react differently than labour intensive sectors. It is therefore relevant to reallocate funds by shifting them from where they are less required toward more demanding areas. The issue of allocation has been discussed extensively in the literature. Most obviously, any reform that will result in higher labor effectiveness requires a thorough design and implementation process. In this thesis, we were limited to simply pointing out at the outcome of such reforms without necessarily providing the technicalities behind. It is also worth mentioning the fact that the success of a reform is associated with a certain probability. Behind the scene, policy makers have to be adamant when selecting the correct type of reform. Also, the maturity of the reform most often carries several uncertainties that need to be accounted for. Improving labor effectiveness could be the results of different types of combined reforms. One may think of a direct assistance to firms in providing more training and better health to their employees throughout direct or indirect subsidies from the government. For example, firms that provide adequate training programs and offer appropriate health packages may qualify for tax rebates. Alternatively, the reform on labor effectiveness may also be more household-based or community-based. Here the reform may target public schools and public hospitals looking at high-quality efficiency ratios.

From our understanding, freeing up the market undeniably induces more competitiveness in different sectors and therefore we can observe an increase of firms seeking to make their employees more productive. Also, more competitive firms in the sectors help increasing tax revenue. With more money available, the government can invest much more in qualitative health and education.

The three sets of freedom reforms that we have implemented using this model are interlinked. Freeing up the market using lower entry cost helps raising the number of firms operating in the sector. More firms translate into more employees and therefore higher disposable income. Also, firms becoming more competitive heave the incentive for firms to provide appropriate training to their employees that make them more cost effective. Further research on this issue is therefore highly recommendable.

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APPENDIX TO CHAPTER 2

Additional figures and tables containing the data analysis of effective labour are included in this appendix. Using the fitted values for labour effectiveness z , as described in table 2.1 and in figure 2.2 in the main text, a new series of effective labour per sector (EL or zL) has been computed and the computed series are included in sectoral growth equations. These estimation results are listed in tables 2.5 to 2.21.

Effective labour

Figure 2.4: The long-run residual (z_2 as dependent variable)

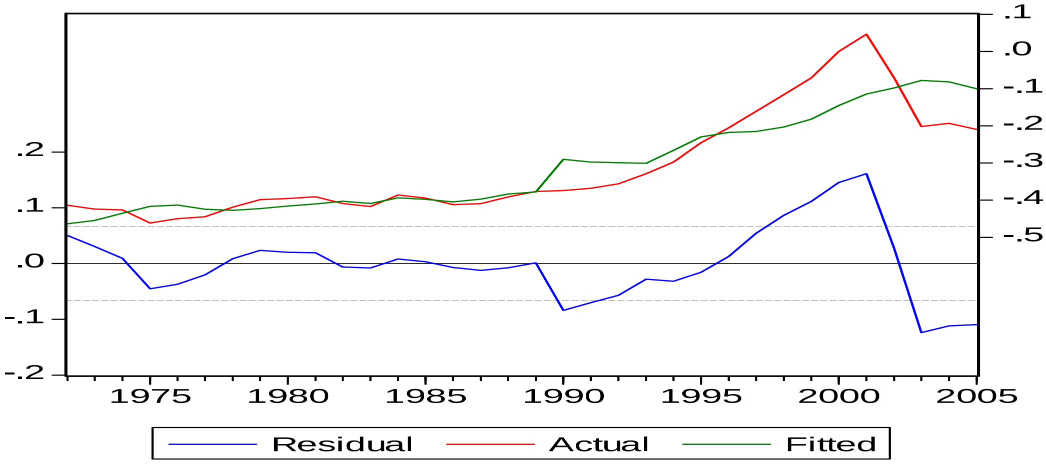


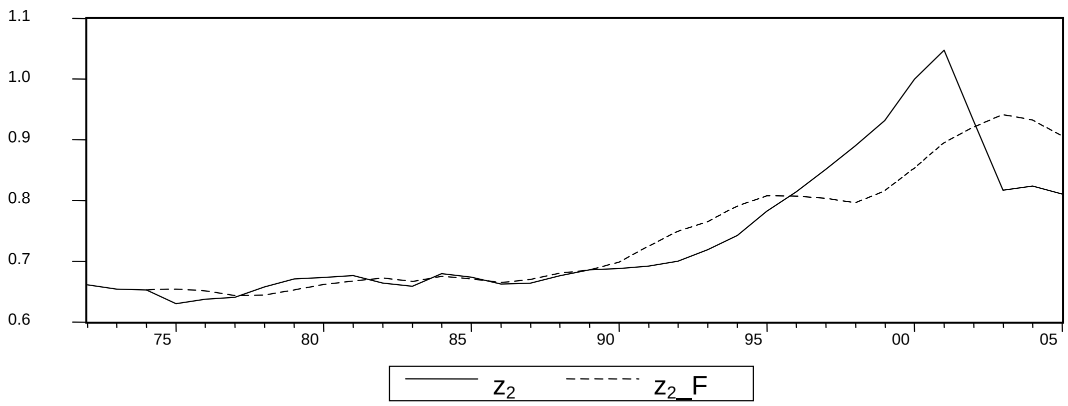
Table 2.5: Cointegration test (z_2 as dependent variable)

Augmented Dickey-Fuller test statistic	**-3.221096****
1% level	-3.653730
5% level	-2.957110
10% level	-2.617434

Table 2.6: Error correction model

Variable	Coefficient	Std. Error	t-Statistic
Difference: D(h)	0.000368	0.000215	1.712202
Difference: D(h(-1))	-0.000228	0.000210	-1.084469
Difference: D(LNz_2(-1))	0.668374	0.170260	3.925610
Difference: D(DUMEDU)	5.76E-05	0.000116	0.495495
Residual(-1)	-0.365407	0.128755	-2.837997
C	-0.000301	0.008363	-0.036005

Figure 2.5: Actual versus Fitted: Model z_2



Sectoral growth equations

Agriculture

Table 2.7: Long-run regression of LNYAGRIC (Log of agricultural output) on capital (LNKAGRIC) and effective labour (LNELAGRIC_SM)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LNKAGRIC	3.967104	2.950402	1.344598	0.2117
LNELAGRIC_SM	24.76007	10.70353	2.313262	0.0460
C	-369.0831	175.9170	-2.098052	0.0653
R-squared	*0.525614*	*Mean dependent var*		*10.17638*
Adjusted R-squared	*0.420195*	*S.D. dependent var*		*0.087889*
Log likelihood	*17.14934*	*F-statistic*		*4.985942*
Durbin-Watson stat	*1.647653*	*Prob(F-statistic)*		*0.034881*

Figure 2.6: Long-run residual (Agriculture)

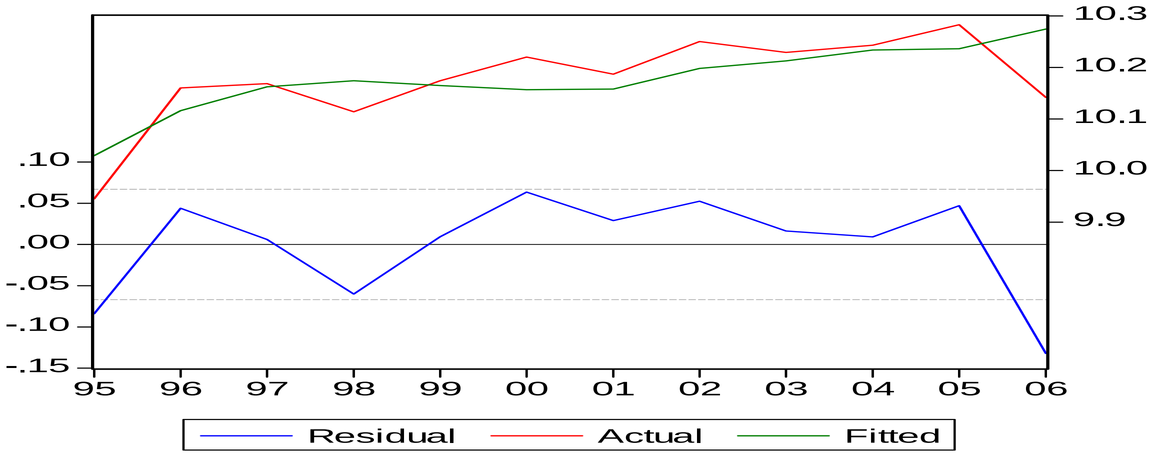


Table 2.8: Cointegration test (Agriculture)

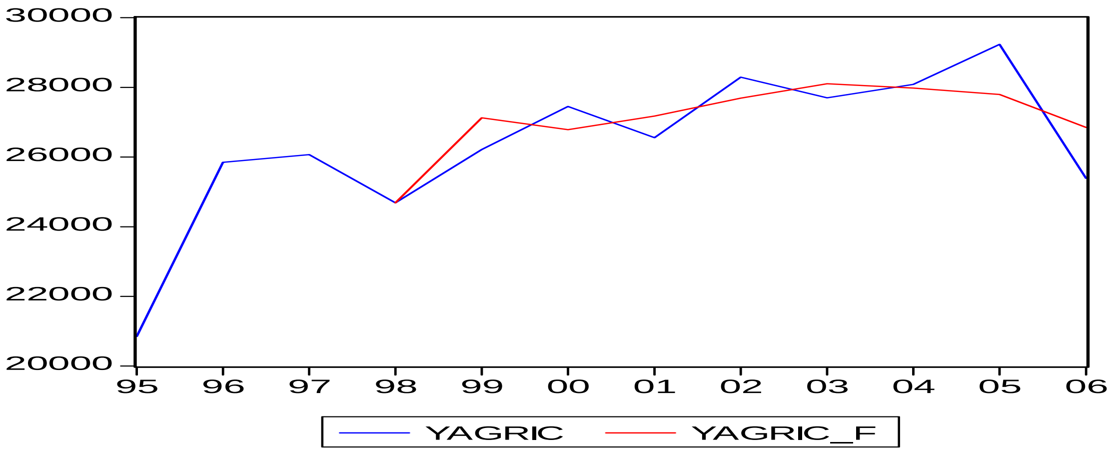
ADF Test Statistic	-1.742080	1% Critical Value*	-4.3260
		5% Critical Value	-3.2195
		10% Critical Value	-2.7557

*MacKinnon critical values for rejection of hypothesis of a unit root.

Table 2.9: Error correction model (Agriculture)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LNYAGRIC(-1))	-0.822767	0.115140	-7.145789	0.0056
D(LNYAGRIC(-2))	-0.359423	0.080557	-4.461708	0.0210
D(LNELAGRIC_SM(-1))	6.42E+08	1.98E+08	3.241459	0.0478
D(LNELAGRIC_SM(-2))	-6.40E+08	1.97E+08	-3.241428	0.0478
RESIDAGRIC(-1)	-0.135101	0.172320	-0.784010	0.4902
C	-1266.425	388.0274	-3.263752	0.0470
R-squared	*0.978361*	*Mean dependent var*		*0.003281*
Adjusted R-squared	*0.942295*	*S.D. dependent var*		*0.041757*

Figure 2.7: Actual (YAGRIC) versus Fitted (YAGRIC_F)



Mining

Table 2.10: Long-run regression of LNYMIN (Log of Mining Output) on capital (LNKMIN) and effective labour (LNELMIN_SM)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LNKMIN	0.735996	0.204247	3.603461	0.0057
LNELMIN_SM	0.094988	0.054368	1.747144	0.1146
C	1.036020	2.804829	0.369370	0.7204
R-squared	*0.592706*	*Mean dependent var*		*11.08793*
Sum squared resid	*0.003989*	*Schwarz criterion*		*-4.549978*
Log likelihood	*31.02723*	*F-statistic*		*6.548518*
Durbin-Watson stat	*0.671088*	*Prob(F-statistic)*		*0.017563*

Figure 2.8: Long-run residual (Mining)

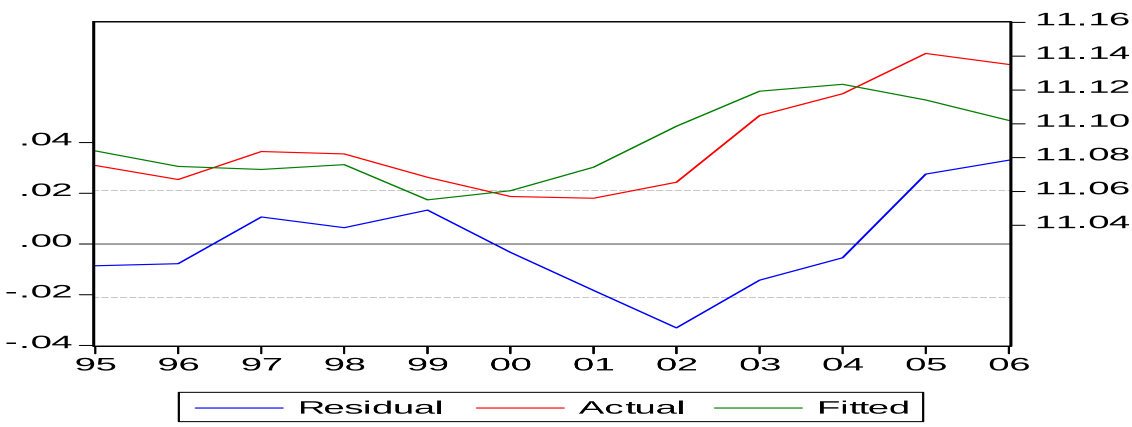


Table 2.11: Cointegration test (Mining)

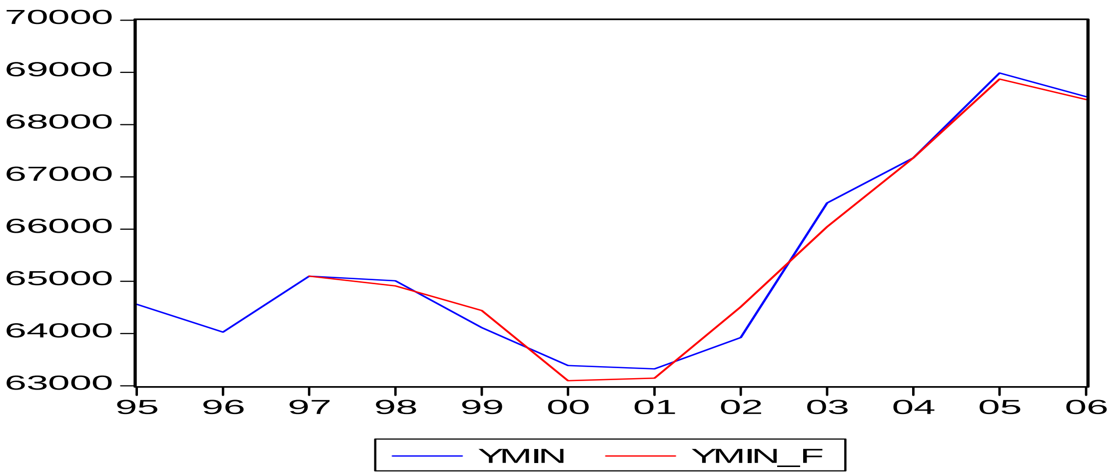
ADF Test Statistic	-1.720712	1% Critical Value*	-4.3260
		5% Critical Value	-3.2195
		10% Critical Value	-2.7557

*MacKinnon critical values for rejection of hypothesis of a unit root.

Table 2.12: Error correction model (Mining)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LNYMIN(-1))	0.361318	0.130047	2.778365	0.0321
D(LNKMIN(-1))	-1.010303	0.296536	-3.407020	0.0144
RESIDMIN(-1)	-1.306875	0.193425	-6.756483	0.0005
C	0.008196	0.002590	3.164653	0.0195
R-squared	*0.906847*	*Mean dependent var*		*0.006803*
Adjusted R-squared	*0.860270*	*S.D. dependent var*		*0.016872*
Log likelihood	*39.02587*	*F-statistic*		*19.47004*
Durbin-Watson stat	*3.177162*	*Prob(F-statistic)*		*0.001705*

Figure 2.9: Actual (YMIN) versus Fitted (YMIN_F)



Construction & Buildings

Table 2.13: Long-run regression of LNYCONSTR (Log of Construction & Buildings Output) on capital (LNKCONSTR) and effective labour (LNELCONSTR_SM)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LNKCONSTR	0.706290	0.043565	16.21225	0.0000
LNELCONSTR_SM	0.349187	0.105169	3.320247	0.0089
C	-0.928966	1.408352	-0.659612	0.5260
R-squared	*0.967806*	*Mean dependent var*		*10.07238*
Adjusted R-squared	*0.960652*	*S.D. dependent var*		*0.186337*
Sum squared resid	*0.012296*	*Schwarz criterion*		*-3.424288*
Durbin-Watson stat	*1.666631*	*Prob(F-statistic)*		*0.000000*

Figure 2.9: Long-run residual (Construction & Buildings)

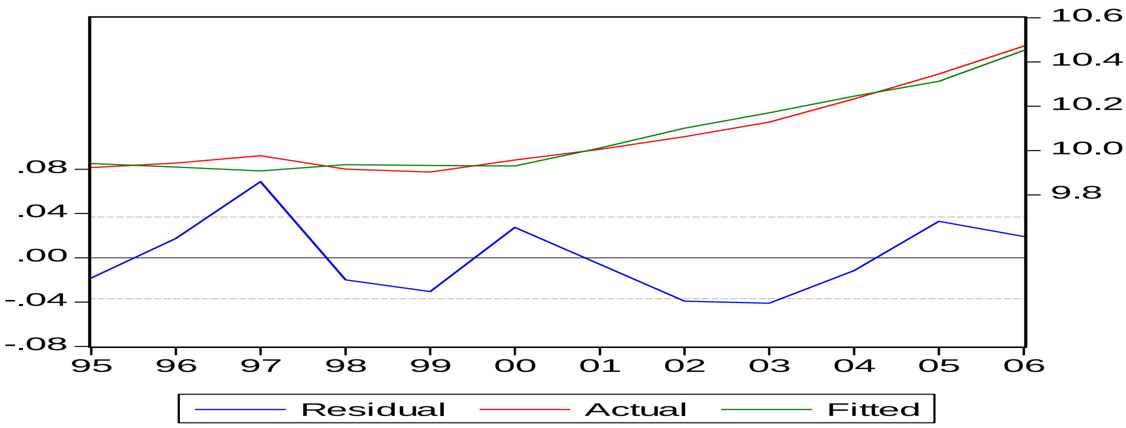


Table 2.14: Cointegration test (Construction & Buildings)

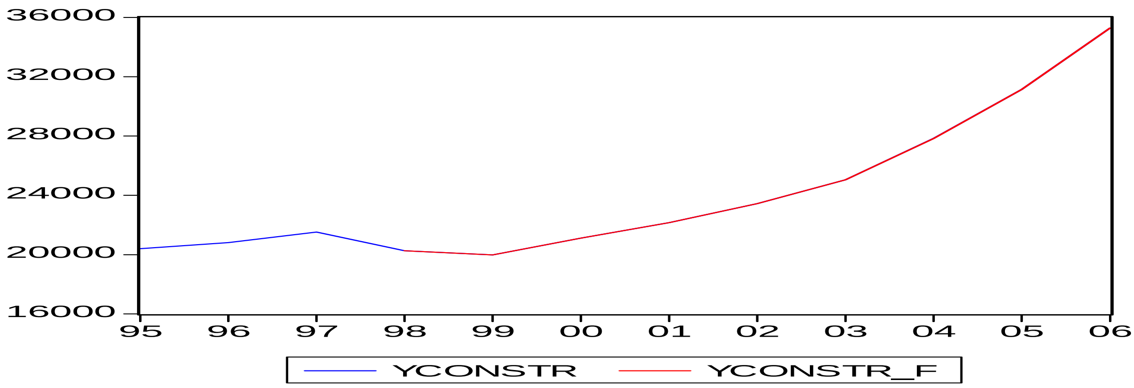
ADF Test Statistic	-3.213112	1% Critical Value*	-4.3260
		5% Critical Value	-3.2195
		10% Critical Value	-2.7557

*MacKinnon critical values for rejection of hypothesis of a unit root.

Table 2.15: Error correction model (Construction & Buildings)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LNYCONSTR(-1))	-0.284763	0.008789	-32.39947	0.0196
D(LNKCONSTR)	0.934876	0.017633	53.01987	0.0120
D(LNKCONSTR(-1))	0.157567	0.010743	14.66754	0.0433
D(LNKCONSTR(-2))	1.363403	0.014705	92.71793	0.0069
D(LNELCONSTR_SM)	-0.120158	0.003464	-34.69203	0.0183
D(LNELCONSTR_SM(-1))	-0.091734	0.002886	-31.78122	0.0200
RESIDCONSTR(-1)	0.123374	0.013700	9.005178	0.0704
C	-0.094840	0.000966	-98.13191	0.0065
R-squared	*0.999996*	*Mean dependent var*		*0.054892*
Adjusted R-squared	*0.999968*	*S.D. dependent var*		*0.060204*
Sum squared resid	*1.17E-07*	*Schwarz criterion*		*-13.36311*
Log likelihood	*68.92291*	*F-statistic*		*35254.85*
Durbin-Watson stat	*2.760296*	*Prob(F-statistic)*		*0.004101*

Figure 2.10: Actual (YCONSTR) versus Fitted (YCONSTR_F)



Transport & Communication

Table 2.16: Long-run regression of LNYCOMTRS (Log of Transport & Communication Output) on capital (LNKCOMTRS) and effective labour (DUMELCOMTRS)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LNKCOMTRS	2.439538	0.224226	10.87983	0.0000
DUMELCOMTRS	0.006470	0.002751	2.351630	0.0432
C	-19.82394	2.842835	-6.973300	0.0001
R-squared	*0.983899*	*Mean dependent var*		*11.32851*
Adjusted R-squared	*0.980321*	*S.D. dependent var*		*0.224358*
Log likelihood	*26.20209*	*F-statistic*		*274.9807*
Durbin-Watson stat	*1.285933*	*Prob(F-statistic)*		*0.000000*

Figure 2.11: Long-run residual (Transport & Communication)

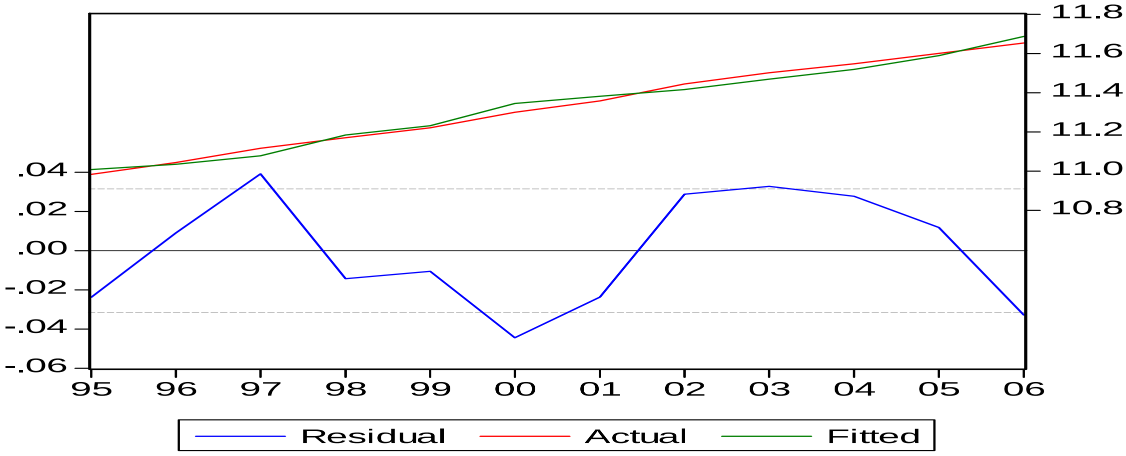


Table 2.17: Cointegration test (Transport & Communication)

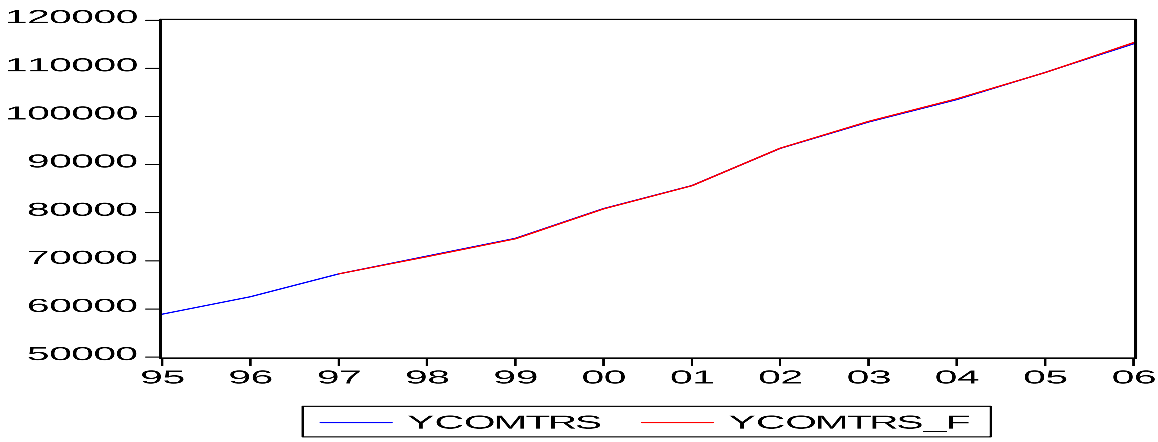
ADF Test Statistic	-2.356290	1% Critical Value*	-4.3260
		5% Critical Value	-3.2195
		10% Critical Value	-2.7557

*MacKinnon critical values for rejection of hypothesis of a unit root.

Table 2.18: Error correction model (Transport & Communication)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LNKCOMTRS)	0.170938	0.086959	1.965723	0.1208
D(LNKCOMTRS(-1))	-1.121195	0.077685	-14.43263	0.0001
D(DUMELCOMTRS)	0.000420	0.000182	2.304277	0.0825
D(DUMELCOMTRS(-1))	-0.003542	0.000254	-13.93497	0.0002
RESIDCOMTRS(-1)	-0.594360	0.042790	-13.89021	0.0002
C	0.087225	0.002241	38.92403	0.0000
R-squared	*0.991675*	*Mean dependent var*		*0.060991*
Adjusted R-squared	*0.981270*	*S.D. dependent var*		*0.013717*
Log likelihood	*53.17145*	*F-statistic*		*95.30140*
Durbin-Watson stat	*1.386433*	*Prob(F-statistic)*		*0.000301*

Figure 2.12: Actual (YCOMTRS) versus Fitted (YCOMTRS_F)



Manufacturing

Table 2.19: Long-run regression of LYMAN (Log of Manufacturing Output) on capital (LNKMAM) and effective labour (LNELMAM_SM)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LNKMAN	0.783062	0.460422	1.700750	0.1232
LNELMAN_SM	0.352829	0.110842	3.183161	0.0111
C	-1.855128	4.719687	-0.393062	0.7034
R-squared	*0.908193*	*Mean dependent var*		*11.98385*
Adjusted R-squared	*0.887792*	*S.D. dependent var*		*0.101936*
Sum squared resid	*0.010494*	*Schwarz criterion*		*-3.582802*
Log likelihood	*25.22417*	*F-statistic*		*44.51591*
Durbin-Watson stat	*1.096677*	*Prob(F-statistic)*		*0.000022*

Figure 2.13: Long-run residual (Manufacturing)

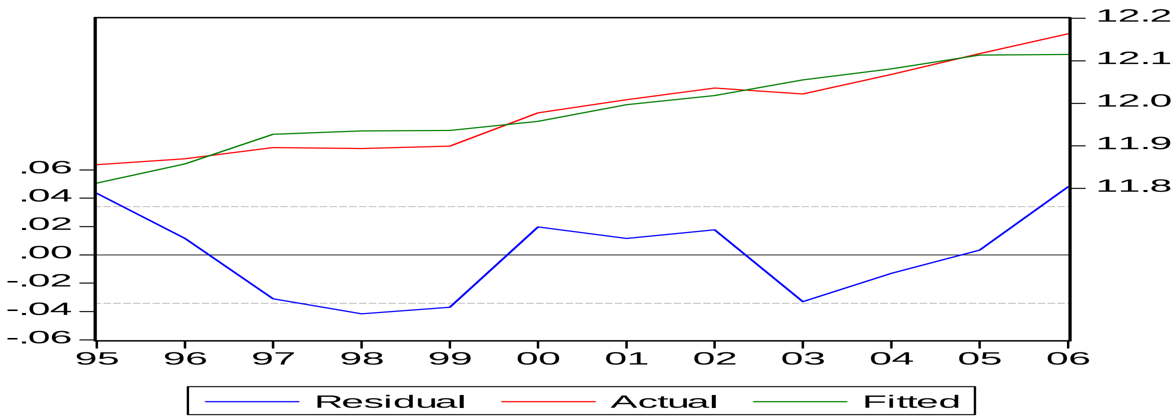


Table 2.20: Cointegration test (Manufacturing)

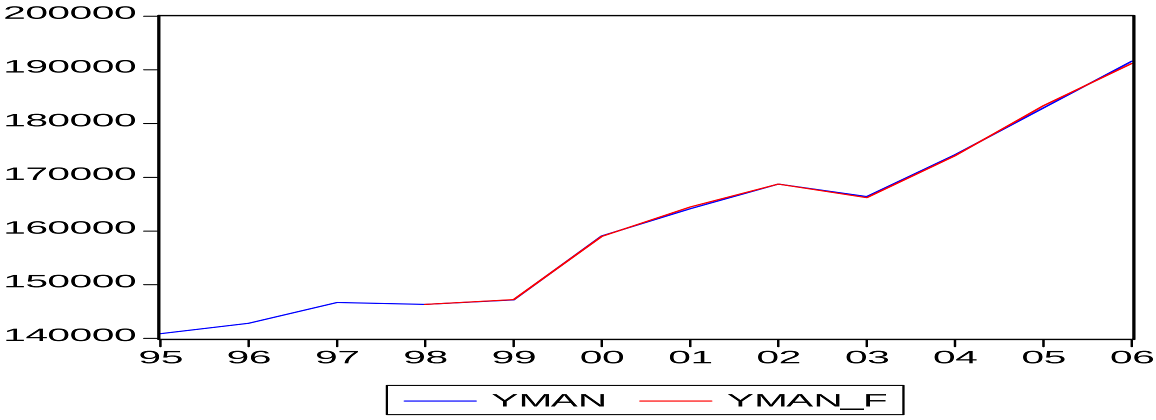
ADF Test Statistic	-2.362856	1% Critical Value*	-4.3260
		5% Critical Value	-3.2195
		10% Critical Value	-2.7557

*MacKinnon critical values for rejection of hypothesis of a unit root.

Table 2.21: Error correction model (Manufacturing)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LNKMAN)	-0.586090	0.269826	-2.172106	0.1620
D(LNKMAN(-1))	2.580581	0.402524	6.410996	0.0235
D(LNKMAN(-2))	-3.287029	0.264823	-12.41218	0.0064
D(LNELMAN_SM(-1))	-0.216757	0.024414	-8.878537	0.0124
D(LNELMAN_SM(-2))	-0.384598	0.024378	-15.77668	0.0040
RESIDMAN(-1)	-0.711648	0.061717	-11.53074	0.0074
C	0.074911	0.003101	24.15642	0.0017
R-squared	*0.996434*	*Mean dependent var*		*0.029692*
Adjusted R-squared	*0.985735*	*S.D. dependent var*		*0.029034*
Sum squared resid	*2.41E-05*	*Schwarz criterion*		*-8.285723*
Log likelihood	*44.97604*	*F-statistic*		*93.13303*
Durbin-Watson stat	*2.616651*	*Prob(F-statistic)*		*0.010661*

Figure 2.14: Actual (YMAN) versus Fitted (YMAN_F)



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APPENDICES TO CHAPTER 3

APPENDIX A: Deriving the RFE (Continuous time)

Considering the three equations: (1) sales supply; (2) sales demand; and (3) entry/exit; the final RFE-DA have been derived as follows:

$$\frac{\dot{S}_S}{S_S} = \theta_1 \frac{\dot{A}}{A} + \left(\frac{\dot{\Phi}}{\Phi} + \frac{\dot{N}}{N} \right) + \theta_2 \frac{\dot{P}_Q}{P_Q} + \theta_3 \frac{\dot{w}}{w} + \theta_4 \frac{\dot{r}}{r} + \sum_{l=1}^{T} \sigma_l \frac{\dot{P}_l}{P_l} + \theta_5 \frac{\dot{z}}{z} + \theta_6 \frac{\dot{\Gamma}}{\Gamma}$$

(2.2.4)

$$\frac{\dot{S}_D}{S_D} = (1-\Delta)\frac{\dot{P}_Q}{P_Q} + \frac{(\dot{BD})}{(BD)} + (\lambda_1 + \Delta)\frac{\dot{P}_Q^e}{P_Q^e} + \chi_{j1}\frac{\dot{Y}_d}{Y_d} + \chi_{j2}\frac{\dot{WY}}{WY} \qquad (2.3.4)$$

$$\frac{\dot{N}}{N} = C_E (S_S - \pi^e) \qquad (2.6.1)$$

Equating equations 2.2.4 and 2.3.4:

$$\theta_1 \frac{\dot{A}}{A} + \left(\frac{\dot{\Phi}}{\Phi} + \frac{N(\dot{\Gamma})}{N(\Gamma)} \right) + \theta_2 \frac{\dot{P}_Q}{P_Q} + \theta_3 \frac{\dot{w}}{w} + \theta_4 \frac{\dot{r}}{r} + \sum_{l=1}^{T} \sigma_l \frac{\dot{P}_l}{P_l} + \theta_5 \frac{\dot{z}}{z} + \theta_6 \frac{\dot{\Gamma}}{\Gamma} = (1-\Delta)\frac{\dot{P}_Q}{P_Q} + \left(\frac{\dot{\mathrm{B}}}{\mathrm{B}} + \frac{\dot{D}}{D} \right)$$

$$+ (\lambda_1 + \Delta)\frac{\dot{P}_Q^e}{P_Q^e} + \chi_1 \frac{\dot{Y}_d}{Y_d} + \chi_2 \frac{\dot{WY}}{WY}$$

(A.1)

Replacing $\frac{\dot{N}}{N}$ in equation A.1 by equation 2.6.1:

$$\frac{\dot{P}_Q}{P_Q} = \frac{C_E}{\theta_2 - 1 + \Delta}.\pi^e - \frac{C_E}{\theta_2 - 1 + \Delta}.S_S + \frac{1}{\theta_2 - 1 + \Delta}.\frac{\dot{D}}{D} + \frac{(\lambda_1 + \Delta)}{\theta_2 - 1 + \Delta}.\frac{\dot{P}_Q^e}{P_Q^e} - \frac{\theta_1}{\theta_2 - 1 + \Delta}.\frac{\dot{A}}{A} - \frac{\theta_3}{\theta_2 - 1 + \Delta}.\frac{\dot{w}}{w}$$

$$- \frac{\theta_4}{\theta_2 - 1 + \Delta}.\frac{\dot{r}}{r} - \frac{\sum_{l=1}^{T}\sigma_l}{\theta_2 - 1 + \Delta}.\frac{\dot{P}_l}{P_l} - \frac{\theta_5}{\theta_2 - 1 + \Delta}.\frac{\dot{z}}{z} - \frac{\theta_6}{\theta_2 - 1 + \Delta}.\frac{\dot{\Gamma}}{\Gamma} + \frac{\chi_1}{\theta_2 - 1 + \Delta}.\frac{\dot{Y}_d}{Y_d} + \frac{\chi_2}{\theta_2 - 1 + \Delta}.\frac{\dot{WY}}{WY}$$

(A.2)

Plugging A.2 and 2.6.1 into 2.2.4can be written in a simplified form with equation 2.3.4 plugged into equation 2.2.4 in order to obtain the following RFE-DA for price and sales supply:

$$\frac{\dot{S}_{Si}}{S_{Si}} = \theta'_{0i} + \theta'_{1i} S_{Si} + \theta'_{2i}\frac{\dot{D}_i}{D_i} + \theta'_{3i}\frac{\dot{A}_i}{A_i} + \theta'_{4i}\frac{\dot{w}_i}{w_i} + \theta'_{5i}\frac{\dot{r}}{r} + \theta'_{6i}\frac{\dot{z}_i}{z_i} + \theta'_{7i}\frac{\dot{Y}_d}{Y_d} + \theta'_{8i}\frac{\dot{WY}}{WY} + \theta'_{9i}\frac{\dot{\Gamma}}{\Gamma} + \sum_{l=1}^{T}\wp_{il}\frac{\dot{P}_{il}}{P_{il}}$$

$$\frac{\dot{P}_{Qi}}{P_{Qi}} = \sigma'_{0i} + \sigma'_{1i} S_{Si} + \sigma'_{2i}\frac{\dot{D}_i}{D_i} + \sigma'_{3i}\frac{\dot{A}_i}{A_i} + \sigma'_{4i}\frac{\dot{w}_i}{w_i} + \sigma'_{5i}\frac{\dot{r}}{r} + \sigma'_{6i}\frac{\dot{z}_i}{z_i} + \sigma'_{7i}\frac{\dot{Y}_d}{Y_d} + \sigma'_{8i}\frac{\dot{WY}}{WY} + \sigma'_{9i}\frac{\dot{\Gamma}}{\Gamma} + \sum_{l=1}^{T}\wp_{il}\frac{\dot{P}_{il}}{P_{il}}$$

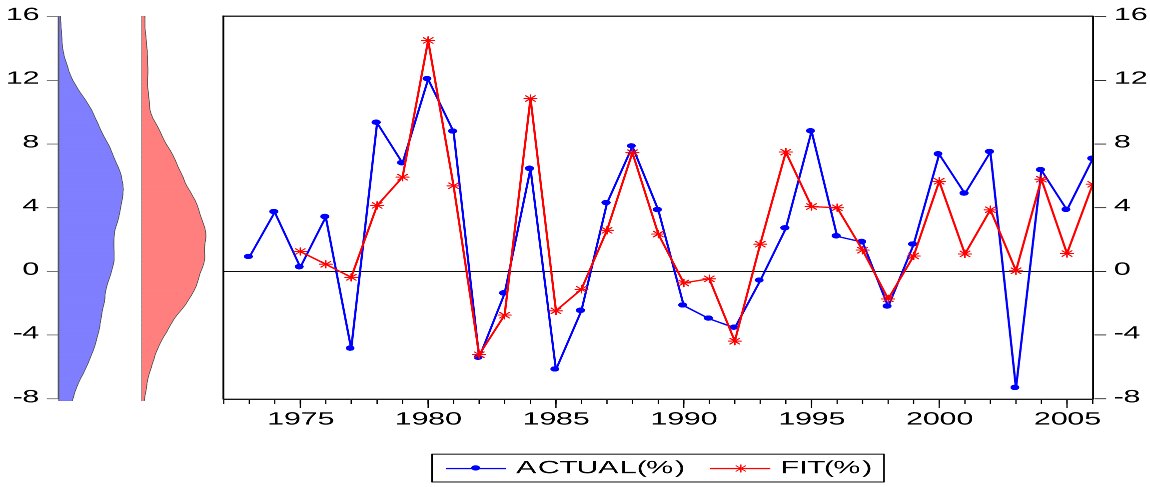
with: $\sum_{l=1}^{T}\wp_{il}\frac{\dot{P}_{il}}{P_{il}} = \frac{(\lambda_1+\Delta)}{\theta_2 - 1 + \Delta}.\frac{\dot{P}^e_{Q_{it}}}{P^e_{Q_{it}}} - \frac{\sum_{l=1}^{T}\sigma_l}{\theta_2 - 1 + \Delta}.\frac{\dot{P}_l}{P_l}$

APPENDIX B

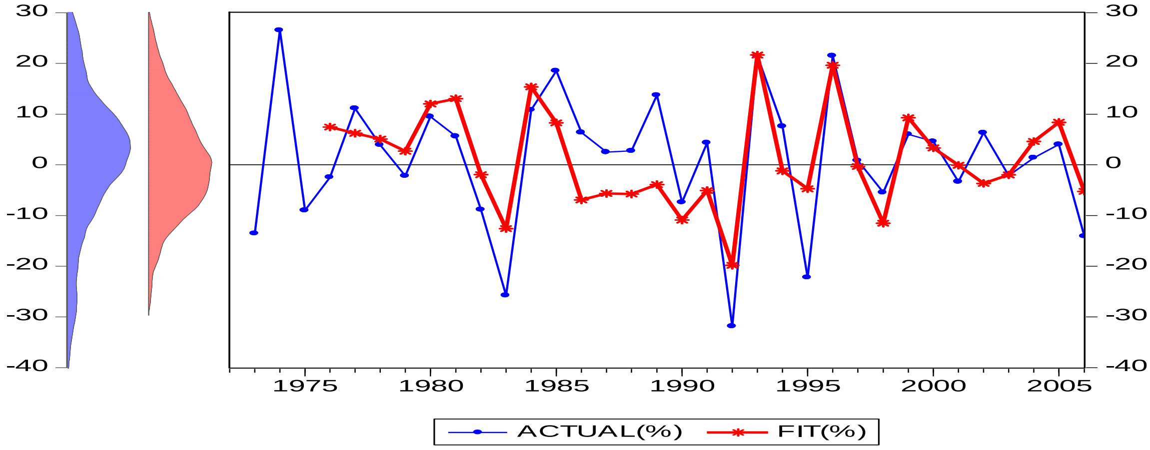
B.1. Model fitness results

All graphs represent ‘actual’ versus ‘fitted’ series of sectoral sales growth: $\ln\left(\frac{S_{i,t}}{S_{i,t-1}}\right)$; including the Kernel Density Estimation at the left. The sample size ranges from 1972 to 2006.

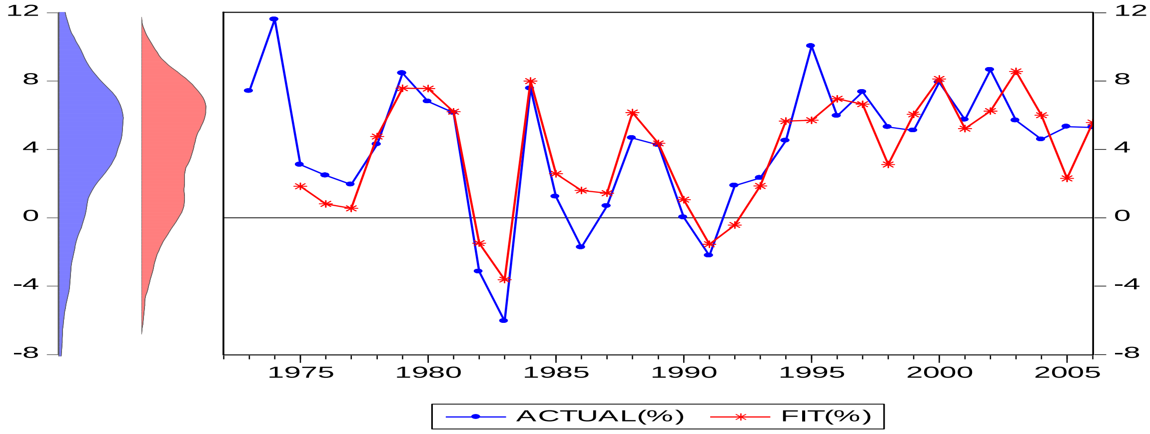
Figure 3.2: Model fitness: Actual versus Fitted Series per sector



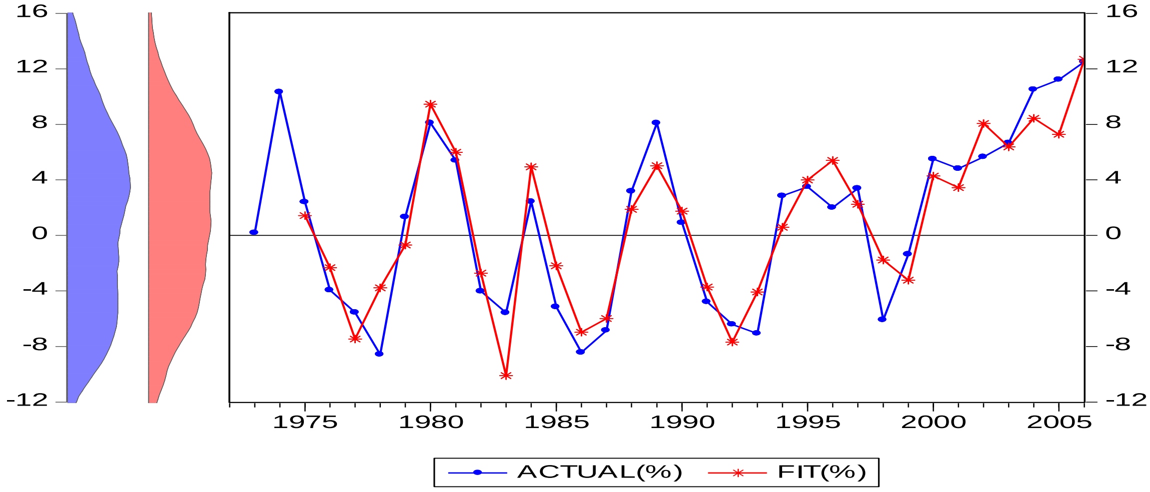
Manufacturing



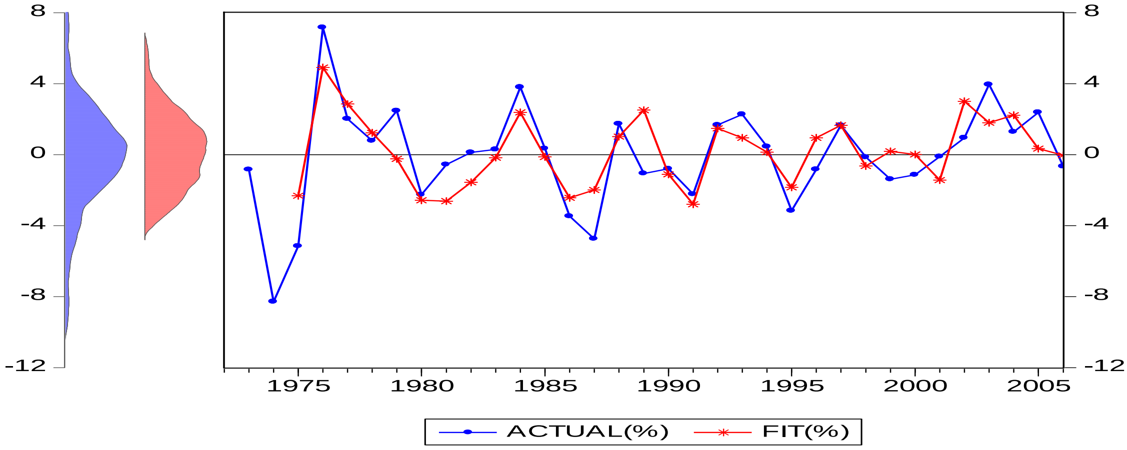
Agriculture



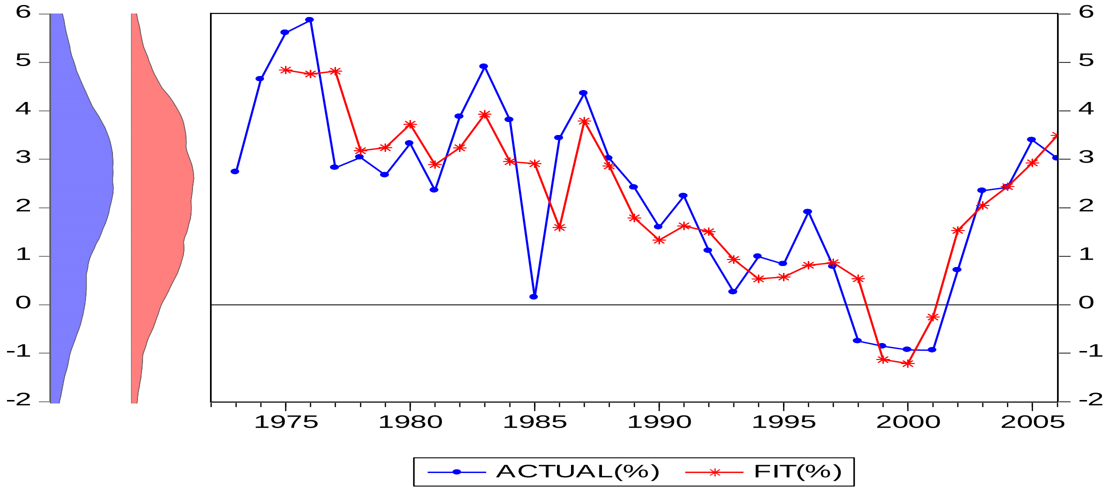
Transport



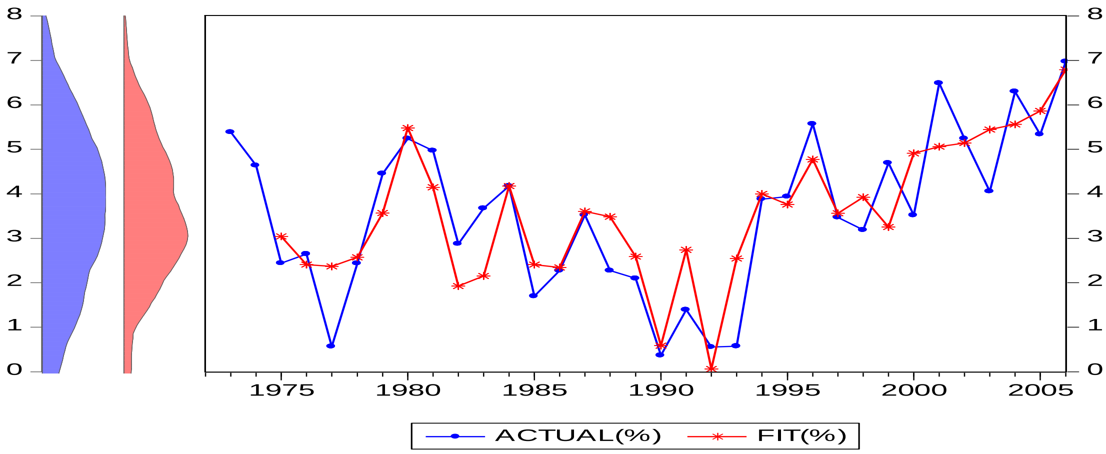
Construction



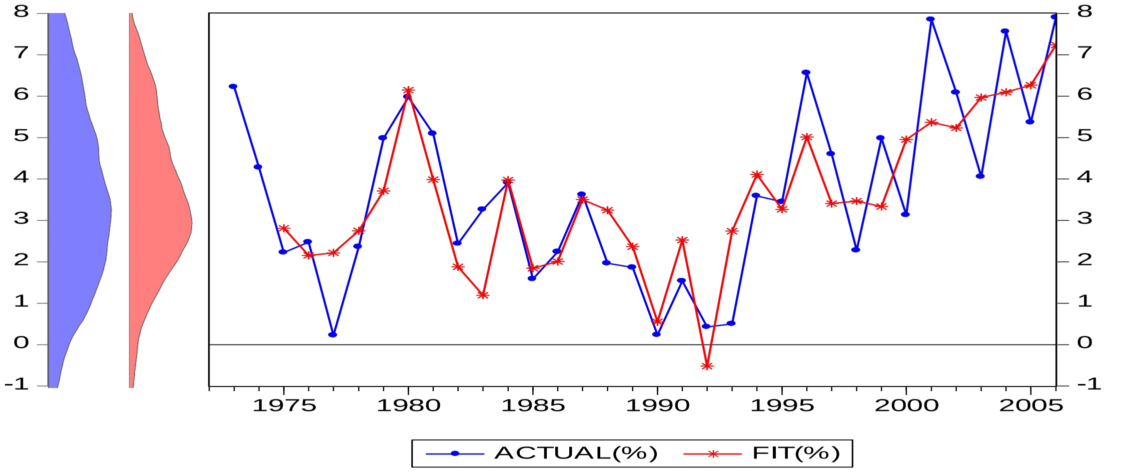
Mining



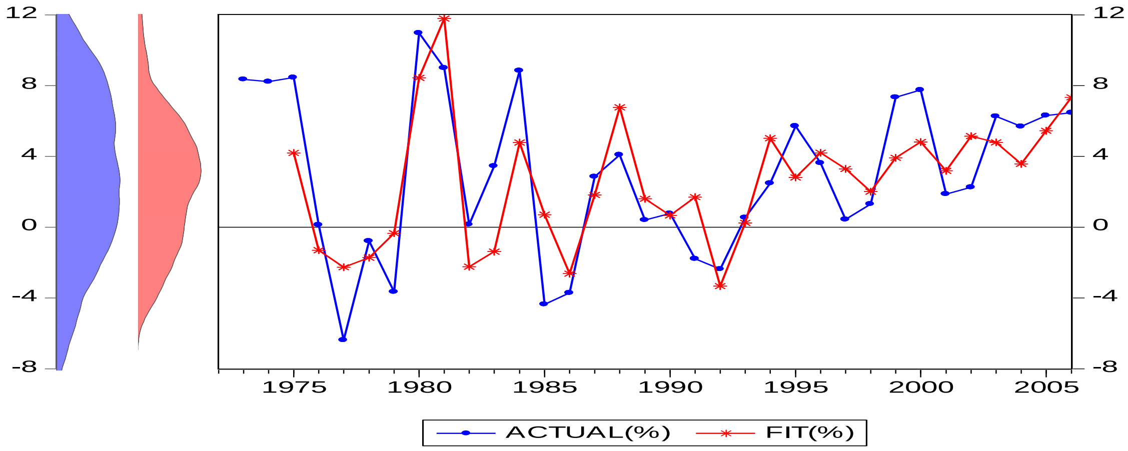
Government



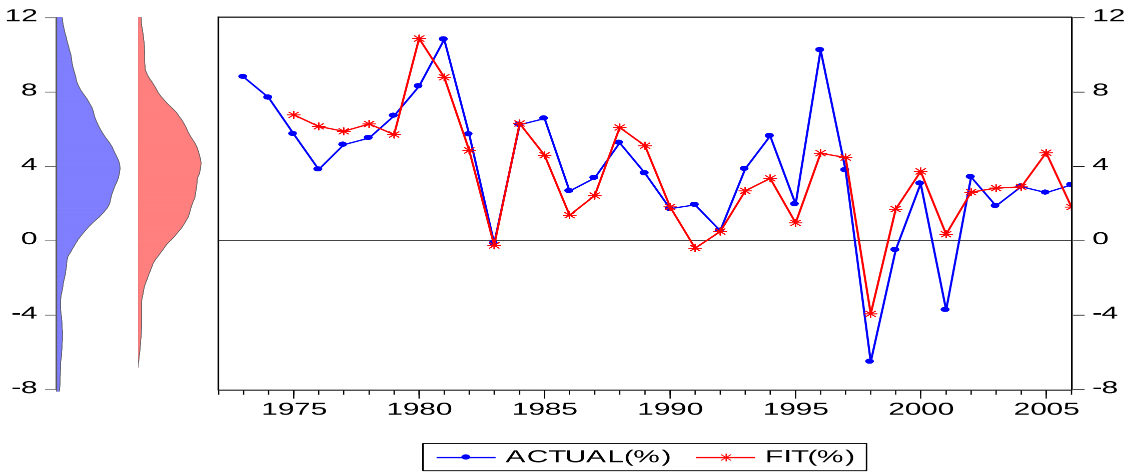
Community services



Financial sector



Wholesale

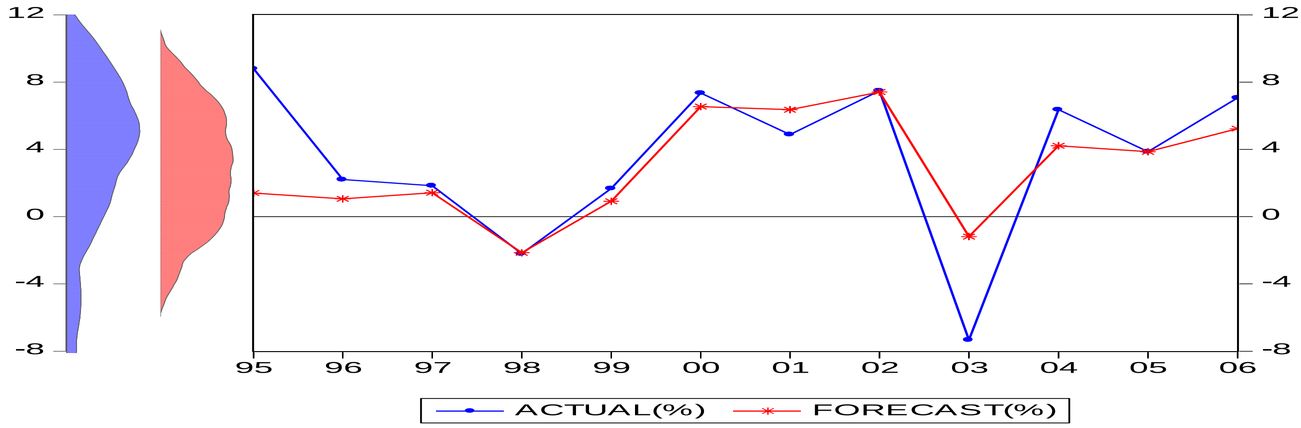


Electricity

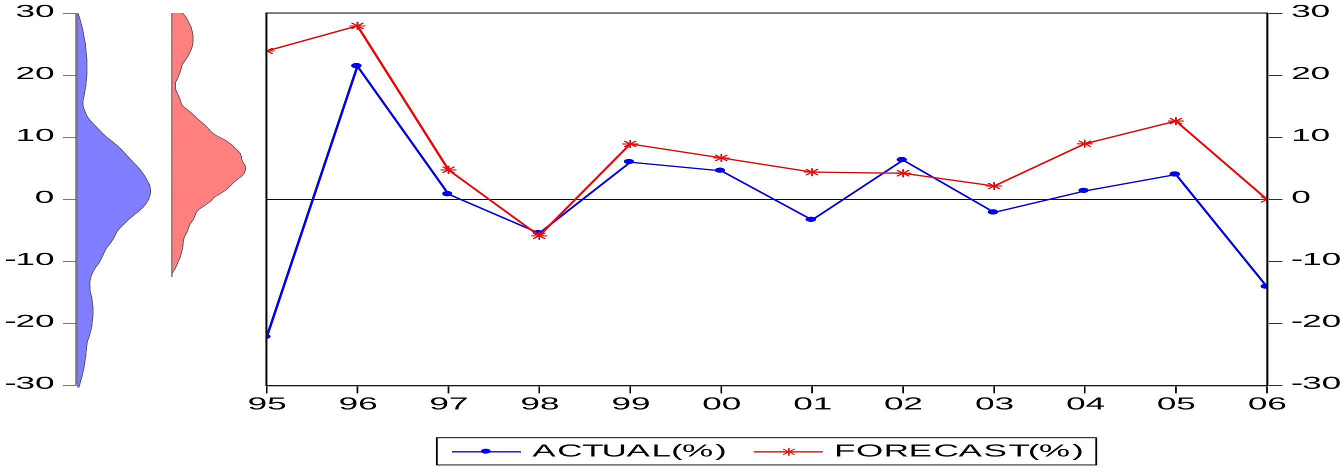
B.2. Model's Prediction Ability: Actual versus Predictions (Forecasts)

In this section, results of the one-year-ahead forecast are provided for individual sectors assessing the forecasting performance of the MMM-DA. Predictions are conducted from 1995 until 2006. Also the figures include the Kernel Density Estimation.

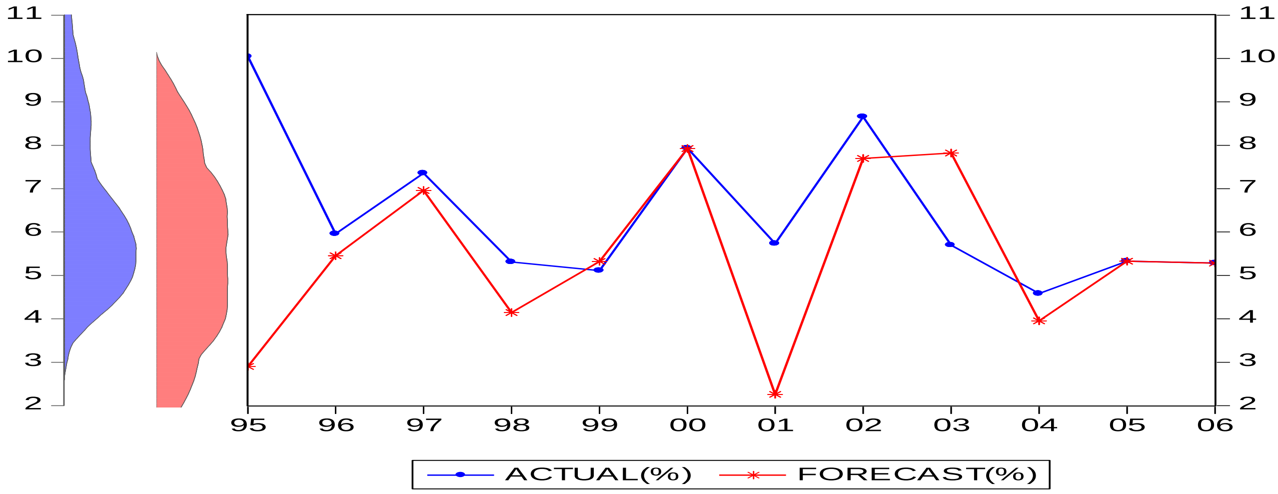
Figure 3.3: Forecasting results: Actual versus Forecasted Series per sector



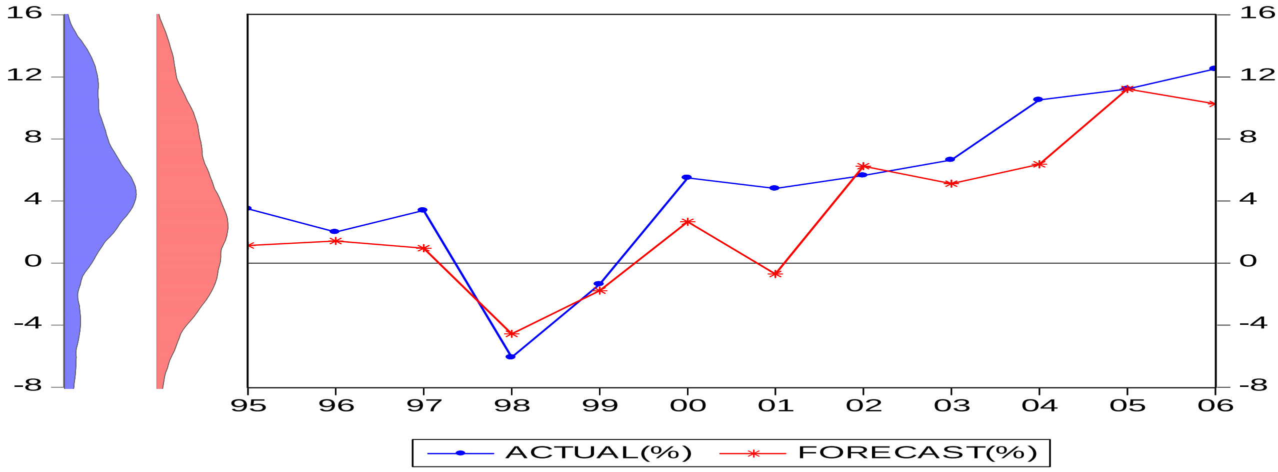
Manufacturing



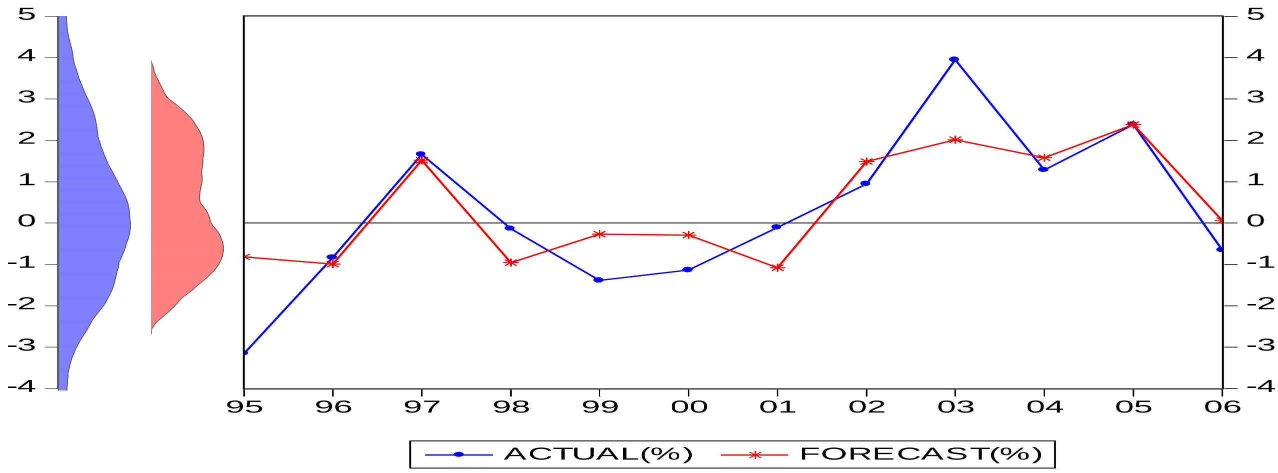
Agriculture



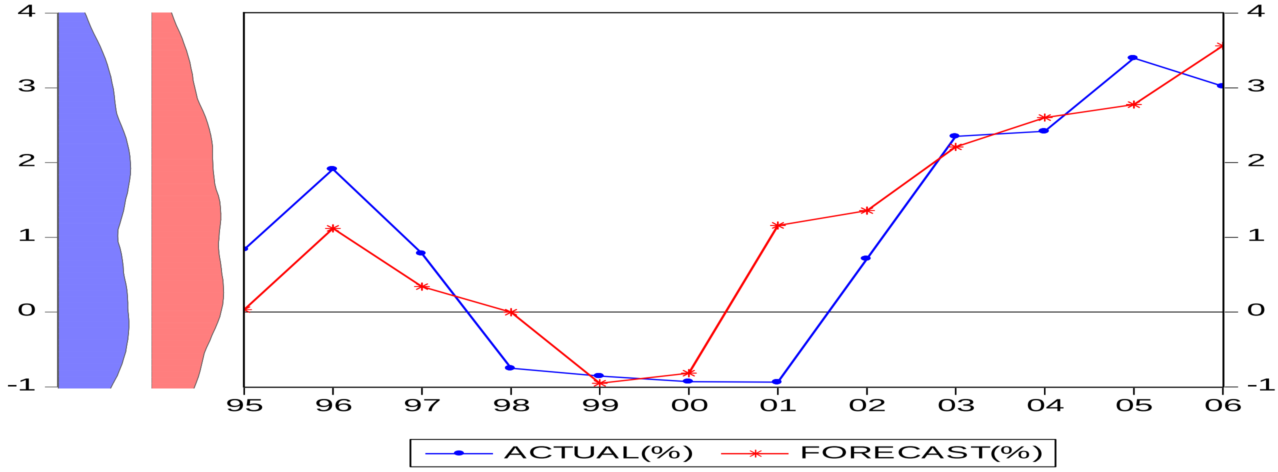
Transport



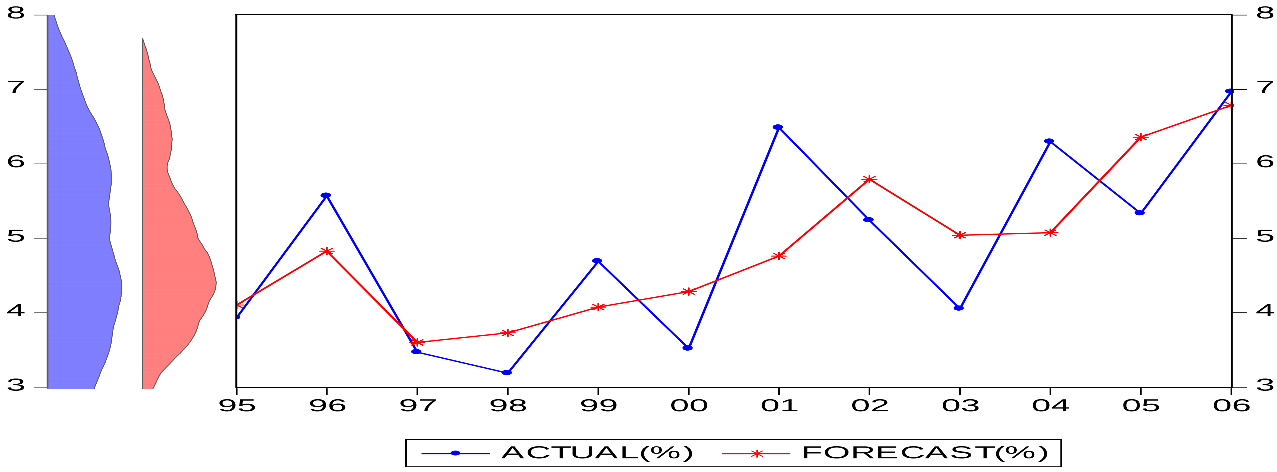
Construction



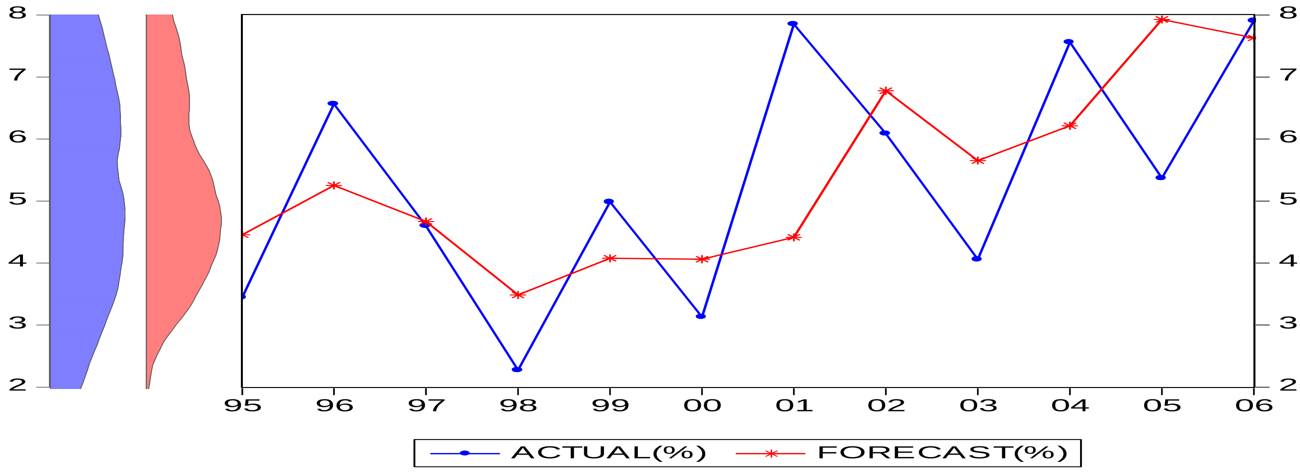
Mining



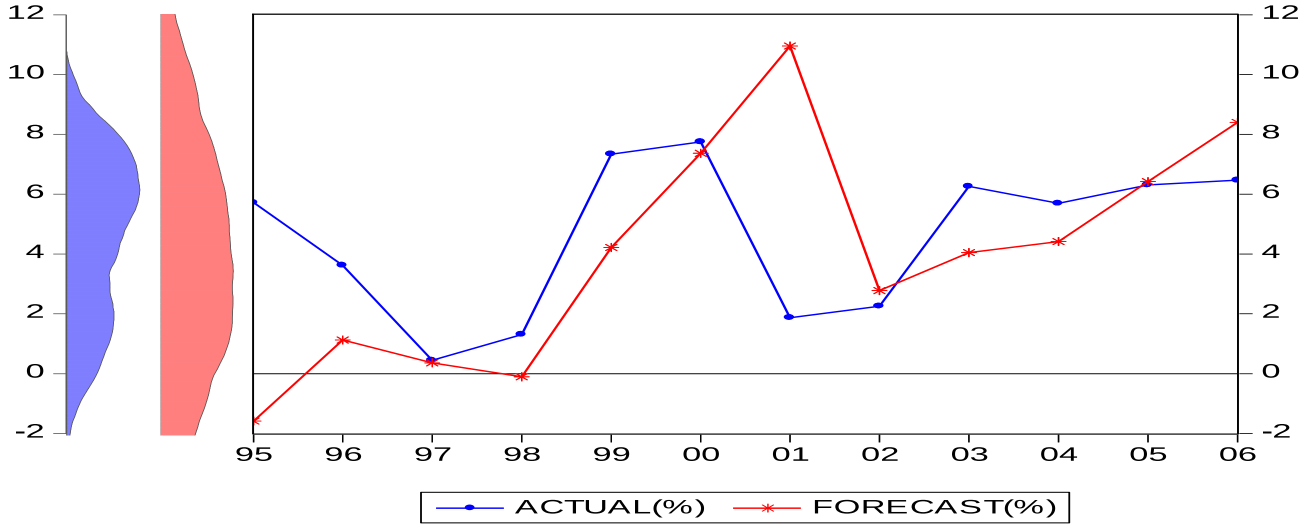
Government



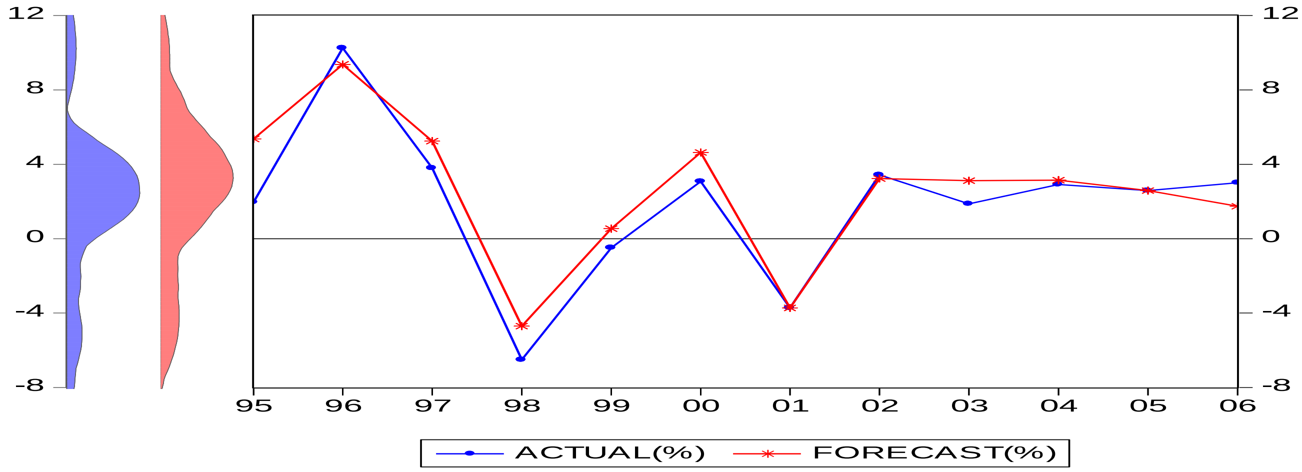
Community services



Financial services



Wholesales

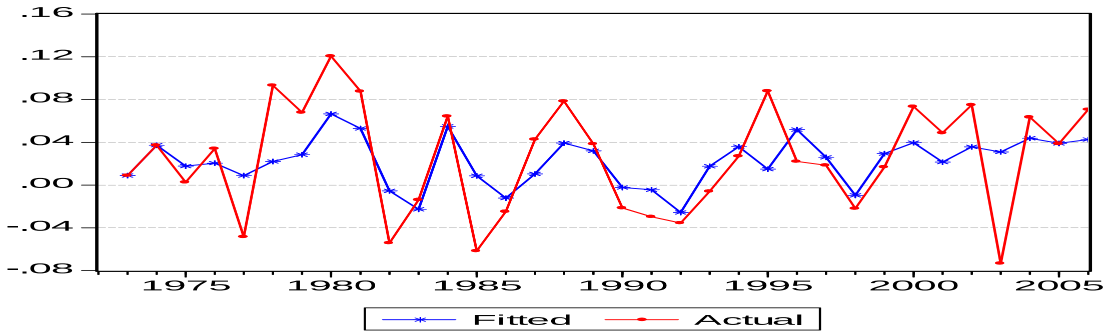


Electricity

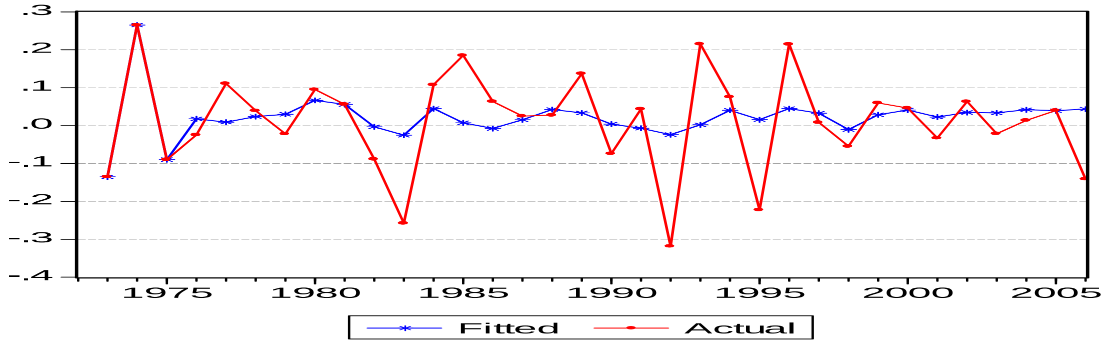
B.3. Model fitness using complete shrinkage

All graphs represent 'actual' versus 'fitted' series of sectoral sales growth using complete shrinkage with the sample size ranging from 1972 to 2006.

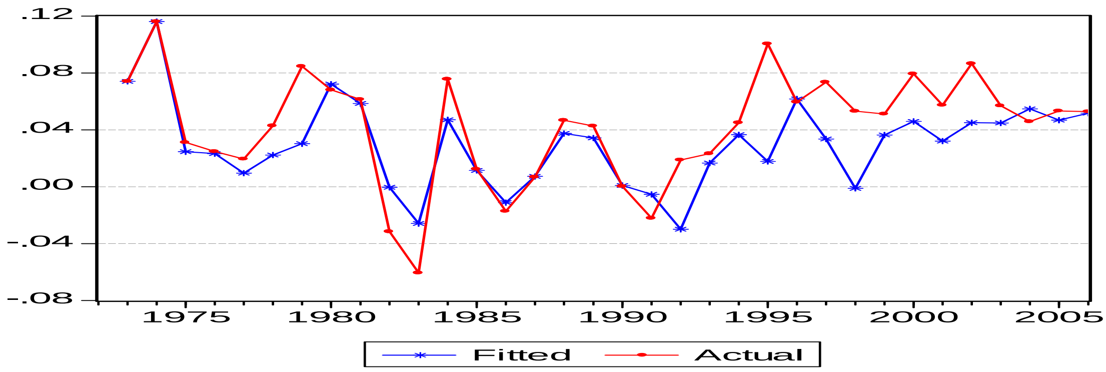
Figure 3.4: Model fitness using complete shrinkage: Actual versus Fitted Series



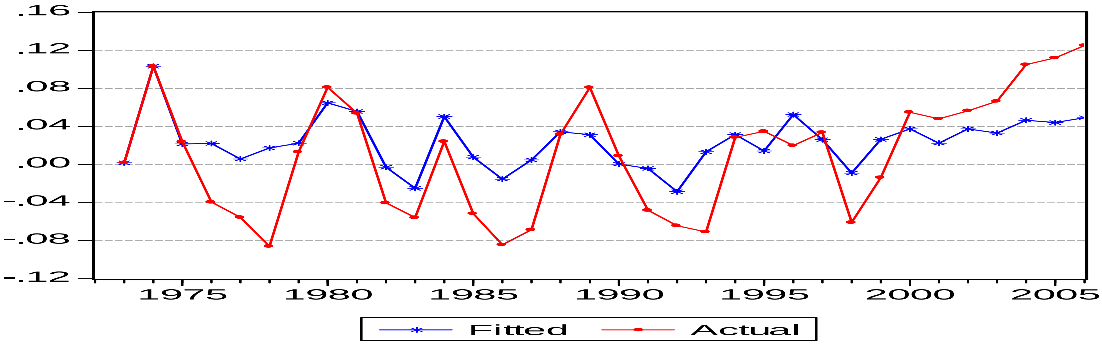
Manufacturing



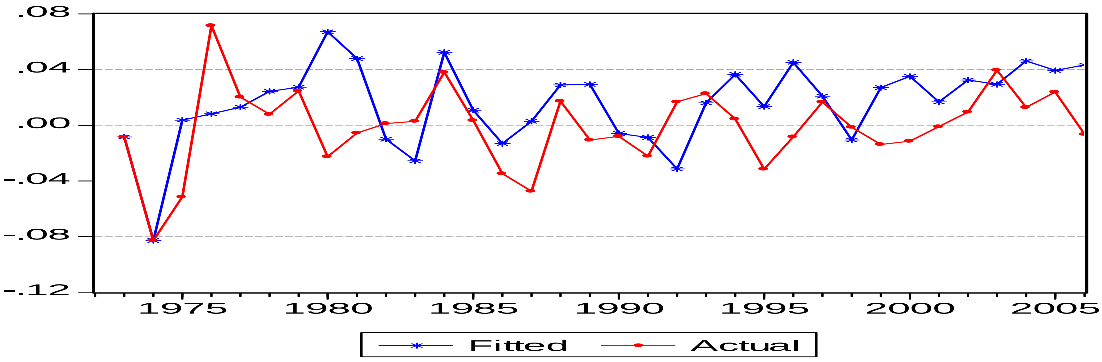
Agriculture



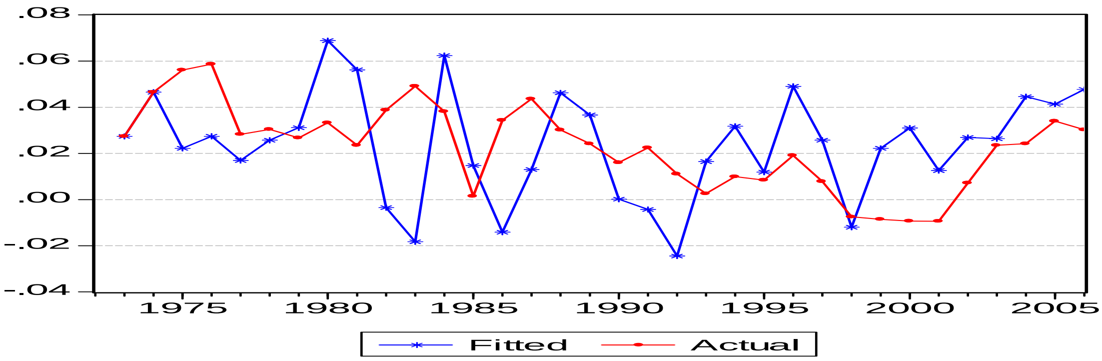
Transport



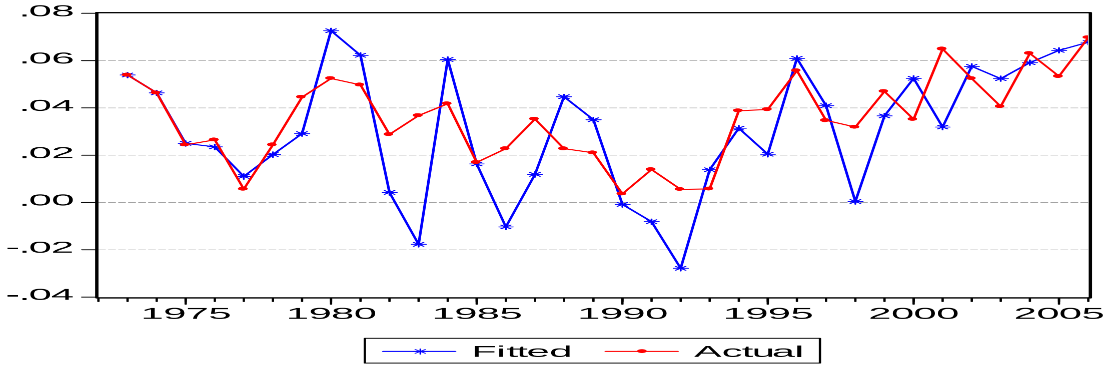
Construction



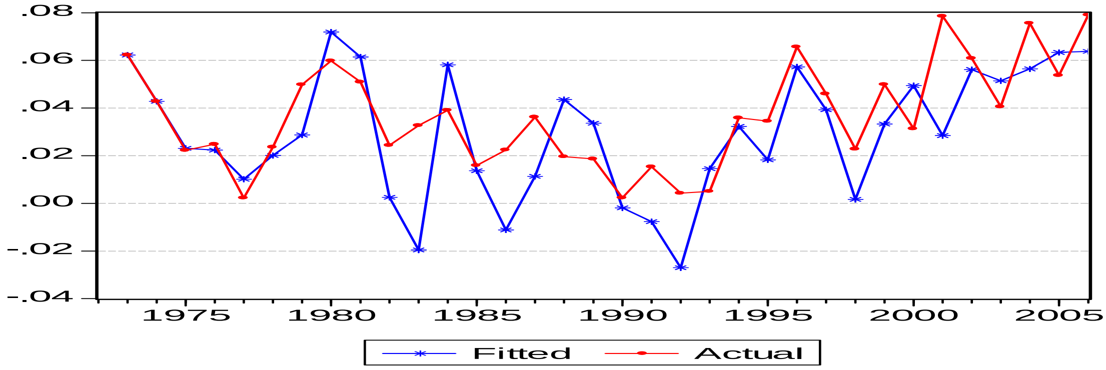
Mining



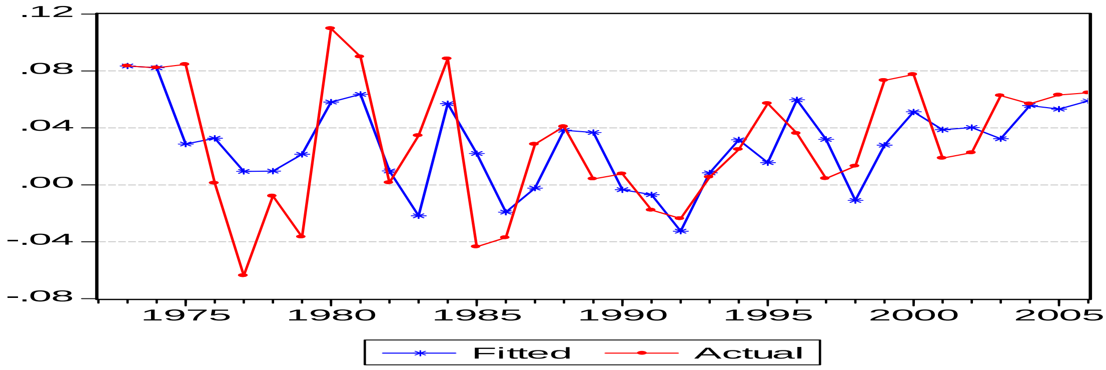
Government



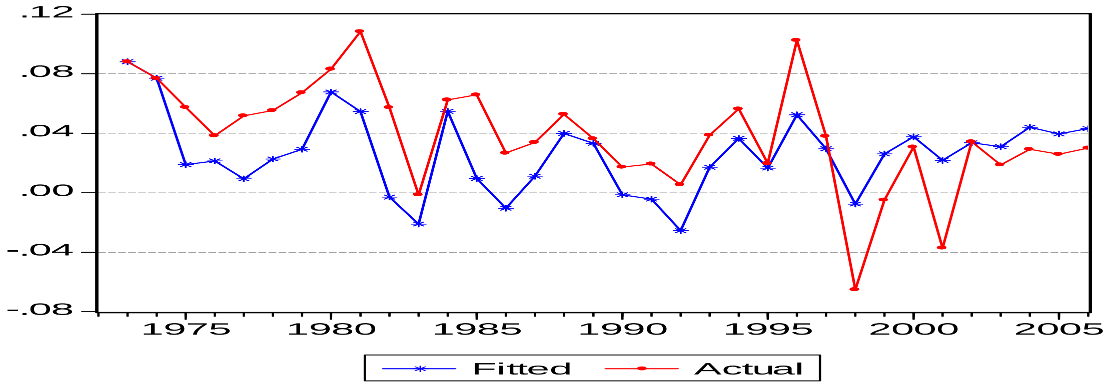
Community



Financial



Wholesales

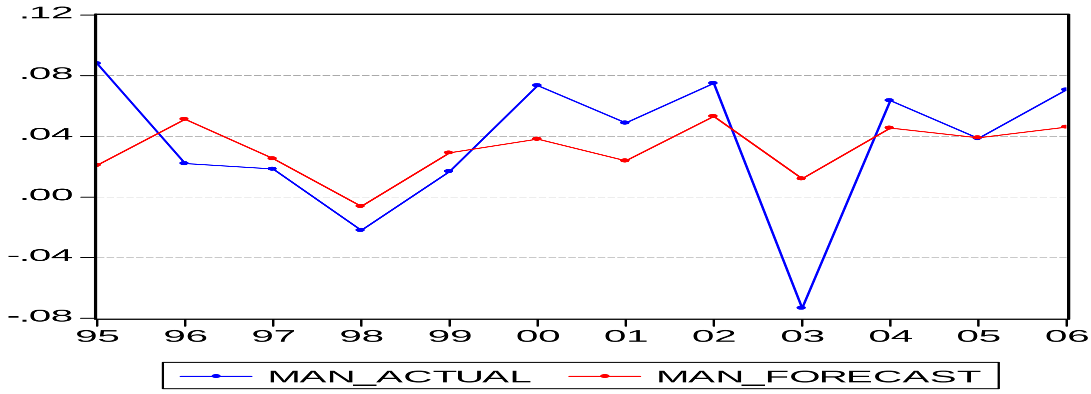


Electricity

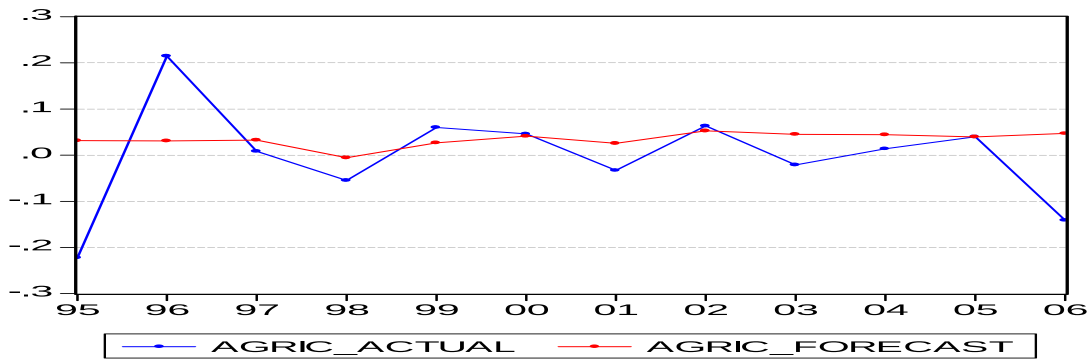
B.4. Predictions (forecasting) using shrinkage technique

This constitutes a repetition of B.2 assuming complete shrinkage.

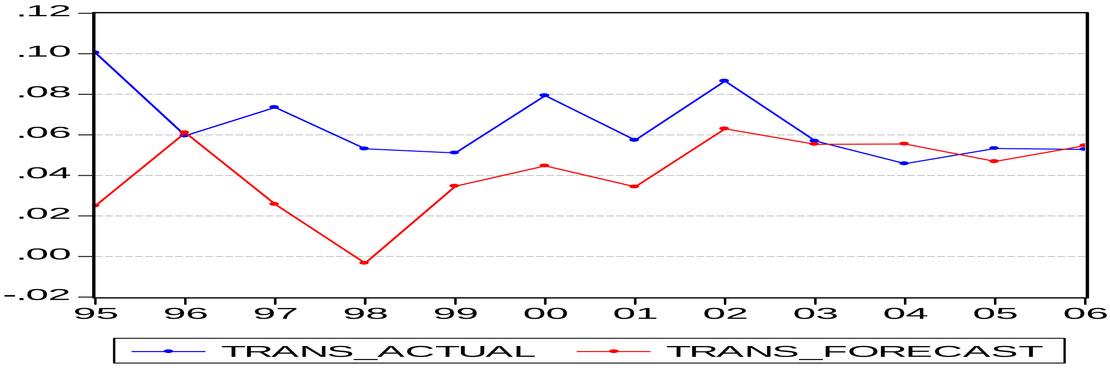
Figure 3.5: Forecasting using complete shrinkage: Actual versus Forecasted Series



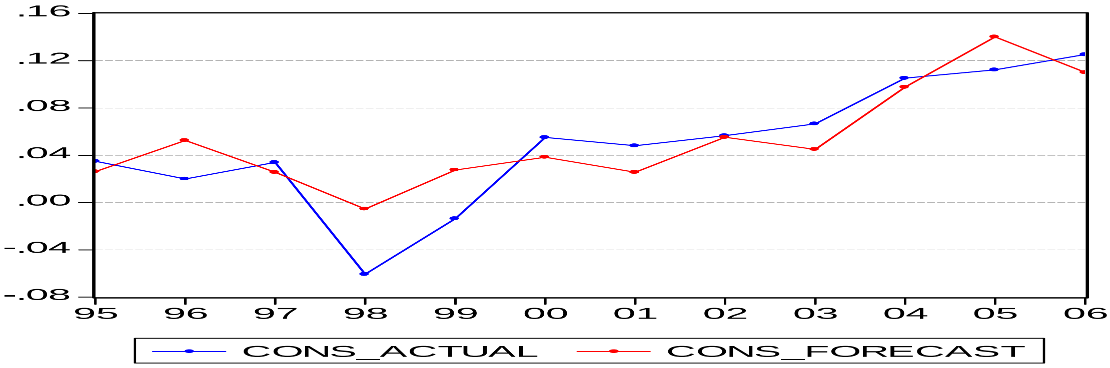
Manufacturing



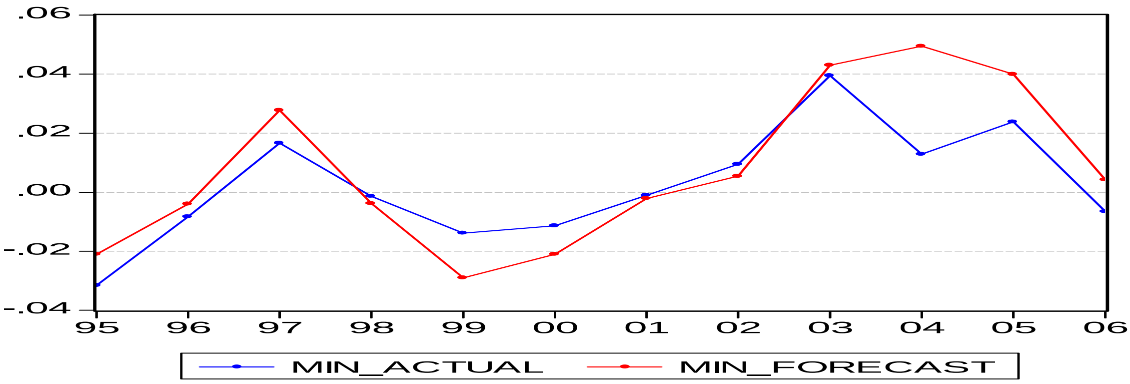
Agriculture



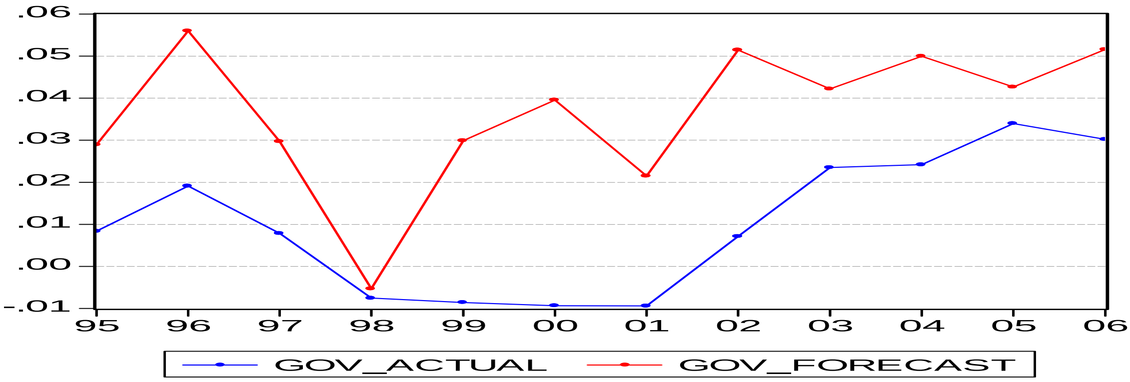
Transport



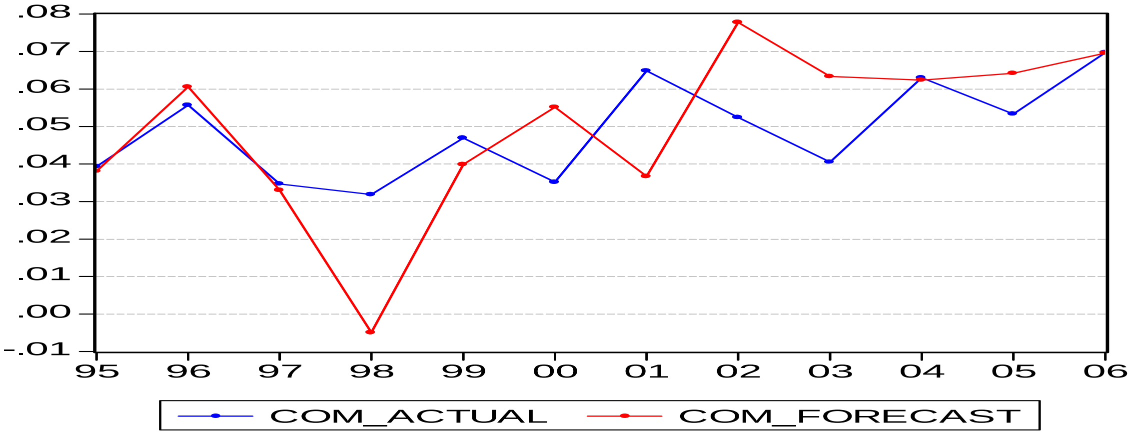
Construction



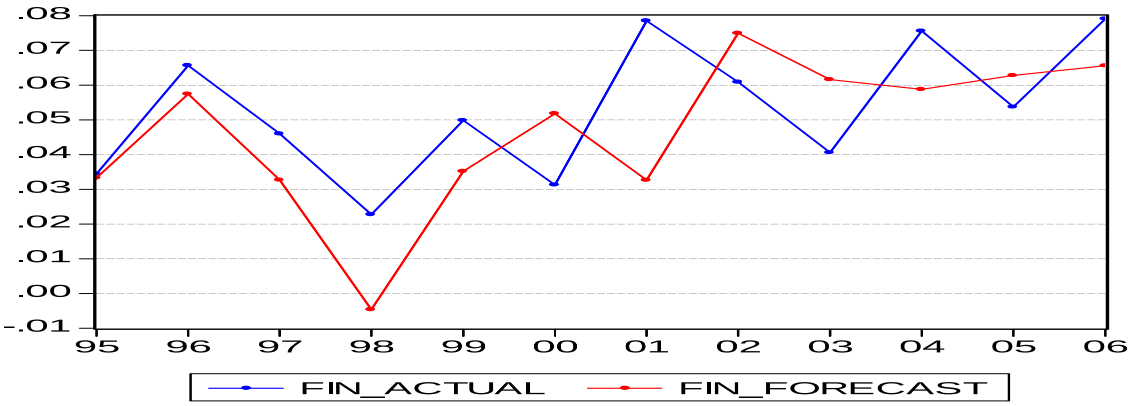
Mining



Government

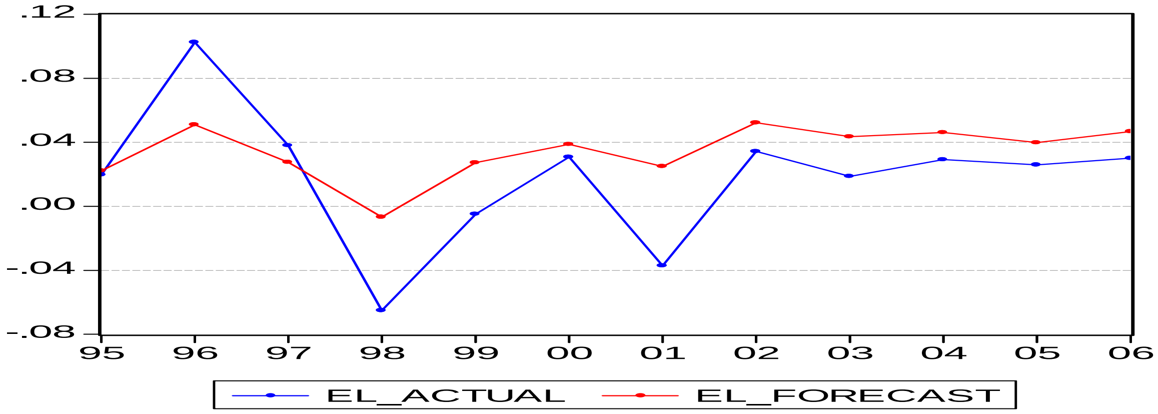


Community



Financial services

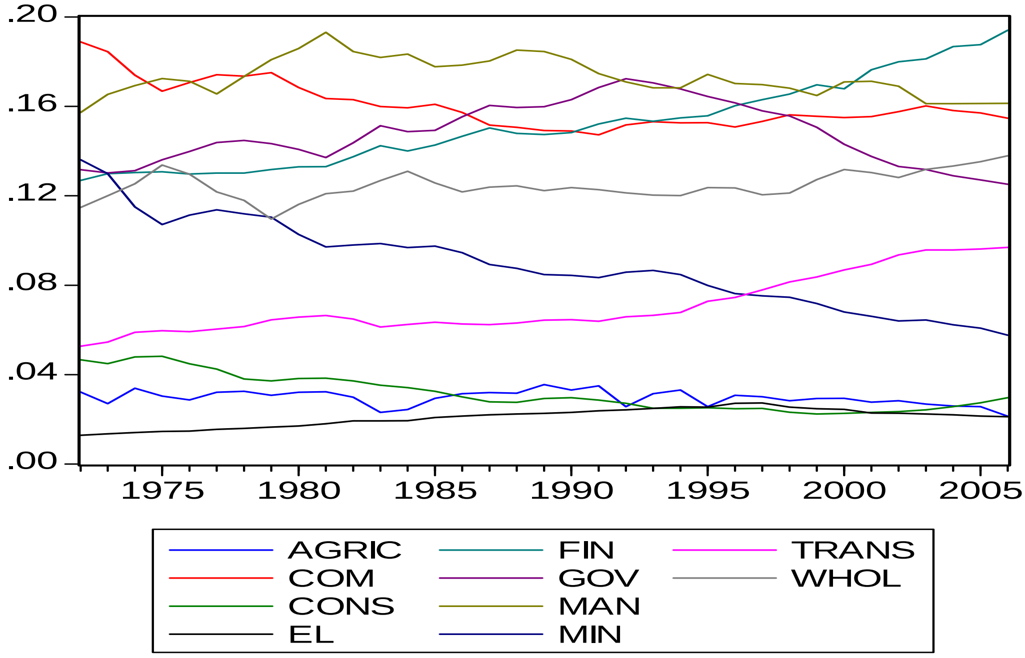
Wholesales



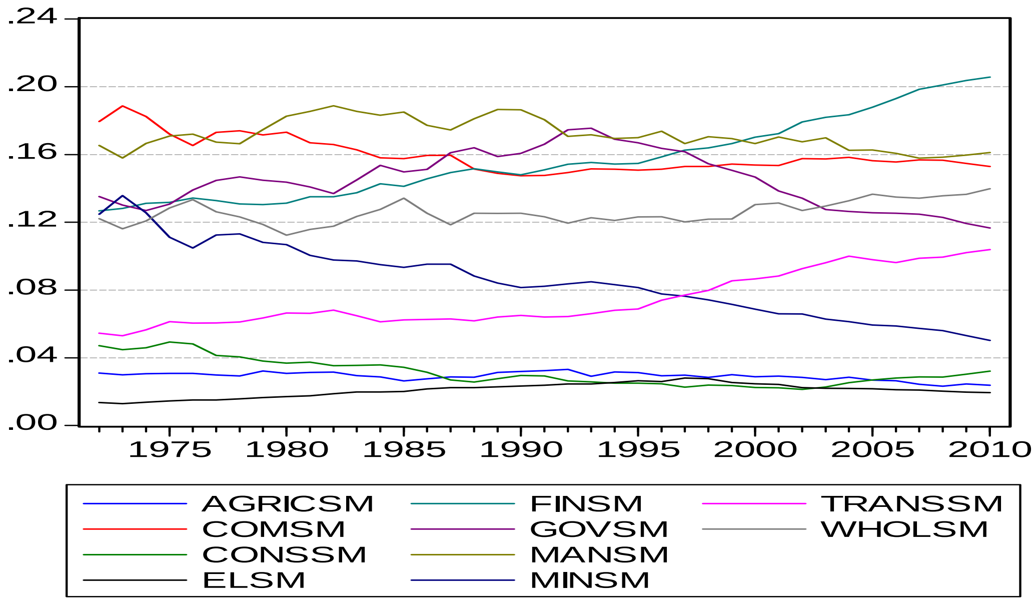
Electricity

APPENDIX C: Sectors' Weights as compared to the National GDP (1972-2006)

C.1. Sectors' shares (Lines)

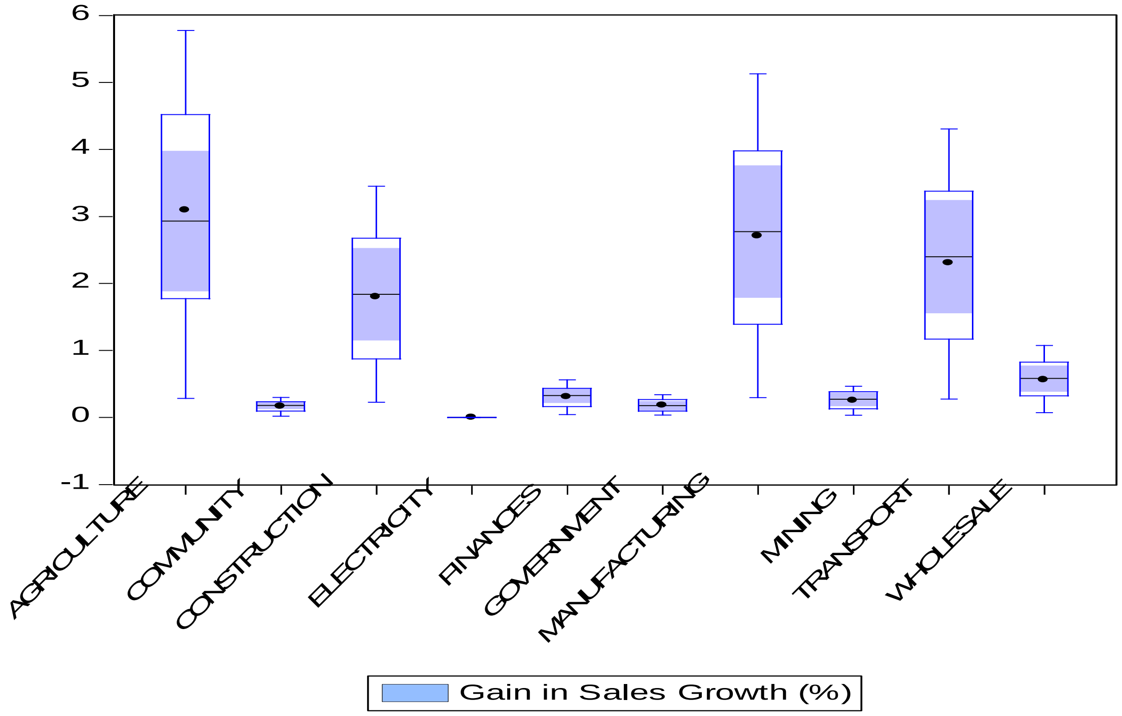


C.2. Weights forecasts (until 2010) using the HWM (Holt Winter Multiplicative) exponential filter

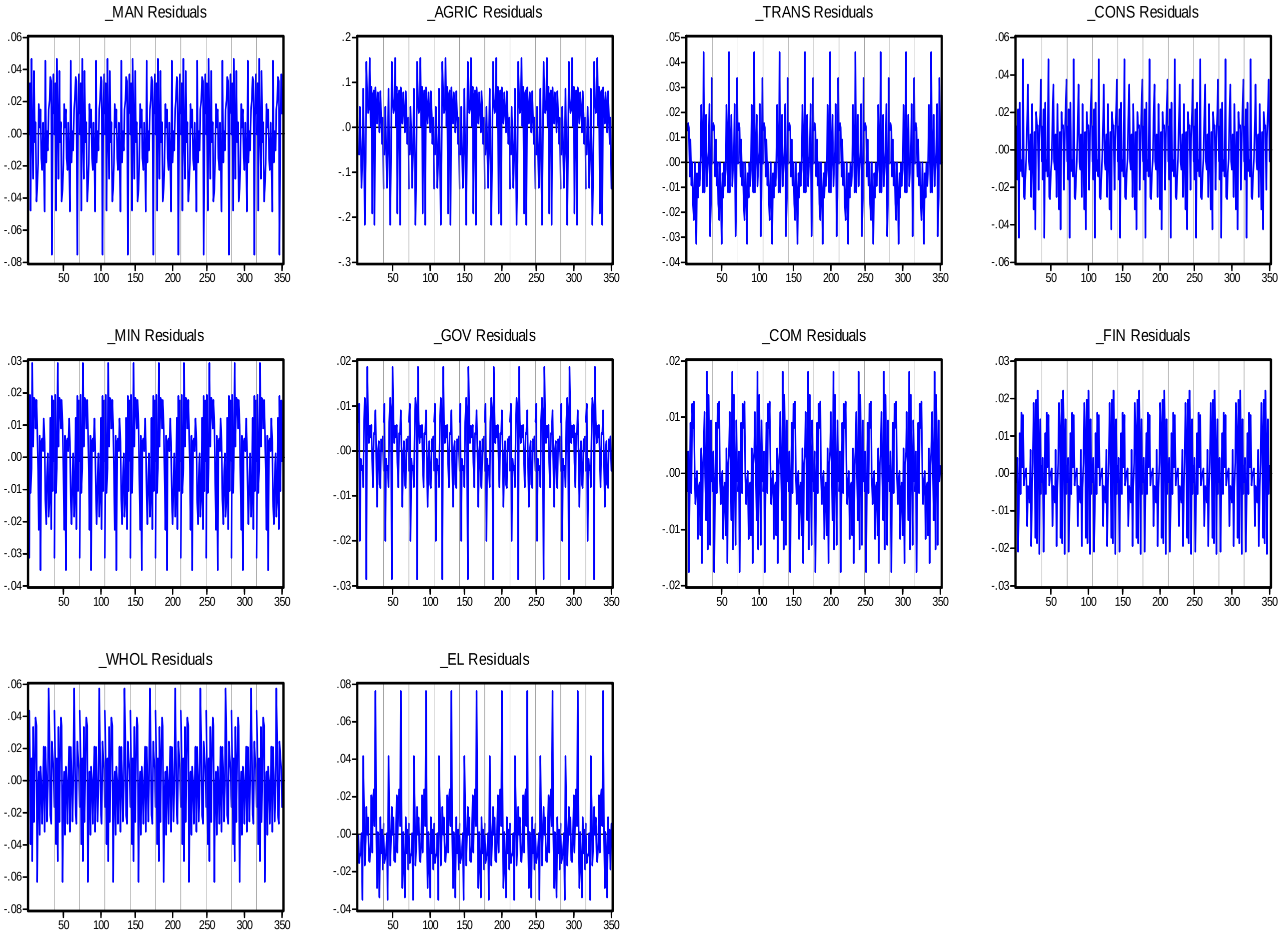


APPENDIX D: POLICY SHOCKS

A 10 percent shock has been applied on selected variables from 1990 onward and the box plot summarizes the reaction across all sectors.



APPENDIX E: RESIDUAL GRAPHS



APPENDIX F: INTRODUCING AN ENTRY COST IN THE MMM: SUR APPROACH (RFE-DA)

At this stage of the MMM-DA, I have introduced a cost of entry charged on each firm willing to enter the sector. In fact, the cost of entry shall be some nonlinear function of several entry requirements. However, in order to palliate to data unavailability, the model makes use of a proxy: other taxes on production by firms. These other taxes constitute additional amount that firms need to pay in addition to the normal operating costs.

Table F.1: Cross section ISUR of $\ln\left(\frac{S_{S,it}}{S_{S,i(t-1)}}\right)$ **using** Γ **and** $\mathbf{Y_d}$ **as demand shifter.**

Variable	Coefficient	Std. Error	t-Statistic	Prob.
_MAN—C	0.207542	0.021395	9.700555	0.0000
_AGRIC—C	0.300368	0.054945	5.466705	0.0000
_TRANS—C	0.032553	0.011304	2.879812	0.0040
_CONS—C	-0.065646	0.016695	-3.932079	0.0001
_MIN—C	0.419283	0.030240	13.86522	0.0000
_GOV—C	0.029906	0.005590	5.349672	0.0000
_COM—C	0.071430	0.005411	13.20020	0.0000
_FIN—C	0.080668	0.006716	12.01062	0.0000
_WHOL—C	-0.030871	0.019135	-1.613343	0.1068
_EL—C	0.056147	0.011835	4.744170	0.0000
_MAN--S_MAN	-0.002006	0.000172	-11.67989	0.0000
_AGRIC--S_AGRIC	-1.28E-05	2.01E-06	-6.396167	0.0000
_TRANS--S_TRANS	-1.75E-07	9.90E-08	-1.772195	0.0765
_CONS--S_CONS	1.83E-06	6.63E-07	2.759967	0.0058
_MIN--S_MIN	-6.47E-06	4.60E-07	-14.08115	0.0000
_GOV--S_GOV	-2.06E-07	3.56E-08	-5.784847	0.0000
_COM--S_COM	-6.13E-08	1.92E-08	-3.198191	0.0014
_FIN--S_FIN	-8.04E-08	3.09E-08	-2.598721	0.0094
_WHOL--S_WHOL	3.28E-07	1.27E-07	2.590682	0.0096
_EL--S_EL	-4.11E-06	3.80E-07	-10.80525	0.0000
_MAN—LNP2_MAN	-0.569565	0.064303	-8.857470	0.0000
_AGRIC—LNP2_AGRIC	0.255161	0.124264	2.053372	0.0401
_TRANS—LNP2_TRANS	-0.399277	0.039123	-10.20568	0.0000
_CONS—LNP2_CONS	-0.099211	0.046252	-2.144983	0.0320
_MIN—LNP2_MIN	0.000681	0.000116	5.865821	0.0000
_GOV—LNP2_GOV	0.247573	0.031723	7.804161	0.0000

_COM—LNP2_COM	-0.132345	0.034300	-3.858430	0.0001
_FIN—LNP2_FIN	-0.082079	0.042999	-1.908865	0.0564
_WHOL—LNP2_WHOL	-0.087528	0.102955	-0.850155	0.3953
_EL—LNP2_EL	-0.120834	0.082076	-1.472218	0.1411
_MAN—LNP3_MAN	0.272028	0.068214	3.987852	0.0001
_AGRIC—LNP3_AGRIC	-0.043427	0.103707	-0.418743	0.6754
_TRANS—LNP3_TRANS	0.147271	0.037980	3.877584	0.0001
_CONS—LNP3_CONS	-0.387414	0.043284	-8.950418	0.0000
_MIN—LNP3_MIN	-0.000274	0.000120	-2.282093	0.0226
_GOV—LNP3_GOV	0.003997	0.040586	0.098486	0.9216
_COM—LNP3_COM	0.032495	0.041286	0.787064	0.4313
_FIN—LNP3_FIN	0.028269	0.051498	0.548936	0.5831
_WHOL—LNP3_WHOL	0.302534	0.123701	2.445694	0.0145
_EL—LNP3_EL	0.196447	0.102173	1.922691	0.0546
_MAN—LNP4_MAN	-0.514556	0.064029	-8.036337	0.0000
_AGRIC—LNP4_AGRIC	-0.171434	0.103312	-1.659382	0.0971
_TRANS—LNP4_TRANS	-0.112284	0.035638	-3.150690	0.0016
_CONS—LNP4_CONS	-0.196563	0.038767	-5.070308	0.0000
_MIN—LNP4_MIN	0.000744	0.000133	5.601234	0.0000
_GOV—LNP4_GOV	-0.032975	0.032810	-1.005048	0.3150
_COM—LNP4_COM	-0.274004	0.033128	-8.271155	0.0000
_FIN—LNP4_FIN	-0.430970	0.041567	-10.36806	0.0000
_WHOL—LNP4_WHOL	-0.647351	0.100001	-6.473438	0.0000
_EL—LNP4_EL	0.020907	0.082841	0.252378	0.8008
_MAN—LNM22_MAN	0.234037	0.033932	6.897243	0.0000
_AGRIC—LNM22_AGRIC	0.085560	0.113318	0.755044	0.4503
_TRANS—LNM22_TRANS	0.097773	0.026767	3.652800	0.0003
_CONS—LNM22_CONS	0.383823	0.031105	12.33971	0.0000
_MIN—LNM22_MIN	0.006271	0.018037	0.347663	0.7281
_GOV—LNM22_GOV	-0.019635	0.011542	-1.701092	0.0890
_COM—LNM22_COM	0.070039	0.011304	6.195866	0.0000
_FIN—LNM22_FIN	0.079343	0.014228	5.576391	0.0000
_WHOL—LNM22_WHOL	0.261961	0.032894	7.963842	0.0000
_EL—LNM22_EL	0.097955	0.028804	3.400699	0.0007
_MAN—LNSP2_MAN	0.015270	0.005216	2.927544	0.0034
_AGRIC—LNSP2_AGRIC	-0.053507	0.018815	-2.843845	0.0045
_TRANS—LNSP2_TRANS	0.009041	0.003585	2.521602	0.0117
_CONS—LNSP2_CONS	-0.015857	0.004310	-3.678708	0.0002
_MIN—LNSP2_MIN	-0.016224	0.002802	-5.790186	0.0000

_GOV—LNSP2_GOV	-0.012492	0.001743	-7.167865	0.0000
_COM—LNSP2_COM	0.004616	0.001665	2.771912	0.0056
_FIN—LNSP2_FIN	0.001021	0.002096	0.487098	0.6262
_WHOL—LNSP2_WHOL	0.032170	0.004765	6.751456	0.0000
_EL—LNSP2_EL	0.000698	0.004382	0.159322	0.8734
_MAN—LNYD1_MAN	1.124663	0.087477	12.85662	0.0000
_AGRIC—LNYD1_AGRIC	2.361450	0.288425	8.187406	0.0000
_TRANS—LNYD1_TRANS	0.805426	0.071744	11.22633	0.0000
_CONS—LNYD1_CONS	1.645082	0.082965	19.82874	0.0000
_MIN—LNYD1_MIN	0.204862	0.044207	4.634188	0.0000
_GOV—LNYD1_GOV	0.150384	0.029837	5.040153	0.0000
_COM—LNYD1_COM	0.261179	0.029313	8.910141	0.0000
_FIN—LNYD1_FIN	0.382525	0.036893	10.36853	0.0000
_WHOL—LNYD1_WHOL	0.615491	0.086535	7.112589	0.0000
_EL—LNYD1_EL	0.688065	0.075281	9.139921	0.0000
_MAN—LNR1_MAN	0.044961	0.007781	5.778412	0.0000
_AGRIC—LNR1_AGRIC	-0.059833	0.025898	-2.310334	0.0209
_TRANS—LNR1_TRANS	-0.020986	0.005755	-3.646744	0.0003
_CONS—LNR1_CONS	0.016031	0.006459	2.481866	0.0131
_MIN—LNR1_MIN	0.001399	0.003929	0.356054	0.7218
_GOV—LNR1_GOV	0.006570	0.002707	2.427113	0.0153
_COM—LNR1_COM	0.009065	0.002686	3.374614	0.0007
_FIN—LNR1_FIN	0.009263	0.003374	2.745563	0.0061
_WHOL—LNR1_WHOL	0.066023	0.007823	8.439108	0.0000
_EL—LNR1_EL	-0.007498	0.006729	-1.114419	0.2652
_MAN—LNW1_MAN	-0.024886	0.014008	-1.776495	0.0758
_AGRIC—LNW1_AGRIC	-0.112130	0.049324	-2.273318	0.0231
_TRANS—LNW1_TRANS	-0.006213	0.010196	-0.609399	0.5423
_CONS—LNW1_CONS	-0.015561	0.011974	-1.299531	0.1939
_MIN—LNW1_MIN	-0.017010	0.007590	-2.241056	0.0251
_GOV—LNW1_GOV	-0.006799	0.004765	-1.426853	0.1537
_COM—LNW1_COM	0.027575	0.004668	5.907230	0.0000
_FIN—LNW1_FIN	0.014807	0.005876	2.519983	0.0118
_WHOL—LNW1_WHOL	0.013395	0.013585	0.986062	0.3242
_EL—LNW1_EL	-0.019852	0.011854	-1.674624	0.0941
_MAN—LNH1_MAN	0.063030	0.049354	1.277109	0.2017
_AGRIC—LNH1_AGRIC	-0.434422	0.158645	-2.738335	0.0062
_TRANS—LNH1_TRANS	-0.061789	0.033713	-1.832796	0.0669
_CONS—LNH1_CONS	-0.015225	0.039407	-0.386358	0.6993

_MIN—LNH1_MIN	0.131804	0.025457	5.177426	0.0000
_GOV—LNH1_GOV	0.102724	0.016123	6.371262	0.0000
_COM—LNH1_COM	0.012771	0.017583	0.726316	0.4677
_FIN—LNH1_FIN	-0.019574	0.022351	-0.875786	0.3812
_WHOL—LNH1_WHOL	-0.128228	0.052941	-2.422071	0.0155
_EL—LNH1_EL	0.056061	0.040388	1.388045	0.1652
_MAN—LNZ1_MAN	0.517990	0.095303	5.435172	0.0000
_AGRIC—LNZ1_AGRIC	-0.421887	0.363651	-1.160143	0.2461
_TRANS—LNZ1_TRANS	0.287233	0.072077	3.985114	0.0001
_CONS—LNZ1_CONS	0.245771	0.095775	2.566141	0.0103
_MIN—LNZ1_MIN	-0.376915	0.057276	-6.580663	0.0000
_GOV—LNZ1_GOV	-0.393812	0.032105	-12.26637	0.0000
_COM—LNZ1_COM	0.069786	0.030122	2.316774	0.0206
_FIN—LNZ1_FIN	0.093974	0.037757	2.488934	0.0129
_WHOL—LNZ1_WHOL	0.552889	0.086423	6.397442	0.0000
_EL—LNZ1_EL	-0.008782	0.078797	-0.111448	0.9113
_MAN—LNΓ1_MAN	0.022903	0.028409	0.806192	0.4202
_AGRIC—LNΓ1_AGRIC	-0.204187	0.084591	-2.413804	0.0158
_TRANS—LNΓ1_TRANS	0.067262	0.020681	3.252400	0.0012
_CONS—LNΓ1_CONS	-0.039554	0.023413	-1.689363	0.0913
_MIN—LNΓ1_MIN	0.076200	0.013092	5.820254	0.0000
_GOV—LNΓ1_GOV	-0.021357	0.009290	-2.298862	0.0216
_COM—LNΓ1_COM	-0.046539	0.009353	-4.975578	0.0000
_FIN—LNΓ1_FIN	-0.038240	0.011756	-3.252825	0.0012
_WHOL—LNΓ1_WHOL	0.043408	0.027649	1.569937	0.1165
_EL—LNΓ1_EL	0.059533	0.023256	2.559866	0.0105
Weighted Statistics				
R-squared	0.857411	**Mean dependent var**		0.844553
Adjusted R-squared	0.851218	**S.D. dependent var**		2.583258
F-statistic	138.4424	**Durbin-Watson stat**		2.193141
Prob(F-statistic)	0.000000			

MAE = 0.84

RMSE = 1.75

Table F.2: Wald coefficients test

Test Statistic	Value	Df	Probability
F-statistic	99.38386	(2, 2970)	0.0000
Chi-square	198.7677	2	0.0000

Null Hypothesis Summary

Normalized Restriction (= 0)	Value	Std. Err.
C(3): lag of order 2	0.032553	0.011304
C(5): lag of order 3	0.419283	0.030240

Table F.3: ISUR of $\ln\left(\frac{S_{S,it}}{S_{S,i(t-1)}}\right)$ using imposed restrictions on the parameters

Variable	Coefficient	Std. Error	t-Statistic	Prob.
_MAN--C	0.022157	0.016447	1.347172	0.1780
_AGRIC--C	0.240582	0.046653	5.156871	0.0000
_TRANS--C	-0.007155	0.009703	-0.737418	0.4609
_CONS--C	-0.093549	0.017009	-5.499839	0.0000
_MIN--C	0.404997	0.032407	12.49717	0.0000
_GOV--C	0.054457	0.004402	12.37242	0.0000
_COM--C	0.046019	0.004412	10.43045	0.0000
_FIN--C	0.054102	0.005703	9.486295	0.0000
_WHOL--C	0.001826	0.016135	0.113192	0.9099
_EL--C	0.059855	0.009441	6.339724	0.0000
_MAN—S_MAN	-0.000749	0.000141	-5.319932	0.0000
_AGRIC—S_AGRIC	-1.08E-05	1.62E-06	-6.647116	0.0000
_TRANS—S_TRANS	-1.59E-07	9.43E-08	-1.688080	0.0915
_CONS—S_CONS	2.50E-06	6.41E-07	3.908054	0.0001
_MIN—S_MIN	-6.30E-06	4.92E-07	-12.80196	0.0000
_GOV—S_GOV	-3.61E-07	2.79E-08	-12.97633	0.0000
_COM—S_COM	1.41E-08	1.58E-08	0.888283	0.3745
_FIN—S_FIN	1.18E-08	2.66E-08	0.443480	0.6574
_WHOL—S_WHOL	-1.08E-08	1.07E-07	-0.101165	0.9194
_EL—S_EL	-3.95E-06	3.01E-07	-13.11604	0.0000
_MAN--LNP3_MAN	-0.018389	0.056709	-0.324265	0.7458
_AGRIC--LNP3_AGRIC	-0.137550	0.081894	-1.679610	0.0931
_TRANS--LNP3_TRANS	-0.077944	0.029230	-2.666597	0.0077
_CONS--LNP3_CONS	-0.514074	0.031707	-16.21339	0.0000
_MIN--LNP3_MIN	-0.000233	0.000146	-1.593624	0.1111

_GOV--LNP3_GOV	0.185260	0.018250	10.15133	0.0000
_COM--LNP3_COM	-0.229821	0.021804	-10.54008	0.0000
_FIN--LNP3_FIN	-0.299377	0.028626	-10.45822	0.0000
_WHOL--LNP3_WHOL	-0.282031	0.069506	-4.057640	0.0001
_EL--LNP3_EL	0.002468	0.051328	0.048084	0.9617
_MAN--LNM22_MAN	0.189303	0.036823	5.140908	0.0000
_AGRIC--LNM22_AGRIC	0.094121	0.118647	0.793284	0.4277
_TRANS--LNM22_TRANS	0.221060	0.027502	8.037994	0.0000
_CONS--LNM22_CONS	0.431060	0.030949	13.92817	0.0000
_MIN--LNM22_MIN	0.014904	0.022914	0.650456	0.5154
_GOV--LNM22_GOV	-0.021538	0.011793	-1.826314	0.0679
_COM--LNM22_COM	0.039208	0.011262	3.481251	0.0005
_FIN--LNM22_FIN	0.030501	0.014649	2.082044	0.0374
_WHOL--LNM22_WHOL	0.209973	0.035424	5.927339	0.0000
_EL--LNM22_EL	0.101473	0.027105	3.743649	0.0002
_MAN--LNSP2_MAN	0.011778	0.005416	2.174398	0.0298
_AGRIC--LNSP2_AGRIC	-0.050288	0.017513	-2.871497	0.0041
_TRANS--LNSP2_TRANS	0.018091	0.003965	4.563249	0.0000
_CONS--LNSP2_CONS	-0.009083	0.004575	-1.985142	0.0472
_MIN--LNSP2_MIN	-0.021101	0.003475	-6.072419	0.0000
_GOV--LNSP2_GOV	-0.006422	0.001721	-3.731331	0.0002
_COM--LNSP2_COM	0.002727	0.001643	1.659701	0.0971
_FIN--LNSP2_FIN	0.000318	0.002137	0.148833	0.8817
_WHOL--LNSP2_WHOL	0.037017	0.005209	7.106177	0.0000
_EL--LNSP2_EL	-0.001267	0.003961	-0.319717	0.7492
_MAN--LNYD1_MAN	1.500029	0.081905	18.31427	0.0000
_AGRIC--LNYD1_AGRIC	2.321036	0.272964	8.503085	0.0000
_TRANS--LNYD1_TRANS	1.103019	0.062980	17.51373	0.0000
_CONS--LNYD1_CONS	1.502805	0.069334	21.67473	0.0000
_MIN--LNYD1_MIN	0.379807	0.051711	7.344788	0.0000
_GOV--LNYD1_GOV	0.000868	0.026982	0.032154	0.9744
_COM--LNYD1_COM	0.309022	0.026141	11.82142	0.0000
_FIN--LNYD1_FIN	0.395932	0.033994	11.64709	0.0000
_WHOL--LNYD1_WHOL	0.426333	0.081584	5.225710	0.0000
_EL--LNYD1_EL	0.704079	0.061845	11.38456	0.0000
_MAN--LNR1_MAN	0.033228	0.008363	3.973049	0.0001
_AGRIC--LNR1_AGRIC	-0.046851	0.027266	-1.718271	0.0858
_TRANS--LNR1_TRANS	-0.037534	0.006376	-5.886482	0.0000
_CONS--LNR1_CONS	0.010630	0.006695	1.587802	0.1124

_MIN--LNR1_MIN	-0.004746	0.004988	-0.951503	0.3414
_GOV--LNR1_GOV	0.004033	0.002793	1.443930	0.1489
_COM--LNR1_COM	0.010063	0.002743	3.668872	0.0002
_FIN--LNR1_FIN	0.009181	0.003568	2.573312	0.0101
_WHOL--LNR1_WHOL	0.062387	0.008597	7.256942	0.0000
_EL--LNR1_EL	-0.000377	0.006530	-0.057795	0.9539
_MAN--LNW1_MAN	0.006762	0.014888	0.454161	0.6497
_AGRIC--LNW1_AGRIC	-0.060738	0.048932	-1.241271	0.2146
_TRANS--LNW1_TRANS	0.024160	0.011099	2.176742	0.0296
_CONS--LNW1_CONS	0.001840	0.012473	0.147491	0.8828
_MIN--LNW1_MIN	-0.023948	0.009447	-2.535062	0.0113
_GOV--LNW1_GOV	0.000261	0.004852	0.053764	0.9571
_COM--LNW1_COM	0.036117	0.004640	7.783115	0.0000
_FIN--LNW1_FIN	0.030418	0.006032	5.042307	0.0000
_WHOL--LNW1_WHOL	0.050534	0.014609	3.459070	0.0005
_EL--LNW1_EL	-0.020360	0.011093	-1.835395	0.0665
_MAN--LNH1_MAN	-0.099905	0.042903	-2.328617	0.0199
_AGRIC--LNH1_AGRIC	-0.418356	0.137081	-3.051897	0.0023
_TRANS--LNH1_TRANS	-0.058437	0.033431	-1.747971	0.0806
_CONS--LNH1_CONS	0.077385	0.040130	1.928380	0.0539
_MIN--LNH1_MIN	-0.019118	0.025701	-0.743873	0.4570
_GOV--LNH1_GOV	0.169798	0.014358	11.82600	0.0000
_COM--LNH1_COM	-0.002042	0.014336	-0.142436	0.8867
_FIN--LNH1_FIN	-0.006005	0.018755	-0.320192	0.7488
_WHOL--LNH1_WHOL	0.065066	0.045311	1.436005	0.1511
_EL--LNH1_EL	0.004117	0.033974	0.121168	0.9036
_MAN--LNZ1_MAN	0.533376	0.100821	5.290348	0.0000
_AGRIC--LNZ1_AGRIC	-0.222763	0.339970	-0.655244	0.5124
_TRANS--LNZ1_TRANS	0.386548	0.077089	5.014334	0.0000
_CONS--LNZ1_CONS	0.396817	0.098007	4.048878	0.0001
_MIN--LNZ1_MIN	-0.495985	0.069993	-7.086246	0.0000
_GOV--LNZ1_GOV	-0.345682	0.033421	-10.34340	0.0000
_COM--LNZ1_COM	0.031246	0.031288	0.998678	0.3180
_FIN--LNZ1_FIN	0.051733	0.040596	1.274334	0.2026
_WHOL--LNZ1_WHOL	0.488901	0.097885	4.994623	0.0000
_EL--LNZ1_EL	-0.015838	0.076466	-0.207128	0.8359
_MAN--LNT1_MAN	-0.010435	0.028267	-0.369165	0.7120
_AGRIC--LNT1_AGRIC	-0.096588	0.087000	-1.110205	0.2670
_TRANS--LNT1_TRANS	-0.019897	0.021752	-0.914695	0.3604

_CONS--LNT1_CONS	-0.102693	0.022054	-4.656375	0.0000
_MIN--LNT1_MIN	0.081649	0.016589	4.921806	0.0000
_GOV--LNT1_GOV	-0.034269	0.009415	-3.639995	0.0003
_COM--LNT1_COM	-0.037831	0.009473	-3.993695	0.0001
_FIN--LNT1_FIN	-0.031786	0.012346	-2.574628	0.0101
_WHOL--LNT1_WHOL	-0.008293	0.029828	-0.278011	0.7810
_EL--LNT1_EL	0.085853	0.022415	3.830139	0.0001
Weighted Statistics				
R-squared	0.870926	**Mean dependent var**		0.846794
Adjusted R-squared	0.866373	**S.D. dependent var**		2.749309
S.E. of regression	1.005012	**Sum squared resid**		3121.051
F-statistic	191.2818	**Durbin-Watson stat**		2.151326
Prob(F-statistic)	0.000000			

MAE = 0.85

RMSE = 2.1

Note: The restriction imposed doesn't seem to bring much improvement on the model's performance.

Table F.4: Wald test: testing restrictions on parameters C4 (lnP3) and C5 (lnP4)

Test Statistic	Value	df	Probability
F-statistic	96.59128	(2, 3090)	0.0000
Chi-square	193.1826	2	0.0000

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(4)	-0.093549	0.017009
C(5)	0.404997	0.032407

Table F.5: SUR estimates with new restrictions on C4 and C5

Variable	Coefficient	Std. Error	t-Statistic	Prob.
_MAN--C	0.059670	0.021641	2.757222	0.0059
_AGRIC--C	0.247667	0.047247	5.241933	0.0000
_TRANS--C	0.026220	0.010275	2.551847	0.0108
_CONS--C	-0.095115	0.018779	-5.064829	0.0000
_MIN--C	0.733153	0.040659	18.03158	0.0000
_GOV--C	0.040302	0.004428	9.101580	0.0000
_COM--C	0.047378	0.004954	9.563238	0.0000
_FIN--C	0.048106	0.006584	7.306451	0.0000
_WHOL--C	0.006882	0.017570	0.391688	0.6953
_EL--C	0.046592	0.010395	4.482232	0.0000
_MAN—S_MAN	-0.000819	0.000176	-4.646826	0.0000
_AGRIC—S_AGRIC	-1.17E-05	1.70E-06	-6.871994	0.0000
_TRANS—S_TRANS	-3.14E-07	8.48E-08	-3.702702	0.0002
_CONS—S_CONS	2.72E-06	7.32E-07	3.716595	0.0002
_MIN—S_MIN	-1.13E-05	6.26E-07	-18.00766	0.0000
_GOV—S_GOV	-2.88E-07	2.58E-08	-11.15009	0.0000
_COM—S_COM	-5.02E-10	1.70E-08	-0.029554	0.9764
_FIN—S_FIN	1.92E-08	2.94E-08	0.653462	0.5135
_WHOL—S_WHOL	-7.53E-08	1.11E-07	-0.677204	0.4983
_EL—S_EL	-3.37E-06	3.19E-07	-10.57176	0.0000
_MAN--LNP2_MAN	-0.260111	0.068058	-3.821934	0.0001
_AGRIC--LNP2_AGRIC	0.159311	0.095806	1.662845	0.0964
_TRANS--LNP2_TRANS	-0.240756	0.032905	-7.316634	0.0000
_CONS--LNP2_CONS	-0.407386	0.039662	-10.27151	0.0000
_MIN--LNP2_MIN	0.001273	0.000169	7.511315	0.0000
_GOV--LNP2_GOV	0.234573	0.015857	14.79332	0.0000
_COM--LNP2_COM	-0.184939	0.021106	-8.762238	0.0000
_FIN--LNP2_FIN	-0.198783	0.028501	-6.974580	0.0000
_WHOL--LNP2_WHOL	-0.241449	0.064913	-3.719560	0.0002
_EL--LNP2_EL	0.060099	0.048683	1.234511	0.2171
_MAN--LNM22_MAN	0.121334	0.038631	3.140825	0.0017
_AGRIC--LNM22_AGRIC	0.087374	0.113013	0.773134	0.4395
_TRANS--LNM22_TRANS	0.173294	0.027583	6.282725	0.0000
_CONS--LNM22_CONS	0.401841	0.037331	10.76437	0.0000
_MIN--LNM22_MIN	-0.006705	0.026058	-0.257325	0.7969
_GOV--LNM22_GOV	-0.019251	0.011080	-1.737471	0.0824

_COM--LNM22_COM	0.028337	0.011308	2.506034	0.0123
_FIN--LNM22_FIN	0.013992	0.014970	0.934699	0.3500
_WHOL--LNM22_WHOL	0.209780	0.034064	6.158482	0.0000
_EL--LNM22_EL	0.077855	0.026605	2.926278	0.0035
_MAN--LNSP2_MAN	0.021176	0.005689	3.722342	0.0002
_AGRIC--LNSP2_AGRIC	-0.054749	0.017172	-3.188309	0.0014
_TRANS--LNSP2_TRANS	0.010450	0.003807	2.744914	0.0061
_CONS--LNSP2_CONS	-0.011203	0.005361	-2.089822	0.0367
_MIN--LNSP2_MIN	-0.006060	0.003934	-1.540627	0.1235
_GOV--LNSP2_GOV	-0.009807	0.001657	-5.917235	0.0000
_COM--LNSP2_COM	0.006319	0.001681	3.759435	0.0002
_FIN--LNSP2_FIN	0.004871	0.002225	2.188688	0.0287
_WHOL--LNSP2_WHOL	0.039717	0.005031	7.894828	0.0000
_EL--LNSP2_EL	-0.000499	0.004049	-0.123213	0.9019
_MAN--LNYD1_MAN	1.087168	0.093857	11.58328	0.0000
_AGRIC--LNYD1_AGRIC	2.598321	0.248433	10.45883	0.0000
_TRANS--LNYD1_TRANS	0.944645	0.062062	15.22102	0.0000
_CONS--LNYD1_CONS	1.084394	0.084594	12.81887	0.0000
_MIN--LNYD1_MIN	0.363823	0.058929	6.173966	0.0000
_GOV--LNYD1_GOV	0.089166	0.025425	3.507072	0.0005
_COM--LNYD1_COM	0.222324	0.026473	8.398308	0.0000
_FIN--LNYD1_FIN	0.288445	0.035150	8.206002	0.0000
_WHOL--LNYD1_WHOL	0.363530	0.080534	4.513973	0.0000
_EL--LNYD1_EL	0.644744	0.063003	10.23358	0.0000
_MAN--LNR1_MAN	0.033972	0.009096	3.734785	0.0002
_AGRIC--LNR1_AGRIC	-0.062370	0.025440	-2.451644	0.0143
_TRANS--LNR1_TRANS	-0.027031	0.006096	-4.434383	0.0000
_CONS--LNR1_CONS	0.022375	0.008078	2.769857	0.0056
_MIN--LNR1_MIN	-0.014228	0.005674	-2.507320	0.0122
_GOV--LNR1_GOV	0.005409	0.002581	2.095758	0.0362
_COM--LNR1_COM	0.003775	0.002710	1.392913	0.1637
_FIN--LNR1_FIN	0.000418	0.003591	0.116429	0.9073
_WHOL--LNR1_WHOL	0.056674	0.008139	6.963400	0.0000
_EL--LNR1_EL	-0.002089	0.006316	-0.330732	0.7409
_MAN--LNW1_MAN	-0.005734	0.016141	-0.355238	0.7224
_AGRIC--LNW1_AGRIC	-0.090168	0.048231	-1.869507	0.0616
_TRANS--LNW1_TRANS	0.015029	0.010630	1.413817	0.1575
_CONS--LNW1_CONS	-0.035653	0.014638	-2.435706	0.0149
_MIN--LNW1_MIN	-0.018224	0.010898	-1.672282	0.0946

_GOV--LNW1_GOV	-0.002125	0.004662	-0.455813	0.6486
_COM--LNW1_COM	0.036431	0.004754	7.663047	0.0000
_FIN--LNW1_FIN	0.030295	0.006292	4.815088	0.0000
_WHOL--LNW1_WHOL	0.053229	0.014345	3.710525	0.0002
_EL--LNW1_EL	-0.023811	0.011153	-2.135007	0.0328
_MAN--LNH1_MAN	-0.021377	0.051250	-0.417109	0.6766
_AGRIC--LNH1_AGRIC	-0.450713	0.136556	-3.300582	0.0010
_TRANS--LNH1_TRANS	-0.012590	0.032846	-0.383308	0.7015
_CONS--LNH1_CONS	0.233127	0.044670	5.218911	0.0000
_MIN--LNH1_MIN	-0.032259	0.029358	-1.098822	0.2719
_GOV--LNH1_GOV	0.137948	0.012664	10.89313	0.0000
_COM--LNH1_COM	0.060358	0.013764	4.385171	0.0000
_FIN--LNH1_FIN	0.073348	0.018425	3.980798	0.0001
_WHOL--LNH1_WHOL	0.136217	0.041885	3.252182	0.0012
_EL--LNH1_EL	0.018625	0.030487	0.610904	0.5413
_MAN--LNZ1_MAN	0.541744	0.109949	4.927242	0.0000
_AGRIC--LNZ1_AGRIC	-0.204062	0.336745	-0.605984	0.5446
_TRANS--LNZ1_TRANS	0.336292	0.072293	4.651800	0.0000
_CONS--LNZ1_CONS	0.605857	0.112646	5.378418	0.0000
_MIN--LNZ1_MIN	-0.562514	0.077709	-7.238724	0.0000
_GOV--LNZ1_GOV	-0.380903	0.032249	-11.81132	0.0000
_COM--LNZ1_COM	0.039167	0.032253	1.214392	0.2247
_FIN--LNZ1_FIN	0.052667	0.042569	1.237210	0.2161
_WHOL--LNZ1_WHOL	0.498702	0.096468	5.169638	0.0000
_EL--LNZ1_EL	-0.053190	0.077667	-0.684845	0.4935
_MAN--LNT1_MAN	0.046983	0.029528	1.591143	0.1117
_AGRIC--LNT1_AGRIC	-0.181823	0.081778	-2.223380	0.0263
_TRANS--LNT1_TRANS	-0.007935	0.020092	-0.394954	0.6929
_CONS--LNT1_CONS	-0.112233	0.026545	-4.227976	0.0000
_MIN--LNT1_MIN	0.089142	0.017973	4.959849	0.0000
_GOV--LNT1_GOV	-0.034087	0.008442	-4.037738	0.0001
_COM--LNT1_COM	-0.052365	0.008986	-5.827371	0.0000
_FIN--LNT1_FIN	-0.051540	0.011942	-4.315870	0.0000
_WHOL--LNT1_WHOL	-0.031571	0.027153	-1.162721	0.2450
_EL--LNT1_EL	0.098936	0.020973	4.717280	0.0000

Weighted Statistics			
R-squared	0.873331	**Mean dependent var**	0.946239
Adjusted R-squared	0.869003	**S.D. dependent var**	2.770911
S.E. of regression	1.002890	**Sum squared resid**	3208.464
F-statistic	201.7778	**Durbin-Watson stat**	1.990235
Prob(F-statistic)	0.000000		

APPENDIX G: PRICE EQUATIONS (WITH ENTRY COST)

Table G.1: Cross section ISUR of price equation with Γ.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
_MAN--C	0.109258	0.012334	8.858137	0.0000
_AGRIC--C	0.200674	0.023663	8.480581	0.0000
_TRANS--C	0.051690	0.008902	5.806397	0.0000
_CONS--C	0.007207	0.010174	0.708354	0.4788
_MIN--C	19.59402	11.77791	1.663624	0.0963
_GOV--C	0.026930	0.004359	6.178600	0.0000
_COM--C	0.029192	0.004144	7.043800	0.0000
_FIN--C	0.029258	0.004152	7.045892	0.0000
_WHOL--C	0.030166	0.004946	6.099392	0.0000
_EL--C	0.024807	0.004077	6.084881	0.0000
_MAN—S_MAN	-0.000635	9.30E-05	-6.820433	0.0000
_AGRIC—S_AGRIC	-4.23E-06	7.78E-07	-5.441587	0.0000
_TRANS—S_TRANS	-4.22E-07	5.74E-08	-7.361696	0.0000
_CONS—S_CONS	4.57E-07	3.00E-07	1.522390	0.1280
_MIN—S_MIN	-0.000368	0.000180	-2.046793	0.0408
_GOV—S_GOV	-1.23E-07	1.47E-08	-8.376776	0.0000
_COM—S_COM	-8.39E-08	8.14E-09	-10.30173	0.0000
_FIN—S_FIN	-1.11E-07	1.08E-08	-10.26587	0.0000
_WHOL—S_WHOL	-1.48E-07	2.18E-08	-6.783265	0.0000
_EL—S_EL	-6.07E-07	6.32E-08	-9.610115	0.0000
_MAN--LNP2_MAN	0.556328	0.037856	14.69600	0.0000
_AGRIC--LNP2_AGRIC	-0.062509	0.046470	-1.345143	0.1787
_TRANS--LNP2_TRANS	0.743295	0.045920	16.18678	0.0000
_CONS--LNP2_CONS	0.752110	0.051495	14.60544	0.0000
_MIN--LNP2_MIN	0.141569	0.045862	3.086834	0.0020
_GOV--LNP2_GOV	0.796238	0.037653	21.14682	0.0000
_COM--LNP2_COM	0.781418	0.036264	21.54820	0.0000
_FIN--LNP2_FIN	0.780703	0.036340	21.48330	0.0000
_WHOL--LNP2_WHOL	0.786956	0.039876	19.73510	0.0000
_EL--LNP2_EL	0.789785	0.036723	21.50627	0.0000
_MAN--LNP3_MAN	-0.381199	0.044072	-8.649402	0.0000
_AGRIC--LNP3_AGRIC	0.025951	0.041847	0.620152	0.5352
_TRANS--LNP3_TRANS	-0.261118	0.044662	-5.846491	0.0000
_CONS--LNP3_CONS	-0.257964	0.049743	-5.185899	0.0000
_MIN--LNP3_MIN	0.141633	0.044563	3.178308	0.0015

_GOV--LNP3_GOV	-0.516770	0.055102	-9.378481	0.0000
_COM--LNP3_COM	-0.504715	0.052864	-9.547380	0.0000
_FIN--LNP3_FIN	-0.504359	0.052835	-9.545990	0.0000
_WHOL--LNP3_WHOL	-0.508308	0.057218	-8.883724	0.0000
_EL--LNP3_EL	-0.511741	0.053812	-9.509807	0.0000
_MAN--LNP4_MAN	-0.043966	0.035717	-1.230954	0.2184
_AGRIC--LNP4_AGRIC	-0.548804	0.039453	-13.91018	0.0000
_TRANS--LNP4_TRANS	0.018003	0.041128	0.437737	0.6616
_CONS--LNP4_CONS	0.063902	0.045625	1.400582	0.1614
_MIN--LNP4_MIN	0.246105	0.051699	4.760364	0.0000
_GOV--LNP4_GOV	0.398564	0.044442	8.968172	0.0000
_COM--LNP4_COM	0.375145	0.042468	8.833556	0.0000
_FIN--LNP4_FIN	0.376150	0.042478	8.855180	0.0000
_WHOL--LNP4_WHOL	0.365229	0.046174	7.909911	0.0000
_EL--LNP4_EL	0.383219	0.043270	8.856533	0.0000
_MAN—LNM22_MAN	0.106281	0.026327	4.036950	0.0001
_AGRIC—LNM22_AGRIC	-0.044100	0.054206	-0.813569	0.4160
_TRANS—LNM22_TRANS	0.265922	0.033686	7.894171	0.0000
_CONS—LNM22_CONS	0.264479	0.037936	6.971740	0.0000
_MIN—LNM22_MIN	30.37939	7.373927	4.119839	0.0000
_GOV—LNM22_GOV	0.027866	0.017539	1.588822	0.1122
_COM—LNM22_COM	0.030443	0.016843	1.807463	0.0708
_FIN—LNM22_FIN	0.030745	0.016791	1.831035	0.0672
_WHOL—LNM22_WHOL	0.032472	0.018027	1.801346	0.0717
_EL—LNM22_EL	0.030701	0.017165	1.788530	0.0738
_MAN--LNSP2_MAN	-0.012321	0.004145	-2.972747	0.0030
_AGRIC--LNSP2_AGRIC	0.038589	0.009014	4.281024	0.0000
_TRANS--LNSP2_TRANS	-0.027355	0.005025	-5.443391	0.0000
_CONS--LNSP2_CONS	-0.025780	0.005648	-4.564202	0.0000
_MIN--LNSP2_MIN	-7.777968	1.142792	-6.806111	0.0000
_GOV--LNSP2_GOV	0.010485	0.002635	3.978731	0.0001
_COM--LNSP2_COM	0.009836	0.002525	3.895138	0.0001
_FIN--LNSP2_FIN	0.009831	0.002517	3.905672	0.0001
_WHOL--LNSP2_WHOL	0.008646	0.002697	3.206097	0.0014
_EL--LNSP2_EL	0.010855	0.002581	4.206041	0.0000
_MAN--LNYD1_MAN	-0.058037	0.073347	-0.791264	0.4289
_AGRIC--LNYD1_AGRIC	-0.512678	0.154083	-3.327286	0.0009
_TRANS--LNYD1_TRANS	-0.261171	0.100336	-2.602979	0.0093
_CONS--LNYD1_CONS	-0.318383	0.112343	-2.834030	0.0046

_MIN--LNYD1_MIN	146.1545	20.13600	7.258369	0.0000
_GOV--LNYD1_GOV	-0.217961	0.048541	-4.490223	0.0000
_COM--LNYD1_COM	-0.212355	0.046611	-4.555906	0.0000
_FIN--LNYD1_FIN	-0.212697	0.046478	-4.576337	0.0000
_WHOL--LNYD1_WHOL	-0.210822	0.049913	-4.223776	0.0000
_EL--LNYD1_EL	-0.221287	0.047521	-4.656632	0.0000
_MAN--LNR1_MAN	0.015644	0.006024	2.597009	0.0095
_AGRIC--LNR1_AGRIC	0.111820	0.012499	8.946414	0.0000
_TRANS--LNR1_TRANS	0.017977	0.007402	2.428584	0.0152
_CONS--LNR1_CONS	0.026058	0.008223	3.168882	0.0015
_MIN--LNR1_MIN	4.677231	1.597822	2.927253	0.0034
_GOV--LNR1_GOV	0.009534	0.003980	2.395134	0.0167
_COM--LNR1_COM	0.008184	0.003830	2.136567	0.0327
_FIN--LNR1_FIN	0.008106	0.003819	2.122337	0.0339
_WHOL--LNR1_WHOL	0.009607	0.004100	2.343397	0.0192
_EL--LNR1_EL	0.008982	0.003895	2.305861	0.0212
_MAN--LNW1_MAN	0.007812	0.011516	0.678347	0.4976
_AGRIC--LNW1_AGRIC	0.023301	0.024088	0.967333	0.3335
_TRANS--LNW1_TRANS	0.021253	0.014301	1.486128	0.1374
_CONS--LNW1_CONS	0.020476	0.016060	1.274970	0.2024
_MIN--LNW1_MIN	-10.23044	3.241618	-3.155967	0.0016
_GOV--LNW1_GOV	0.024599	0.007794	3.156126	0.0016
_COM--LNW1_COM	0.024142	0.007484	3.225925	0.0013
_FIN--LNW1_FIN	0.024409	0.007461	3.271465	0.0011
_WHOL--LNW1_WHOL	0.023160	0.008012	2.890437	0.0039
_EL--LNW1_EL	0.024887	0.007626	3.263322	0.0011
_MAN--LNH1_MAN	0.066698	0.034608	1.927264	0.0540
_AGRIC--LNH1_AGRIC	0.135573	0.070372	1.926508	0.0541
_TRANS--LNH1_TRANS	0.110581	0.041868	2.641176	0.0083
_CONS--LNH1_CONS	0.031886	0.046689	0.682938	0.4947
_MIN--LNH1_MIN	22.50610	10.23523	2.198886	0.0280
_GOV--LNH1_GOV	0.045496	0.022900	1.986762	0.0470
_COM--LNH1_COM	0.069489	0.022217	3.127798	0.0018
_FIN--LNH1_FIN	0.072773	0.022228	3.273963	0.0011
_WHOL--LNH1_WHOL	0.060584	0.024018	2.522507	0.0117
_EL--LNH1_EL	0.047821	0.022360	2.138719	0.0325
_MAN--LNOP1_MAN	-0.010967	0.005749	-1.907595	0.0565
_AGRIC--LNOP1_AGRIC	-0.012243	0.011894	-1.029331	0.3034
_TRANS--LNOP1_TRANS	0.005760	0.007506	0.767438	0.4429

_CONS--LNOP1_CONS	0.005111	0.008421	0.606975	0.5439
_MIN--LNOP1_MIN	-16.89719	1.641766	-10.29208	0.0000
_GOV--LNOP1_GOV	-0.006558	0.003912	-1.676285	0.0938
_COM--LNOP1_COM	-0.006413	0.003757	-1.707145	0.0879
_FIN--LNOP1_FIN	-0.006333	0.003745	-1.690744	0.0910
_WHOL--LNOP1_WHOL	-0.007719	0.004023	-1.918619	0.0551
_EL--LNOP1_EL	-0.006776	0.003828	-1.770077	0.0768
_MAN--LNT1_MAN	0.107442	0.020353	5.278926	0.0000
_AGRIC--LNT1_AGRIC	0.318080	0.040363	7.880583	0.0000
_TRANS--LNT1_TRANS	-0.145871	0.026382	-5.529238	0.0000
_CONS--LNT1_CONS	-0.101352	0.029301	-3.458966	0.0005
_MIN--LNT1_MIN	-15.67584	5.243171	-2.989762	0.0028
_GOV--LNT1_GOV	0.092441	0.013234	6.985058	0.0000
_COM--LNT1_COM	0.084375	0.012775	6.604883	0.0000
_FIN--LNT1_FIN	0.083338	0.012753	6.534972	0.0000
_WHOL--LNT1_WHOL	0.090211	0.013707	6.581316	0.0000
_EL--LNT1_EL	0.088973	0.012957	6.866585	0.0000
Weighted Statistics				
R-squared	0.937490	**Mean dependent var**		1.332331
Adjusted R-squared	0.934775	**S.D. dependent var**		3.051673
S.E. of regression	0.779374	**Sum squared resid**		1804.050
F-statistic	345.2887	**Durbin-Watson stat**		2.230358
Prob(F-statistic)	0.000000			

MAE = 1.33

RMSE = 3.05

Note: As expected, it is noticeable that in most cases, rise in the growth of entry cost leads to higher growth in the price level (more inflation).

APPENDIX H: INTRODUCING THE WORLD INCOME

It will be incomplete and rather unrealistic to present a full-fledged MMM-DA model with omission of the world impact on sectors' production activities. Since most countries, with higher emphasis on South Africa, are actively engaged in international trade through high volumes of exports and imports, the world income must appear in the sales equations. It will probably be a much better option with more accuracy to introduce the disposable income of the main exporting countries. However, as the number of export partners is high and export partners might change over time, it is simpler to consider the world income. In several occasions, the use of world income to capture the countries export volumes has produced reliable results. Therefore I have run another set of seemingly unrelated regressions for both sales and price with world income includes.

Table H.1: ISUR of Sales Supply using IY (International Income)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
_MAN—C	0.224956	0.021429	10.49787	0.0000
_AGRIC—C	0.253067	0.058538	4.323097	0.0000
_TRANS—C	0.059333	0.010222	5.804525	0.0000
_CONS—C	-0.074249	0.017190	-4.319215	0.0000
_MIN—C	0.290666	0.030559	9.511691	0.0000
_GOV—C	0.032408	0.005586	5.801359	0.0000
_COM—C	0.076694	0.005416	14.16027	0.0000
_FIN—C	0.085535	0.006669	12.82603	0.0000
_WHOL—C	-0.031086	0.019206	-1.618580	0.1056
_EL—C	0.049232	0.011890	4.140538	0.0000
_MAN—S_MAN	-0.002129	0.000170	-12.55533	0.0000
_AGRIC—S_AGRIC	-1.06E-05	2.09E-06	-5.047176	0.0000
_TRANS—S_TRANS	-5.55E-07	9.35E-08	-5.941307	0.0000
_CONS—S_CONS	2.06E-06	6.72E-07	3.069566	0.0022
_MIN—S_MIN	-4.69E-06	4.58E-07	-10.24115	0.0000
_GOV—S_GOV	-2.21E-07	3.54E-08	-6.245820	0.0000
_COM—S_COM	-7.18E-08	1.89E-08	-3.793242	0.0002
_FIN—S_FIN	-8.25E-08	3.02E-08	-2.730396	0.0064
_WHOL—S_WHOL	3.47E-07	1.26E-07	2.745026	0.0061
_EL—S_EL	-3.58E-06	3.80E-07	-9.419370	0.0000

_MAN--LNP2_MAN	-0.609981	0.064195	-9.502055	0.0000
_AGRIC--LNP2_AGRIC	0.494675	0.135652	3.646641	0.0003
_TRANS--LNP2_TRANS	-0.381977	0.035226	-10.84370	0.0000
_CONS--LNP2_CONS	-0.077373	0.046706	-1.656607	0.0977
_MIN--LNP2_MIN	0.000357	0.000108	3.305762	0.0010
_GOV--LNP2_GOV	0.244255	0.032141	7.599470	0.0000
_COM--LNP2_COM	-0.170525	0.034826	-4.896430	0.0000
_FIN--LNP2_FIN	-0.128519	0.043356	-2.964279	0.0031
_WHOL--LNP2_WHOL	-0.108209	0.105465	-1.026018	0.3050
_EL--LNP2_EL	-0.127471	0.083399	-1.528439	0.1265
_MAN--LNP3_MAN	0.278969	0.067344	4.142455	0.0000
_AGRIC--LNP3_AGRIC	-0.238034	0.103536	-2.299060	0.0216
_TRANS--LNP3_TRANS	0.009701	0.036044	0.269143	0.7878
_CONS--LNP3_CONS	-0.420762	0.045419	-9.263986	0.0000
_MIN--LNP3_MIN	-0.000616	0.000112	-5.480574	0.0000
_GOV--LNP3_GOV	-0.023384	0.040823	-0.572805	0.5668
_COM--LNP3_COM	0.061498	0.041495	1.482047	0.1384
_FIN--LNP3_FIN	0.078785	0.051318	1.535217	0.1248
_WHOL--LNP3_WHOL	0.312087	0.126057	2.475768	0.0134
_EL--LNP3_EL	0.280659	0.103178	2.720159	0.0066
_MAN--LNP4_MAN	-0.521400	0.063636	-8.193533	0.0000
_AGRIC--LNP4_AGRIC	-0.150309	0.101553	-1.480098	0.1390
_TRANS--LNP4_TRANS	-0.166523	0.032253	-5.162959	0.0000
_CONS--LNP4_CONS	-0.204335	0.038895	-5.253448	0.0000
_MIN--LNP4_MIN	0.000845	0.000122	6.946741	0.0000
_GOV--LNP4_GOV	-0.012779	0.032633	-0.391604	0.6954
_COM--LNP4_COM	-0.279908	0.032800	-8.533884	0.0000
_FIN--LNP4_FIN	-0.444830	0.040759	-10.91363	0.0000
_WHOL--LNP4_WHOL	-0.634444	0.100180	-6.333009	0.0000
_EL--LNP4_EL	-0.024955	0.082140	-0.303810	0.7613
_MAN—LNM22_MAN	0.237641	0.033878	7.014672	0.0000
_AGRIC—LNM22_AGRIC	0.039306	0.113311	0.346885	0.7287
_TRANS—LNM22_TRANS	0.080794	0.024445	3.305177	0.0010
_CONS—LNM22_CONS	0.386094	0.031017	12.44767	0.0000
_MIN—LNM22_MIN	0.003638	0.016281	0.223476	0.8232
_GOV—LNM22_GOV	-0.021130	0.011535	-1.831809	0.0671
_COM—LNM22_COM	0.069237	0.011194	6.185230	0.0000
_FIN—LNM22_FIN	0.077869	0.013879	5.610644	0.0000
_WHOL—LNM22_WHOL	0.258536	0.032877	7.863839	0.0000

_EL—LNM22_EL	0.094225	0.028198	3.341523	0.0008
_MAN--LNSP2_MAN	0.015886	0.005198	3.055854	0.0023
_AGRIC--LNSP2_AGRIC	-0.073847	0.019246	-3.837105	0.0001
_TRANS--LNSP2_TRANS	0.007072	0.003289	2.150195	0.0316
_CONS--LNSP2_CONS	-0.015397	0.004293	-3.586984	0.0003
_MIN--LNSP2_MIN	-0.018756	0.002547	-7.363221	0.0000
_GOV--LNSP2_GOV	-0.012285	0.001740	-7.060539	0.0000
_COM--LNSP2_COM	0.005128	0.001651	3.105742	0.0019
_FIN--LNSP2_FIN	0.001584	0.002047	0.773509	0.4393
_WHOL--LNSP2_WHOL	0.032535	0.004769	6.821593	0.0000
_EL--LNSP2_EL	-0.000554	0.004294	-0.128946	0.8974
_MAN--LNYD1_MAN	1.115951	0.087781	12.71295	0.0000
_AGRIC--LNYD1_AGRIC	2.495401	0.287618	8.676084	0.0000
_TRANS--LNYD1_TRANS	0.801753	0.065529	12.23513	0.0000
_CONS--LNYD1_CONS	1.654479	0.083192	19.88739	0.0000
_MIN--LNYD1_MIN	0.077666	0.041593	1.867300	0.0620
_GOV--LNYD1_GOV	0.141292	0.029851	4.733169	0.0000
_COM--LNYD1_COM	0.258900	0.029077	8.903974	0.0000
_FIN--LNYD1_FIN	0.389040	0.036093	10.77896	0.0000
_WHOL--LNYD1_WHOL	0.616917	0.086578	7.125584	0.0000
_EL--LNYD1_EL	0.738397	0.073956	9.984290	0.0000
_MAN--LNR1_MAN	0.044700	0.007766	5.756105	0.0000
_AGRIC--LNR1_AGRIC	-0.046768	0.025897	-1.805892	0.0710
_TRANS--LNR1_TRANS	-0.027422	0.005301	-5.172934	0.0000
_CONS--LNR1_CONS	0.015562	0.006432	2.419454	0.0156
_MIN--LNR1_MIN	0.002223	0.003545	0.627128	0.5306
_GOV--LNR1_GOV	0.007016	0.002704	2.594764	0.0095
_COM--LNR1_COM	0.008772	0.002657	3.300782	0.0010
_FIN--LNR1_FIN	0.009018	0.003290	2.741070	0.0062
_WHOL--LNR1_WHOL	0.066356	0.007817	8.488567	0.0000
_EL--LNR1_EL	-0.006670	0.006594	-1.011542	0.3118
_MAN--LNW1_MAN	-0.026972	0.014039	-1.921146	0.0548
_AGRIC--LNW1_AGRIC	-0.139676	0.049359	-2.829816	0.0047
_TRANS--LNW1_TRANS	0.006860	0.009440	0.726763	0.4674
_CONS--LNW1_CONS	-0.011345	0.012030	-0.943078	0.3457
_MIN--LNW1_MIN	-0.018097	0.006853	-2.640654	0.0083
_GOV--LNW1_GOV	-0.005745	0.004769	-1.204526	0.2285
_COM--LNW1_COM	0.026978	0.004630	5.827013	0.0000
_FIN--LNW1_FIN	0.013365	0.005741	2.328066	0.0200

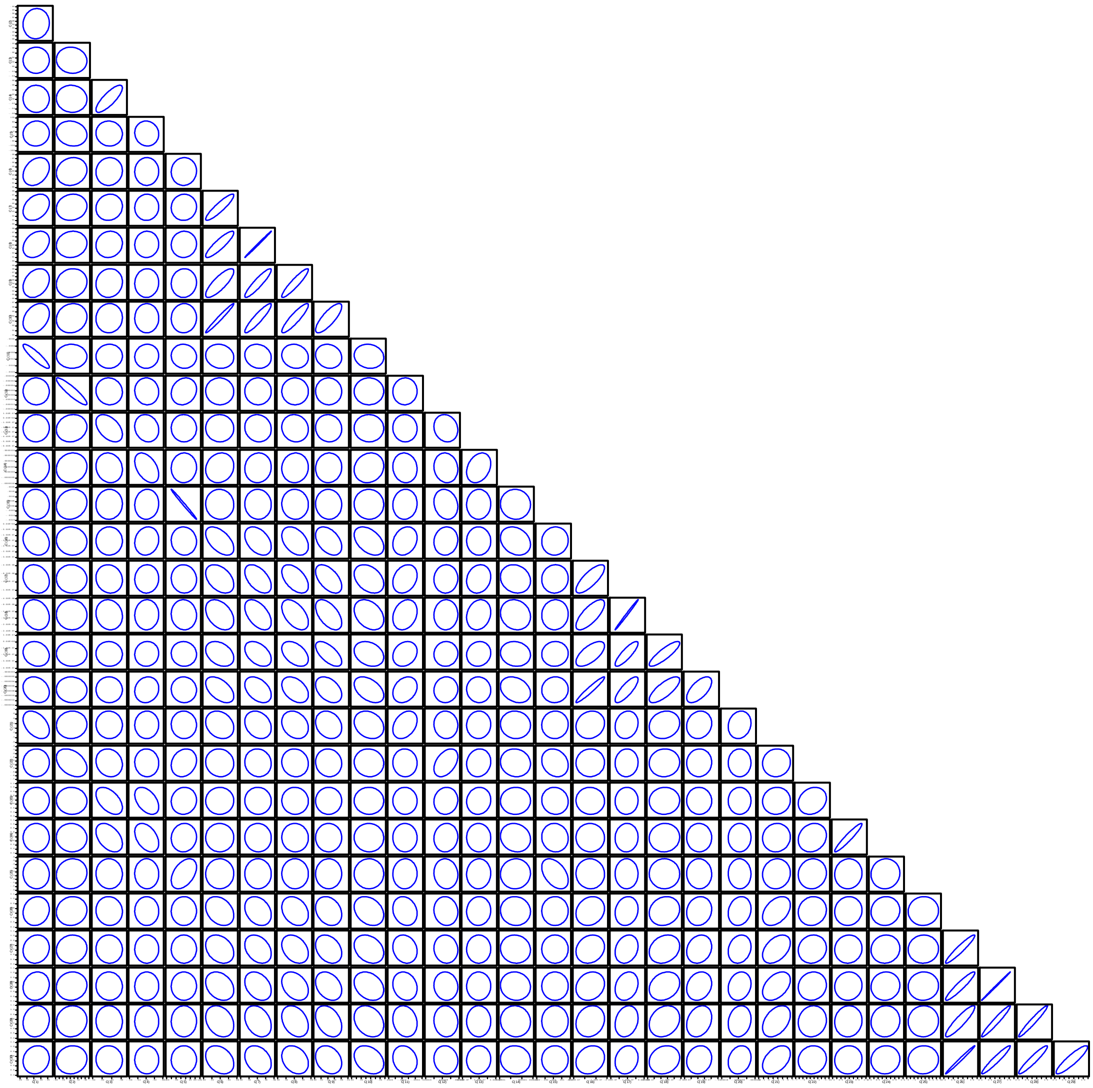
_WHOL--LNW1_WHOL	0.013647	0.013597	1.003643	0.3156
_EL--LNW1_EL	-0.024318	0.011616	-2.093510	0.0364
_MAN--LNH1_MAN	0.067467	0.050926	1.324801	0.1853
_AGRIC--LNH1_AGRIC	-0.563717	0.159377	-3.537007	0.0004
_TRANS--LNH1_TRANS	0.043358	0.032583	1.330698	0.1834
_CONS--LNH1_CONS	-0.013542	0.039743	-0.340742	0.7333
_MIN--LNH1_MIN	0.230470	0.024793	9.295697	0.0000
_GOV--LNH1_GOV	0.103868	0.016604	6.255652	0.0000
_COM--LNH1_COM	0.008211	0.017835	0.460375	0.6453
_FIN--LNH1_FIN	-0.035093	0.022365	-1.569106	0.1167
_WHOL--LNH1_WHOL	-0.138401	0.054243	-2.551516	0.0108
_EL--LNH1_EL	0.017981	0.040922	0.439401	0.6604
_MAN--LNZ1_MAN	0.478053	0.104373	4.580226	0.0000
_AGRIC--LNZ1_AGRIC	-0.659462	0.371712	-1.774124	0.0761
_TRANS--LNZ1_TRANS	0.436403	0.068880	6.335665	0.0000
_CONS--LNZ1_CONS	0.309575	0.103168	3.000693	0.0027
_MIN--LNZ1_MIN	-0.157804	0.057676	-2.736054	0.0063
_GOV--LNZ1_GOV	-0.382706	0.035122	-10.89637	0.0000
_COM--LNZ1_COM	0.038282	0.032632	1.173122	0.2408
_FIN--LNZ1_FIN	0.029684	0.040275	0.737033	0.4612
_WHOL--LNZ1_WHOL	0.517185	0.094631	5.465277	0.0000
_EL--LNZ1_EL	-0.148677	0.084427	-1.761018	0.0783
_MAN--LNT1_MAN	0.021862	0.028719	0.761233	0.4466
_AGRIC--LNT1_AGRIC	-0.212278	0.085433	-2.484732	0.0130
_TRANS--LNT1_TRANS	0.032712	0.019237	1.700453	0.0892
_CONS--LNT1_CONS	-0.040241	0.023409	-1.719037	0.0857
_MIN--LNT1_MIN	0.072577	0.011877	6.110674	0.0000
_GOV--LNT1_GOV	-0.020087	0.009435	-2.128844	0.0333
_COM--LNT1_COM	-0.044286	0.009374	-4.724508	0.0000
_FIN--LNT1_FIN	-0.032620	0.011616	-2.808050	0.0050
_WHOL--LNT1_WHOL	0.048924	0.028035	1.745074	0.0811
_EL--LNT1_EL	0.075409	0.023212	3.248678	0.0012
_MAN--LNIY1_MAN	-0.123157	0.098768	-1.246941	0.2125
_AGRIC--LNIY1_AGRIC	0.143401	0.355053	0.403885	0.6863
_TRANS--LNIY1_TRANS	0.675974	0.071508	9.453065	0.0000
_CONS--LNIY1_CONS	0.158558	0.087655	1.808876	0.0706
_MIN--LNIY1_MIN	0.501851	0.051933	9.663517	0.0000
_GOV--LNIY1_GOV	0.016838	0.034046	0.494558	0.6209
_COM--LNIY1_COM	-0.086361	0.032943	-2.621544	0.0088

_FIN--LNIY1_FIN	-0.161830	0.040845	-3.962082	0.0001
_WHOL--LNIY1_WHOL	-0.084989	0.096755	-0.878390	0.3798
_EL--LNIY1_EL	-0.272744	0.082762	-3.295523	0.0010
Weighted Statistics				
R-squared	0.871534	**Mean dependent var**		0.824852
Adjusted R-squared	0.865501	**S.D. dependent var**		2.714652
S.E. of regression	0.995575	**Sum squared resid**		2933.859
F-statistic	144.4680	**Durbin-Watson stat**		2.228069
Prob(F-statistic)	0.000000			

MAE = 0.02

RMSE = 1.93

APPENDIX I: CONFIDENCE ELLIPSE OF THE ISUR MODEL



APPENDIX J: Simplified RE Models

Assuming that exogenous variables grow at a constant rate, basic simulations could be performed on a shorter scale of the RFE model including only two lag terms.

$$\ln\left(\frac{S_{St}}{S_{S(t-1)}}\right) \approx \theta_0' + \theta_1' S_{S(t-1)} + \theta_2' \ln\left(\frac{P_{Q_{(t-1)}}}{P_{Q_{(t-2)}}}\right) \quad (1)$$

$$\ln\left(\frac{P_{Q_t}}{P_{Q_{(t-1)}}}\right) \approx \sigma_0' + \sigma_1' S_{S(t-1)} + \sigma_2' \ln\left(\frac{P_{t-1}}{P_{t-2}}\right) \quad (2)$$

$$\ln\left(\frac{w_t}{w_{t-1}}\right) \approx \lambda_0' + \lambda_1' S_{S(t-1)} + \lambda_2' \ln\left(\frac{P_{t-1}}{P_{t-2}}\right) \quad (3)$$

$$\ln\left(\frac{r_t}{r_{t-1}}\right) \approx \tau_0' + \tau_1' S_{S(t-1)} + \tau_2' \ln\left(\frac{P_{t-1}}{P_{t-2}}\right) \quad (4)$$

Basic simulation results of the RFEM:

In this annex, estimated values are assigned to the parameters and the variables (Sales and Prices) to observe the path of these series. The main idea is to point out cyclical patterns that may exist in observed series. When there is no much cyclical effect in the observed variables, the use of RFEM is justifiable. At this stage the same exercise can be performed for all sectors since the RFEM is meant to be disaggregated (RFEM-DA).

a) Manufacturing

Eq. 1: LNSSM_1 = -0.0009847859031*SSM(-1) - 0.5033795221*LNPPM_2 + 0.1586284306

(0.1608) *(0.0968)* *(0.0568)*

Eq. 2: LNPPM_1 = -0.0008523978602*SSM(-1) + 0.4960335134*LNPPM_2 + 0.1212367301

(0.0242) *(0.0031)* *(0.0071)*

Eq. 3: LNWM_1 = -0.002792525535*SSM(-1) - 0.3888479686*LNPPM_2 + 0.3461588654

(0.1212) *(0.6083)* *(0.1011)*

Eq. 4: LNKM_1 = -0.001528755074*SSM(-1) - 0.1852182113*LNPPM_2 + 0.1797313182

(0.0012) *(0.3209)* *(0.0012)*

Eq. 5: R_1 = -0.003593766788*SSM(-1) + 0.06682734982*LNPPM_2 + 1.338182137

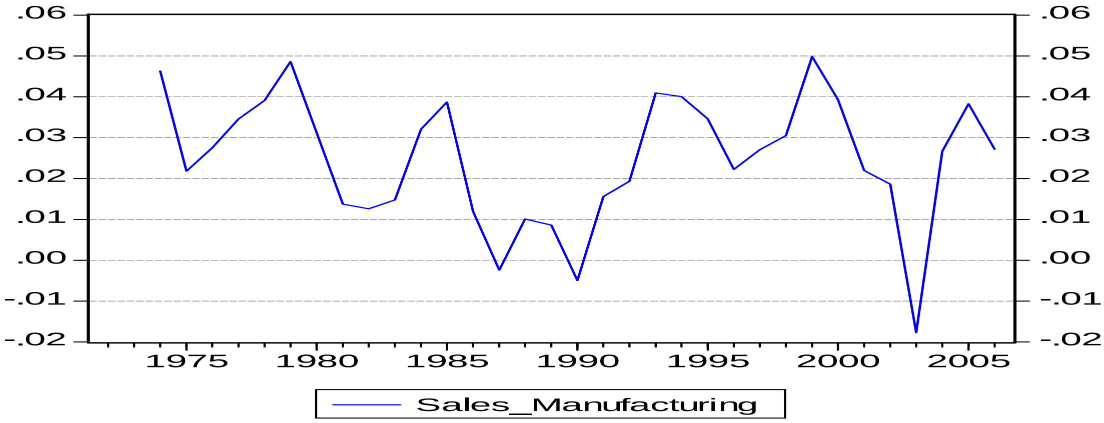
Eq. 6: LNLM_1 = -0.0004352825144*SSM(-1) + 0.155023422*LNPPM_2 + 0.02242745282

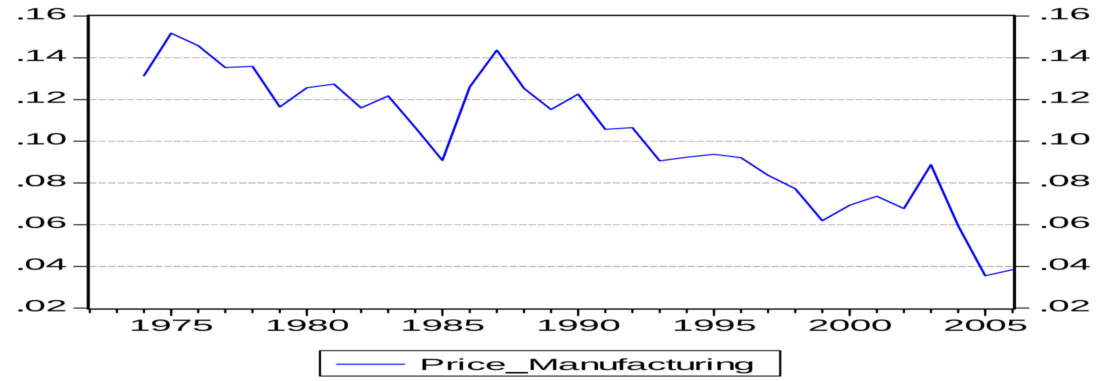
(0.2547) *(0.3425)* *(0.6124)*

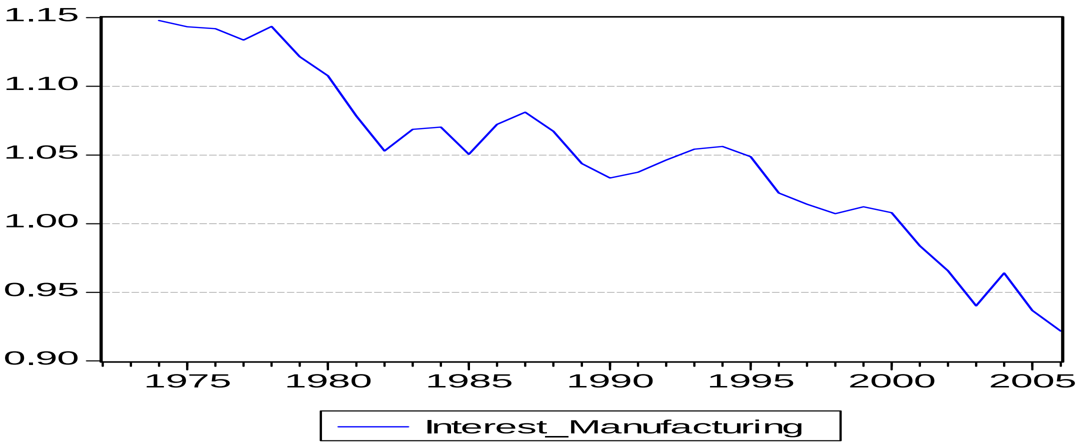
Eq. 7: LNM2_1 = 0.0004810211352*SSM(-1) + 0.4820351334*LNPPM_2 + 0.06256657467

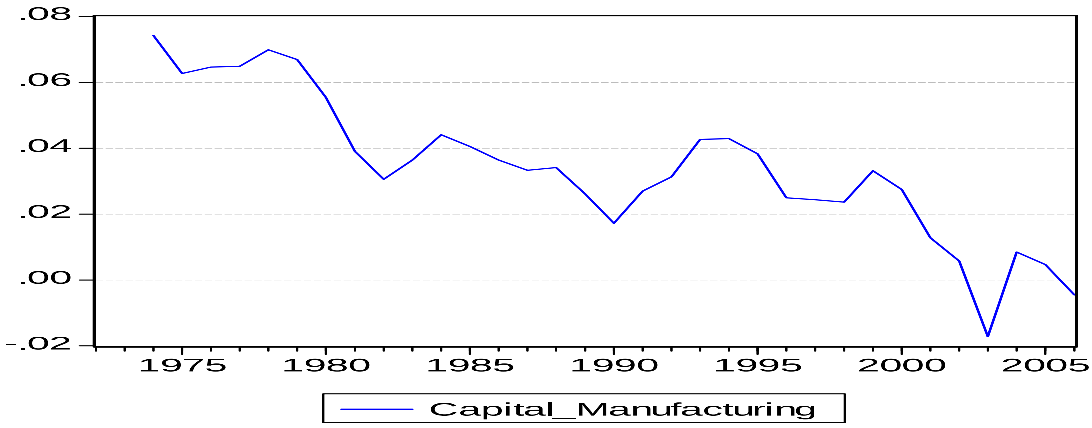
(0.5373) *(0.1551)* *(0.4127)*

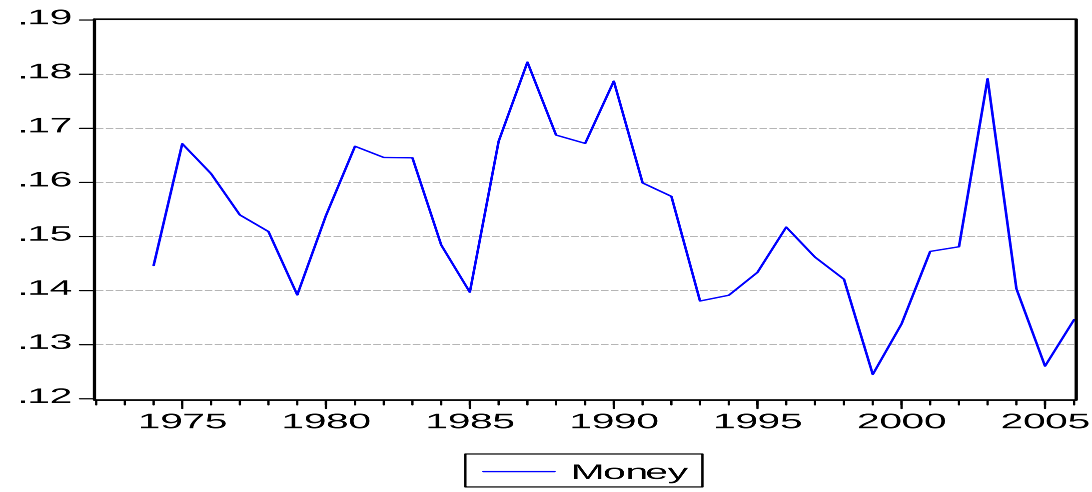
N.B. p-values in brackets











Sales and Money (*M2*) present very high cyclicality while other variables present a relatively smooth trend. It is therefore reasonable to use REM-DA to model Sales as price variables have a smooth trend.

b) Agriculture (Forestry and Fishing)

For all the sectors, trends for, *r*, *w*, and *M2* remain the same since those variables are assumed to be given in the aggregate economy.

Eq. 1: LNSA_1 = -1.449860774e-005*SA(-1) + 0.0164657689*LNPPA_2 + 0.3478267106

(0.0205) *(0.09657)* *(0.0342)*

Eq. 2: LNPPA_1 = -3.37661597e-006*SA(-1) + 0.1969181616*LNPPA_2 + 0.1472646375

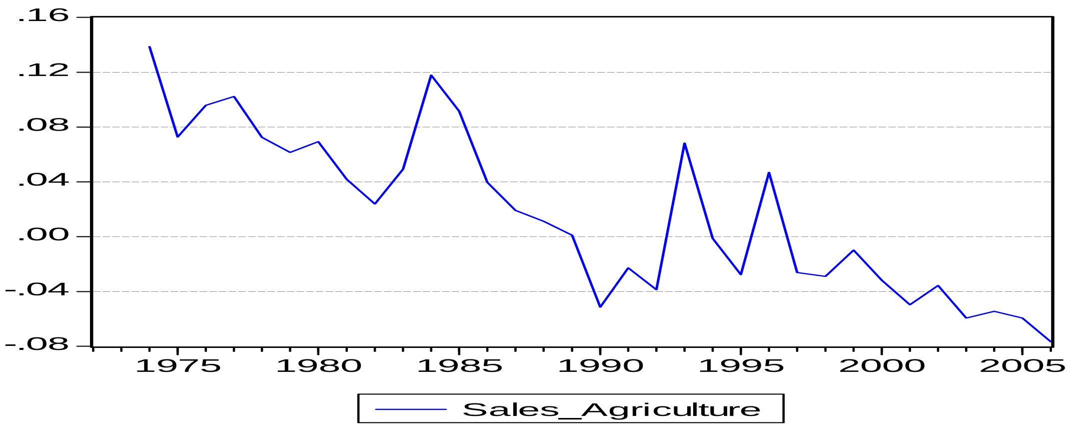
(0.2554) *(0.2997)* *(0.0655)*

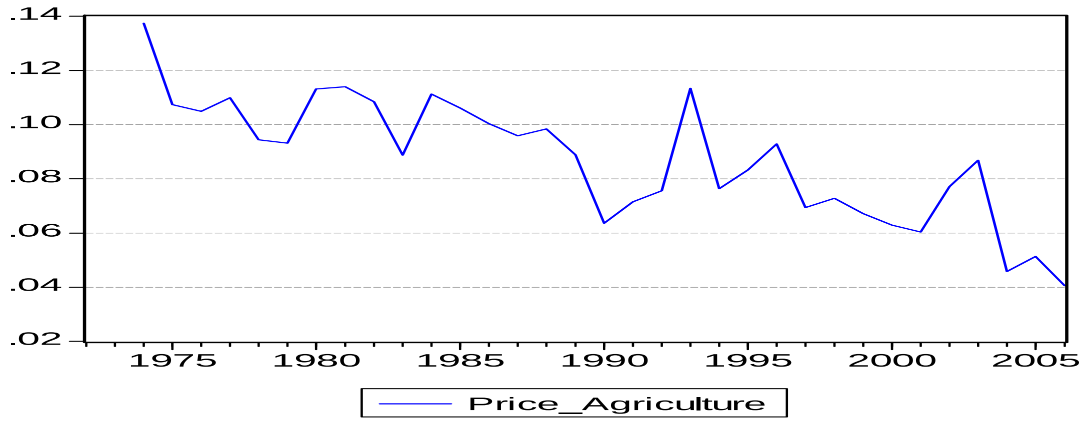
Eq. 3: LNKA_1 = -1.325510058e-006*SA(-1) + 0.0419274253*LNPPA_2 + 0.02485148132

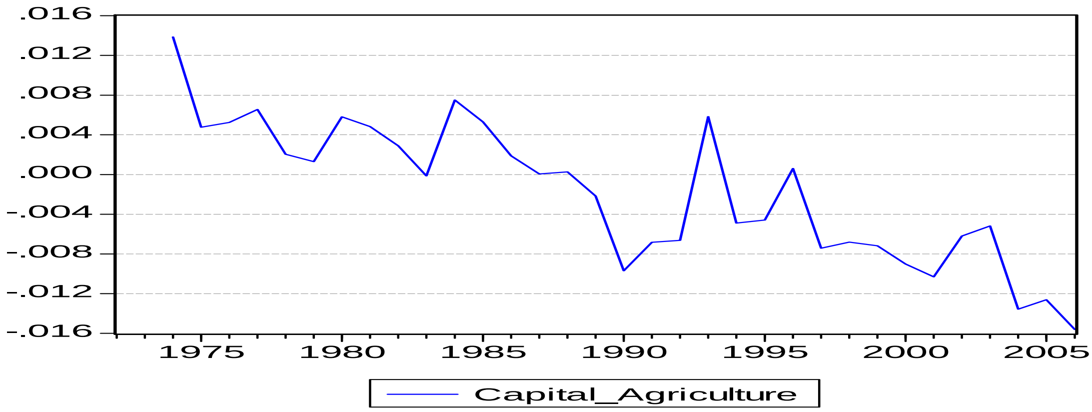
(0.1643) *(0.4870)* *(0.3204)*

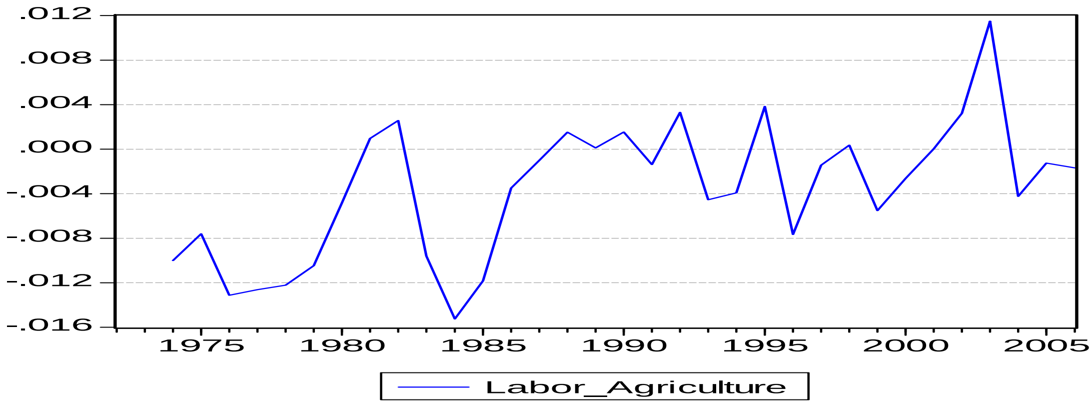
Eq. 4: LNLA_1 = 1.692814882e-006*SA(-1) + 0.06753534749*LNPPA_2 - 0.0484189937

(0.0237) *(0.1483)* *(0.0151)*









c) Transport & Communication

Eq. 1: LNST_1 = -3.700812868e-007*ST(-1) - 0.5426323197*LNPT_2 + 0.1080195039

(0.2443) *(0.0002)* *(0.0003)*

Eq. 2: LNPT_1 = -6.718483327e-007*ST(-1) + 0.5125717372*LNPT_2 + 0.07728127176

(0.0950) *(0.0036)* *(0.0249)*

Eq. 3: LNKT_1 = -4.389509723e-008*ST(-1) + 0.05838576312*LNPT_2 + 0.01884831656

(0.8458) *(0.5358)* *(0.3237)*

Eq. 4: LNLT_1 = 2.090997945e-007*ST(-1) - 0.004150279626*LNPT_2 + 0.008136863475

(0.0385) *(0.9185)* *(0.3238)*

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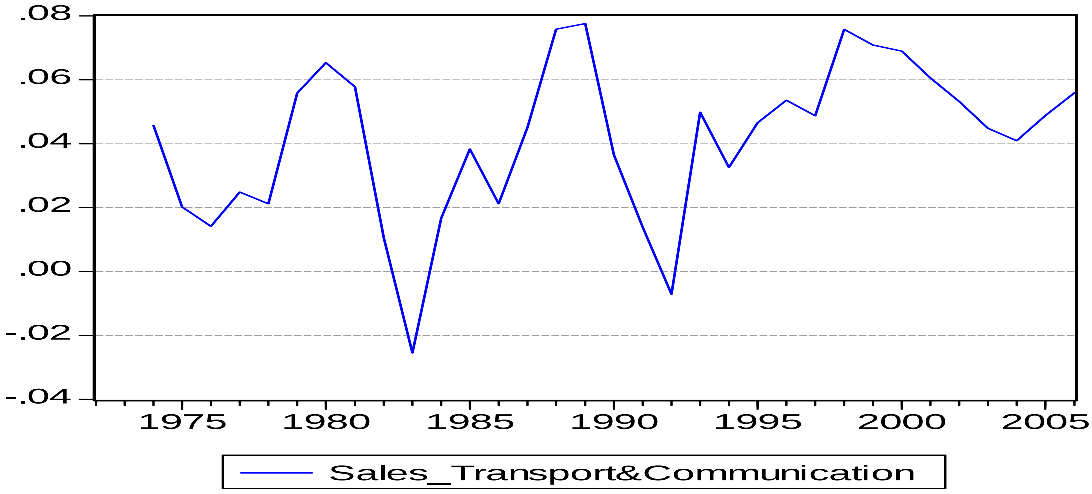
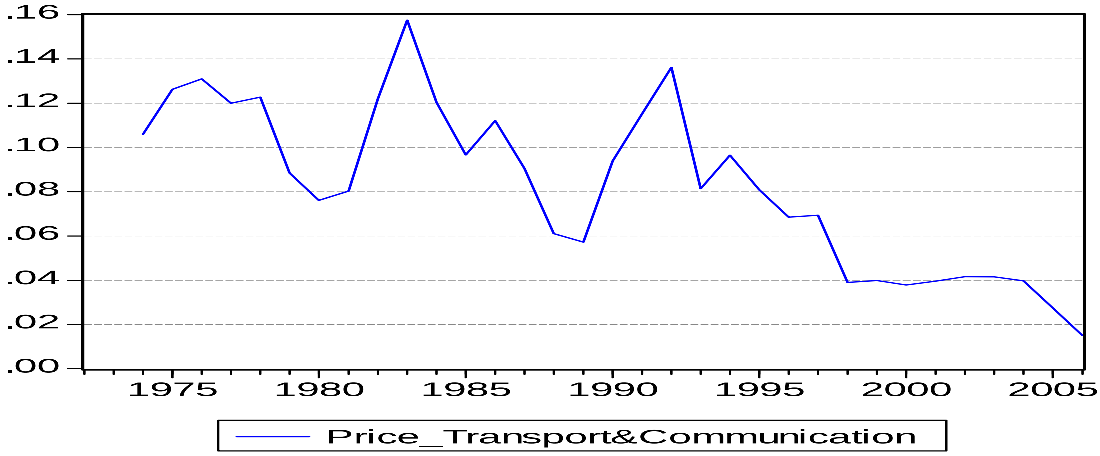
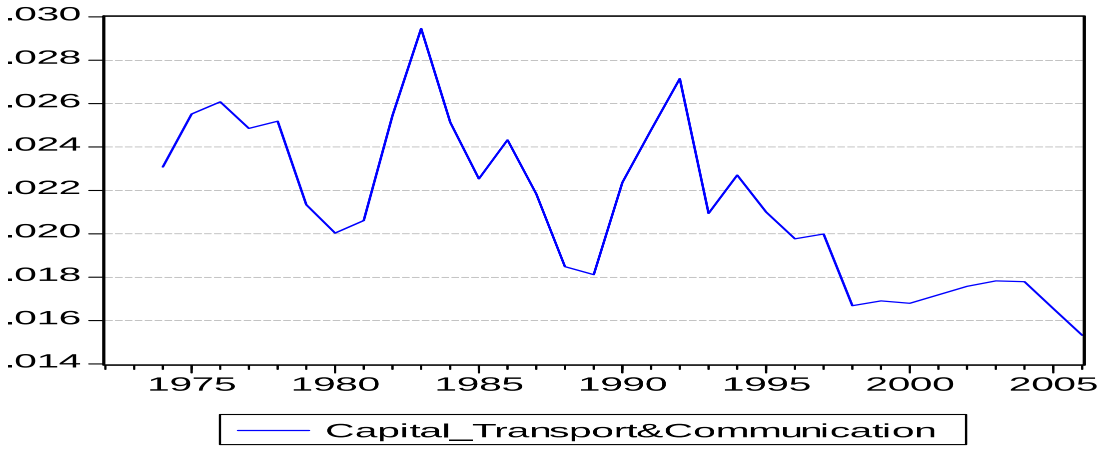
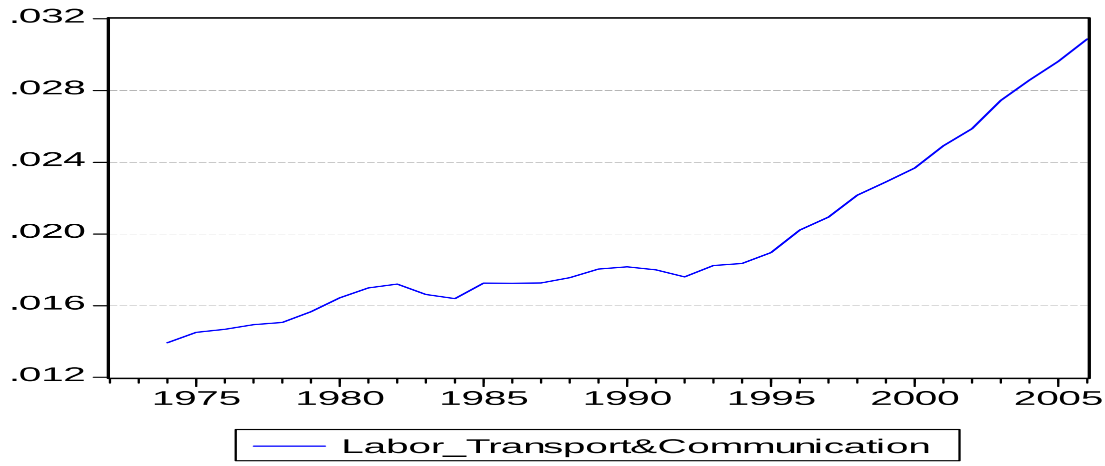








d) Construction and Buildings

Eq. 1: LNSC_1 = 7.161355464e-006*SC(-1) - 0.8320176048*LNPC_2 - 0.08593985072

(0.0335) (0.0001) (0.2441)

Eq. 2: LNPC_1 = 3.193400134e-006*SC(-1) + 0.6192166157*LNPC_2 - 0.04440839336

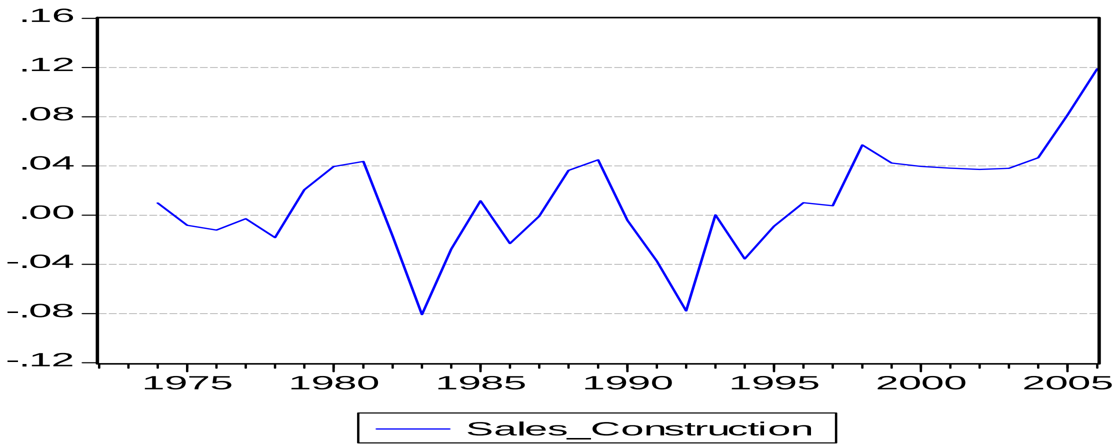
(0.2266) (0.0001) (0.4516)

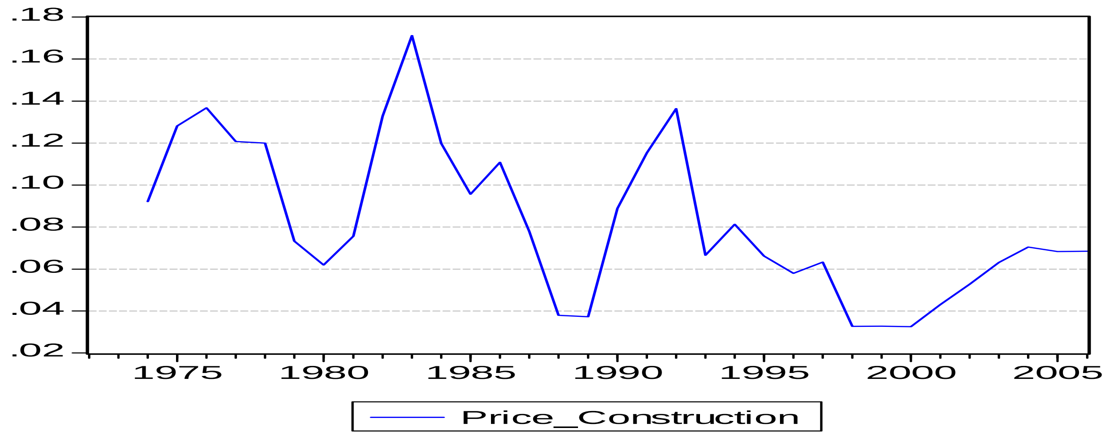
Eq. 3: LNKC_1 = 1.166027905e-005*SC(-1) - 0.4080082195*LNPC_2 - 0.2081604311

(0.0003) (0.0135) (0.0026)

Eq. 4: LNLC_1 = 1.484073785e-005*SC(-1) - 0.7906665729*LNPC_2 - 0.2770053927

(0.0349) (0.0408) (0.0768)




.18
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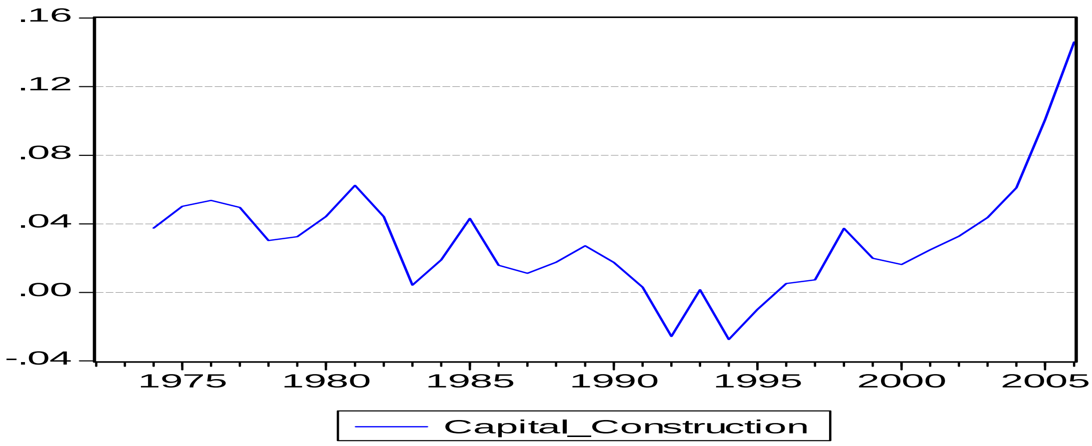
2005
Price_Construction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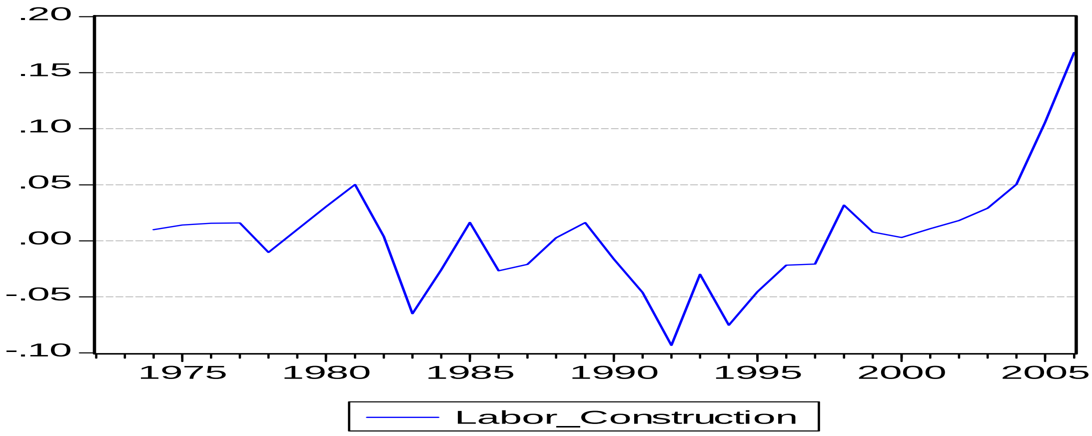
Capital_Construction


.20
.15
.10
.05
.00
-.05
-.10
1975
1980
1985
1990
1995
2000
2005
Labor_Construction

e) Mining

The price variable used here is the 'mining shares prices of non-gold products'. The employment figures used here represent the employment in the private sector.

Eq. 1: LNSMIN_1 = -6.931141118e-006*SMIN(-1) + 0.001028145553*PMIN_2 + 0.458080001

(0.0026) *(0.1274)* *(0.0027)*

Eq. 2 : PMIN_1 = 0.0004849421609*SMIN(-1) + 0.1922480139*PMIN_2 - 33.18159384

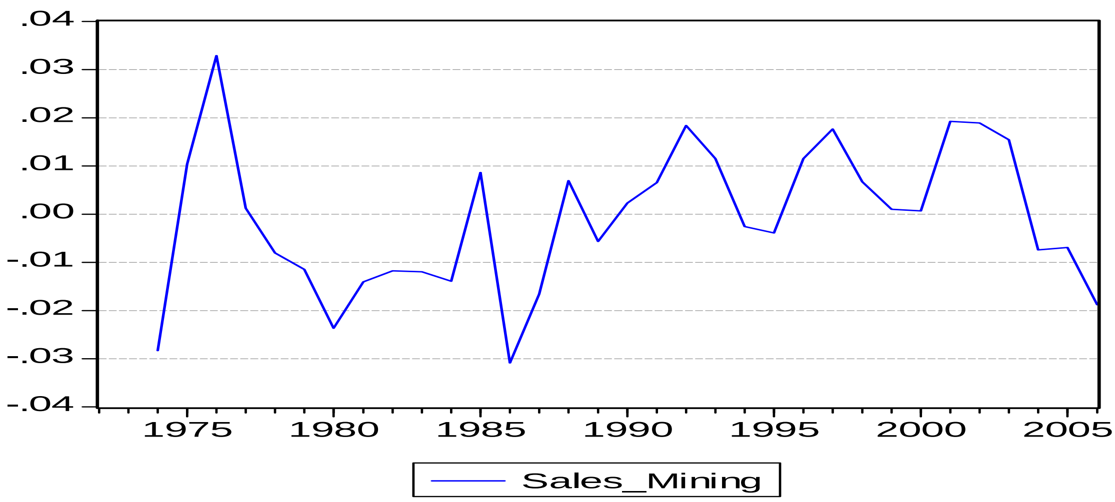
(0.4396) *(0.3256)* *(0.4251)*

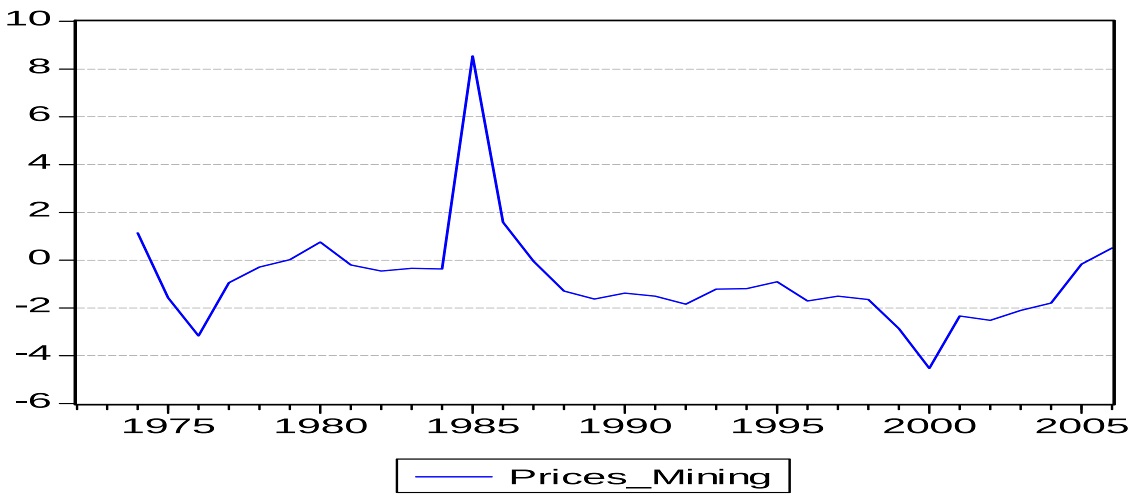
Eq. 3 : LNLMIN_1 = 7.302960494e-006*SMIN(-1) + 0.0004871534778*PMIN_2 - 0.4966305143

(0.0852) *(0.7051)* *(0.0778)*

Eq. 4 : LNKMIN_1 = 3.599247421e-006*SMIN(-1) + 0.0002943772211*PMIN_2 - 0.2056379422

(0.2063) *(0.7362)* *(0.2743)*





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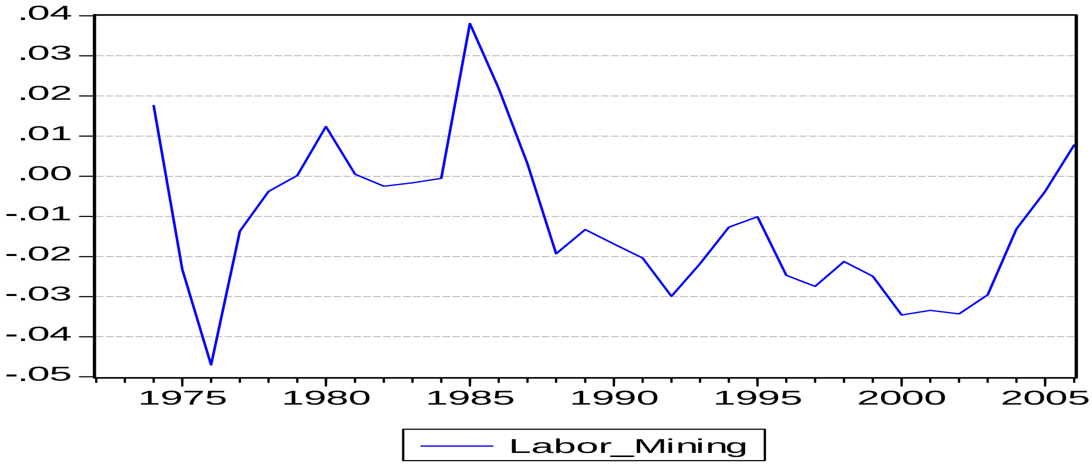
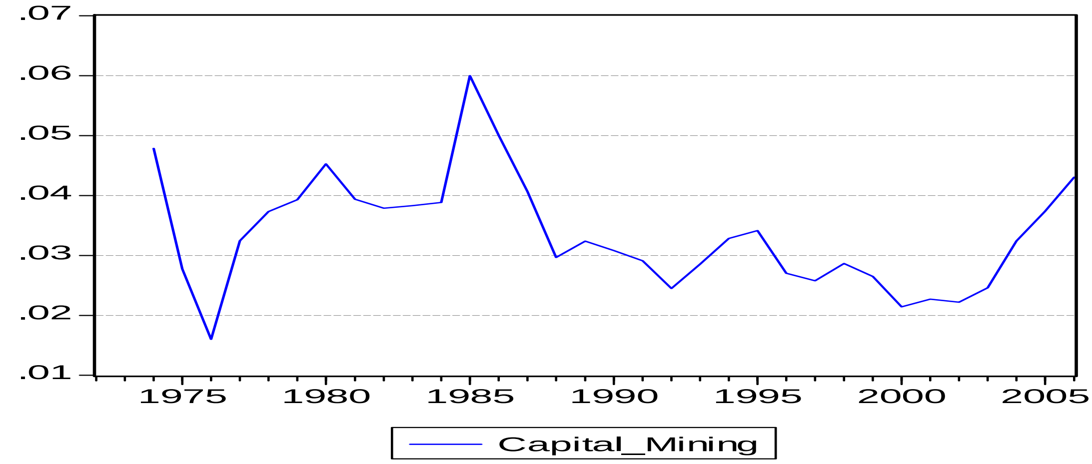




For other sectors with no specific data on price and wage the normal 'consumer price index' is used with the wage for non-agricultural sectors.

APPENDIX K: WALD TEST ON SECOND MODEL APPROXIMATION

Table K.1: Wald test: testing restrictions on parameters C2 ($S_{i(t-1)}$) and C3 ($S_{i(t-2)}$) from SUR 1

Test Statistic	Value	df	Probability
F-statistic	3.657067	(1, 179)	0.0574
Chi-square	3.657067	1	0.0558

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(2)	0.260753	0.136352

Note: According to the Wald test that we performed, we can only impose a restriction on C2, and not on other parameters.

APPENDIX L: CONFIDENCE ELLIPSE FOR MODEL 2

Figure L.1: Confidence ellipse for ISUR (Sales Supply)

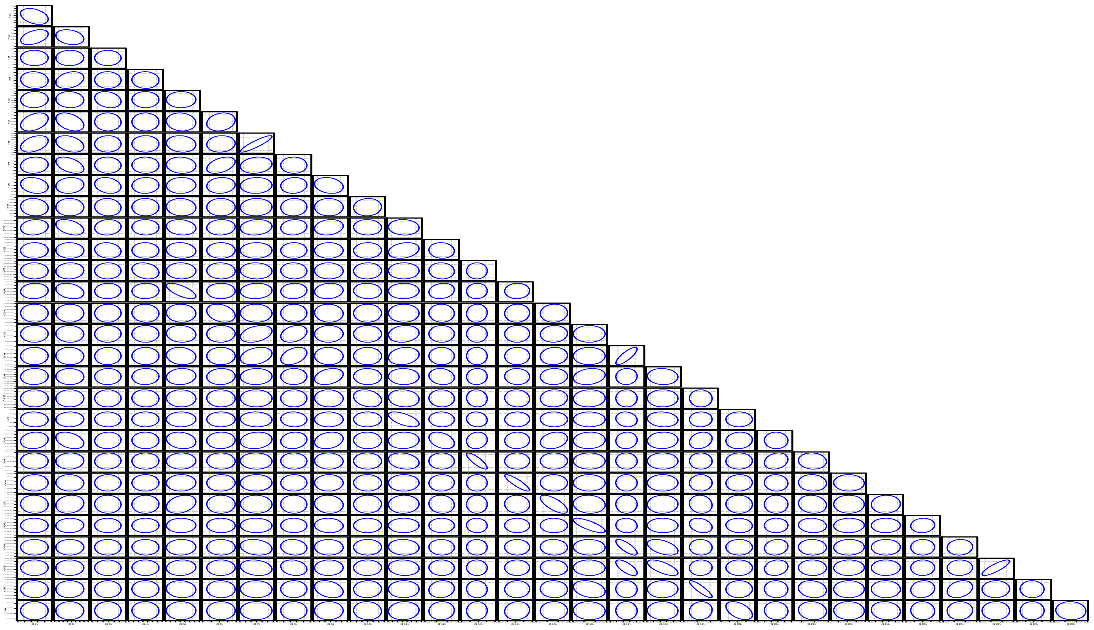
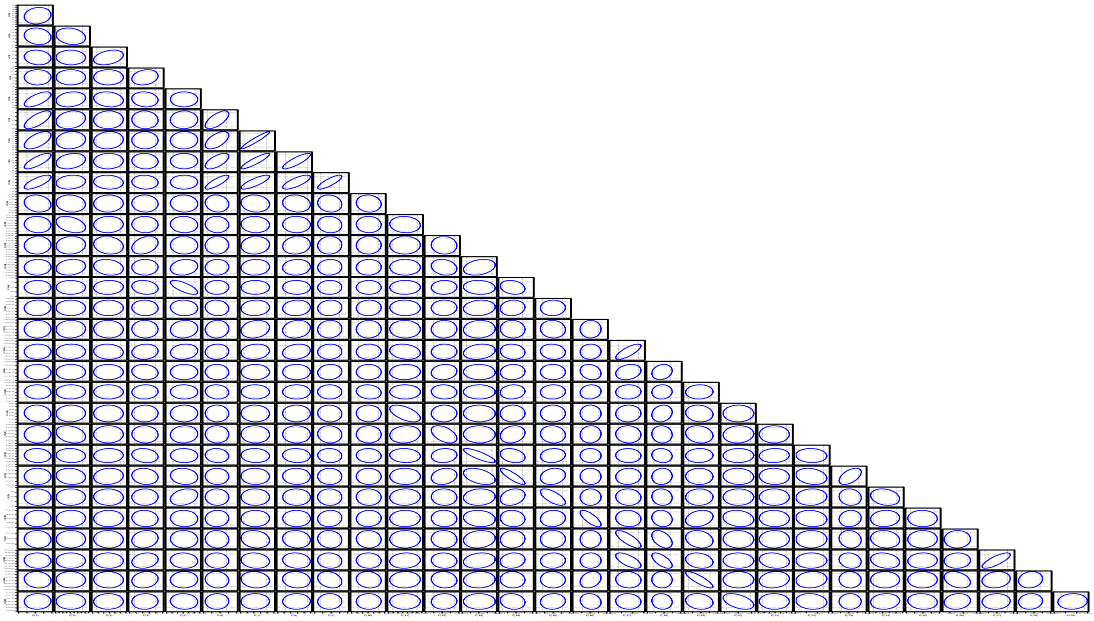


Figure L.2: Confidence ellipse for ISUR (Price)



APPENDIX M: A DISCUSSION ON THE MARKET OF MEDICAL SERVICES WITH VARYING QUALITY OF INPUTS (Rosen's Quality Model)

a) The choice between public health coverage and private health insurance plans

A fruitful advise on the choice between the two types of policies will be determined by the results showing how effective are the public expenditures on enhancing household production activities. To this regard, many may reflexively argue that private health insurance coverage is more efficient while more expensive than public health coverage. And as the quality of health outcomes matters most, the general opinion tends to support private health insurance rather than public coverage. Some evidence, mainly on the US health system, could be garnered as to orientate the debate. Many analysts have suggested that the US congress makes use of federal funds to assist in subsidizing the purchase of private health coverage. Reliable evidence has been used to indicate that public coverage is less expensive than the private one and offers comparable service though[34].

In relation to the adverse selection problem, people with insurance coverage are more likely to present higher incidence of sickness requiring higher care as compared to those without insurance. And administration of private health insurance cost more than public coverage.

Returning to the issue of efficiency, the low cost of public insurances attract more demanders and that renders the delivery of public health less effective. Individuals with public coverage find it more difficult to be timely served as medical practitioners are overwhelmed by a large demand.

The present annex does not address into much detail the question of choice between public and private health insurance systems. It rather includes both aspects in the model.

b) The supply model of medical services

Model requirements (see Abowd, 1977):

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[34] Center on Budget and Policy Priorities, May 2007.

- The model must acknowledge and account for the fact that all inputs (medical doctors; nurses; prescription drugs;…) have nontrivial differences regarding outputs for which they contribute to the production;
- The market of medical services is also subject to generic equilibrium principles: the market price is determined by the interaction between supply and demand. However both agents (suppliers or demanders) will not necessarily trade at the same price. Whenever the market clearing mechanism is not achieved, the equilibrium position is obtained through rationing;

It is important to understand that market of medical services will often follow unusual behavioural patterns due to major role played by the external forces such as subsidies (local or international). Policy simulations should therefore be more realistic and they will often differ from market conditions. In this discussion of the market of medical services (M) we make use of the underlying foundations of the implicit market (Rosen, 1974). We allow differences in the output quality. There will be different levels of competition at the different quality levels determining different prices. That is specified under a hedonic function. In the model specification, both distributions (demand and supply) are endogenous. The demand for quality MS is a monotonic function of medical inputs such as: medical doctors; nurses; etc. The econometric analysis of this extension of our model could focus on measuring the impact of these input variables on MS.
We refer to a quadratic spline function with stable functional form over years allowing parameters to vary from year to year as a result of changes in the market conditions. The model ties up a quality index to the input producing MS and to the type of health coverage received.

It is important to mention some restrictions set in this model. No household can simultaneously buy the same input with different quality levels in order to enjoy a price lower than the one for intermediate inputs. In fact, our model should be able to address issues such as the impact that more public coverage or more private coverage will have on the production of MS, on the production of health characteristics and on the household's utility. The differences existing between public coverage and private coverage are highlighted in the setting of our constraints.

The supply model:

$$N_i(c_i;q_i;\mathrm{O}_i)=M_i[\psi_i(o_i;h_i)] \quad (1)$$

Where:

- N_i is a function continuously differentiated twice, convex and strictly increasing. It represents the 'production possibility frontier' assuming M_i fixed;
- c_i is the number of consultations (visits) to a medical doctor or any other type of medical practitioner that a household has made during a considered period;
- q_i is an index of quality of medical doctors or medical practitioners;
- O_i represents other activities established between the household and health facilities;
- M_i is a function continuously differentiated twice, concave and strictly increasing. It represents an isoquant assuming N_i fixed;
- ψ_i represents the vector of input variables.

This joint production function has the advantage of allowing an unlimited number of outputs for the same number of inputs. c_i can be purchased at a price P_{c_i}. The theory of profit maximization will be easily applicable in this case. The household will choose output based on equalizing the price ratio (output prices) with the rate of product transformation. It is assumed that even high quality doctors or other types of medical practitioners can be interested in residual income.

I also assume that any liabilities regarding tax are included in the expenditures on other activities.

The objective function:

$$Y=Max\sum_{t=1}^{\infty}\lambda^t U(c_{it};q_{it};\mathrm{O}_{it}) \quad (2)$$

Subject to:

$$P_{c_{i(t+1)}}=P_{c_{it}}c_{it}+I_{c_t} \quad (3)$$

$$P_{o_{i(t+1)}}o_{i(t+1)}=P_{o_{it}}o_{it}+I_{o_t} \quad (4)$$

$$P_{\psi_{i(t+1)}}\psi_{i(t+1)}=P_{\psi_{it}}(1-\delta_\psi)\psi_{it}+I_{\psi_{it}} \quad (5)$$

$$W_{(t+1)}=(1+r)W_t+s_t+P_{c_{it}}c_{it}+P_{\mathrm{O}_{it}}\mathrm{O}_{it}-I_{c_t}-I_{\mathrm{O}_t} \quad (6)$$

Where:

- W_{t+1} is the wealth generated by the household in period (t+1);
- r is the market rate of return on wealth;
- s is the accruing subsidies.

Prices and subsidies can be assumed to be exogenous to the decision making units. The health care units are assumed to be subject to some sort of competition when it comes to factor markets and output markets.

Functional form of the supply model:

$$Max\left\lfloor U(cit;qit;\mathrm{O}it)+\lambda Y(W_{t+1};P_{c_{(t+1)}};P_{\mathrm{O}_{(t+1)}};P_{\psi_{(t+1)}};\delta_{\psi_{(t+1)}})\right\rfloor \tag{7}$$

Subject to:

$$P_{c_{i(t+1)}}=P_{c_{it}}c_{it}+I_{c_t} \tag{8}$$

$$P_{o_{i(t+1)}}o_{i(t+1)}=P_{o_{it}}o_{it}+I_{o_t} \tag{9}$$

$$P_{\psi_{i(t+1)}}\psi_{i(t+1)}=P_{\psi_{it}}(1-\delta_{\psi})\psi_{it}+I_{\psi_{it}} \tag{10}$$

$$W_{(t+1)}=(1+r)W_t+s_t+P_{c_{it}}c_{it}+P_{\mathrm{O}_{it}}\mathrm{O}_{it}-I_{c_t}-I_{\mathrm{O}_t} \tag{11}$$

Y must be proven to be: existent; unique; and concave.

First order conditions:

$$(1)\ \frac{\partial U}{\partial c_i}+\lambda\frac{\partial Y}{\partial W}(Pc_i)-\chi\frac{\partial N_i}{\partial c_i}=0 \tag{12}$$

$$(2)\ \frac{\partial U}{\partial q_i}+\lambda\frac{\partial Y}{\partial W}.\frac{\partial W}{\partial ci}.\frac{\partial c_i}{\partial q_i}-\chi\frac{\partial N_i}{\partial q_i}=0 \tag{13}$$

$$(3)\ \frac{\partial U}{\partial \mathrm{O}_i}+\lambda\frac{\partial Y}{\partial W}P_{\mathrm{O}_i}-\chi\frac{\partial N_i}{\partial \mathrm{O}_i}=0 \tag{14}$$

$$(4)\ -\lambda\frac{\partial Y}{\partial W}+\lambda\frac{\partial Y}{P_{c_{(t+1)}}}.\frac{1}{P_{c_{(t+1)}}}=0 \tag{15}$$

$$(5)\ -\lambda\frac{\partial Y}{\partial W}+\lambda\frac{\partial Y}{\partial P_{\mathrm{O}_{(t+1)}}}.\frac{1}{P_{\mathrm{O}_{(t+1)}}}=0 \tag{16}$$

$$(6)\ -\lambda\frac{\partial Y}{\partial W}+\lambda\frac{\partial Y}{\partial P_{\psi_{(t+1)}}}.\frac{1}{P_{\psi_{(t+1)}}}=0 \qquad (17)$$

With $\frac{\partial Y}{\partial W}$ being the measure of the incremental value of wealth.

Having agreed that Y is concave, $\frac{\partial Y}{\partial W}$ will be decreasing as compared to W. A rise of W leads the discounted utility to increase at a decreasing rate.

In addition to what was said earlier, some features of the model need to be highlighted:

- The $\vec{\psi}_i^{\,*}$ is determined solely by the vector market price $\vec{P}_{\psi_i}$ and the two other prices (P_{c_i}, P_{O_i});
- χ is the multiplier or the shadow value of N_i expressed in terms of units of utility and it is function of $\frac{\partial Y}{\partial W}$. Therefore χ cannot be identically considered as maximizing shadow present value of N_i.

The demand model:

In this subsection, I discuss the household's demand model for medical services with consideration the quality of MS offered. Once again, I make use of a lifetime wealth function. The household demands for quality MS to enhance it's number of working hours for n years. It is important to indicate that the household makes use of its labour time added to the quality of medical services acquired during a year in order to produce household characteristics (human capital) for the following year. In fact, quality MS is demanded for the production of household's health characteristics that is later translated into human capital for the economic growth function.

$$H_{t+1}=(1-\delta_H)H_t+\phi(g;q)H_t \qquad (18)$$

Where:

- r is the return on human capital;
- W_t is the wealth in year t (opportunity wage in year t);
- H_t is the human capital stock in year t;
- g is the portion of market time allocated to the production of H_t;
- σ is the vector of medical fees parameters.

H_t enters the production of human capital using Hicks neutral, implying that H_t is much more efficient in producing H_t.

The demand model can therefore be formulated as follows:

$$F_1(H_1;R_1;\sigma_1;c_1;\delta_H) = Max\sum_{t=1}^{n}\lambda^t r_t H_t(1-g_t) - \left[P_{tH}(q_t;\sigma_t) + f\right]I_{tH} \qquad (19)$$

Constraints:

(1) $H_{t+1} = (1-\delta_H)H_t + \phi(g;q)H_t$ (20)

(2) $r_{t+1} = r(r_t)$ (21)

(3) $\sigma_{t+1} = \sigma(\sigma_t)$ (22)

The functional form equation:

$$F_1(H_1;R_1;\sigma_1;c_1;\delta_H) = \underset{q,g}{Max}\left\{R_1 H_1(1-g) - [P_{1H}(q_1;\sigma_1) + c]I_{1H} + \lambda F_2(H_2;R_2;\sigma_2;c_2;\delta_H)\right\} \qquad (23)$$

APPENDIX N: OPTIMISING THE LIFE CYCLE HOUSEHOLD UTILITY FUNCTION INTRODUCING THE PROBABILITY OF DEATH

The comprehension of how households work at maximizing their utility over live, scientifically termed the life cycle utility optimization, has captivated several economic thinkers. In fact, the literature has provided clear evidence that the individual (assumed to be rational to some extent) follows a certain pattern (behaviour) when it comes to its decision to consume. The theory of consumer behaviour constitutes a seminal reference to this regard[35]. Friedman and several others conducted prominent researches in order to provide a clearer understanding of the consumer behaviour on a life cycle basis.

The permanent income hypotheses as well as the relationship between measured consumption and measured income have been tested on several data sets. The results obtained have not always confirmed Friedman's theory. However, better explanations of the concept have been garnered in the literature over years. From the various empirical studies conducted to test the income-consumption relationship, rather obvious correlation between the two was found. The magnitude of estimates was revealed to be less likely predictable and understandable according to the theory though. Consumption couldn't be solely related to income. It was therefore well understood that both consumption and income include permanent components that determine the individual's consumption habits.

Nowadays, the income-consumption relationship is much better explained and the weight of evidence in favour of the permanent income hypothesis remains consistent. The notion of permanent wealth, also initiated by Friedman, appears to be much more convenient and realistic as compared to income since wealth is a much broader and more comprehensive tool used in the consumer's behaviour. The consumer, while taking consumption decisions, faces different forms of uncertainty, making the optimization problem more complex. The use of stochastic processes combined with other sequential optimization methods have come up to enhance the life cycle utility

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[35] Milton Friedman: "A theory of the Consumption Function", Princeton University Press, Princeton, 1957.

problem. More recent literature on the topic has made major contributions on how to deal with issues such as: additive separability; risk; etc.

Assuming that the consumer understands the life cycle process, she (he) organizes her (his) stream of expenditures in an appropriate manner. Therefore, the consumer borrows or lends timely with aim to stabilize expenditures over years and earn interests on transferring wealth across time periods. It is relevant to understand that, during starvation, consumption decisions are taken differently. Considering that nearly 2.8 billion people in the world live on less than $ 2 a day, consumer behaviour under poverty can not be overlooked in the literature[36]. The level of uncertainty is much higher and the consumer is unsecured about the future. She (he) is constantly facing the probability that calamities or major disasters can occur and cause sudden death in the life cycle. It highly determines the consumer's consumption pattern on every period. If an individual, who is already under starvation, is aware that she (he) is more likely to die after one period, she (he) will tend to consume most of her wealth under the current period and not save for the next periods. The utility problem in this case will be different. The consumer will have to face two types of probabilistic utilities: (1) a utility with probability to live until the end of the cycle; (2) a utility with probability to die before the end of the cycle. Under rationality assumption, the consumer will opt for the expected utility of the two kinds.

In this annex, we aim at using this argument to enrich the literature on some canonical elements previously omitted or rather not explicitly stated. The question of uncertainty in life-cycle optimization has been addressed without explicit consideration that the consumer facing low confidence in the future will take decisions based on the probability to live until the end of the cycle.

Admittedly, the emergence of all sorts of life insurance policies tends to embody the consumer with less fear in future consumption planning. However, most wealthy individuals, who have less probability to die, are the one that can afford life coverage,

[36] Here we refer to the poverty headcount ratio using the purchasing power parity as published by 'Global Impact' (2007 annual report). In Sub-Saharan Africa, more than 70 % of the population lives with less than $ 2 a day.

while poor people, who usually have very high death probability, can not afford to be covered.

Fisher is among the pioneers in the design of the intertemporal consumption theory. Nevertheless, the use of discounted utility functions goes back as far as the 50s with economists such as Modigliani and Ando. Friedman's life-cycle models advertised a flat lifetime consumption, assuming that people save when income is high and borrow during low income periods, keeping a balanced and flat life-cycle consumption. Several critics arose against Friedman's life-cycle model in the sense that the model overlooked different facts [37]. People don't always save enough during pick income periods and they often consume much less during downturn in order to borrow less.
Various alterations have hence been suggested to render Friedman's theory more consistent with the data evidence. Although, one of the alterations suggested to Friedman's model consisted of having a changing utility function or making use of state space modelling[38]. The present annex makes use of non-changing utility function to prove how previous results could be seriously biased by omitting the probability of death in life cycle optimization.

In the more recent literature, the behavioural life cycle hypothesis has been suggested, associating individual's emotions in decision making process. Kahneman (2002) was awarded a Nobel Prize for his contribution to the use of psychology to explain consumer behavior under the so called: Prospect Theory. There are undoubtedly inaccurate results that were obtained in the use of expected utility. The introduction of discount factors in life-cycle models has been of great impact, and the use of hyperbolic discount factors is quoted among the major contribution though. It was rather unrealistic to use a linear discount factor. The discounting process can go much faster or much slower depending on how close is the future considered. Herrnstein (1961) was among the first to introduce the 'Melioration Theory', which is a theory borrowed from behavioral sciences to explain utility discounting over time.
The point we intend to prove at this stage is pretty obvious and justifiable by behavioral economics. As Herrnstein (1961) made use of hyperbolic discounting

[37] See Courant et al. (1984).
[38] State-space models allow the estimated parameters to vary at different states.

theory to explain that time lags between payoffs and the size of payoffs have joint significance in consumer behavior, this annex aims to describe how the probability of sudden death also matters in consumers behavior. The higher is the probability of death, the less the consumer will be willing to save for future periods. While applied to poor populations living under starvation, this argument supports the idea that the consumer will not ‘underconsume’ in the current period and yet will not be willing to save for future periods. However, she (he) might still borrow from future periods without guaranty to repay.

A simple 2 period life cycle model

Assuming that a consumer has the following 2 period utility function:

$$U(C_1;C_2) = kC_1^{\alpha_1}C_2^{\alpha_2}$$

where: - U : Utility

- C_1 : Consumption in period one;
- C_2 : Consumption in period two;
- α_1 : share of life cycle wealth spent during period one;
- α_2 : share of life cycle wealth spent during period two.

The consumer is subject to a life cycle constraint:

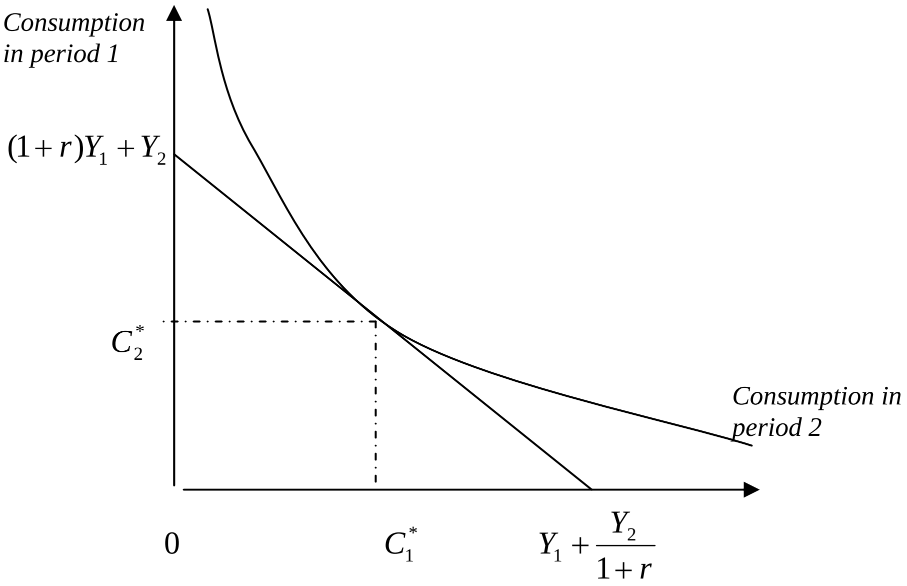
$$W_1 = Y_1 + \frac{Y_2}{1+r} \quad \text{or} \quad W_2 = Y_1(1+r)Y_1 + Y_2$$

where:

- W_1 : maximum amount the consumer can spend in period 1 if she (he) plans to consume nothing in period 2;
- W_2 : maximum amount the consumer can spend in period 2 if she (he) spends nothing in period 1;
- Y_1 : expected financial resources for period 1;
- Y_2 : expected financial resources for period 2;
- r : interest rate capturing the valuation or devaluation of the financial resources across time.

a) Under certainty

Graph:



This graph represents the utility optimization problem under certainty. Therefore the consumer's utility is maximized as follows:

$$U^*(C_1^*;C_2^*)=kC_1^{\alpha_1}C_2^{\alpha_2}$$

b) Under uncertainty

The reality of life provides sufficient evidence that the consumer (individual or household) may die after period 1 and never reach period 2 for unforeseen reasons such as calamities. Importantly the consumer is aware of that aspect and therefore takes consumption decisions accordingly. The 2-period problem will therefore be based on expected utility function obtained by the some of two probabilistic outcomes: (1) the consumer die after period 1; (2) the consumer survives until the end of period 2.

b.1) The outcome is 'death after the first period'

If the outcome is death after period 1, the problem will be reformulated as follows:

$$U(C_1;C_2)=kC_1^{\alpha_1}C_2^{0}$$

$$U(C_1)=kC_1^{1}$$

Subject to the constraint:

$$W_1=Y_1$$

In this case $\alpha_2 = 0$ and $\alpha_1 = 1$ since the consumer uses the total wealth during period 1 and nothing is left for the second period. The consumer will limit her (him) expenditures during period 1 based on the income earned in that period only. Provide that she (he) decides to borrow extra money from what she could earn if she (he) could live and work during period two, taking advantage of asymmetric information, the lending institutions will not easily allocate her (him) any loan as her outcome is to die after first period. She will be restricted to only consume what she earns during period one unless she (he) can really take advantage of an uninformed lender. Now the reality is that there is only a probability ρ that this outcome occurs. ρ is the probability to die before the next period and $(1-\rho)$ the probability to live. In the consumer's expected utility function, $\rho U(C_1)$ will be the first component.

b.2) The outcome is 'no death after period 1'

Should the outcome be that the consumer survives until the end of period 2, the probabilistic outcome will be:

$$(1-\rho)U^*(C_1^*;C_2^*) = kC_1^{\alpha_1}C_2^{\alpha_2}$$

Where the constraint will be:

$$W_1 = Y_1 + \frac{Y_2}{1+r}$$

As the consumer is confident that she (he) will survive until period 2, she (he) will allocate the share α_1 of the total available resources (life wealth) to the consumption in period 1.

b.3) The consumer's expected utility

The use of probability in the optimization outcomes has transformed the life cycle model into a set of stochastic equations. Earlier work on the issue suggested the use of expected utility to be maximized. Notwithstanding, repetitive and very constructive critics have been made against the use of expected utility functions. The theory of expected utility functions made consistent progress to assuage the criticism by introducing more realistic approaches such as: the hyperbolic discount factor; etc. In this case we have initiated the discussion by the simple 2 period model where the discounting process does not constitute a major concern.

The expected utility is therefore the following:

$$EU = \rho U^{*}(C_1^{**}) + (1-\rho)U^{*}(C_1^{*};C_2^{*})$$

Sequential optimization using Bellman equations

Sequential optimization is undoubtedly one of the most appropriate methods to be used in this case; however, it has not been used extensively in the literature mainly because of its complexity. This new aspect of sudden death that has been brought up earlier in our model affects the optimization process mainly when we move from one period to the next one. This section mainly focuses on evidence that omitting death at that level includes bias in the results. It is clearly considered that ρ is not constant over time (see fig.1). It is unrealistic to assume that household members' probability of death remains constant across age. Ageing together with other factors raise the value of ρ while technological progress tends to reduce the probability of death and render it less U-shaped (horizontal).

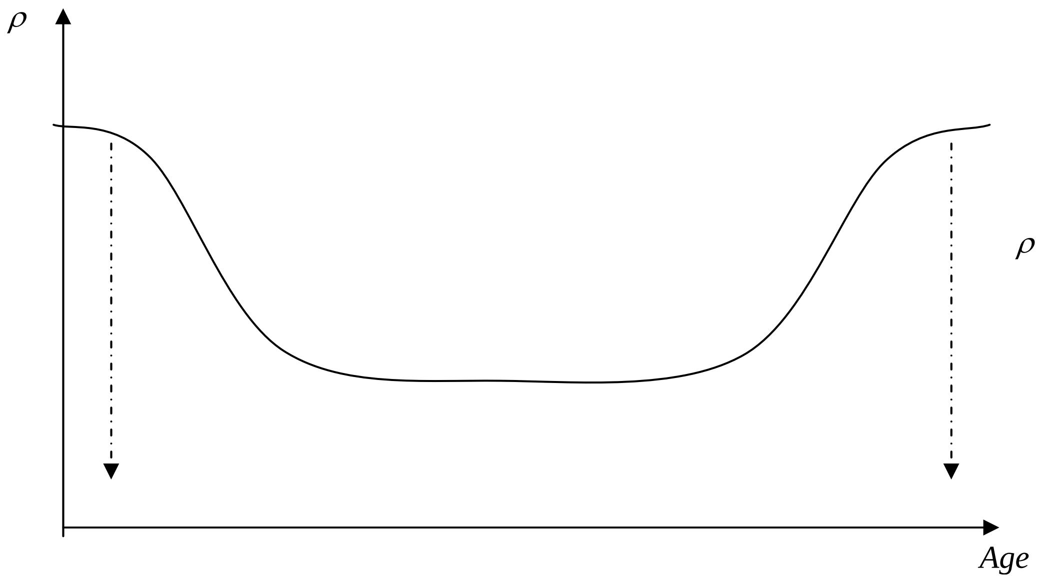


Fig. b.1

An interesting point of discussion to rise at this level will be to determine whether ρ is a 'Brownian Motion'. From the very first approximation, that can be easily assumed. The probability of death is constantly in movement with both diffusions and osmotic movements during an individual's life; although it follows a general U-shaped trend (see fig.1). ρ is random and uncertain reason why it is assumable to be a 'brownian motion'. It is therefore possible to derive a steady state for the distribution function associated to the Brownian motion.

The horizontal ρ' represents an ideal probability curve, lower than ρ, that is obtained with technological progress. Technological progress and improvement in health care reduce both infant and elderly mortality, making probability of death much less affected by age. The death of one member is dependent (correlated) with the death of other household members.

The traditional model for sequential optimization (Bellman Equations) using expected utility is stated as follows:

$$V^*(C_0) = \max_{\substack{\{C_{t+1}\}_{t=0}^{\infty} \\ C_{t+1}\in\Omega(C_t)}} \left\{ \sum_{t=0}^{\infty} \beta^t EU(C_t; C_{t+1}) \right. \tag{1}$$

$$V^*(C_0) = \max_{C_1\in\Omega(C_0)} \left\{ EU(C_0;C_1) + \max_{\{Ct+1\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t EU(C_t;C_{t+1}) \right\} \tag{2}$$

$$V^*(C_0) = \max_{C_1\in\Omega(C_0)} \left\{ EU(C_0;C_1) + \beta \max_{\{C_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t EU(C_{t+1};C_{t+2}) \right\} \tag{3}$$

$$V^*(C_0) = \max_{C_1\in\Omega(C_0)} \left\{ EU(C_0;C_1) + \beta V^*(C_1) \right\} \tag{4}$$

Equation 4 is used to derive the functional form:

$$V(C_0) = \max_{C_1\in\Omega(C_0)} \left\{ EU(C_0;C_1) + \beta V(C_1) \right\} \tag{5}$$

Equation 5 is used to determine V^* since one of the optimality principles states that $V = V^*$

$$V^*(C_0) > \sup_{\substack{\{C_{t+1}\}_{t=0}^{\infty} \\ C_{t+1}\in\Omega(C_t)}} \sum_{t=0}^{\infty} \beta^t EU(C_t;C_{t+1}) \tag{6}$$

Rigorous principles of optimality need to be followed when it comes to solve the functional form equation. V^* is the unique and optimal value in solving the functional form. Therefore, in our household model specification all C_i^* are all optimal and have unique solution for the functional form.

At first, we introduce ρ in the sequential optimization and later on we bring up more updates on its functional characteristics.

$$\rho = n.\varpi \tag{7}$$

where:

- $\varpi(t)$: the probability that one of the active member of the household dies;
- n: number of active members of the household.

The literature has implicitly approached this issue using dynamic optimization of utility under uncertainty as we mentioned earlier. The dynamic process did not account for the fact that ρ varies with age of active household members. Therefore, our Bellman model can be reformulated as follows:

$$V^*(C_0) = \max_{\substack{\{C_{t+1}\}_{t=0}^{\infty} \\ C_{t+1}\in\Omega(C_t)}} \left\{ \sum_{t=0}^{\infty} \beta^t EU(C_t;C_{t+1}) \right\} \tag{8}$$

$$V^*(C_0) = \max_{C_1\in\Omega(C_0)} \left\{ EU(C_0;C_1) + (1-\rho(t)) \max_{\{Ct+1\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t EU(C_t;C_{t+1}) \right\} \tag{9}$$

$$V^*(C_0) = \max_{C_1\in\Omega(C_0)} \left\{ EU(C_0;C_1) + (1-\rho(t))\beta \max_{\{C_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t EU(C_{t+1};C_{t+2}) \right\} \tag{10}$$

$$V^*(C_0) = \max_{C_1\in\Omega(C_0)} \left\{ EU(C_0;C_1) + (1-\rho(t))\beta V^*(C_1) \right\} \tag{11}$$

Equation 11 is used to derive the functional form:

$$V(C_0) = \max_{C_1\in\Omega(C_0)} \left\{ EU(C_0;C_1) + (1-\rho(t))\beta V(C_1) \right\} \tag{12}$$

Equation 12 is used to determine V^* since one of the optimality principles states that $V = V^*$

$$V^*(C_0) > (1-\rho(t)) \sup_{\substack{\{C_{t+1}\}_{t=0}^{\infty} \\ C_{t+1}\in\Omega(C_t)}} \sum_{t=0}^{\infty} \beta^t EU(C_t;C_{t+1}) \tag{13}$$

Setting that: $0 < \rho < 1$; the value of V^*, assuming that $\rho = 0$, as it is the case in most studies, will be smaller than the V^* obtained when $\rho \neq 0$. The larger is the time period considered, the bigger is the gap between the two solutions. However changes in the value of ρ, as the household members grow older, reduce or increase the bias.