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Selective Feature Preserved Elastic Surface Registration in Complex Geometric Morphology

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Selective Feature Preserved Elastic Surface Registration in Complex Geometric Morphology

by

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Abstract

Title:	Selective Feature Preserved Elastic Surface Registration in Complex Geometric Morphology
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Deforming a complex generic shape into a representation of another complex shape is investigated. An initial study is done on the effect of cranial shape variation on masticatory induced stress. A finite element analysis is performed on two different skull geometries. One skull geometry has a prognathic shape, characterised by jaws protruding forward, while the other has a non-prognathic form.

Comparing the results of the initial finite element analyses, the effect of an undesired variation in shape and topology on the resulting stress field is observed. This variation in shape and topology can not be attributed to the cranial shape variation that is investigated. This means that the variation in the masticatory induced stress field that is due to the relative degree in prognathism can not be quantified effectively.

To best compare results, it would be beneficial to have a computational domain for the different skull geometries that have one-to-one correspondence. An approach to obtain a computational domain that represents various geometries with the exact same mesh size and connectivity between them does exist. This approach involves deforming a generic mesh to represent different target shapes.

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This report covers an introductory study to register and deform a generic mesh to approximately represent a complex target geometry. Various procedures are investigated, implemented and combined to specifically accommodate complex geometries like that of the human skull.

A surface registration procedure is implemented and combined with a feature registration procedure. Feature lines are extracted from the surface representation of each skull as well as the generic shape. These features are compared and an initial deformation is applied to the generic shape to better represent the corresponding features on the target.

Selective feature preserved elastic surface registration is performed after the initial feature based registration. Only the registration to surfaces of featureless areas and matched feature areas are allowed along with user selected areas during surface registration.

The implemented procedures have various aspects that still require improvement before the desired study regarding prognathism's effect on masticatory induced stress could truly be approached pragmatically. Focus is only given to the use of existing procedures while the additional required improvements could be addressed in future work. It is however required that the resulting discretised domain obtained in this initial study be of sufficient quality to be used in a finite element analysis (FEA).

The implemented procedure is illustrated using the two original skull geometries. Symmetric versions of these geometries are generated with a one-to-one correspondence map between them. The skull representations are then used in a finite element analysis to illustrate the appeal of having computational domains with a consistent mapping between them. The variation in the masticatory induced stress field due to the variation in cranial shape is illustrated using the consistent mapping between the geometries as part of this example.

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Contents

Nomenclature

a	-	Lower scale bound in the reformulated ICP.
b	-	Upper scale bound in the reformulated ICP.
c	-	Counter, indicating the c^{th} triangle patch on the target shape surface mesh.
c_k	-	Correspondence between a generic and data shape.
$\mathbf{c}$	-	Vector containing the rotation, reflection and scale variables used in the reformulated ICP.
d	-	Counter, indicating the d^{th} triangle patch on the generic shape surface mesh.
$\mathbf{d}$	-	Distance from a point to it's registered location.
D	-	Distance.
$\mathbf{D}_j$	-	Set of linear bases of a diagonal matrix. The only non-zero entry is $D_{jj} = 1$.
E	-	Young's modulus.
$\mathbf{E}_j$	-	Linearised bases of the special orthogonal group representation of an invertible matrix.
f	-	Smoothing parameter in the elastic surface registration procedure.
f_i	-	Function evaluation at location i.
F	-	Force.
$\mathcal{F}$	-	Implicit surface $\mathcal{F}(\mathbf{x}) = 0$.
h	-	Positive increasing function for determining element quality.
H	-	Reference plane.
i	-	Counter. $i = 1, 2, .., k_{\max}$ where $k_{\max}$ is the maximum number of iterations for example.

$\mathbf{I}$	-	Identity matrix.
$\mathbf{J}$	-	Jacobian matrix.
k	-	Iteration.
L	-	A line.
$\mathbf{m}$	-	The points on the generic shape.
$\mathbf{m_t}$	-	Point correspondence of the target on the generic shape translated so it's centroid is at the origin of the Cartesian coordinate axis.
M	-	Moment.
$\mathbf{M}_{b,b}$	-	Matrix containing evaluations of a radial basis function.
$\mathcal{M}$	-	Model or generic shape.
n	-	Number of neighbours used in the elastic registration procedure.
$\mathbf{n}$	-	Unit normal.
N	-	Indicates size. N_p is the number of points in the target shape and N_m the number of points in the model shape for example.
p	-	Linear polynomial.
p_i^j	-	Portion of points on one line L_i registered to line L'_j
$\mathbf{p}$	-	The points on the target shape.
$\mathbf{p}_i$	-	A specific point.
$\mathbf{p_t}$		Target shape $\mathbf{p}$ after it is translated so the centroid is at the origin of the Cartesian coordinate axis
$\mathbf{P}_b$	-	Matrix containing boundary coordinates.
$\mathcal{P}$	-	Data or target shape.
q_m	-	Element quality of tetrahedron m.
$\mathbf{Q}$	-	Matrix. $\mathbf{Q} = \mathbf{J}\mathbf{W}^{-1}$ when determining element quality.
r	-	Radius.
r_j	-	Rotation variables in the reformulated ICP.
$\mathbf{r}$	-	The registration location. $\mathbf{r}_{w_j}$ is the possible registered location of point $\mathbf{w}_j$ onto a target shape for example.
$\mathbf{R}$	-	Rotation Matrix.
$\mathbb{R}$	-	Real number indicator. $\mathbb{R}^3$ indicates a tensor consisting of three real numbers.
s_j	-	Scale variables in the reformulated ICP.

S^{k-1}	-	Deformation applied to $\mathcal{W}^{k-1}$ to better approximate the target.
S_i	-	Shape index.
$\mathbf{S}$	-	Scale Matrix.
$\mathbf{t}$	-	Translation vector.
T	-	Transformation.
T_h	-	Threshold, used when pruning false lines.
T_m	-	Indicates size. T_m is the number of triangles in the model shape
T_p	-	Indicates size. T_p is the number of triangles in the target shape.
$\mathcal{T}$	-	A tetrahedron.
u_j	-	Reflection variables in the reformulated ICP.
$\mathbf{U}$	-	Reflection matrix.
$\mathbf{w}$	-	The points on the deformable surface.
$\mathbf{W}$	-	Jacobian matrix that maps the tetrahedron $\mathcal{T}_R$ to tetrahedron $\mathcal{T}_I$
$\mathcal{W}$	-	Deformable surface in the elastic registration procedure.
$\mathbf{x}$	-	Nodal coordinates

Greek Symbols

α	-	Coefficients used in radial basis function interpolation
$\boldsymbol{\beta}$	-	Coefficients of the linear polynomial when using RBF interpolation.
γ	-	Smoothing parameter in the elastic registration procedure
δ	-	Shift variable used in positive increasing function.
ε	-	The average distance of point set correspondence in the ICP procedure.
	OR	The average total deformation applied to the deformable surface during elastic registration.
ε_T	-	Tolerance.
ζ	-	Machine epsilon or tolerance ($0 < \zeta \ll 1$).
κ	-	Principal curvature.
λ	-	Eigenvalue.
μ	-	Average shape index.
ν	-	Poisson's ratio.
ξ	-	Compact radial basis function scaling factor.
$\boldsymbol{\varpi}$	-	Principal curvature direction.
σ_0	-	Smoothing parameter in the elastic registration procedure
σ_m	-	Determinant of the matrix $\mathbf{S}_m$.
τ	-	Curvature derivative or extremality coefficient.
ϕ	-	Radial basis function.

Superscripts

-1	-	Inversion.
k	-	Iteration counter. $k-1$ indicates the previous iteration.
T	-	Tensor transpose.

Subscripts

$0, 1, 2, ...$	-	Used where the number represents the index within a list or set.
i	-	Quantity in list defining the target shape. $i \in \{1, 2, ..., N_p\}$
$i, j, k, ...$	-	Used where indicial notation is used and summation is implied.
j	-	Quantity in list defining the generic shape. $j \in \{1, 2, ..., N_m\}$
k	-	Iteration counter. $k-1$ indicates the previous iteration.
m	-	Indicates a value related to the generic shape $\mathcal{M}$.
p	-	Indicates a value related to the target shape $\mathcal{P}$.
x, y, z	-	Indicates coordinates in the x -, y - and z- axis.

Mathematical Symbols and Operators

$\in$	-	Indicates membership of a set.
Σ	-	Summation.
$\lvert . \rvert$	-	Frobenius norm.
$\lVert . \rVert$	-	Euclidean distance.
$\det(.)$	-	Determinant of a matrix.
$\mathrm{tr}(.)$	-	Trace of a matrix.
∂	-	Partial derivative.
∇	-	Gradient.

Acronyms and Abbreviations

ba	-	Basion - Landmark position on the human skull.
CT	-	Computed Tomography.
FEA	-	Finite Element Analysis.
FEM	-	Finite Element Model.
FSI	-	Fluid Structure Interaction.
GI	-	Gnathic index - The distance ratio of the lines connecting the basion landmark to the prostion and nasion landmarks on the human skull. This ratio is expressed as a percentage quantity.
ICP	-	Iterative Closest Point - Procedure used in rigid registration.
LST	-	Local Structure Tensor.
MLS	-	Moving Least Squares.
MRI	-	Magnetic Resonance Imaging.
n	-	Nasion - Landmark position on the human skull.
OC	-	Occipital condyles - Condyles at the foramen magnum where the skull articulates with the spinal column.
PCA	-	Principal Component Analysis - Statistical analysis to determine the principal modes of variation within sample data.
pr	-	Prostion - Landmark position on the human skull.
RBF	-	Radial Basis Function - Interpolation function used to interpolate a scalar quantity known at select positions within spatial data.
TMJ	-	Temporomandibular joint - Joint connecting the mandible to the skull.
TPS	-	Thin Plate Spline - A type of radial basis function.

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