CHAPTER TWO

SYNCHRONOUS AND ASYNCHRONOUS CDMA MODELS

This chapter contains the mathematical signal and channel models that are to be used to analyze multiuser detection methods. We will limit ourselves to the baseband case for simplicity. The analysis contained here is largely based on the approach followed by Verdu in [31] and Rappaport in [32].

2.1 THE CDMA SIGNAL MODEL

Consider a CDMA channel that is shared by K simultaneous users. Each user is assigned a signature waveform. For user k, the waveform is denoted by

$$s_k(t) = \sum_{n=0}^{N-1} a_k(n)\, p(t - nT_c), \quad 0 \leq t \leq T \tag{2.1}$$

where $\{a_k(n), 0 \leq n \leq N-1\}$ is a pseudo-noise sequence, consisting of N chips that take the values $\{\pm 1\}$ and $p(t)$ is a pulse of duration T_c, where T_c is a chip interval. Without loss of generality, we assume that all K signature waveforms have unit energy, i.e.

$$\|s_k(t)\| = \int_0^T s_k(t)dt = 1. \tag{2.2}$$

The cross correlations (or inner products) between pairs of signature waveforms play an important role in the metrics for the signal detector and on its performance. We define the cross correlations between two arbitrary signature waveforms for the synchronous case.

$$\rho_{kj} = \langle s_k, s_j \rangle = \int_0^T s_k(t)s_j(t)dt \tag{2.3}$$

Note that by the Cauchy-Schwartz inequality and (2.2) we have

$$|\rho_{kj}| = \langle s_k, s_j \rangle \leq \|s_k\| \, \|s_j\| = 1. \tag{2.4}$$

Let us also define the cross correlation matrix,

$$\mathbf{R} = \{\rho_{kj}\} \tag{2.5}$$

which has diagonal elements equal to one and is symmetric nonnegative definite, because for any K-vector $\mathbf{c} = (c_1, \dots, c_K)^T$ we have

$$\mathbf{c}^T\mathbf{R}\mathbf{c} = \left\| \sum_{k=1}^{K} c_k s_k \right\|^2 \geq 0. \tag{2.6}$$

Therefore the cross correlation matrix $\mathbf{R}$ is positive definite if and only if the signature waveforms $\{s_1, \dots, s_K\}$ are linearly independent.
Concerning the asynchronous case, Figure 2.1 shows a schematic representation of the cross correlation between two synchronous users.

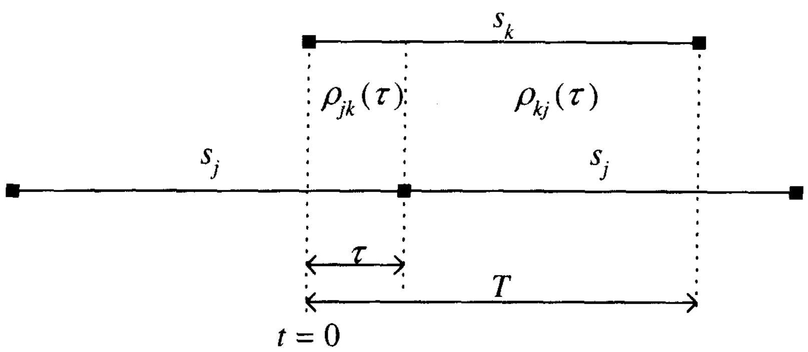

Figure 2.1: Schematic representation of the cross correlation between two synchronous users

As can be seen in Figure 2.1, we must define two cross correlations between every pair of signature waveforms that depends on τ, the offset between the two signatures. If $k < j$, we write the cross correlations as

$$\rho_{kj}(\tau) = \int_{\tau}^{T} s_k(t)s_j(t-\tau)dt \tag{2.7}$$

$$\rho_{jk}(\tau) = \int_{0}^{\tau} s_k(t)s_j(t+T-\tau)dt \tag{2.8}$$

where $t \in [0, T]$, and T denoted the signature waveform length in seconds.

2.2 DISCRETE-TIME SYNCHRONOUS MODELS

Multiuser detectors commonly have a front end which has the task of obtaining a discrete time process from a received continuous waveform $y(t)$. Generally, continuous to discrete conversion can be done by correlating $y(t)$ with deterministic signals. In communication theory, there are two types of deterministic signals of interest. These are matched signature waveforms (matched filters) and orthonormal signals.

The basic K-user CDMA model, consisting of the sum of antipodally modulated synchronous signature waveforms embedded in AWGN is given by

$$y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \sigma n(t), \qquad t \in [0, T] \tag{2.9}$$

where A_k is the received signal amplitude of the kth user, $b_k \in \{\pm 1\}$ is the bit transmitted by the kth user, $s_k(t)$ is the deterministic signature waveform of user k, $n(t)$ is the white Gaussian noise component with unit power spectral density, and σ the noise variance.

2.2.1 MATCHED FILTER OUTPUTS

Using equations (2.3) and (2.9), we can express the matched filter output of the kth user as

$$y_k = A_k b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + n_k \tag{2.10}$$

where

$$n_k = \sigma \int_0^T n(t) s_k(t) dt \tag{2.11}$$

is a Gaussian random variable with zero mean and variance equal to σ^2, since by (2.2), $s_k(t)$ has unit energy. We refer to n_k as the noise component of user k.

If we express (2.10) in vector matrix notation, we obtain

$$\mathbf{y} = \mathbf{RAb} + \mathbf{n} \tag{2.12}$$

where $\mathbf{R}$ is the normalized cross correlation matrix, $\mathbf{y} = [y_1, \ldots, y_K]^T$, $\mathbf{b} = [b_1, \ldots, b_K]^T$ and $\mathbf{A} = \text{diag}\{A_1, \ldots, A_K\}^T$. The vector $\mathbf{n}$ is a zero mean Gaussian random vector with a covariance matrix equal to

$$E[\mathbf{nn}^T] = \sigma^2 \mathbf{R} \tag{2.13}$$

It will later be shown that no information relevant to demodulation is lost by the bank of matched filters. This means that $y(t)$ can be replaced with $\mathbf{y}$ without loss of optimality.

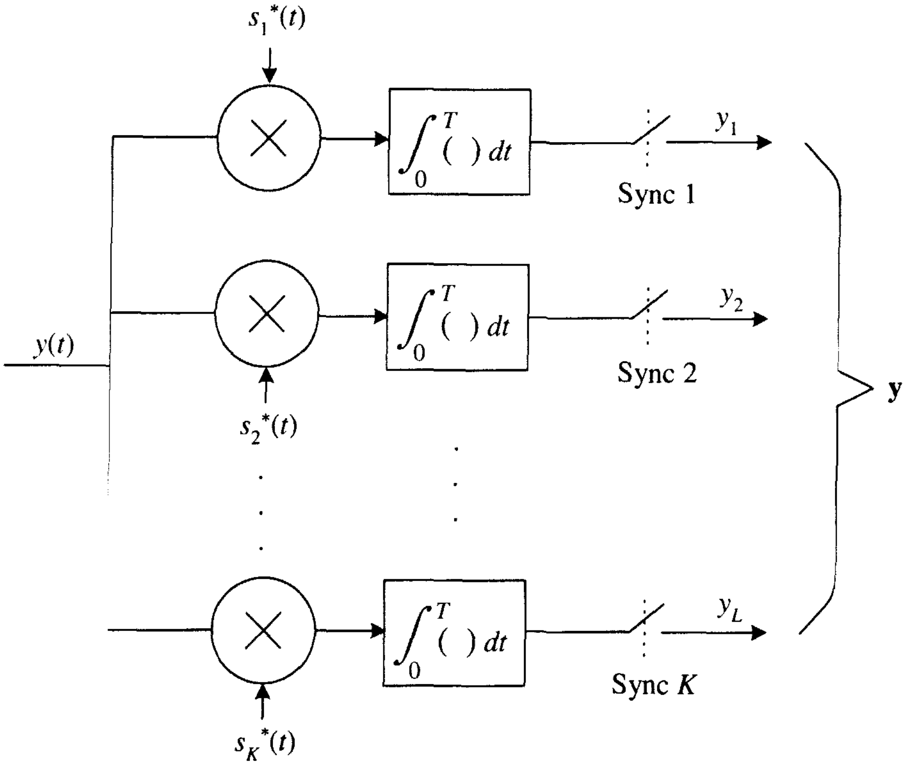

Figure 2.2: Block diagram illustration of the complex matched filter receiver

The unnormalized cross correlation matrix whose (j, k) elements is given by $\langle A_j s_j, A_k s_k \rangle$, is written as

$$\mathbf{H} = \mathbf{ARA} \tag{2.14}$$

When the receiver front end consists of a bank of matched filters, we have seen that we can replace the model in (2.9) with the linear Gaussian vector matrix model in (2.12). The same model can be generalized to encompass complex numbers. The only difference is that the output of the matched filter is given by

$$y_k = \langle y, s_k^* \rangle = \int_0^T y(t) s_k^*(t) dt \tag{2.15}$$

where * denotes the complex conjugate. This means that the cross correlation values are given by

$$\rho_{kj} = \int_0^T s_k^*(t) s_j(t) dt \tag{2.16}$$

yielding the same model as in (2.12) encompassing complex values,

$$\mathbf{y} = \mathbf{RAb} + \mathbf{n} \tag{2.17}$$

where the correlation matrix $\mathbf{R}$ is in Hermitian form. $\mathbf{A}$ is a complex diagonal matrix and n is a complex valued Gaussian vector with independent real and imaginary components and with a covariance matrix equal to $2\sigma^2\mathbf{R}$.

In the complex valued asynchronous case, the cross correlation values are given by

$$\rho_{kj}(\tau) = \int_{\tau}^{T} s_k(t)s_j^*(t-\tau)dt \tag{2.18}$$

and

$$\rho_{jk}(\tau) = \int_{0}^{\tau} s_k(t)s_j^*(t+T-\tau)dt. \tag{2.19}$$

2.2.2 WHITENED MATCHED FILTER MODEL

Notice that the noise between users is correlated in the standard discrete time synchronous model. This causes difficulty in the evaluation of performance of the various multiuser detection techniques. We can correct this with the use of a whitening filter as described below.

Proposition 2.1 *(Cholesky Factorization) For every positive definite Hermitian matrix* $\mathbf{R}$*, there exists a unique lower triangular matrix* $\mathbf{F}$ *(i.e.* $F_{ik} = 0$ *for* $i < k$*) with positive diagonal elements such that*

$$\mathbf{R} = \mathbf{F}^H\mathbf{F}$$

where $\mathbf{F}^H$ *denotes the Hermitian (complex conjugate) transpose of F.*

For brevity we shall denote the inverse of a Hermitian transpose of a matrix by

$$\left(\mathbf{F}^H\right)^{-1} \stackrel{\text{def}}{=} \mathbf{F}^{-H} \tag{2.20}$$

If the matched filter outputs $\mathbf{y}$ are processed by the matrix $\mathbf{F}^{-H}$, called a whitening filter, we obtain the whitened matched filter model

$$\begin{aligned} \bar{\mathbf{y}} &= \mathbf{F}^{-H}\mathbf{y} \\ &= \mathbf{F}^{-H}\mathbf{F}^H\mathbf{F}\mathbf{A}\mathbf{b} + \mathbf{F}^{-H}\mathbf{n} \\ &= \mathbf{F}\mathbf{A}\mathbf{b} + \bar{\mathbf{n}} \end{aligned} \tag{2.21}$$

where $\bar{y}_k$ contains contributions for users $1 \ldots k$, but not from users $k+1 \ldots K$. The covariance matrix of $\bar{\mathbf{n}}$ is

$$\begin{aligned} E[\bar{\mathbf{n}}\bar{\mathbf{n}}^H] &= 2\sigma^2\mathbf{F}^{-H}\mathbf{R}\mathbf{F}^{-1} \\ &= 2\sigma^2\mathbf{F}^{-H}\mathbf{F}^H\mathbf{F}\mathbf{F}^{-1} \\ &= 2\sigma^2\mathbf{I} \end{aligned} \tag{2.22}$$

where $\mathbf{I}$ is the identity matrix. As the name suggests, the whitened matched filter causes the noise components to be independent as in (2.22).

2.2.3 ORTHONORMAL PROJECTIONS

In the previous two models, the dimensionality of the vectors in (2.12) and (2.17) is equal to the number of users. In some situations (such as when the signature waveforms of some interferers are unknown) other models (with possibly different dimensionality) are useful. A receiver utilizing orthonormal projections is termed a correlation receiver (Figure 2.3).

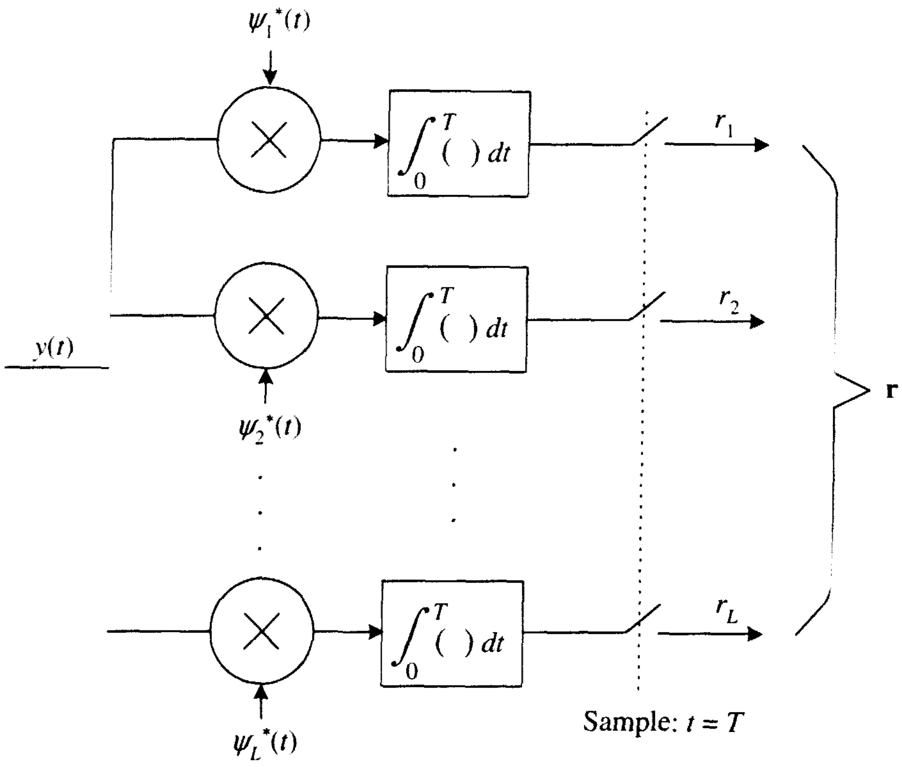

Figure 2.3: Block diagram description of the orthonormal projection correlation receiver

Let $\{\psi_1, \ldots, \psi_L\}$ be a set of L complex orthonormal signals defined on $[0, T]$. The complex *signature vector* $\mathbf{s_k}$ of the kth user is the L dimensional representation of s_k on the basis $\{\psi_1, \ldots, \psi_L\}$. That is to say the lth component of the column vector $\mathbf{s}_k$ is

$$s_{kl} = \int_0^T s_k(t)\psi_l^*(t)dt \tag{2.23}$$

Furthermore, we define the lth component of the vector $\mathbf{r}$ as

$$r_l = \int_0^T y(t)\psi_l^*(t)dt \tag{2.24}$$

The column vector can then be written as

$$\begin{aligned} \mathbf{r} &= \sum_{k=1}^{K} A_k b_k \mathbf{s}_k + \sigma \mathbf{m} \\ &= \mathbf{SAb} + \sigma \mathbf{m} \end{aligned} \tag{2.25}$$

where $\mathbf{m}$ is an L dimensional complex Gaussian vector with independent unit variance components. Now we introduce a $L \times K$ matrix of complex signature vectors

$$\begin{aligned} \mathbf{S} &= \begin{bmatrix} \mathbf{s}_1 & \cdots & \mathbf{s}_K \end{bmatrix} \\ &= \begin{bmatrix} s_{11} & \cdots & s_{K1} \\ \vdots & \ddots & \vdots \\ s_{1L} & \cdots & s_{KL} \end{bmatrix} \end{aligned} \tag{2.26}$$

The bits of the different users are uncorrelated, resulting in a covariance matrix equal to

$$\begin{aligned} E[\mathbf{r}\mathbf{r}^H] &= 2\sigma^2 \mathbf{I} + 2\sum_{k=1}^{K} A_k^2 \mathbf{s}_k \mathbf{s}_k^H \\ &= 2\sigma^2 \mathbf{I} + 2\mathbf{S}\mathbf{A}^2\mathbf{S}^H \end{aligned} \tag{2.27}$$

The finite dimensional model in (2.25) holds regardless of whether the L orthonormal signals $\{\psi_1, \ldots, \psi_L\}$ span the signature waveforms $\{s_1, \ldots, s_K\}$. An example of a set of orthonormal signals that span the signature waveforms is a DS-CDMA system where L is equal to the number of chips per symbol and the orthonormal signals are the delayed chip waveforms $\psi_i = p(t-(i-1)T_c)$.

If the signature waveforms are spanned by $\{\psi_1, \ldots, \psi_L\}$, then the $K \times K$ cross correlation matrix simply becomes

$$\mathbf{R} = \mathbf{S}^H \mathbf{S} \tag{2.28}$$

Furthermore

$$||\mathbf{s}_k|| = 1 \tag{2.29}$$

and all the information contained in $\mathbf{y}$ is also contained in $\mathbf{r}$, because the matched filter outputs can be expressed as a linear combination of the components of $\mathbf{r}$, i.e.

$$\mathbf{y} = \mathbf{S}^H \mathbf{r} \tag{2.30}$$

2.3 DISCRETE-TIME ASYNCHRONOUS MODELS

For a simplified notation, we shall label the users chronologically. We assume without loss of generality that $\tau_1 \leq \tau_2 \leq \ldots \leq \tau_K$. If we generalize (2.9) to the complex asynchronous case, the complex asynchronous CDMA model becomes

$$y(t) = \sum_{k=1}^{K}\sum_{i=1}^{M} A_k b_k[i] s(t - iT - \tau_k) + \sigma n(t) \tag{2.31}$$

taking into account that the users send a complex bitstream $b_k[-M], \ldots, b_k[0], \ldots, b_k[M]$. The length of the packets transmitted by each user is assumed to be equal to $(2M + 1)$.

In the context of this model, users initiate and terminate their transmissions within T time units from each other. This presupposes some form of block synchronism, if not symbol synchronism. This assumption allows us to focus on the offsets modulo T, and does not impact on the generality of the analysis, because of typically large values of M. Figure 2.4 shows the symbol epochs for three asynchronous users in a case where $M = 1$.

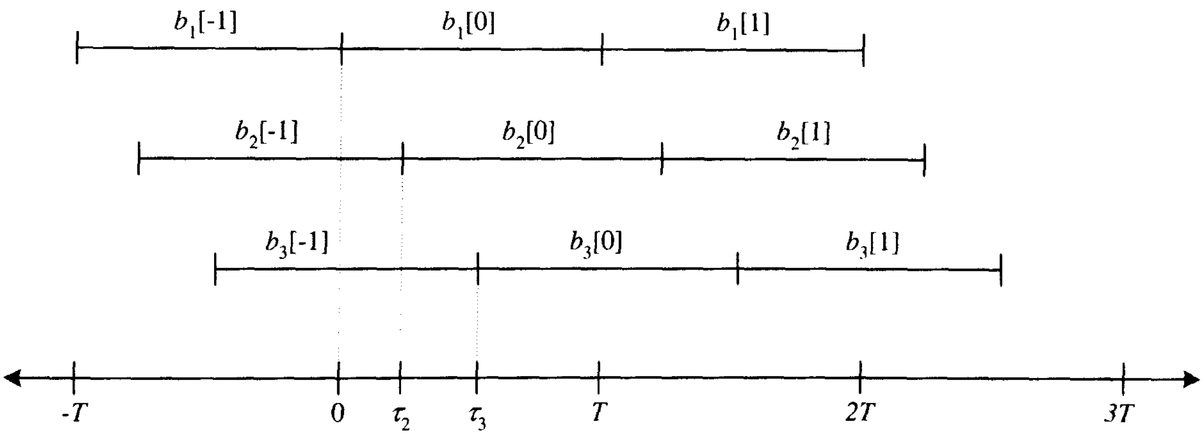

Figure 2.4: Schematic representation of the symbol epochs for three users if $M = 1$

The synchronous channel corresponds to the special case of (2.31) where $\tau_1 = \tau_2 = \ldots = \tau_K = 0$.

2.3.1 INTERSYMBOL INTERFERENCE

Consider the special case in which all the complex received amplitudes and all the complex signature waveforms are equal, i.e. $A_1 = A_2 = \ldots = A_K$ and $s_1 = s_2 = \ldots = s_K$, and in which the offsets satisfy

$$\tau_k = \frac{(k-1)T}{K}. \tag{2.32}$$

The asynchronous model then becomes

$$\begin{aligned} y(t) &= \sum_{k=1}^{K}\sum_{i=-M}^{M} Ab_k[i]s\left(t - iT - \frac{(k-1)T}{K}\right) + \sigma n(t) \\ &= \sum_{j} Ab[j]s\left(t - \frac{jT}{K}\right) + \sigma n(t) \end{aligned} \tag{2.33}$$

where we have denoted $b[iK + k - 1] = b_k[i]$. The channel in (2.33) is, in fact, the single user white Gaussian channel with intersymbol interference. ISI is a phenomenon encountered in both synchronous and asynchronous CDMA systems. It may be due to a frequency selective (discrete multipath) channel or partial response signalling (to increase the signature time bandwidth product). We will discuss the frequency selective (or multipath) channel later in this chapter, as it is commonly encountered in the mobile channel. We will also derive the discrete multipath channel from the continuous time dispersion channel filter model.

2.3.2 ASYNCHRONOUS VECTOR MATRIX MODEL

When using (2.31) with (2.18) and (2.19), the matched filter outputs can be expressed as

$$\begin{aligned} y_k[i] &= A_k b_k[i] \\ &\quad + \sum_{j<k} A_j b_j[i+1]\rho_{kj} + \sum_{j<k} A_j b_j[i]\rho_{jk} \\ &\quad + \sum_{j>k} A_j b_j[i]\rho_{kj} + \sum_{j>k} A_j b_j[i-1]\rho_{jk} \\ &\quad + n_k[i] \end{aligned} \tag{2.34}$$

where

$$n_k[i] = \sigma \int_{\tau_k+iT}^{\tau_k+iT+T} n(t)s_k^*(t - iT - \tau_k)dt. \tag{2.35}$$

The first line of equation (2.34) is the desired information. The second line is the interference due to earlier users and the third line represents the interference due to later users. We can write equation (2.34) in matrix form,

$$\begin{aligned} \mathbf{y}[i] &= \mathbf{R}^H[1]\mathbf{A}\mathbf{b}[i+1] + \mathbf{R}[0]\mathbf{A}\mathbf{b}[i] \\ &\quad + \mathbf{R}[1]\mathbf{A}\mathbf{b}[i-1] + \mathbf{n}[i] \end{aligned} \tag{2.36}$$

where the zero mean Gaussian process $\mathbf{n}[i]$ has the autocorrelation matrix

$$E[\mathbf{n}[i]\mathbf{n}^H[j]] = \begin{cases} 2\sigma^2\mathbf{R}^H[1] & \text{if} j = i+1 \\ 2\sigma^2\mathbf{R}[0] & \text{if} j = i \\ 2\sigma^2\mathbf{R}[1] & \text{if} j = i-1 \\ 0 & \text{otherwise,} \end{cases} \tag{2.37}$$

and the complex valued matrices $\mathbf{R}[0]$ and $\mathbf{R}[1]$ are defined by

$$R_{jk}[0] = \begin{cases} 1 & \text{if} j = k \\ \rho_{jk} & \text{if} j < k \\ \rho_{kj} & \text{if} j > k \end{cases} \tag{2.38}$$

and

$$R_{jk}[1] = \begin{cases} 0 & \text{if} j \geq k \\ \rho_{kj} & \text{if} j < k \end{cases} . \tag{2.39}$$

For example, in the three user case

$$\mathbf{R}[0] = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} \\ \rho_{12} & 1 & \rho_{23} \\ \rho_{13} & \rho_{23} & 1 \end{bmatrix}, \tag{2.40}$$

$$\mathbf{R}[1] = \begin{bmatrix} 0 & \rho_{21} & \rho_{31} \\ 0 & 0 & \rho_{32} \\ 0 & 0 & 0 \end{bmatrix}. \tag{2.41}$$

The vector matrix discrete time model in (2.36) can be represented in the z-transform domain

$$\mathbf{S}(z) = \mathbf{R}^H[1]z + \mathbf{R}[0] + \mathbf{R}[1]z^{-1}. \tag{2.42}$$

This means we can also represent (2.42) as the combined asynchronous correlation matrix

$$\mathbf{R}_a = \begin{bmatrix} \mathbf{R}[0] & \mathbf{R}^H[1] & 0 & \cdots & 0 \\ \mathbf{R}[1] & \mathbf{R}[0] & \mathbf{R}^H[1] & \cdots & \vdots \\ 0 & \mathbf{R}[1] & \ddots & \vdots & 0 \\ \vdots & \vdots & \cdots & \mathbf{R}[0] & \mathbf{R}^H[1] \\ 0 & \cdots & 0 & \mathbf{R}[1] & \mathbf{R}[0] \end{bmatrix}. \tag{2.43}$$

The z domain model is depicted in Figure 2.5, where $\bar{\mathbf{n}}[i]$ is independent Gaussian with covariance matrix $2\sigma^2\mathbf{I}$.

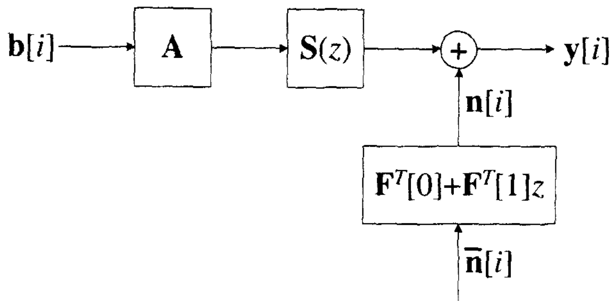

Figure 2.5: Block diagram of the z domain vector matrix model of equations (2.36) and (2.42)

Note that if the signature waveforms have a duration larger than T, then the model has to be generalized to incorporate crosscorrelation matrices $\mathbf{R}[2], \ldots, \mathbf{R}[L]$, where L is the length of the intersymbol interference. The choice of the $K \times K$ matrices $\mathbf{F}[0]$ and $\mathbf{F}[1]$ in Figure 2.5 is governed by the following proposition:

Proposition 2.2 *(z Transform Model Cholesky Factorization) The complex valued matrix* $\mathbf{S}[z]$ *in (2.42) can be expressed as*

$$\mathbf{S}(z) = \left[\mathbf{F}[0] + \mathbf{F}[1]z\right]^H \left[\mathbf{F}[0] + \mathbf{F}[1]z^{-1}\right] \tag{2.44}$$

where $\mathbf{F}[0]$ *is lower triangular and* $\mathbf{F}[1]$ *is upper triangular with zero diagonal such that*

$$\mathbf{R}[0] = \mathbf{F}^H[0]\mathbf{F}[0] + \mathbf{F}^H[1]\mathbf{F}[1] \tag{2.45}$$

$$\mathbf{R}[1] = \mathbf{F}^H[0]\mathbf{F}[1] \tag{2.46}$$

$$\det \mathbf{F}[0] = \exp\left(\frac{1}{2}\int_0^1 \log\left(\det \mathbf{S}\left(e^{j2\pi f}\right)\right) df\right) \tag{2.47}$$

Furthermore, if $\det \mathbf{S}(e^{j2\omega}) > 0$ *for all* $\omega \in [-\pi, \pi]$, *then* $[\mathbf{F}[0] + \mathbf{F}[1]z^{-1}]^{-1}$ *is causal and stable.*

As with (2.21), if the vector sequence of matched filter outputs is fed into the filter $[\mathbf{F}[0]+\mathbf{F}[1]z^{-1}]^{-1}$, the output sequence is given by

$$\bar{\mathbf{y}}[i] = \mathbf{F}[0]\mathbf{A}\mathbf{b}[i] + \mathbf{F}[1]\mathbf{A}\mathbf{b}[i-1] + \bar{\mathbf{n}}[i] \tag{2.48}$$

where as with (2.22), $\bar{\mathbf{n}}[i]$ is independent Gaussian with covariance matrix $2\sigma^2\mathbf{I}$.

As with the synchronous case, alternative finite dimensional models can be used with a set of orthonormal waveforms that span the signature space, i.e. all the signature waveforms and their delayed versions. In a direct sequence spread spectrum system, this can be accomplished by chip matched filters sampled at the chip rate times the number of users. Nevertheless, for approximately band-limited chip waveforms, it is sufficient to sample at the Nyquist rate.

2.4 THE FADING MOBILE CHANNEL MODEL

We will now consider mobile channel models for the evaluation of multiuser detection methods and CDMA transmission. Mobile channels are dominated by a phenomenon called *fading*. Fading is the variation in signal strength over a period of time. We will mostly concern ourselves with *small scale* fading. Small scale fading is rapid signal strength variation over time or distance. We will assume the large scale fading (due to shadowing) to be quasi-stationary, and thus less relevant to our comparative evaluation of multiuser detection schemes.

There are two main types of small scale fading in a mobile channel. The first is fading due to multipath, and the other is fading due to Doppler spread. Multipath delay causes time dispersion and frequency selective fading, while Doppler spread causes frequency dispersion and time selective fading. We can subdivide multipath fading into two more components.

Flat Fading - In this case, the bandwidth of the signal is smaller than the bandwidth of the channel. This also means that the delay spread is smaller than the symbol period. The spectral characteristics of the transmitted signal is preserved at the receiver, thus no inter symbol interference (ISI) is introduced.

Frequency Selective Fading - Here, the bandwidth of the signal is greater than the bandwidth of the channel. Furthermore, the delay spread is greater than the symbol period. Frequency selective fading introduces time dispersion between the symbols, introducing ISI.

Fading based on Doppler spread, can also be subdivided into two categories. These are fast fading and slow fading.

Fast Fading - In this case, the channel has a large Doppler spread. Furthermore, the coherence time is smaller than the symbol period. Here, the channel variations occur faster that the baseband signal variations.

Slow Fading - In contrast with fast fading, the channel has a small Doppler spread. This means that the coherence time is greater than the symbol period, and channel variations appear to be slower than baseband signal variations.

Another mobile channel effect is Doppler shift due to the relative motion between transmitter and receiver. This is a carrier frequency and velocity dependent frequency offset on each of the multipath components. This effect is taken into account in Clarke's model, which we will discuss now.

2.4.1 RAYLEIGH FADING DUE TO DOPPLER SPREAD - CLARKE'S MODEL

Clarke developed a model to deduce the statistical characteristics from the scattered electromagnetic fields of the received signal at the mobile receiver [33]. In this model, the envelope of the received E-field E_z is the square root of the sum of two squared Gaussian random variables. By random variable transformation, we have that the received signal envelope of a certain propagation path has a Rayleigh distribution given by

$$f_R(r) = \begin{cases} r^2 \exp\left(\frac{-r^2}{2}\right), & 0 \leq r \leq \infty. \\ 0, & r < 0. \end{cases} \tag{2.49}$$

To evaluate the probability of error of a CDMA detector (or any other digital communication detector) in a Rayleigh fading channel, the signal-to-noise ratio γ must be averaged over all possible fading signal amplitudes. That is to say

$$P_{e,R} = \int_0^\infty P_e(\gamma)p(\gamma)d\gamma, \tag{2.50}$$

where $P_e(\gamma)$ is the Gaussian channel error probability for an arbitrary modulation at a specific value of signal-to-noise ratio γ, and $P_{e,R}$ is the error probability for the Rayleigh faded signal.

2.4.2 MULTIPATH TIME DISPERSION MODEL

The mobile channel can be modelled as a linear filter. This means that the small scale variations of the mobile radio channel can be characterized by the impulse response of the mobile channel. The impulse response model of the mobile channel is useful, since it may be used to predict and compare the performances of many mobile communication systems under many different conditions.

To show that the mobile channel can be modelled as a linear filter with a time varying response, consider the case in Figure 2.6 where the time variation is only due to the motion of the mobile. We assume that receiver moves along the ground at some constant speed v. For a fixed position d, the channel between the receiver and the transmitter can be modelled as a linear time invariant system.

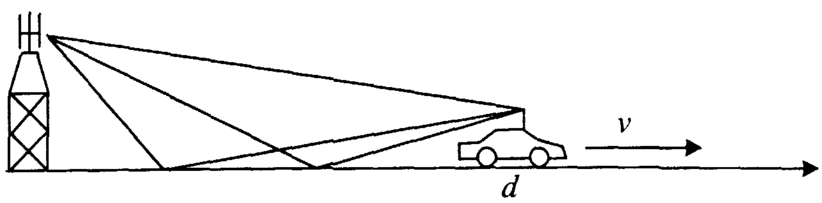

Figure 2.6: The mobile radio channel as a function of time and space.

Due to different multipath waves that differ from position to position, the impulse response of the linear time invariant system should be a function of the position of the receiver. Let the channel impulse response of user k be depicted by $h_k(d,t)$. The effect of time dispersion on the basic CDMA model is that the signature waveform seen at the receiver is the complex convolution

$$\tilde{s}_k(d,t) = s_k(t) \otimes h_k(d,t) = \int_{-\infty}^{\infty} s_k(\tau)h_k(d,t-\tau)d\tau. \tag{2.51}$$

For a causal system, $h_k(d,t) = 0$ for $t < 0$. This results in the following equation

$$\tilde{s}_k(d,t) = \int_0^t s_k(\tau)h_k(d,t-\tau)d\tau. \tag{2.52}$$

If the receiver moves at a constant speed v, the position of the receiver can be expressed as

$$d = vt. \tag{2.53}$$

Substituting (2.53) in equation (2.52), we obtain

$$\tilde{s}_k(vt,t) = \int_0^t s_k(\tau)h_k(vt,t-\tau)d\tau \tag{2.54}$$

We assume v to be constant with respect to symbol time T, that is $\tilde{s}(vt,t)$ is only a function of t. Therefore, (2.53) can be written as

$$\tilde{s}(vt,t) = \int_0^t s_k(\tau)h_k(vt,t-\tau)d\tau = s_k(t) \otimes h_k(vt,t) = s_k(t) \otimes h_k(d,t) \tag{2.55}$$

The kth user impulse response $h_k(t,\tau)$ completely characterizes the channel as a function of both t and τ. The variable t represents the time variations due to motion, and τ represents the channel multipath delay for a fixed value of t. The output of the "channel filter" for user k is given by

$$\tilde{s}_k(t) = \int_0^t s_k(\tau)h_k(t,\tau)d\tau = s_k(t) \otimes h_k(t,\tau) \tag{2.56}$$

2.4.2.1 DISCRETE TIME CHANNEL IMPULSE RESPONSE

We can divide the multipath delay axis τ of every user's impulse response into discrete bins. These equal time delay segments are called *excess delay* bins. Each bin has a time delay width of $\tau_{p+1} - \tau_p$, where τ_0 is equal to the time instant of the first arriving signal and equal to τ_k. The first bin from τ_0 to τ_1 has a bin width of $\Delta\tau$, as with all the other bins. This means that $\tau_0 = 0$, $\tau_1 = \Delta\tau$, and $\tau_p = p\Delta\tau$, for $p = 0$ to $p = P-1$, where P represents the total number of equally spaced multipath components. The size of $\Delta\tau$ determines the time delay resolution of the channel model. The useful frequency span of the model is shown to be $1/(2\Delta\tau)$. This means that signals of maximum bandwidth $1/(2\Delta\tau)$ can be evaluated using this model. We assume that the delay resolution is equal for all users. There are terms that apply to this model that have to be briefly discussed. The first is *excess delay*, being the relative delay of the pth multipath component to the first arriving component, and is denoted by τ_p. The *maximum excess delay* is given by $P\Delta\tau$.

The received signal consists of a series of attenuated, time delayed and phase shifted versions of the transmitted signals. The impulse response of user k can be expressed as

$$h_k(t,\tau) = \sum_{p=0}^{P-1} A_{k,p}(t,\tau)\exp\left[j(2\pi f_{d_k}\tau_p + \phi_{k,p}(t,\tau))\right]\delta(\tau - \tau_p) \tag{2.57}$$

where $A_{k,p}(t,\tau)$ and τ_p are the real amplitudes at time t and excess delays, respectively, of the pth multipath component of user k. The phase term $(2\pi f_{d_k}\tau_p + \phi_{k,p}(t,\tau))$ represents the phase shift, due to a doppler shift f_{d_k} and other channel effects (such as Rayleigh fading), of the pth multipath component at time t of user k. The phase term can be combined and represented by the term $\theta_{k,p}(t,\tau)$. Figure 2.7 is a graphical representation of equation (2.57) for a certain channel at different times t. Note that some of the excess delay bins (or multipath components) may have a zero amplitude.

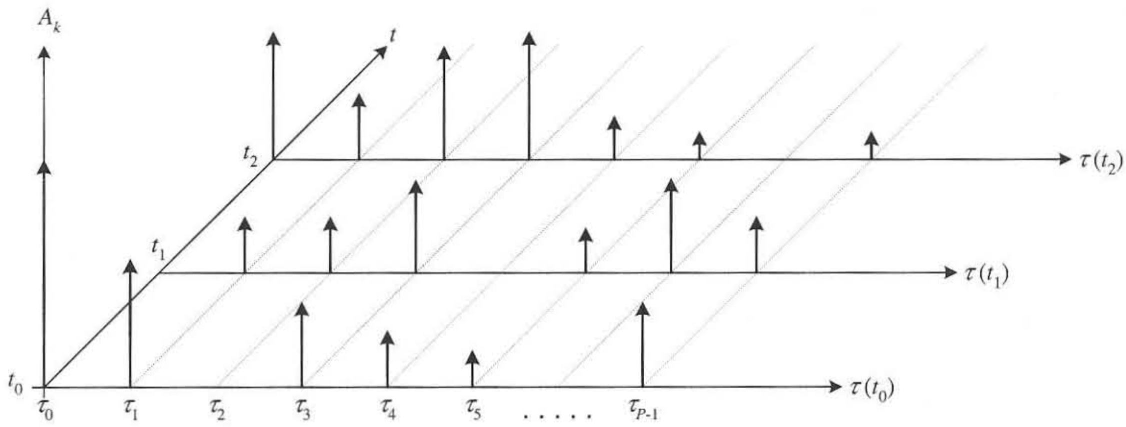

Figure 2.7: The time varying discrete time impulse response model for a specific multipath radio channel.

If the channel impulse response is assumed to be time invariant, or is at least stationary in a wide sense, then the baseband channel impulse response can be simplified to

$$h_k(t) = \sum_{p=0}^{P-1} A_{k,p}\exp(-j\theta_{k,p})\delta(\tau - \tau_p). \tag{2.58}$$

It is evident that the discrete time channel impulse response is the summation of a series of impulses, each with a different phase. Applying this to the CDMA case we have the sum of the complex convolutions of the different user signature waveforms with their respective frequency selective channels.

Since we are considering multiple delayed versions of the received signals, we will use an asynchronous equation (2.31). Considering only the ith bit,

$$y(t)[i] = \sum_{k=1}^{K} \sum_{p=0}^{P-1} A_{k,p} b_k[i] s(\tau - \tau_p) \exp(-j\theta_{k,p}) + \sigma n(t). \tag{2.59}$$

where P is the number of equally spaced multipath components.

2.4.2.2 THE CDMA UPLINK AND DOWNLINK CHANNELS

The above model provides for the uplink situation, where the transmission from each mobile to a central base station has a different impulse response. The downlink situation (base station to mobile) in which the channel impulse response pertaining to all the users are equal, i.e., $h_1(t) = h_2(t) = \ldots = h_K(t)$, leads to the simplification that the contribution of all users can be added, before passing through a single channel $h(t)$.

2.5 SUMMARY

This chapter supplies the mathematical background to thoroughly analyze all the multiuser detection schemes contained in this dissertation. The concept of multiuser interference in terms of cross-correlation coefficients is presented here. Discrete synchronous and asynchronous baseband CDMA signal and channel models are introduced in vector matrix notation. The fading mobile channel is also presented in this chapter. The Rayleigh fading and multipath time dispersion models applied to CDMA channels are also introduced.