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Blind Multi-user Cancellation using the Constant Modulus Algorithm

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Advisor: Professor L.P. Linde

Johan Pieter de Villiers

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SUMMARY

Constant Modulus Detectors for Blind Multiuser Detection

by

J.P. de Villiers

Advisor: Professor L.P. Linde

Department of Electrical, Electronic and Computer Engineering

Master of Engineering (Electronic)

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Multiuser detection in direct sequence code division multiple access (DS-CDMA) systems has received much attention in recent years. This activity can be attributed to the fact that DS-CDMA systems are to be used in third generation (3G) cellular networks. Cellular operators have already paid billions of dollars for 3G licences, and are serious about ensuring effective use of available channel resources. It is for this reason that methods for increasing channel capacity are being vigorously researched. Multiuser detection can increase effective channel usage significantly, and so save the operators large amounts of money. The multiuser detection problem is discussed in the following paragraph.

In a DS-CDMA channel, each user is separated from the other users by his/her unique signature waveform. These signature waveforms are in practice quasi-orthogonal, i.e. exhibiting small amounts of cross correlation between the signature waveforms of different users. This has an adverse effect on performance, and introduces what is termed as *multiuser interference* when demodulated with a conventional matched filter detector. The effect of multiuser interference is especially visible in a channel consisting of unequal power users. Multiuser detection techniques concern themselves with the minimization of multiuser interference. Many techniques have been considered, of which the linear adaptive detectors offer good performance while employing relatively simple structures. A well known linear adaptive detector is the adaptive minimum mean square error (MMSE) detector.

This detector offers a significant improvement when compared with the conventional matched filter detector. Furthermore, it is adaptive, and is able to follow slow variations in the channel. The adaptive MMSE detector has the disadvantage that training sequences need to be transmitted to allow the detector to initially converge. In addition, if large channel fluctuations occur, the detector has to be retrained. These training sequences sacrifice valuable bandwidth, and are undesirable. This poses the need for blind detectors, that are able to adaptively tune out multiuser interference without the need for training sequences. The application of the widely used blind equalization constant modulus algorithm (CMA) to blind multiuser detection is attempted in this dissertation.

Direct application of the constant modulus algorithm to blind multiuser detection poses two distinct problems. The first is the fact that detecting any other user rather than the desired user may yield a constant modulus signal. This means that the standard constant modulus detector may lock onto any of the active users in the channel. The second problem is that some of the desired user's signal may be cancelled, even if the receiver locks onto the desired user component. Both of these problems may be solved by implementing a linear constraint which restricts the constant modulus detector to operate only on the subspace orthogonal to the desired user component. This detector is called the linearly constrained constant modulus (LCCM) detector. This detector exhibits performance equalling that of the MMSE detector subject to the fact that the desired user component is greater than a fixed value. This limitation may be a problem, especially in the case where the different users may have greatly varying amplitudes, such as in a mobile fading channel. The linearly constrained differential constant modulus (LCDCM) detector is the solution to this problem. The LCDCM detector penalizes any deviation in signal modulus from one sample to the next, whereas the LCCM detector penalizes any deviation in signal modulus from a constant value. The LCDCM detector has no limitation on minimum desired user amplitude, and convergence of the adaptation algorithm is assured.

In this dissertation, the two constant modulus multiuser detectors are analyzed, evaluated and compared with the MMSE detector. Existing signal and channel models are expanded to encompass the complex valued multipath DS-CDMA channels. For the first time, a global convexity condition is derived for the LCCM detector cost function. Simulation results for different channel types are generated and discussed. These channel types range from the additive white Gaussian noise (AWGN) channel to multipath fading channels.



OPSOMMING

Konstante Modulus Detektors vir Blinde Multigebruiker Deteksie
deur

J.P. de Villiers

Studieleier: Professor L.P. Linde

Departement Elektriëse, Elektroniese en Rekenaar Ingenieurswese

Meester in Ingenieurswese (Elektronies)

Sleutelwoorde: Blinde Multigebruikerdeteksie, Lineêr Beperkte Konstante Modulus Algoritme, Lineêr Beperkte Differensiële Konstante Modulus Algoritme, Blinde Vereffening, DS-CDMA

Multigebruiker-deteksie in direkte sekvensie kodeverdeling multi-toegang (DS-KVMT) stelsels het baie aandag in die laaste paar jare ontvang. Hierdie aktiwiteit kan toegeskryf word aan die feit dat DS-KVMT stelsels in derde generasie (3G) sellulêre netwerke gebruik gaan word. Sellulêre operateurs het reeds biljoene dollar betaal vir 3G lisensies. Dit is om hierdie rede dat metodes om kanaalkapasiteit te vermeerder met so baie toewyding nagevors word. Multigebruiker-deteksie kan kanaalverbruik merkwaardig vermeerder, en kan dus die operateurs groot hoeveelhede geld bespaar. Die multigebruiker-deteksie-probleem word in die volgende paragraaf bespreek.

In 'n DS-KVMT kanaal word elke gebruiker van 'n ander een geskei d.m.v. sy/haar unieke identifikaasiegolfvorm. Hierdie identifikaasiegolfvorms is in die praktyk kwasi-ortogonaal, m.a.w. klein hoeveelhede kruiskorrelasie bestaan tussen die identifikaasiegolfvorms van die verskillende gebruikers. Hierdie eienskap het 'n nadelige effek op werkverrigting, en stel die kanaal bloot aan *multigebruiker-oorvleueling* wanneer dit d.m.v. 'n aangepaste filter gedemoduleer word. Die effek van multigebruiker oorvleueling is veral sigbaar in 'n kanaal wat uit gebruikers met ongelyke drywing bestaan. Multigebruiker deteksie tegnieke poog om die hoeveelheid multigebruiker oorvleueling binne 'n kanaal te minimeer. Baie tegnieke is al oorweeg, waarvan die lineêr aanpasbare detektors goeie werkverrigting lewer terwyl dit van 'n eenvoudige struktuur gebruik maak. 'n Bekende lineêr aanpasbare detektor is die minimum gemiddelde kwadraat fout (MGKF) detektor. Hierdie detektor bied 'n merkwaardige

verbetering wanneer vergelyk word met die konvensionele aangepaste filter detektor. Verder, is dit ook aanpasbaar, en het die vermoë om stadige kanaalveranderinge te volg. Die aanpasbare MGKF detektor het die nadeel dat opleidingsekwensies nodig is om die detektor aanvanklik toe te laat om te konvergeer. Daarbenewens, as groot kanaalvariasies plaasvind, moet die detektor weer geleer word. Hierdie opleidingsekwensies offer duursame bandwydte op, en is dus ongewens. Dit stel die behoefte aan blinde detektors daar, wat aan kan pas om multigebruiker oorvleueling te minimeer sonder 'n behoefte aan opleidingsekwensies. Die toepassing van die algemeen gebruikte blinde vereffening konstante omhulling algoritme op blinde multigebruiker deteksie word in hierdie verhandeling aangespreek.

Die direkte toepassing van die konstante omhulling algoritme op blinde multiverbruiker deteksie bring twee spesifieke probleme mee. Die eerste is deteksie van enige ander gebruiker buiten die gewenste gebruiker sal ook 'n konstante omhulling lewer. Dit beteken dat die gewone konstante omhulling detektor kan sluit op enige van die aktiewe gebruikers in die kanaal. Die tweede probleem is dat 'n gedeelte van die gewenste gebruiker se in uitgekanselleer kan word, selfs al sluit die ontvanger op die gewenste gebruiker komponent. Beide hierdie probleme kan opgelos word deur gebruik te maak van 'n lineêre beperking, wat die werking van die konstante omhulling detektor beperk tot die subruimte ortogonaal tot die gewenste gebruiker komponent. Hierdie detektor word die lineêre beperkte konstante omhulling (LBKO) detektor genoem. Hierdie detektor lewer dieselfde werkverrigting as die MGKF detektor, onderhewig aan die beperking dat die gewenste gebruiker komponent groter as 'n spesifieke waarde is. Hierdie beperking mag 'n probleem wees, veral in gevalle waar die verskillende gebruikers grootliks varieërende amplitudes mag hê, soos in 'n mobiele deinde kanaal. Die lineêre beperkte differensiële konstante omhulling (LBDKO) detektor is die oplossing tot hierdie probleem. Die LBDKO detektor penaliseer enige afwyking in seinomhulling vanaf een monster tot die volgende, terwyl die LBKO detektor enige afwyking in omhulling vanaf 'n konstante waarde penaliseer. Dit het die gevolg dat die LBDKO detektor geen minimum beperking op gewenste gebruiker amplitude het nie, en dat konvergensie van die aanpassingsalgoritme verseker is.

In hierdie verhandeling word daar 'n analise en evaluasie van die twee konstante omhulling detektors gedoen, en word hulle met mekaar vergelyk, asook die MGKF detektor. Bestaande sein en kanaal modelle word uitgebrei om komplekse waarde multipad DS-KVMT kanale te akkomodeer. Vir die eerste keer word 'n globale konveksiteitsvoorwaarde vir die LBKO detektor kostefunksie afgelei. Simulasie resultate vir verskillende kanaaltipes word gegenereer en bespreek. Hierdie kanaaltipes wissel van die sommeerbare wit Gaussiese ruis (SWGR) kanaal tot multipad deinde kanale.



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LIST OF ABBREVIATIONS AND ACRONYMS

A

AWGN - Additive White Gaussian Noise

B

BEP - Bit Error Probability

BER - Bit Error Rate

BPSK - Binary Phase Shift Keying

C

CDMA - Code Division Multiple Access

CMA - Constant Modulus Algorithm

CM - Constant Modulus

D

DD - Decision Directed

DF - Decision Feedback

DFE - Decision Feedback Equalizer

DS - Direct Sequence

DSP - Digital Signal Processor

E

EVD - Eigenvalue Decomposition

F

FIR - Finite Impulse Response

FPGA - Field Programmable Gate Array

G

GCL - General Chirp Like (sequences)



I

- IC - Interference Cancellation
- ISI - Inter Symbol Interference

J

- JD - Joint Detection

L

- LCCM - Linearly Constrained Constant Modulus
- LCCMA - Linearly Constrained Constant Modulus Algorithm
- LCDCM - Linearly Constrained Differential Constant Modulus
- LCDCMA - Linearly Constrained Differential Constant Modulus Algorithm
- LCMV - Linearly Constrained Minimum Variance
- LCMVA - Linearly Constrained Minimum Variance Algorithm
- LMS - Least Mean Square

M

- MAI - Multiple Access Intereference
- MF - Matched Filter
- MIMO - Multiple Input Multiple Output
- MISO - Multiple Input Single Output
- ML - Maximum Likelihood
- MLSE - Maximum Likelihood Sequence Estimation
- MMSE - Minimum Mean Square Error
- MOE - Minimum Output Energy
- MSE - Mean Square Error
- MUD - Multi User Detection

N

- MAI - Narrow Band Interference

R

- RU - Root of Unity

S

- SD - Sequence Detection
- SNR - Signal to Noise Ratio
- SSD - Single Symbol Detection
- SISO - Single Input Single Output



- SIR - Signal to (Noise and) Interference Ratio
SUD - Single User Detection
SVD - Singular Value Decomposition

T

- TDMA - Time Division Multiple Access

V

- VA - Viterbi Algorithm

Z

- ZF - Zero Forcing



LIST OF SYMBOLS

- A** - Matrix of which the diagonal contains all the users' amplitudes
- A - The single user channel amplitude
- $a_k(n)$ - Pseudo noise sequence of the signature waveform of user k
- A_k - The received signal amplitude of user k
- \tilde{A}_k - The complex valued amplitude of user k due to a phase θ_k
- $A_{k,l}$ - The received signal amplitude of the l th multipath component of user k
- \bar{A}^2 - The statistical average of the squared amplitudes of all interfering users
- b** - Vector of all K users' transmitted bits
- b_k - The bit transmitted by the k th user
- \hat{b} - The bit decision of a single user channel
- \hat{b}_k - The bit decision of the k th user
- $b_k[i]$ - The i th bit transmitted by the k th asynchronous user
- c** - An arbitrary K -vector $\mathbf{c} = (c_1, \dots, c_K)^T$.
- c_k - Complex waveform of duration T used for a linear transform
- c_k^s - Subspace of c_k spanned by the signature waveforms s_1, \dots, s_K
- c_k^o - Subspace of c_k spanned orthogonal to the signature waveforms s_1, \dots, s_K
- C** - Covariance matrix of the received signal vector **r**
- f_{d_k} - The Doppler frequency of user k
- $e_k(\sigma)$ - Effective energy of user k
- E_b - Energy per bit
- \mathbf{F}^{-H} - Whitening filter

-
- G - The subset of interferers with the partially open eye condition satisfied
- \bar{G} - The subset of interferers with the closed eye condition satisfied
- $h_k(t)$ - Linear time invariant channel impulse response of user k
- $h_k(t, \tau)$ - Linear channel impulse response of user k
- H_i - The i th hypothesis out of a total of m hypotheses
- $\mathbf{H}(J)$ - The Hessian matrix of the cost function J
- \mathbf{I} - Identity matrix
- J - Joint cost function of all K users
- J_k - Cost function of user k
- J_{\min} - Minimum mean square error of the Wiener filter
- K - The number of simultaneous users
- L - The number of dimensions or orthonormal signals in the orthonormal detector model
- M - Asynchronous user packet length is equal to $(2M + 1)$
- \mathbf{M} - Arbitrary linear transform on the received signal vector
- $\bar{\mathbf{M}}$ - Optimum linear transformation in a minimum mean square error sense
- $\tilde{\mathbf{M}}$ - Scaled version of an optimum linear transformation in a MMSE sense
- $\tilde{\mathbf{M}}_a$ - Asynchronous channel optimum linear transformation in a MMSE sense
- $\bar{\mathbf{n}}$ - Vector of K whitened (uncorrelated) noise components
- \mathbf{n} - The vector which contains the noise components of all K matched filter outputs
- n_k - The unnormalized inner product between $n(t)$ and the signature waveform of user k
- $n_k[i]$ - The i th bit asynchronous noise contribution of user k
- $n(t)$ - White Gaussian noise with unit power spectral density
- N - The length (or spreading gain) of the pseudo noise sequence for each signature waveform
- N_0 - The one sided noise spectral density
- $\mathcal{N}(\mu, \sigma^2)$ - Notation for a Gaussian distribution with mean μ and variance σ^2
- \mathbf{p} - The cross correlation vector between the vector \mathbf{r} and the desired response b_1



- $p(t)$ - The chip waveform of duration T_c
- P - Number of equally spaced discrete multipath components
- \mathcal{P} - Maximum allowable bit error probability in power-tradeoff region
- P_e - Single user error probability in a Gaussian channel
- $P_{e,R}$ - Error probability in a Rayleigh fading channel
- $P_e(\sigma, k)$ - Error probability in a Gaussian channel for user k
- $\tilde{P}_e(\sigma, k)$ - Gaussian approximation of error probability in a Gaussian channel for user k
- $P_e^F(\sigma, k)$ - Error probability in a Rayleigh fading channel for user k
- $P_e^{\mathbf{t}_k}$ - Error probability of user k due to a linear transform \mathbf{t}_k
- $Q(\cdot)$ - The Q -function as defined in Appendix A
- $R_p(\tau)$ - Autocorrelation function of an arbitrary rectangular chip signature waveform
- \mathbf{r} - The output vector of a correlation receiver
- \mathbf{R} - Synchronous cross correlation matrix
- $\mathbf{R}[0]$ - Asynchronous cross correlation matrix as defined in (2.38)
- $\mathbf{R}[1]$ - Asynchronous cross correlation matrix as defined in (2.39)
- \mathcal{R} - An arbitrary interval on the real line
- R_i - The i th decision region out of a total of m regions
- $s(t)$ - Unit energy deterministic signature waveform in a single user channel
- $s_k(t)$ - Unit energy deterministic signature waveform of user k
- $\tilde{s}_k(t)$ - Linear time dispersive channel signature waveform response of user k
- \mathbf{s}_k - Signature vector of user k
- \mathbf{S} - Signature matrix containing the signature vectors of K users
- $S(z)$ - z -transform transfer function of the asynchronous cross correlations
- T - Symbol duration equals $L \times T_c$
- T_c - Chip duration
- \mathbf{v} - Arbitrary tap weight vector of a linear detector



- $\bar{\mathbf{v}}$ - MMSE optimum tap weight vector
- $x_i(t)$ - An arbitrary deterministic energy function defined on an interval \mathcal{R} on the real line
- \mathbf{y} - Vector of K matched filter outputs
- $\bar{\mathbf{y}}$ - Vector of K whitened matched filter outputs
- y_k - Matched filter output of user k
- $y_k[i]$ - Matched filter output of the i th bit of asynchronous user k
- $y(t)$ - Received signal embedded in AWGN in the multipath case
- Y - Decision statistic of $y(t)$
- $\nabla_{m_{vw}}$ - Element of the v th row and w th column of the complex gradient operator matrix $\nabla_{\mathbf{M}}$
- $\nabla_{\mathbf{M}}(J)$ - Complex gradient matrix of the cost function J
- α - Arbitrary real scalar used as desired modulus in the constant modulus type detector
- $\phi_{k,l}(t, \tau)$ - Phase shift of the l th multipath component due to fading and other channel effects
- β - Ratio of number of users to spreading gain (K/N)
- φ - A deterministic energy signal of duration T
- γ - Signal-to-noise ratio (SNR)
- γ_{ck} - Signal-to-interference ratio (SIR) of user k
- η_k - Asymptotic multiuser efficiency of user k
- $\hat{\eta}_k$ - Multiuser efficiency of user k
- $\bar{\eta}_k$ - Near-far resistance of user k
- η_k^F - Asymptotic multiuser efficiency of user k in a Rayleigh fading channel
- μ_{\max} - Maximum step size to ensure convergence of the LMS algorithm
- $\theta_{k,l}$ - Combined phase term of the l th multipath component of user k
- Θ - The parameter to be inferred in a statistical inference problem
- ρ_{kj} - Synchronous cross correlation between the signature waveforms of user k and user j
- $\rho_{kj}(\tau)$ - Asynchronous cross correlation between two signature waveforms as defined in (2.7)
- $\rho_{jk}(\tau)$ - Asynchronous cross correlation between two signature waveforms as defined in (2.8)



- σ^2 - Additive white Gaussian noise variance
- $\Delta\tau$ - The excess delay bin spacing
- τ_l - The relative delay of the l th multipath bin of any user
- τ_k - The time delay of asynchronous user k
- ω - Worst asymptotic multiuser efficiency among all users
- Ω - Covariance matrix of the interference
- $\tilde{\Omega}$ - Covariance matrix of the interference in the multipath case
- ψ_l - The l th orthonormal signal out of a set of L