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### **Appendix A**

**Diversity and Combining Techniques** 

#### **A-I Diversity**

#### **A-1.I Polarisation Diversity**

In mobile radio environments, signals transmitted on orthogonal polarisations exhibit decorrelated fading and, therefore, offer potential for diversity combining. Polarisation diversity can be obtained either by explicit or implicit techniques. Note that with polarisation, only two diversity branches are available as opposed to space diversity where several branches can be obtained using multiple antennas. In explicit polarisation diversity, the signal is both transmitted and received in two orthogonal polarisations. For a fixed total transmit power, the power in each branch will be 3dB lower than if single polarisation was used. In the implicit polarisation technique, the signal is launched in a single polarisation antenna, but is received with cross-polarised antennas. The propagation medium couples some energy into the cross-polarisation plane. Polarisation diversity can be very useful at the base station and also for hand held terminals which are subjected to varying handset orientation. Under such condition, polarisation diversity is an obvious way of improving link quality [85].

The mobile hand set can be held at random orientations during a call. This results in energy being launched with varying polarisation angles ranging from vertical to horizontal. This further increases the advantage of cross-polarised antennas at the base station since at least one of the two antennas will be well matched to the polarisation of the launched signal. An extensive study of the subject, use of the technique and improvement of system performance has been shown in the literature [86-90].

#### **A-I.2 Angle Diversity**

In situations where the angle spread is very high, as in the case of indoor or at the mobile unit in urban locations, signals collected from multiple nonoverlapping beams offer low-



fade correlation with balanced power in the diversity branches. Clearly, since directional beams imply use of antenna aperture, angle diversity is closely related to the space diversity. Angle diversity has been utilised in indoor wireless local area networks (LANs), where its use allows substantial increase in LAN throughputs.

#### **A-1.3 Frequency Diversity**

Another technique to obtain decorrelated diversity branches is to transmit the same signal over different frequencies. The frequency separation between carriers should be larger than the coherence bandwidth. The coherence bandwidth depends on the multipath delay spread of the channel. The larger the delay spread is, the smaller the coherence bandwidth and the more closely the frequency diversity channels can be spaced. Clearly, frequency diversity is an explicit diversity technique and needs additional frequency spectrum.

#### **A-l.4 Path Diversity**

A sophisticated form of diversity is based on using a signal bandwidth much larger than the channel coherence bandwidth, as is used in the so-called direct sequence spread spectrum modulation techniques. This modulation scheme is used in the CDMA mobile networks. Spread spectrum signals can resolve multipath arrivals as long as the path delay are separated by at least one *chip* period. If the signal in each path shows low-fade correlation, as is usually the case, these paths offer a valuable source of diversity.

In CDMA, diversity gain provided by the multiple paths (and other diversity branches, if any) not only reduces transmit power needs but also increases the number of users that can be supported per cell for a given bandwidth.

#### **A-1.5 Time Diversity**

In mobile communication channels, the mobile motion, together with scattering in the vicinity of the mobile, causes time selection fading of the signal with Rayleigh fading statistics for the signal envelope. Signal fade levels separated by the coherence time show low correlation and can be used as diversity branches if the same signal can be transmitted at multiple instants separated by the coherence time. The coherence time depends on the



Doppler spread of the signal, which in tum is a function of the mobile speed and the carrier frequency.

One fundamental drawback with a time diversity approach is the delay needed to collect the repeated or interleaved transmissions. If the coherence time is large, as, for example, when the vehicle is slow moving, the required delay becomes too large to be acceptable for interactive voice conversation.

#### **A-2 Combining Techniques**

#### **A-2.t Selection Combining**

Selection combining is the simplest of all techniques analysed here. A block diagram of selection combining is shown in Figure. A.I, and it is similar to Figure 2.4, where *m*  demodulators are used to provide m diversity branches whose gains are adjusted to provide the same average SNR for each branch. As can be seen in Figure. A.I, if all branches have the same noise power, the amplitude of the output from the combiner is simply the strongest signal. **It** is possible to improve the performance of selection combining to be very close to that of MRC, by selecting the M strongest output, instead of selecting only the strongest one [91,92]. The technique has been extensively studied in the literature for different scenarios by considering different channel models, and different modulation schemes to find the best optimum performance for selection combining [93-96].



Figure A.I: Selection Combining.



**In** [97], new results show better performance of selection combining by considering moreaccurate signal-*plus*-noise (S+N Selection) as compared to the traditional models based on choosing the branch with the largest signal-to-noise ratio  $(SNR)$ . The performance improved even further by increasing the number of branches.

#### **A-2.2 Feedback diversity**

Feedback or Scanning diversity is very similar to selection diversity except that instead of always using the best of  $M$  signals, the  $M$  signals are scanned in a fixed sequence until one is found to be above a predetermined threshold. This signal is then received until it falls below threshold and the scanning process is again initiated. The resulting fading statistics are somewhat inferior to those obtained by the other methods but the advantage with this method is that it is very simple to implement, only one receiver is required. A block diagram of this method is shown in Figure A.2.



Figure A.2: Feedback diversity.

#### **A-2.3 Switched Combining**

The disadvantage with selection combination is that the combiner is unable to monitor all N branches simultaneously. This requires N independent receivers, which is expensive and complicated. An alternative is to apply switched combining. Here only one receiver is required, and it is only switched between branches when the SNR on the current is lower than some predefined threshold. This is a 'switch and stay' combiner.

The performance is less than in selection combining, since unused branches may have SNRs higher than the current branch if the current SNR exceeds the threshold. The



threshold therefore has to be carefully set in relation to the mean power on each branch, which must also be estimated with good accuracy.

#### **A-2.4 Equal Gain Combining**

Since both selection and switching combining receive the signal on only one branch at a given time, the signal energy in the other branches is wasted. One way to improve on this is to add the signals from all the branches. If this were done directly on the complex signals, however, the random real and imaginary components would combine incoherently, resulting in the fading statistics at the combiner input (although a greater total power). To provide any useful diversity, the signals must be co-phased so that they add coherently; the noise on each branch is independent and randomly phased, henoe it adds only incoherently. This process is shown in Figure A.3, where each branch is multiplied by a complex phasor  $(e^{-j\theta I})$  having a phase- $\theta_i$ , where  $\theta_i$  is the phase of the channel associated with branch i. The resultant signals then all have zero phase.

The EG combiner is of considerable interest for various reason. It offers performance comparable to that of the optimal maximal Ratio Combiner (MRC) with much greater simplicity than the MRC, making it hardware feasible and cost viable [98,34].







Equal gain CEQ) combining is a powerful means to combat multipath fading and has been widely adopted in practice due to its good performance and ease of implementation, however exact calculation of error performance over different fading channels is a complex computation. A one step procedure that leads to an exact solution is presented in (99].

G,

Appendix-B



### **Appendix B**

#### **B-1 Small Scale Fading**

Small-scale fading, or simply *jading,* is used to describe the rapid fluctuation of the amplitude of a radio signal over short period of time or travel distance, so that *large-scale*  path loss effect may be ignored. Fading is caused by interference between two or more versions of the transmitted signal, which arrive at the receiver at different times. These waves, called *multipath,* combine at the receiver antenna to give a resultant signal which can vary widely in amplitude and phase. Depending on the distribution of the intensity and relative propagation time of the waves and the bandwidth of the transmitted signal [2].

Multipath in the radio channel creates small-scale fading effects. The three most important effects are:

- Rapid changes in signal strength over a small travel distance or time interval.
- Random frequency modulation due to varying Doppler shifts on different multipath signals.
- Time dispersion (echoes) caused by multipath propagation delays.

Many physical factors in the radio propagation channel influence small-scale fading. These include the following:

- Multipath propagation
- Speed of the mobile
- Speed of surrounding objects
- The transmission bandwidth of the signal

When the received signal is made up of multipath reflective rays plus a significant line-ofsight (nonfaded) component, the envelope amplitude due to small-scale fading has a Rician pdf, and is referred to as *Rician Fading.* The nonfading component is called the *specular component.* As the amplitude of the specular component approaches zero, the Rician pdf



approaches a Rayleigh pdf [49]. It is important to talk about some basic concept about fading as discussed below before we look at the different statistical fading models.

*(a) Flat Fading* 

If the mobile radio channel has a constant gain and linear phase response over a bandwidth, which is greater than the bandwidth of the transmitted signal, then the received signal will undergo *flat fading.* 

Frequency-nonselective or flat fading, degradation occurs whenever *fo>W.* Hence, all of the signal's spectral components will be affected by the channel in a similar manner (e.g., fading or no fading). Flat-fading does not introduce channel-induced lSI distortion, but performance degradation can still be expected due to loss in SNR whenever the signal is fading. In order to avoid channel-induced lSI distortion, the channel is required to exhibit flat fading by ensuring that

$$
f_q > W \approx \frac{1}{T_s} \tag{B-1}
$$

For the flat-fading case, where  $f_0 > W$  (or  $T_m < T_s$ ), as mobile radio changes its position, there will be times when the received signal experiences frequency-selective distortion even through  $f_0 > W$ . Whenever this occurs, the base band pulse will be especially mutilated by deprivation of its DC document. Even though a channel is categorised as flat fading (base on rms relationship), it can still manifest frequency-selective fading on occasions. It is fair to say that a mobile radio channel, classified as having flat fading degradation, cannot exhibit flat fading all time. By comparison, it should be clear that in Figure B .1 (a), the fading is independent of the position of the signal band, and frequency-selective fading occurs all the time, not just occasionally [49].

Viewed in the time-delay domain, a channel is said to exhibit *frequency-nonselective* or *flat fading* if  $T_m < T_s$ . In this case, all of the receiver multipath components of a symbol arrive within the symbol time duration; hence, the components are not resolvable [100].

#### *(b) Frequency Selective Fading*

If the channel possesses a constant-gain and linear phase response over a bandwidth that is smaller than the bandwidth of transmitted signal, then the channel creates *frequency selective fading* on the received signal. Frequency selective fading is due to time dispersion

Appendix-B



of the transmitted symbols within the channel. Thus the channel includes *intersymbol interference (lSI).* 



Figure **B.l:** Matrix illustrating type of fading experienced by a signal as a function of (a) Symbol period and (b) baseband signal bandwidth.

A channel is referred to as frequency-selective of  $f_0 < I/T_s \approx W$ , where the symbol rate,  $I/T_s$ . is nominally taken to be equal to the signal bandwidth *W.* Frequency-selective fading distortion occurs whenever a signal's spectral components are not all affected equally by the channel. Some of the signal's spectral components, falling outside the coherence bandwidth, will be affected differently (independently) compared to those components contained within the coherence bandwidth. This occurs whenever  $f_0 < W$ .

When viewed in the time-delay domain, a channel is said to exhibit *frequency-selective*  fading if  $T_m > T_s$  (the delay time is greater than the symbol time). This condition occurs



whenever the received multipath component has a symbol extend beyond the symbol's time duration, and is causing channel-included intersymbol interference [100].

#### (c) *Fast Fading*

Depending on how rapidly the transmitted baseband signal changes as compared to the rate of change of the channel, a channel may be classified either as a *fast fading* or *slow fading*  channel. **In** a *fast fading channel,* the channel impulse response changes within the symbol duration. That is, the coherence time of the channel is smaller than the symbol period of the transmitted signal. A *flat fading, fast fading* channel is a channel in which the amplitude of the delta function varies faster than the rate of change of the transmitted baseband signal. **In** the case of a *frequency selective, fast fading* channel, the amplitudes, phases, and time delays of any one of the multipath components vary faster than the rate of changes of the transmitted signal.

When viewed in the domain, a channel is referred to as fast fading whenever  $T_0 < T_s$ , where  $T_0$  is the channel coherence time and  $T_s$  is the symbol time. Fast fading describes a condition where the time duration for which the channel behaves in a correlated manner is short compared to the time duration of a symbol. Therefore, it can be expected that the fading character of the channel will change several times during the time a symbol is propagating [100].

#### (d) *Slow Fading*

**In** a *slow fading channel,* the channel impulse response changes at a rate much slower than the transmitted baseband signal  $s(t)$ .

Viewed in the time domain, a channel is generally referred to as introducing *slow fading* if  $T_o > T_s$ . Here, the time duration for which the channel behaves in a correlated manner is long compared to the symbol time [100].

#### **B-2 Rayleigh Fading Distribution**

**In** mobile radio channel, the Rayleigh distribution is commonly used to describe the statistical time varying nature of the receiver envelope of a flat fading signal, or the

#### Appendix-B



envelope of an individual multipath component. It is well known that the envelope of the sum of two quadrature Gaussian noise signals obeys a Rayleigh distribution. The Rayleigh distribution has a probability density function (pdf) given by (B.2). Rayleigh distribution has been widely used in the literature to model fading channels for diversity reception [49, 100-108].

$$
p(r) = \left\{\frac{r}{\sigma^2} exp\left(-\frac{r^2}{2\sigma^2}\right)\right\} \quad (0 \leq r \leq \infty)
$$
  
= 0 \qquad (r < 0)

where  $\sigma$  is the rms value of the received signal voltage before *envelope detection*,  $\sigma^2$  is the time-average power of the received signal *before* envelope detection and *r* is the variable in this case volts, that is *Rayleigh* distributed.

Rayleigh fading distribution can be effectively used to model multipath fading signals and combining the different paths for diversity reception. It has also been shown in literature that diversity combining of independent *Rayleigh-faded* signals offers significant improvement in the signal level.

As with temporal or frequency diversity methods, they help to combat fading by resolving several fully or partially decorrelated fading channels. Since it is unlikely that these channels will go through a deep fade at the same time, higher average received SNR results when the output of the branches are combined [109]. The system model here is based on diversity combining of correlated signals, as can be seen in Chapter 5, where *maximalratio combining* is used to combine the signals.

#### **B-3 Rician Fading Distribution**

When there is a dominant stationary (nonfading) signal component present, such as a lineof-sight propagation path, the small-scale fading envelope distribution is Rician. As the dominant signal becomes weaker, the composite signal resembles a noise signal which has an envelope that is Rayleigh. The Rician distribution degenerates to a Rayleigh distribution when the dominant component fades away. The Rician distribution is given by

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$$
p(r) = \left\{\frac{r}{\sigma^2} e^{-\frac{(r^2 + A^2)}{2\sigma^2}} I_o\left(\frac{Ar}{\sigma^2}\right)\right\} \qquad \text{for } (A \ge 0, r \ge 0)
$$
\n
$$
= 0 \qquad \text{for } (r < 0)
$$
\n(8-3)

where the parameter A denotes the peak amplitude of the dominant signal and  $I_0$  ( $\bullet$ ) is the modified Bessel function of the first kind and zero-order. The Rician distribution is often described in terms of a parameter  $K$  which is defined as the ratio between the deterministic signal power and the variance of the multipath. It is given by  $K = A^2/(2\sigma^2)$  or in terms of dB as

$$
K (dB) = 10 log \frac{A^2}{2 \sigma^2} dB
$$
 (B - 4)

The parameter  $K$  is known as the Rician factor and completely specifies the Rician distribution. As  $A\rightarrow 0$ ,  $K\rightarrow -\infty$  dB, and as the dominant path decreases in amplitude, the Rician distribution degenerates to a Rayleigh distribution.

Signal fading, which is due to the multipath propagation, is one of the main transmission impairment in cellular mobile radio transmission. It has been shown in the literature that in some microcellular systems, the fading amplitude of the received signal has Rician probability density function. And the distribution has been used to model the fading channel for a microdiversity in [110] and to model a macrodiversity cellular model in [111]. It is observed that fading (either Rayleigh or Rician) has little effect on the *diversity gain* of local-mean-based macrodiversity system. The diversity gain is the same, but Rayleigh fading is always worse than Rician fading. A more effective model to use is the *Nakagami-m distribution.* 

Appendix C



# **Appendix C**

### **Q-function [2 pp 593-597]**

### **C-l Q-function**

$$
Q(z) = \int_{z}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy
$$
 (C-1)

Important properties of  $Q(z)$ 

$$
Q(-z) = 1 - Q(z) \tag{C-2}
$$

$$
Q(\theta) = \frac{1}{2} \tag{C-3}
$$

#### **C-2 The** *erf* **and** *erfc* **functions**

The error function is defined as

$$
erf(z) = \frac{2}{\sqrt{\pi}} \int_{0}^{z} e^{-x^{2}} dx
$$
 (C-4)

And the complementary error function (erfc) is defined as

$$
erfc(z) = \frac{2}{\sqrt{\pi}} \int_{z}^{\infty} e^{-x^2} dx
$$
 (C-5)

The erfc function is related to the erf function by

$$
erfc(z) = 1 - erf(z) \qquad (C-6)
$$

The Q-function is related to the *erf* and *erfc* function by

$$
Q(z) = \frac{1}{2} \left[ 1 - erf\left(\frac{z}{\sqrt{2}}\right) \right] = \frac{1}{2} erfc\left(\frac{z}{\sqrt{2}}\right)
$$
 (C-7)

$$
erfc(z) = 2Q(\sqrt{2}z)
$$
 (C-8)

$$
erf(z) = 1 - 2Q(\sqrt{2}z)
$$
 (C-9)

Appendix E



### Appendix D

#### D-l Bit Error Rate of Frequency Hopping Spread Spectrum.

In FH-SS systems, several users independently hop their carrier frequencies while BFSK modulation. If two users are not simultaneously utilising the same frequency band, the probability of error for BFSK can be given by

$$
P_e = \frac{1}{2} exp\left(-\frac{E_b}{N_o}\right) \qquad (D-1)
$$

However, if two users transmit simultaneously in the same frequency band, a collision, or "hit", occurs. In this case it is reasonable to assume that the probability of error is 0.5. The over all probability of bit error can be modelled to incorporate the probability of hit as well.

#### D-2 BER For Some Common Modulation Schemes in AWGN.

$$
P_{e,FSK} = \frac{1}{2\Gamma} \qquad \text{(coherent FSK)} \tag{D-2}
$$

$$
P_{e, DPSK} = \frac{1}{2\Gamma} \qquad (Differential PSK)
$$
 (D-3)

$$
P_{e, NCFSK} = \frac{I}{\Gamma} \qquad (noncoherent orthogonal binary FSK) \qquad (D-4)
$$

For GMSK, the expression for BER in the AWGN (Additive white Gaussian noise) channel when evaluated Equation (2.9) yields a Rayleigh fading BER of

$$
P_{e,GMSK} = \frac{1}{2} \left( I - \sqrt{\frac{\delta \Gamma}{\delta \Gamma + 1}} \right) \approx \frac{1}{4 \delta \Gamma} \qquad \text{(coherent GMSK)} \tag{D-5}
$$

Where

$$
\delta \cong \begin{cases} 0.68 & \text{for} \quad BT = 0.25 \\ 0.85 & \text{for} \quad BT = \infty \end{cases} \qquad (D-6)
$$

BT is the 3dB-bandwidth-bit duration product.

Appendix E



# **Appendix E**

### E **Mathematical Formulas [51]**

$$
{}_{I}F_{I}(a,b;x) = \frac{\Gamma(b)}{\Gamma(a)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)}{\Gamma(b+n)} \frac{x^{n}}{n!}
$$
 (E-la)

$$
{}_{I}F_{I}(a,b;x) = \frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)} \int_{a}^{t} e^{xt} t^{a-t} (1-t)^{b-a-1} dt
$$
 (E-lb)

$$
{}_{I}F_{I}(a,a;z) = e^{z} \qquad (E-2)
$$

$$
{}_{2}F_{I}(a,b;c;z)=\frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}\sum_{n=0}^{\infty}\frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)}\frac{x^{n}}{n!}\qquad(E-3a)
$$

$$
{}_{2}F_{l}(a,b;c;z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_{0}^{l} t^{b-l} (1-t)^{c-b-l} (1-tx)^{-a} dt
$$
  
Re(c) > Re(b) > 0 (E-3b)

$$
\int_{0}^{\infty} t^{b-l} e^{-st} {}_{l}F_{l}(a,c;kt) dt = \Gamma(b) s^{-b} \cdot {}_{2}F_{l}(a,b; c; \frac{k}{s})
$$
  
Re(b) > 0, Re(s) > 0, Re(s) > Re(k), |s| > |k| (E-4)

$$
{}_2F_1(a,b;b;z) = (1-z)^{-a}
$$
 (E-5)

f co r (s + *a* )2 *F1* ( *l,* s + *a,.* s + *l,.* ) *x e* s-1 -focr( *a,x* )d *x* - - -----------'- <sup>1</sup>+ fJ *o* s(1 + fJ *)s+a*  Re( fJ ) > 1, Re(s) > 0 (E -6)

$$
\Gamma\left(n+\frac{1}{2}\right) = \sqrt{\pi} 2^{-2n+1} \frac{\Gamma(2n)}{\Gamma(n)}, n = 0,1,2,\dots
$$
 (E-7)

Appendix E



$$
Q(\sqrt{2ax}) = \frac{1}{2\sqrt{\pi}}\Gamma\left(\frac{1}{2},ax\right) \qquad (E-8a)
$$

$$
=\frac{1}{2}-\sqrt{\frac{ax}{\pi}}e^{-ax}iF_i\left(1,\frac{3}{2};-ax\right) \qquad \qquad (E-8b)
$$

$$
P(a,x) = \Gamma(a) \left[ 1 - e^{-x} \sum_{n=0}^{a-1} \frac{x^n}{n!} \right], a = 1,2,...
$$
 (E-9)

$$
\int_{a}^{1} x^{a-1} (1-x)^{b-1} (1-\beta x)^{c} e^{-\mu x} dx = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} \Phi_{1}(a,c,a+b,\beta,-\mu) \qquad (E-10)
$$

$$
\varPhi_{i}(a,b,c,x,y)=\frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{\Gamma(a+i+j)\Gamma(b+j)}{\Gamma(c+i+j)i!j!} \cdot x^{i}y^{i},\left|x\right|<1,\left|y\right|<1\left(E-II\right)
$$

$$
F_2(a; b_1, b_2; c_1, c_2; x, y) = \frac{1}{\Gamma(a)} \int_a^{\infty} t^{a-1} e^{-t} \cdot F_1(b_1, c_1; xt) \cdot F_1(b_2, c_2; yt) dt \quad (E-12)
$$

$$
F_2(a;b_1,b_2;c_1,c_2;x,y)
$$
  
= 
$$
\frac{\Gamma(c_1)\Gamma(c_2)}{\Gamma(a)\Gamma(b_1)\Gamma(b_2)}
$$
  

$$
\sum_{i=o}^{\infty} \sum_{j=0}^{\infty} \frac{\Gamma(a+i+j)\Gamma(b_1+i)\Gamma(b_2+j)}{\Gamma(c_1+i)\Gamma(c_2+j) i! j!} \cdot x^i y^j, |x|+|y| < 1
$$
 (E-13)



# **Appendix F**

#### **F** Determinant for  $k \times k$  matrix

If we define the following  $k \times k$  matrix as:

$$
A = \begin{pmatrix} 1-a & -ab & \cdots & -ab \\ -ab & 1-a & \cdots & -ab \\ -ab & -ab & \cdots & -ab \\ -ab & -ab & \cdots & 1-a \end{pmatrix}_{k \times k}
$$
 (F - 1)

Then we can write

$$
det(A) = [1 - a(1-b)]^{k-1} \cdot [1 - a(1-b+bk)] \qquad (F-2)
$$