Chapter 5

Conformal Array Synthesis as the Intersection of Sets

5.1 Introductory Remarks

Conformal antenna arrays offer a number of advantages [129]. On high speed aircraft flush mounted radiators eliminate the need for radomes, and thus also the problems associated with radomes, such as radome heating, ablation, bore sight errors and side lobe degradation. Printed circuit microwave antennas are also both light weight and robust. The other advantage is improved scanning (360° if the array elements are arranged on a circular surface compared to a planar array which is limited to about 140°). However, practical implementation problems of conformal arrays are significant and include difficulty in finding optimum element positions and polarisation matching.

Conformal array synthesis is very difficult for two reasons. Firstly the array elements do not all point in the same direction, and may not have the same polarisation. Secondly, the body on which the antenna is mounted may have a marked influence on the element patterns. The element patterns may differ substantially even for identical elements placed at different locations on the host. This means that the element pattern for each element must be computed separately, and must be accounted for in the synthesis stage of the antenna development.

Very few conformal array synthesis methods exist and most of these methods involve general optimisation which do not take advantage of the peculiar array radiation pattern properties. In the linear and planar arrays synthesis problem the array elements are assumed to be identical in all respects (directivity, polarisation and orientation), and thus the element pattern could be factored from the radiation pattern. This simplification does not carry over to conformal arrays due to the nature of the conformal array, consequently the synthesis techniques for linear and planar arrays are not transferable to conformal arrays.

The array synthesis problem can be stated as the search for an array factor that

minimises the error between the desired and the synthesised patterns. We can define a set as all the excitations that produce a radiation pattern of the desired magnitude in a specified direction. If we have a number of these sets, each at different directions in the far-field, then the intersection of all these sets, if it exists, is the solution of the synthesis problem. The chapter starts with a discussion on the evolution of the array synthesis problem as a problem of searching for the intersection of sets.

The synthesis problem can also be seen as the intersection of only two sets, the one set is all the possible radiation patterns for the given array geometry (including constraints to enable practical realization of the array), and another set as all the radiation patterns that comply with the desired radiation pattern characteristics. The synthesis problem is then finding the intersection of these two sets. Alternatively one can view the synthesis problem as the intersection of the set of all the excitations within the excitation constraints and the set of all excitations with the required radiation pattern characteristics. The intersection between the sets is found by iteratively projecting between the sets. This approach, using only two sets in the excitation or radiation pattern space to synthesise arrays with an arbitrary geometry, is explored in Section 5.3. The formulation including not only constraints on the radiation pattern but also on the excitations. Relaxation is presented to increase the rate of convergence.

Section 5.4 deals with the details of the numerical implementation of the pattern and excitation constraints. The representation of the element patterns in the software is addressed.

Any non linear numerical method may converge to a local minimum. This is an often overlooked problem. In Section 5.5 some options to select a starting pattern will be discussed. A novel method to compute a starting point as close to the global minimum as possible will then be presented in Section 5.5.3. The proposed method uses genetic algorithm to optimise the phase distribution of the field pattern.

A number of case studies are conducted in Section 5.6 to gauge the performance of various options, such as the selection of backward projector, the selection of pattern angles and the particular choice of the initial values. The importance of accurate element patterns will also be shown.

A detailed comparison with other recently published synthesis methods is discussed in Section 5.7.

The proposed method is illustrated in Section 5.8 by means of a number of examples of the various array types and radiation pattern requirements. The examples are chosen to give insight into the application of the method.

Finally, some observations are made and general conclusions are drawn in Section 5.9.

The notation and nomenclature of Chapter 2 will be used, with only the most important equations repeated for clarity.

5.2 Evolution of the Array Synthesis as the Intersection of Sets

5.2.1 Alternating Orthogonal Projections

A recursive algorithm for the restoration of images from diffraction plane data was first proposed by Gerchberg and Saxton [130]. Youla showed that the technique was equivalent to what he called *alternating orthogonal projections* [131]. The application of the method of alternating orthogonal projections to array synthesis was first proposed by Prasad [106], and subsequently extended by Ng [132, 133]. They both considered an array of isotropic elements with an arbitrary geometry. The method can be used for field synthesis only, and both authors applied only null constraints.

Consider a complex Euclidean space $\mathcal{H}$ with elements $\vec{A}$, $\vec{x}$ and $\vec{y}$. The inner-product is denoted by $(\vec{x}, \vec{y})$ and the length (or norm) of $\vec{x}$ is defined as

$$\|\vec{x}\| = \sqrt{(\vec{x}, \vec{x})} = \vec{x}\vec{x}^H \geq 0 \tag{5.1}$$

where the $[\,]^H$ denotes the adjoint or complex conjugate transpose.

Let $\mathcal{R}$ be any linear subspace in $\mathcal{H}$, and $\mathcal{R}^\perp$ its orthogonal complement, thus $\mathcal{H} = \mathcal{R} \oplus \mathcal{R}^\perp$. Any element $\vec{A}$ possesses a unique decomposition

$$\vec{A} = \vec{x} + \vec{y} \tag{5.2}$$

with $\vec{x} \in \mathcal{R}$ and $\vec{y} \in \mathcal{R}^\perp$. Since $\vec{x}$ and $\vec{y}$ are elements in orthogonal spaces, they are orthogonal, thus $(\vec{x}, \vec{y}) = 0$. The two linear operators $\mathbf{P}$ and $\mathbf{Q}$, defined by $\vec{x} = \mathbf{P}\vec{A}$ and $y = \mathbf{Q}\vec{A}$, are the associated orthogonal projection operators projecting $\vec{A}$ onto $\mathcal{R}$ and $\mathcal{R}^\perp$ respectively. The properties $\mathbf{P}^2 = \mathbf{P}$, $\mathbf{Q}^2 = \mathbf{Q} = \tilde{I} - \mathbf{P}$, $\mathbf{P} = \mathbf{P}^H$ and $\mathbf{Q} = \mathbf{Q}^H$ are well known (where $\tilde{I}$ is the identity matrix).

The problem we are faced with is: if $\vec{A} \in \mathcal{H}$ belongs to a known subspace $\mathcal{R}_2$, with only its orthogonal projection $\vec{x} = \mathbf{P}_1\vec{A}$ onto the known subspace $\mathcal{R}_1$ given, can $\vec{A}$ be reconstructed from $\vec{x}$ and the projection operators?

Let $\mathbf{P}_1$, $\mathbf{Q}_1$, $\mathbf{P}_2$ and $\mathbf{Q}_2$ denote the projection operators projecting $\vec{A}$ onto $\mathcal{R}_1$ $\mathcal{R}_1^\perp$, $\mathcal{R}_2$ $\mathcal{R}_2^\perp$, respectively. Since $\vec{A} \in \mathcal{R}_2$, $\vec{A} = \mathbf{P}_2\vec{A}$ and

$$\vec{x} = \mathbf{P}_1\vec{A} = (\tilde{I} - \mathbf{Q}_1)\vec{A} = (\tilde{I} - \mathbf{Q}_1\mathbf{P}_2)\vec{A}. \tag{5.3}$$

The vector $\vec{A}$ is uniquely determined by $\vec{x}$ if, and only if, the inverse operator $\tilde{C} = (\tilde{I} - \mathbf{Q}_1\mathbf{P}_2)^{-1}$ exists. Youla [134] showed that if either

$$\mathcal{R}_2 \cap \mathcal{R}_1^\perp = \{0\}$$

or

$$\angle(\mathcal{R}_2, \mathcal{R}_1^\perp) > 0$$

where $\cap$ is the set intersection, $\{0\}$ is the zero vector and $\angle(\mathcal{R}_2, \mathcal{R}_1^{\perp})$ is the angle between the linear subspaces $\mathcal{R}_2$ and $\mathcal{R}_1^{\perp}$; then the sequence $\{\vec{A}_k\}$ generated by

$$\vec{A}_{k+1} = \vec{x} + \mathbf{Q}_1\mathbf{P}_2\vec{A}_k \quad \text{for } k = 1 \to \infty, \quad \vec{A}_1 = \vec{x} \tag{5.4}$$

converges to $\vec{A}$ in norm and is strictly monotone. The recursive formula (5.4) is called the method of alternating orthogonal projections. The name is derived from the geometrical interpretation of the iterative process of projecting between orthogonal subspaces [106].

In the array synthesis context (and notation used in this thesis), $\vec{A}$ is the excitation vector and $\vec{B}_m$ is the space vector (the mth row of the radiation matrix $\tilde{B}$ (2.6)). Let $\vec{B}_d$ be the space vector in the desired main beam direction and $\vec{B}_1, \vec{B}_2, \ldots, \vec{B}_M$ the space vector in the directions where the radiation pattern is required to exhibit nulls. The synthesis problem in this is then: can the excitation vector $\vec{A}$ be reconstructed such that

$$\vec{B}_d\vec{A} = 1 \quad \text{and} \quad \vec{B}_m\vec{A} = 0 \quad \text{for } m = 1, 2, \ldots, M. \tag{5.5}$$

Prasad [106] conjectured an iterative algorithm for the sequence $\{\vec{A}_k\}$,

$$\vec{A}_{k+1} = \vec{B}_d + \mathbf{Q}_1(\mathbf{P}_1\mathbf{P}_2 \ldots \mathbf{P}_M)\vec{A}_k \quad \text{for } k = 1 \to \infty \tag{5.6}$$

with $\vec{A}_1 = \vec{B}_d$ and the operators (in array pattern synthesis terms)

$$\mathbf{Q}_1 = \tilde{I} - \vec{B}_d^H(\vec{B}_d\vec{B}_d^H)^{-1}\vec{B}_d \tag{5.7}$$

and

$$\mathbf{P}_m = \tilde{I} - \vec{B}_m^H(\vec{B}_m\vec{B}_m^H)^{-1}\vec{B}_m \tag{5.8}$$

Ng [132] developed a similar but simpler algorithm,

$$\vec{A}_{k+1} = \vec{B}_d + \mathbf{Q}_1\mathbf{P}_2\vec{A}_k \quad \text{for } k = 1 \to \infty \tag{5.9}$$

with $\vec{A}_1 = \vec{B}_d$. The operators $\mathbf{Q}_1$ and $\mathbf{P}_2$ are

$$\mathbf{Q}_1 = \tilde{I} - \vec{B}_d^H(\vec{B}_d\vec{B}_d^H)^{-1}\vec{B}_d \tag{5.10}$$

(the same as by Prasad) and

$$\mathbf{P}_2 = \tilde{I} - \tilde{B}^H(\tilde{B}\tilde{B}^H)^{-1}\tilde{B} \tag{5.11}$$

where $\tilde{B}$ is the radiation matrix in all the directions where nulls are desired. Ng also derived a lower boundary on the rate of convergence [133],

$$\|\xi_{k+1}\| \leq \cos^k\left[\angle(\mathcal{R}_2, \mathcal{R}_1^{\perp})\right] \|\vec{A} - \vec{B}_d\|. \tag{5.12}$$

Generally the iteration will be terminated when the error ξ_k is within design specification ξ

$$\|\vec{A} - \vec{A}_k\| = \xi_k \leq \xi \tag{5.13}$$

The method of orthogonal projections was applied, by Prasad [106] and Ng [133], to phased arrays of arbitrary array geometries, with interfering sources at fixed positions in the far field (in the null directions). They both considered only isotropic elements. The excitation vector ($\vec{A}$ in the notation above) is constructed in an appropriate "excitation subspace". The recursive algorithm starts from the known projection $\vec{B}_d$ of the excitations in a linear subspace in the main beam peak direction. The number of interference sources, or directions in which the radiation pattern level may be constrainted, must be less than the total number of elements of the array. Stated differently, the number of sets (or rows of the radiation matrix) M is restricted to a maximum of the number of array elements N ($M \leq N$). Since the sets are convex convergence to the global minimum is guaranteed.

The most severe restriction of the method is that the operators must be linear, thus the method can not be used for a general pattern synthesis as the operators associated with power synthesis are not linear. In addition, no constraints on the excitations can be implemented.

5.2.2 Successive Projections

Limitations on the type of constraints that could be imposed with alternating orthogonal projections led to improvements resulting in the method of successive projections. The method was used in field synthesis of linear arrays [135] as well as in the design of two-dimensional digital FIR filters [136]. It has also been applied to power pattern synthesis by Poulton [107, 108].

The method of successive projection is to find an intersection of a system of sets. Suppose we have a system of M sets, $\mathcal{C}_m$, $m=1,2,\ldots,M$, in a real or complex Euclidean space. Associated with each set $\mathcal{C}_m$ is a projection operator $\mathbf{P}_m$. In general, for all closed sets (convex and non-convex) we call $\vec{x} \triangleq \mathbf{P}\vec{y}$ the projection of $\vec{y}$ ($\vec{y} \notin \mathcal{C}_m$) onto $\mathcal{C}_m$ if $\vec{x} \in \mathcal{C}_m$ and if

$$\|\vec{x} - \vec{y}\| = \inf_{\vec{v} \in \mathcal{C}_1} \|\vec{y} - \vec{v}\|. \tag{5.14}$$

The projections are thus defined as

$$\mathbf{P}_m(\vec{x}, \mathcal{C}_m) = \|\vec{x} - \vec{y}\| = \inf_{\vec{v} \in \mathcal{C}_m} \|\vec{x} - \vec{v}\|. \tag{5.15}$$

In other words, the projector onto the subset $\mathcal{C} \in \mathcal{H}$ is the operator mapping a point $\vec{y} \in \mathcal{H}$ into the point $\vec{x} \in \mathcal{C} \in \mathcal{H}$, which is the nearest point of $\mathcal{C}$ to $\vec{y}$. The point $\vec{x}$ is called the projection of $\vec{y}$ onto $\mathcal{C}$. We can make the following remarks:

- a projection always exists for a closed set and

- the projection as defined in (5.15) is a unique point if $\mathcal{C}_m$ is a convex set.

The problem is then to find a common point of the system of sets $\vec{A} = \mathcal{C}_1 \in \mathcal{C}_2 \in \ldots \in \mathcal{C}_M$. The method of successive projection consist in constructing a sequence $\vec{A}_0, \vec{A}_1, \ldots, \vec{A}_k$ where $\vec{A}_0$ is arbitrary. The reconstruction algorithm is

$$\vec{A}_{k+1} = \mathbf{P}_1 \mathbf{P}_2 \ldots \mathbf{P}_M \vec{A}_k \tag{5.16}$$

or the relaxed version

$$\vec{A}_{k+1} = \mathbf{T}_1 \mathbf{T}_2 \ldots \mathbf{T}_M \vec{A}_k \tag{5.17}$$

where

$$\mathbf{T}_m = \tilde{I} + \lambda_m (\mathbf{P}_m - \tilde{I}) \quad m = 1, 2, \ldots, M \tag{5.18}$$

and $\tilde{I}$ is the identity operator and λ_m is the relaxation parameter. Provided that the following two conditions hold [134]:

1. $\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_M$ are closed convex sets with a nonempty intersection $\mathcal{C}_0 \triangleq \bigcap_{m=1}^{M} \mathcal{C}_m$

   and

2. $0 < \lambda_m < 2, \; i = 1, 2, \ldots, M$

the algorithm converges to a point in $\mathcal{C}_0$, the intersection of all the sets. The order in which the sets are selected for the successive projections is arbitrary, but in general the method of projection on to the most remote set is used, as this will increase the convergence rate.

Any number of linear inequalities of the form $(\vec{Q}, \vec{A}) \leq \alpha$ can be easily included in the method as each just forms another set $\mathcal{C}_l = \{\vec{A} : (\vec{Q}_l, \vec{A}) \leq \alpha_l\}$ (a half space) with projection of $\vec{A}$ onto the set $\mathcal{C}_l$

$$\mathbf{P}(\vec{x}, \mathcal{C}_l) = \begin{cases} \vec{A}, & \text{if } (\vec{Q}_l, \vec{A}) \leq \alpha_l \\ \vec{A} - \left[(\vec{Q}_l, \vec{A}) - \alpha_l \right] \dfrac{\vec{Q}_l}{\|\vec{Q}_l\|^2}, & \text{if } (\vec{Q}_l, \vec{A}) > \alpha_l. \end{cases} \tag{5.19}$$

When applied to antenna arrays; a set is formed by all the excitations that produce a radiation pattern of the required magnitude characteristic (a set of inequality constraints)

$$\mathcal{C}_m = \{\vec{A} : \|S_m - |\vec{B}_m \vec{A}|\| \leq \varepsilon_m\} \tag{5.20}$$

where ε_m is the toleration or the maximum allowable deviation from the desired pattern level in the (θ_m, ϕ_m)-direction, S_m is the desired pattern level in the (θ_m, ϕ_m)-direction,

$\vec{B}_m$ is the mth row of the radiation matrix in the far-field direction (θ_m, ϕ_m) and $\vec{A}$ is the excitation vector.

Any arbitrary excitation vector $\vec{A}_{m-1} \notin \mathcal{C}_m$ can be projected on $\mathcal{C}_m$, giving $\vec{A}_m \in \mathcal{C}_m$. The projections are carried out successively until the intersection is found. The order in which the sets are selected for the successive projections is arbitrary, but in generally the projection on to the most remote set (the set with largest distance between the current point and that set) is used. Elmikati and Elsohly [135] used the method of successive projection (without relaxation) to synthesise symmetrical linear arrays with real excitations. This is a field synthesis problem, thus all the sets $\mathcal{C}_m$ are convex, and converge is guaranteed.

Applying the projection definition (5.15) at any step is equivalent to finding the excitation vector that minimises [136]

$$J = \|\vec{A}_{k+1} - \vec{A}_k\|^2 + \alpha \left[(S_m - |\vec{B}_m \vec{A}_k|) - \varepsilon_m \right] \tag{5.21}$$

where α is the Lagrange multiplier, and solving for α

$$\alpha = \frac{2 \left[\|S_m - |\vec{B}_m \vec{A}_k|\| - \varepsilon_m \right]}{\|\vec{B}_m\|^2}. \tag{5.22}$$

Abo-Taleb and Fahmy [136] used these results to design two-dimensional digital filters. In the FIR filter scenario all the sets $\mathcal{C}_m$ are convex, thus the method will converge the global minimum from any arbitrary starting point.

However, in power synthesis the sets are not convex, and convergence is not guaranteed. Levi and Stark [105] showed that in many cases good convergence can be obtained with non-convex sets, but a starting point close to the intersection is prerequisite. The method of successive projections was applied to power pattern synthesis by Poulton [107, 108], using relaxation and the Lagrange multiplier, with good results.

5.2.3 Generalised Projections

As mentioned previously, the sets involved in power pattern synthesis are non-convex. More recently the generalised projections (also called alternating projections) technique has been used as the basis for a power synthesis method [110, 137, 138] which take into account the non-convexness of the sets involved. The generalised projection method allows constraints not only on the radiation pattern but also on the excitations (eg. dynamic range or smoothness), something that is most advantageous when having to deal with mutual coupling effects. The method searches for the intersection of two sets by iteratively projecting between the sets.

The two sets are, $\mathcal{C}_1$ the set of all radiation patters possible with the considered array geometry and $\mathcal{C}_2$ a set of real non-negative functions that satisfy the constraints on the radiation pattern, with projections P_1 and P_2 associated with the sets respectively. The

constraints on the radiation power pattern can be expressed as a "mask" giving the upper S_U and lower S_L values in each pattern direction. We are searching for the excitation that will minimise

$$\xi = \|S - |F|\|^2 \tag{5.23}$$

where S is the function describing the desired radiation pattern. This is done by applying the reconstruction algorithm (5.16) for two sets

$$\vec{F}_{k+1} = \mathbf{P}_1\mathbf{P}_2\vec{F}_k. \tag{5.24}$$

Franceschetti et al. [137] proposed the use of the Fast Fourier transform (FFT) of order M to facilitate efficient computation. The projections then are

$$\mathbf{P}_1 = (\text{FFT})\,\tau_A\,(\text{FFT})^{-1} \tag{5.25a}$$

$$\mathbf{P}_2 = \tau_F \tag{5.25b}$$

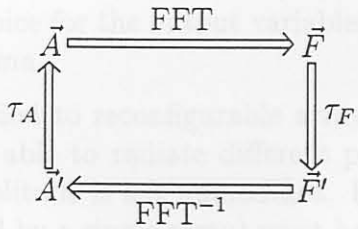
where

$$\tau_A : a_n = \begin{cases} a'_n & \text{if } 0 \leq n \leq N \\ 0 & \text{if } n < 0 \text{ or } n \geq N+1. \end{cases} \tag{5.26}$$

and

$$\tau_F : f'_m = \begin{cases} \frac{f_m}{|f_m|} S_{Um} & \text{if } |f_m| > S_{Um} \\ f_m & \text{if } S_{Lm} \leq |f_m| \leq S_{Um} \\ \frac{f_m}{|f_m|} S_{Lm} & \text{if } |f_m| < S_{Lm}. \end{cases} \tag{5.27}$$

The mapping of the radiation pattern into the mask is done in such a way that the phase is unchanged, only the magnitude changes. This is equivalent to the Gerchberg-Saxton [130] algorithm widely used in image reconstruction:



To incorporate constraints on the excitation, set $\mathcal{C}_1$ can be reduced to a set $\mathcal{C}'_1$ of all radiation patters possible with the array geometry and excitation constraints, $\mathcal{C}'_1 \subset \mathcal{C}_1$. Excitation constraints, amplitude and phase, can be imposed by adding an additional mapping τ'_C and τ''_C respectively [109],

$$\tau'_C : a_n = \begin{cases} \frac{a'_n}{|a'_n|} A_{max} & \text{if } |a'_n| > A_{max} \\ a'_n & \text{if } A_{min} \leq |a'_n| \leq A_{max} \\ \frac{a'_n}{|a'_n|} A_{min} & \text{if } |a'_n| < A_{min} \end{cases} \tag{5.28}$$

where A_{max} and A_{min} are the maximum and minimum allowable magnitude of the excitations; and

$$\tau_C'' : a_n' = \begin{cases} |a_n''| \cos\left(\angle a_n'' - \alpha_{max}\right) e^{\alpha_{max}} & \text{if } \angle a_n'' > \alpha_{max} \\ a_n'' & \text{if } \alpha_{min} \leq \angle a_n'' \leq \alpha_{max} \\ |a_n''| \cos\left(\angle a_n'' - \alpha_{min}\right) e^{\alpha_{min}} & \text{if } \angle a_n'' > \alpha_{min} \end{cases} \tag{5.29}$$

where α_{max} and α_{min} are the maximum and minimum allowable phase of the excitations. The projection onto $\mathcal{C}_1'$ is then

$$\mathbf{P}_1' = (\text{FFT})\, \tau_C' \tau_C'' \tau_A\, (\text{FFT})^{-1}. \tag{5.30}$$

The order of the operators $\tau_C' \tau_C''$ are important, exchanging them will not result in a projection onto $\mathcal{C}_1'$.

However, since the sets are non-convex, convergence (even if an intersection exists) is not guaranteed to a global minimum. One may end up in a local minima and not be able to get out, this is called a trap. Thus, a proper selection of the starting point is of the utmost importance, as a good starting point (or warm start) will be close enough to the final solution to avoid falling into a trap. Another potential problem is tunnels which lead to extremely slow convergence. Traps and tunnels are discussed in detail in Section 5.3. The choice of a good starting point should include the constraints on the excitations, for example if there is a constraint on the dynamic range of the excitations a good starting point may be obtained by a phase only synthesis where the amplitude of each element is fixed.

Use of the fast Fourier transform and the inverse fast Fourier transform (although computationally very fast) limits the application of the method to uniformly spaced linear and planar arrays. The method was extended to the synthesis of non-uniformly spaced arrays [72]. By using singular value decomposition as part of the projection onto $\mathcal{C}_1$ the method has been extended to conformal array synthesis [112]. Bucci et al. [113] also adopted a non-standard choice for the output variables in the generalised projection method and to avoid local minima.

The method was also extended to reconfigurable arrays by phase-only control [139, 140], that is an array which is able to radiate different patterns by changing only the excitation phases while the amplitude is left unmodified. For this type of reconfigurable array Q array patterns (radiated by a single array) must be synthesised at the same time in order to achieve the best common amplitude distribution. The first set $\boldsymbol{\mathcal{C}_1}$ is the set of all the Q-tuples of array factors that belong to the Cartesian product

$$\boldsymbol{\mathcal{C}_1} = \underbrace{\mathcal{C}_1^1 \times \mathcal{C}_1^2 \times \cdots \times \mathcal{C}_1^Q}_{Q\text{times}} \tag{5.31}$$

and satisfy the additional requirement

$$|a_{n,1}| = |a_{n,2}| = \cdots = |a_{n,Q}| \quad \text{for } n = 1, 2, \ldots, N. \tag{5.32}$$

The second set $\mathcal{C}_2$ is the Cartesian product of all the radiation pattern masks

$$\mathcal{C}_2 = \underbrace{\mathcal{C}_2^1 \times \mathcal{C}_2^2 \times \cdots \times \mathcal{C}_2^Q}_{Q\text{times}}. \tag{5.33}$$

The intersection $\mathcal{C}_1 \cap \mathcal{C}_2$ is the solution to this synthesis problem. Bucci et al. [140] derived the necessary projections.

5.3 The Synthesis Problem as an Intersection of Two Sets

The sets involved in the array synthesis problem are usually not convex sets. In general, very little can be said about the reconstruction algorithm when the projections are not onto convex sets. However, when only two sets are involved a number of interesting conclusions can be made. The restriction of two sets is not so serious, as two or more sets can be combined to form single more complex sets for which it is still possible to derive a projection operator [105].

In this section the synthesis of an arbitrary array will be viewed as the search for the intersection of two excitation sets. Unlike the previous formulations which all consider sets in the "pattern space" this formulation considers sets in the "excitation space" as well as sets in the "pattern space". In the "excitation space" the one set is the set of all possible excitations satisfying the excitation constraints; and the other set is the set of all excitations that produce radiation patterns that satisfy the radiation pattern requirements. In the "pattern space" the one set is the set of all radiation patterns possible with the given geometry; and the other set is the set of radiation patterns satisfying the pattern constraints. Both these formulation allow for both field and power pattern synthesis.

All the equations in this section are for the general two-dimensional case of an array of arbitrary sources at arbitrary positions. The two dimensions refer to the pattern angles (θ, ϕ). These expressions can be simplified for the less general cases (eg. linear or planar arrays). The simplification will be given at the relevant example in the Section 5.8. All the necessary equations defined or derived previously will be repeated for convenience.

5.3.1 The Transformation Operators Between the Excitation and Pattern Spaces

To transform from the excitation space to the pattern space will be referred to as the forward operator; and the transformation from the radiation pattern space to the excitation space will be referred to as the backward operator.

The radiation pattern has the form of a discrete Fourier transformation; the Fourier transformation can be used to calculate the radiation pattern from an excitation and the

inverse Fourier transformation to calculate the excitation from a radiation pattern. The use of the Fourier and inverse Fourier transformation pair is proposed in references [137, 110]. Implementing the Fourier transformation by using the fast Fourier transformation (FFT) is computationally very fast. However, Fourier transformations can only be applied to equi-spaced linear and planar arrays and can not take element patterns into account. This limitation makes Fourier transformation useless for conformal array synthesis.

The Forward Operator

The radiation pattern of an arbitrary array in the direction (θ, ϕ) is (as already defined in equation (2.2)),

$$F(\theta,\phi) = \sum_{n=1}^{N} a_n \, E_n(\theta,\phi) \, e^{\mathrm{j}k(x_n \sin\theta\cos\phi + y_n \sin\theta\sin\phi + z_n\cos\theta)} \tag{5.34}$$

where (x_n, y_n, z_n) is the position; $E_n(\theta,\phi)$ is the element pattern and a_n is the relative constrainted complex excitation of the nth array element; and N is the total number of elements in the array.

The radiation pattern in M different far-field directions can be written in vector notation

$$\vec{F} = \tilde{B}\,\vec{A} \tag{5.35}$$

where $\vec{A} = [a_1, a_2, \ldots, a_N]^T$ is the excitation vector, $\vec{F} = [F_1, F_2, \ldots, F_M]^T$ is the pattern vector and $\tilde{B}$ is the radiation matrix with matrix elements

$$b_{mn} = E_n(\theta_m,\phi_m) \, e^{\mathrm{j}k(x_n \sin\theta_m\cos\phi_m + y_n \sin\theta_m\sin\phi_m + z_n\cos\theta_m)}.$$

The mth row vector of the radiation matrix $\tilde{B}$ is $\vec{B}_m$, thus $F_m = \vec{B}_m\vec{A}$ is the value of the two dimensional radiation pattern in direction (θ_m, ϕ_m). The elements of $\tilde{B}$ are a function of the array geometry (which is fixed for a particular synthesis problem) and the selected far-field directions.

The columns of the rectangular matrix $\tilde{B}$ are linearly independent. The vector $\vec{F} = \tilde{B}\vec{A}$ is a vector in the space spanned by the columns of $\tilde{B}$.

The Backward Operator

The backward operator is needed to calculate an excitation from a function in the pattern space and is the inverse of the radiation matrix. There are two problems in determining the inverse of the radiation matrix:

1. The perturbed or constrainted radiation pattern is not in the space spanned by the columns of $\tilde{B}$; thus there will not be an excitation that can produce the perturbed radiation pattern exactly.

2. The radiation matrix $\tilde{B}$ is generally over determined (more equations than unknowns).

A number of possibilities for the pseudo-inverse of the radiation pattern exist. The pseudo-inverse of matrix $\tilde{B}$ can be determined using least squares:

$$\tilde{C} = \left(\tilde{B}^H \tilde{B}\right)^{-1} \tilde{B}^H. \tag{5.36}$$

The method of least squares, also referred to as maximum likelihood, minimises $\|\vec{F} - \tilde{B}\vec{A}\|$. Matrix $\tilde{C}$ is also called the *general inverse* or the *general reciprocal* of matrix $\tilde{B}$. The maximum likelihood inverse can be used for arbitrary arrays of arbitrary radiators in a three-dimensional space. One drawback of least squares is that all the equations (the far-field in a particular direction) have equal importance. This means that a point outside the radiation pattern mask that must move to fit in the mask is just as important as a point that is already in the radiation pattern mask (and for which some movement can be allowed as long as it stays in the mask). Singular value decomposition, proposed by Mazzarella and Panariello [112], will yield the same result as least squares, as it minimises the same function.

Weighted least squares will overcome the shortcomings of least squares. The pseudo-inverse of the radiation matrix using weighted least squares is:

$$\tilde{C} = \left(\tilde{B}^H \tilde{W} \tilde{B}\right)^{-1} \tilde{B}^H \tilde{W} \tag{5.37}$$

where $\tilde{W}$ is a diagonal weighting matrix. The inverse minimises $\|\tilde{W}(\vec{F} - \tilde{B}\vec{A})\|$; and is unique. Weighted least squares allows certain directions (constraints) to be emphasised more than others, and can be used to synthesise arbitrary arrays of arbitrary radiators in a three-dimensional space. The advantage and disadvantage of the weighted least squares pseudo-inverse is investigated in Section 5.6.1

5.3.2 The Sets Associated with the Synthesis Problem

In the synthesis problem we can identify four different sets, two sets in the "pattern space" and two sets in the "excitation space":

1. The set of radiation patterns satisfying the pattern constraints.
2. The set of excitations satisfying the excitation constraints.
3. The set of all radiation patterns possible with the given array geometry and possible excitations.
4. The set of excitations that produce radiation patterns within the pattern requirements.

Let us first define the sets and associated projectors.

Radiation pattern sets:

Pattern space $\mathcal{P}$ is a complex Euclidean space with subsets $\mathcal{C}_f$ and $\mathcal{C}_{f'}$. The set of all radiation patterns possible with the array geometry and any allowable excitation is

$$\mathcal{C}_f = \left\{ \mathcal{F} : F(\theta,\phi) = \sum_{n=1}^{N} a_n \, E_n(\theta,\phi) \, e^{\mathrm{j}k(x_n \sin\theta\cos\phi + y_n \sin\theta\sin\phi + z_n\cos\theta)} \right\} \tag{5.38}$$

with $a_n \in \mathcal{C}_a$ that will be defined in equation (5.43). In the matrix notation defined in Section 5.3.1 the set is

$$\mathcal{C}_f = \left\{ \mathcal{F} : \vec{F} = \tilde{B}\vec{A} \right\}. \tag{5.39}$$

$\mathcal{C}_{f'}$ is the set square integratable functions (real, non-negative for power pattern synthesis) that fulfil the constraints on the radiation pattern, which is generally defined by a power pattern mask with $S_L(\theta,\phi)$ the lower limit and $S_U(\theta,\phi)$ the upper limit,

$$\mathcal{C}_{f'} = \left\{ \mathcal{F}' : S_L(\theta,\phi) \leq |F'(\theta,\phi)|^2 \leq S_U(\theta,\phi) \right\} \tag{5.40}$$

or for a discrete far-field

$$\mathcal{C}_{f'} = \left\{ \mathcal{F}' : S_{Lm} \leq |f'_m|^2 \leq S_{Um} \right\}. \tag{5.41}$$

It is important to note that $\mathcal{C}_{f'}$ is a non-convex set.

Excitation sets:

Excitation space $\mathcal{E}$ is a complex Euclidean space with subsets $\mathcal{C}_a$ and $\mathcal{C}_{a'}$. Let $\mathcal{C}_{a'}$ be the set of all possible excitations that will produce an acceptable radiation pattern

$$\mathcal{C}_{a'} = \left\{ \mathcal{A}' \leftrightarrow \mathcal{F}' : S_L \leq |F'|^2 \leq S_U \right\} \tag{5.42}$$

with S_U and S_L the upper and lower limits of the required radiation pattern.

The second set in the excitation space is the set of all the excitations satisfying the excitation constraints. These constraints can be on the magnitude as well as the phase of the excitations

$$\mathcal{C}_a = \{ \mathcal{A} : A_{Ln} \leq |a_n| \leq A_{Un};\ \alpha_{Ln} \leq \angle a_n \leq \alpha_{Un} \} \tag{5.43}$$

where $\angle$ denotes the argument of a complex number. A_{Un} denotes the upper limit and A_{Ln} the lower limit of the excitation amplitude of the nth element excitation; and α_{Un} and α_{Ln} the upper and lower limits of the phase of the nth element excitation. Depending on the excitation constraints $\mathcal{C}_a$ may or may not be convex.

5.3.3 The Projections between Related Sets

A projection is defined as the mapping of a point outside a set to the point just inside the set nearest to the original point (5.15). However, the sets involved in power pattern synthesis are not convex; the following remarks can be made regarding non-convex sets [105]:

- There may be a set of points that satisfy the definition of the projection (5.15);
- The closeness of a set is a sufficient condition for the existence of a projection onto the set.

Since some of the sets, as defined in the previous section are non-convex, a procedure for uniquely choosing one of the possible points is needed, and is usually achieved through satisfying another condition. This eliminates the ambiguity that would otherwise result from non-singleton projection points.

The Projector from $\mathcal{C}_f$ to $\mathcal{C}_{f'}$

A number of radiation field pattern have the same power pattern; apart from the trivial associations $|F| = |-F| = |F^*|$, for every complex zero z_0 of F

$$|F(u)| = \left| [F(u)] \left[\frac{z_0^* e^{ju} - 1}{e^{ju - z_0}} \right] \right|.$$

As the set is non-convex the most appropriate projection must be selected; for power pattern synthesis the phase of the radiated field must be preserved. The mapping of the radiation pattern to fit in the prescribed magnitude while preserving its phase distribution is

$$\tau_F : f'_m = \begin{cases} \frac{f_m}{|f_m|}\sqrt{S_{Um}} & \text{if } |f_m|^2 > S_{Um} \\ f_m & \text{if } S_{Lm} \leq |f_m|^2 \leq S_{Um} \\ \frac{f_m}{|f_m|}\sqrt{S_{Lm}} & \text{if } |f_m|^2 < S_{Lm}. \end{cases} \tag{5.44}$$

The projector is

$$\mathrm{P}_F = \tau_F \tag{5.45}$$

and the unique projection from any point $\vec{F}$ in pattern space $\mathcal{P}$ onto the set of accepable radiation patterns $\mathcal{C}_{f'}$ is

$$\vec{F}' \in \mathcal{C}_{f'} : \vec{F}' = \tau_F \vec{F} = \mathrm{P}_F \vec{F} \tag{5.46}$$

Note that constraining the radiation pattern to have a prescribed magnitude is not equivalent to projecting onto a convex set.

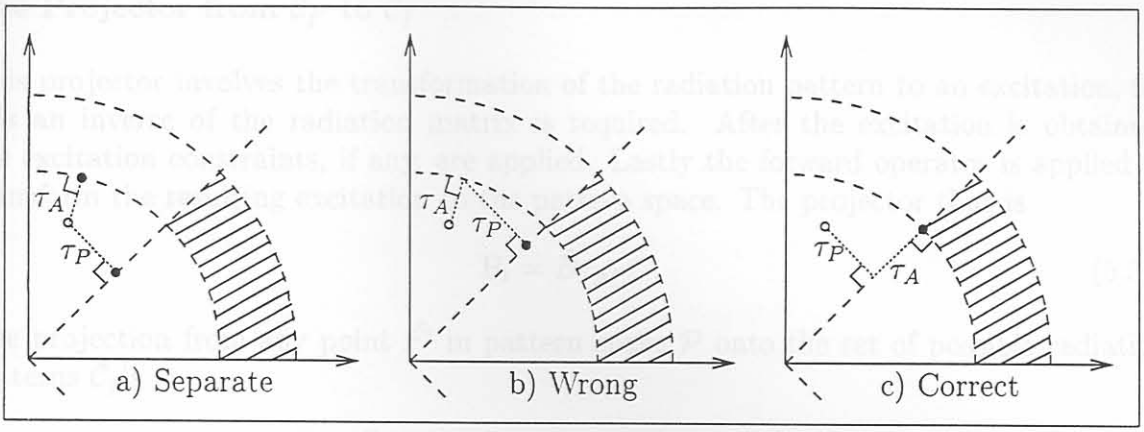


Figure 5.1: Application of the excitation projection: a) amplitude and phase separately, b) combined but wrong as the result is not in the defined set (the shaded region) and c) correct as the result is just inside the set.

The Projector from $\mathcal{C}_{a'}$ to $\mathcal{C}_a$

This projection operator is in essence the application of the excitation constraints. Excitation constraints can be imposed by mapping the excitation phase angle while keeping the excitation amplitude fixed

$$\tau_P : a_n = \begin{cases} |a'_n| \cos\left(\angle a'_n - \alpha_{Un}\right) e^{\alpha_{Un}} & \text{if } \angle a'_n > \alpha_{Un} \\ a'_n & \text{if } \alpha_{Ln} \leq \angle a'_n \leq \alpha_{Un} \\ |a'_n| \cos\left(\angle a'_n - \alpha_{Ln}\right) e^{\alpha_{Ln}} & \text{if } \angle a'_n > \alpha_{Ln} \end{cases} \tag{5.47}$$

and mapping the excitation amplitude while keeping the excitation phase angle fixed,

$$\tau_A : a_n = \begin{cases} \frac{a'_n}{|a'_n|} R_{Un} & \text{if } |a'_n| > R_{Un} \\ a'_n & \text{if } R_{Ln} \leq |a'_n| \leq R_{Un} \\ \frac{a'_n}{|a'_n|} R_{Ln} & \text{if } |a'_n| < R_{Ln}. \end{cases} \tag{5.48}$$

for each excitation. The projector is the combined phase and magnitude mapping

$$\mathbf{P}_A = \tau_{AP} = \tau_A \tau_P. \tag{5.49}$$

It is important to note that the order of the projections can not be interchanged otherwise the resulting projection will not be on $\mathcal{C}_a$. This is illustrated in Figure 5.1. Both the amplitude and phase are constrainted; the shaded region in each of the figures is the set defined by the constraints. The open circle indicate the position of a'_n and the filled circle the position of a_n. Figure 5.1a) show the mapping separately. If the mapping are combined in the wrong order the "projection" is not on to set $\mathcal{C}_a$, as shown in Figure 5.1b). The correct order of the mappings to form the projection is displayed in Figure 5.1b), a_n is in $\mathcal{C}_a$. The correct projection, projecting any point $\vec{A}'$ in excitation space $\mathcal{E}$ onto the set of acceptable excitations $\mathcal{C}_a$, is

$$\vec{A} \in \mathcal{C}_a : \vec{A} = \tau_A \tau_P \vec{A}' = \mathbf{P}_A \vec{A}'. \tag{5.50}$$

The Projector from $\mathcal{C}_{f'}$ to $\mathcal{C}_f$

This projector involves the transformation of the radiation pattern to an excitation; for this an inverse of the radiation matrix is required. After the excitation is obtained, the excitation constraints, if any, are applied. Lastly the forward operator is applied to transform the resulting excitation to the pattern space. The projector thus is

$$\mathrm{P}_1 = \tilde{B}\tau_{AP}\tilde{C}. \tag{5.51}$$

The projection from any point $\vec{F}'$ in pattern space $\mathcal{P}$ onto the set of possible radiation patterns $\mathcal{C}_f$is

$$\vec{F} \in \mathcal{C}_f : \vec{F} = \tilde{B}\tau_{AP}\tilde{C}\vec{F}' = \mathrm{P}_1\vec{F}'. \tag{5.52}$$

Depending on the excitation constraints, this may not be a projection onto a convex set.

The Projector from $\mathcal{C}_a$ to $\mathcal{C}_{a'}$

Using the forward operator the excitation is transformed to the pattern space, where the radiation pattern constraints are applied. The constrainted radiation pattern is the transformed back the the excitation space by means of the back operator.

$$\mathrm{P}_2 = \tilde{C}\tau_F\tilde{B}. \tag{5.53}$$

The projection, projecting any point $\vec{A}$ in excitation space $\mathcal{E}$ onto the set of possible excitations $\mathcal{C}_a$, is

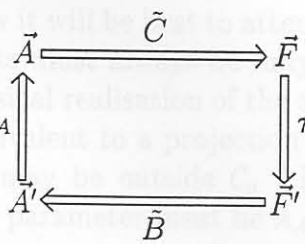
$$\vec{A}' \in \mathcal{C}_{a'} : \vec{A}' = \tilde{C}\tau_F\tilde{B}\vec{A} = \mathrm{P}_2\vec{A}. \tag{5.54}$$

This is not a projection onto a convex set.

If the radiation pattern obtained with the forward operator satisfy the radiation constraints (if τ_F in a unit operator) then $\vec{A}' = \vec{A}$.

5.3.4 Synthesis Algorithm

The synthesis problem is solved when the simultaneous intersection of the sets in both the excitation and pattern spaces is found. The synthesis problem may be presented as a modified Gerchberg-Saxton algorithm:

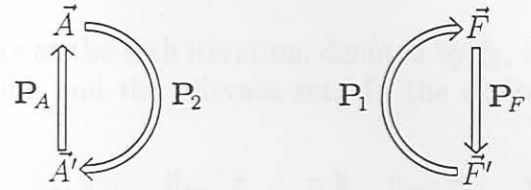


The synthesis problem may be attempted in the excitation space or the pattern space:

$$\begin{aligned} \text{Excitation Space:} \quad & \vec{A}_{k+1} = \mathbf{P}_A \mathbf{P}_2 \vec{A}_k \\ \text{Pattern Space:} \quad & \vec{F}_{k+1} = \mathbf{P}_1 \mathbf{P}_F \vec{F}_k \end{aligned} \tag{5.55}$$

where $\vec{A}_k$ is the estimate of $\vec{A}$ at the kth iteration of the synthesis; and $\vec{F}'_k$ is the estimate of $\vec{F}'$ at the kth iteration of the synthesis. The reconstruction algorithms then are:

Excitation space: Pattern space:



The columns of the rectangular radiation matrix $\tilde{B}$ are linearly independent. The vector $\vec{F} = \tilde{B}\vec{A}$ is a vector in the space spanned by the columns of $\tilde{B}$. However, the perturbed vector $\vec{F}' = \tau_F \vec{F}$ is not in the space spanned by the columns of $\tilde{B}$. As a result the excitations obtained with $\vec{A}' = \tilde{C}\vec{F}'$ will not yield $\vec{F}'$ exactly. This is due to the fact that $\tilde{B}\tilde{C} \neq \tilde{C}\tilde{B}$. Since the projectors are in general not unitary, the projectors $\mathbf{P}_1$ and $\mathbf{P}_2$ are in general not metric projectors.

5.3.5 Relaxation

Relaxation can be used as a convergence accelerator. The relaxed version of (5.55) is

$$\begin{aligned} \text{Excitation Space:} \quad & \vec{A}_{k+1} = \mathbf{T}_A \mathbf{T}_2 \vec{A}_k \\ \text{Pattern Space:} \quad & \vec{F}_{k+1} = \mathbf{T}_1 \mathbf{T}_F \vec{F}_k \end{aligned} \tag{5.56}$$

where

$$\mathbf{T}_i = 1 + \lambda_i(\mathbf{P}_i - 1) \quad i = 1, 2, A, F \tag{5.57}$$

with λ_i the relaxation parameters.

The synthesis can be executed on either the radiation pattern space or the excitation space. If relaxation is applied in the pattern space a pattern may be obtained that does not have an excitation due to the non-convex sets involved. On the other hand if relaxation is applied in the excitation space a physical realizable excitation will always be obtained. From this point of view it will be best to attempt the synthesis in the excitation space. The excitation constraints must always be fully applied as these constraints are generally used to ensure the physical realisation of the array. Since the application of the excitation constraints is not equivalent to a projection onto a convex set, the excitation $\vec{A}_{k+1}$ obtained with relaxation may be outside $\mathcal{C}_a$ (thus not satisfying the excitation constraints); thus the relaxation parameter msut be $\lambda_A = 1$.

Thus the synthesis algorithm used in this thesis is

$$\vec{A}_{k+1} = \mathrm{P}_A \mathrm{T}_2 \vec{A}_k \quad \text{with } \mathrm{T}_2 = 1 + \lambda_2(\mathrm{P}_2 - 1). \tag{5.58}$$

Since the excitation constraints are enforced (projection on a non-convex set), $\lambda_A = 1$, the current excitation will be realizable.

5.3.6 Performance Evaluation

The performance measure at the k-th iteration, denoted by ξ_k, is the sum of the distances between the k-th iteration and the relevant set. In the excitation space the summed-distance error is

$$\xi_k = \xi(\vec{A}_k) = \left\|\mathrm{P}_A \vec{A}_k - \vec{A}_k\right\| + \left\|\mathrm{P}_2 \vec{A}_k - \vec{A}_k\right\| \tag{5.59}$$

and in the pattern space

$$\xi_k = \xi(\vec{F}_k) = \left\|\mathrm{P}_1 \vec{F}_k - \vec{F}_k\right\| + \left\|\mathrm{P}_F \vec{F}_k - \vec{F}_k\right\|. \tag{5.60}$$

Although the projectors P_1 and P_2 are both idempotent ($P^2 = P$), they are in general not unitary; and thus not metric projectors. The error reduction property

$$\xi_{k+1} \leq \xi_k. \tag{5.61}$$

is not ensured for the recursion given in equation (5.55) or its relaxed version (5.56). This does not destroy the possibility of using the proposed algorithm.

The synthesis problem is solved when the simultaneous intersection of the sets in both the excitation and pattern spaces is found. Let us then define the performance measure as the summed-distance in both spaces

$$\xi_k = \left\|\mathrm{P}_F \vec{F}_k - \vec{F}_k\right\| + \left\|\mathrm{P}_1 \vec{F}_k - \vec{F}_k\right\| + \left\|\mathrm{P}_A \vec{A}_k - \vec{A}_k\right\| + \left\|\mathrm{P}_2 \vec{A}_k - \vec{A}_k\right\| \tag{5.62}$$

In the previous section we argued that it would be best to attempt synthesis in the excitation space. Substituting the projector and keeping in mind that in the k-th iteration $\vec{F}_k = \tilde{B}\tau_A \vec{A}_k$

$$\xi_k = \left\|\tau_F \tilde{B}\tau_A \vec{A}_k - \tilde{B}\tau_A \vec{A}_k\right\| + \left\|\tau_A \vec{A}_k - \vec{A}_k\right\| + \left\|\tilde{C}\tau_F \tilde{B}\vec{A}_k - \vec{A}_k\right\|. \tag{5.63}$$

The second term in (5.62) is zero since $\tilde{C}\tilde{B} = \tilde{I}$. If $\lambda_A = 1$, as previously motivated, then the performance measure reduce to

$$\begin{aligned} \xi_k &= \left\|\tau_F \tilde{B}\vec{A}_k - \tilde{B}\vec{A}_k\right\| + \left\|\tilde{C}\tau_F \tilde{B}\vec{A}_k - \vec{A}_k\right\| \\ &= \|\vec{F}'_k - \vec{F}_k\| + \|\vec{A}'_k - \vec{A}_k\| \end{aligned} \tag{5.64}$$

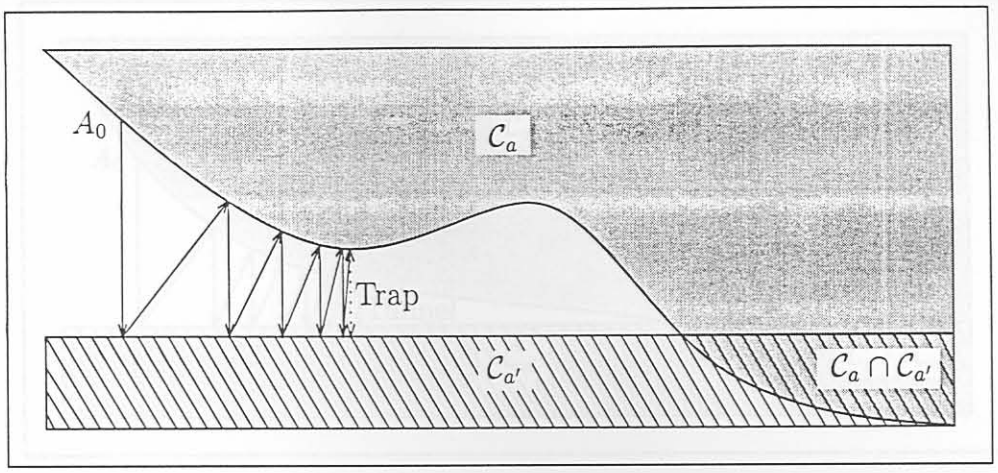


Figure 5.2: Illustrating traps, starting from $\vec{A}_0$ the sequence $\{\vec{A}_k\}$ converges to the trap, instead of the true solution belonging to $\mathcal{C}_a \cap \mathcal{C}_{a'}$

since $\tau_A \vec{A}_k = \vec{A}_k$. If the first term in equation (5.64) is zero the second term will also be zero since $\vec{A}'_k = \tilde{C}\vec{F}'_k = \tilde{C}\vec{F}_k = \tilde{C}\tilde{B}\vec{A}_k = \vec{A}_k$. This can also be deduced from the definitions of the sets, $\mathcal{C}_f$ is the set of all radiation patterns obtainable with all the allowable excitations. Thus, from an engineering point of view it is only important to know how well the current excitation's radiation pattern meets the pattern requirements.

$$\xi_k = \xi(\vec{A}_k) = \left\| \mathbf{P}_F \tilde{B}\vec{A}_k - \tilde{B}\vec{A}_k \right\| = \left\| \mathbf{P}_F \vec{F}_{k+1} - \vec{F}_{k+1} \right\|. \tag{5.65}$$

5.3.7 Traps and Tunnels

The synthesis problem is finding the intersection of the specified sets. However, since the sets are non-convex, convergence to a global maximum, even if an intersection between the relevant sets exists, can not be guaranteed. The reconstruction may end up in a local minima and not be able to proceed to the global minimum, this is called a trap. Mathematically the condition for a trap is when the error does not decrease from iteration to iteration and is larger than zero,

$$\xi_{k+1} = \xi_k > 0 \tag{5.66}$$

The condition can also be stated as

$$\begin{aligned} \text{Excitation Space:} \quad & \mathbf{P}_A \mathbf{P}_2 \vec{A}_k = \vec{A}_k, \quad \vec{A}_k \neq \vec{A} \\ \text{Pattern Space:} \quad & \mathbf{P}_1 \mathbf{F}_F \vec{F}_k = \vec{F}_k, \quad \vec{F}_k \neq \vec{F}. \end{aligned} \tag{5.67}$$

A trap is graphically presented in Fig. 5.2.

Another potential problem is tunnels which lead to extremely slow convergence. A tunnel is when the reduction in error from iteration to iteration is very small,

$$\xi_{k+1} \leq \xi_k, \quad \text{but} \quad \xi_{k+1} \approx \xi_k. \tag{5.68}$$

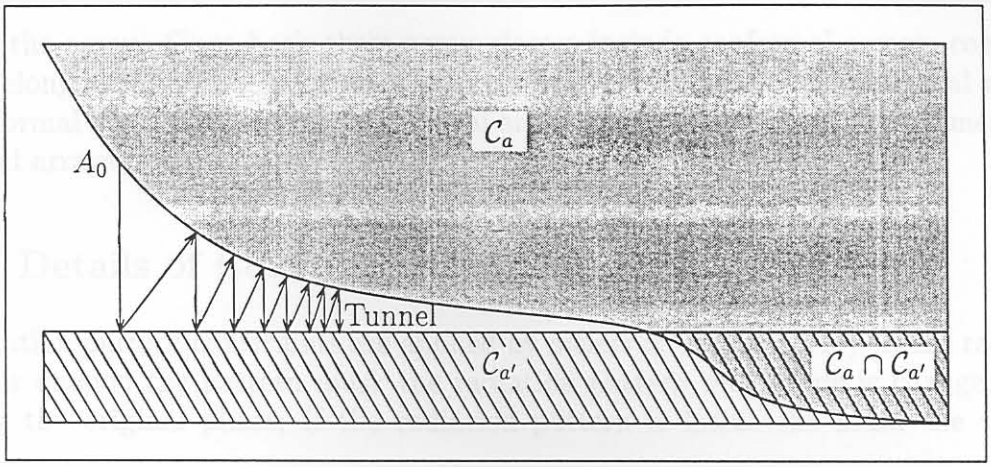


Figure 5.3: Illustrating tunnels, starting from $\vec{A}_0$ the sequence $\{\vec{A}_k\}$ converges to the true solution belonging to $C_a \cap C_{a'}$ through a long tunnel.

The effect of a tunnel is illustrated in Fig. 5.3. Relaxation, discussed in the previous section, can be used to alleviate this problem.

As stated earlier, a proper selection of the starting point is of the utmost importance, as a good starting point will be close enough to the final solution to avoid falling into a trap. Selecting a good starting point is not simple, the selection should take into account all the constraints. Various methods have been investigated, as discussed in Section 5.5.

5.4 Implementation Detail

In this section the computer program implementation of the intersection of sets synthesis technique will be discussed in some detail. However, before we embark on this discussion we need to define some terms.

Arrays with an array pattern of only one angular variable (say ϕ) will be referred to as one dimensional pattern arrays, or simply "one dimensional arrays". This class of arrays includes all linear arrays as well as conformal arrays where we are only interested in the pattern in the plane of the array.

Two dimensional pattern arrays, or simply "two dimensional arrays", are arrays with radiation patterns with two angular variables, (θ, ϕ). This class of arrays will include planar arrays where we are interested in a pattern in half-space; and conformal arrays where the radiation patterns are functions of both (θ, ϕ).

Although some arrays may seem to be a two dimensional array at first glance due to the array geometry, one has to consider the radiation pattern in order to classify the array. If the radiation pattern is of interest only in one plane, or dependent on only one angular variable, the array should be classified as a one dimensional array. An example of such an one dimensional array is an circular array which is used to scan 360° in the