Chapter 4

The Synthesis of Planar Array Difference Distributions

4.1 Introduction

The Zolotarev polynomial distribution [42, 44] or its modification [45, 46] can be used for the synthesis of linear arrays with optimum difference pattern performance. The Cheng-Tseng method [81], though powerful in the synthesis of planar sum patterns, cannot be extended to planar difference patterns. While some spreading-out procedure can be used with linear array techniques for the synthesis of planar arrays [127], this spreading-out relies on a certain amount of experienced guesswork which, while relatively straightforward for sum pattern synthesis, is not at all simple for difference patterns. The synthesis method proposed here uses the transformation based technique discussed in the previous chapter as part of the method. The sum pattern forthcoming from the transformation based synthesis is multiplied by a difference pattern to add the appropriate null loci to the sum pattern; thus the proposed synthesis method relies on the convolution synthesis method as one of its steps. The convolution synthesis method is discussed in Section 2.7.3.

The planar array difference pattern is an odd function about the difference axis. The zero at the difference null can be factored out (let us call this the *factored difference pattern*) from the planar array difference pattern to leave a *factored sum pattern* (an even function). Thus the planar array difference pattern can be written as the product of the factored difference pattern f_d and the factored sum pattern f_s, while the excitation of the final planar array will be the convolution of the factored difference array excitation and the factored sum excitation. The array giving the factored sum pattern we will call the *interim sum array*, while that giving the factored difference pattern will be called the *interim difference array*.

The essential idea of the synthesis method is as follows: a set of excitations for the interim sum planar array is obtained using the transformation based synthesis method.

This is then convolved with the excitations of the interim difference array, to give the final planar array difference excitations.

4.2 The Synthesis Procedure Step by Step

For a specified planar array geometry, the procedure involves a number of steps:

1. Determine the best geometry for the interim difference array. There is no clearcut way to choose the geometry of the interim difference array, but it must be kept as small as possible while ensuring that the geometry of the interim sum array is such that it can be synthesised with the transformation based synthesis technique. It will typically be in the shape of the array unit cell. Once the geometry of the interim difference array is set, the interim sum array geometry is obtained by a de-convolution of the planar array and interim difference array geometries.

2. Determine the geometry of the interim sum array by de-convolving the interim difference array from the final planar array.

3. Decide on the number of transformation coefficients to use, and determine which of these must be set to zero to obtain the correct array lattice and boundary.

4. Synthesise the difference distribution of the archetypal linear array. The number of elements depends on the number of elements needed for the prototype linear array and the size of the interim difference array. The prototype linear array size in turn depends on the number of transformation coefficients and the size of the interim sum array.

5. Factor out the zero(s) corresponding to the zero(s) of the factored difference pattern from the archetypal linear array factor to give the prototype linear array factor and excitation.

6. Calculate the transformation coefficients. The design requirements can be minimum beamwidth in all cuts, maximum directivity or maximum bore sight slope.

7. Compute the interim sum array using the transformation based synthesis method. The prototype linear array and the transformation coefficients are obtained in the previous steps.

8. The interim difference array excitations are selected in such a way that the zeros are in the correct positions. It is easy to calculate these positions from the transformation function and the known location of the zero removed from the archetypal linear array.

9. The final planar array excitations are obtained by a convolution of the interim sum array and the interim difference array. On the pattern plane this is equivalent to multiplying the interim sum array factor and the interim difference array factor.

The method uses a linear array synthesis technique appropriate for difference patterns as one of its steps to enable the synthesis of planar arrays with difference patterns in a selected principal plane pattern cut. The sidelobes are all equal to or below those of the archetypal linear array, but not unnecessarily low, since this would cause an unwanted increase in the width of the difference pattern principal lobes. The technique in effect provides a structured procedure for spreading-out the collapsed distributions, thereby eliminating any guesswork that may be required. The method, including the choice of the interim difference array, is best demonstrated by a few examples.

4.3 Examples of Planar Array Difference Distributions

Two examples will be presented. First we will look at the synthesis of a rectangular (both lattice and boundary) array with an even number of elements in both principal planes. The second example is of a hexagonal array with 5 rings. Each step in the design procedure will be discussed.

Illustrative Example #1

We choose a rectangular grid planar array with 20 by 20 elements (that is $M=N=10$). Assume half wavelength inter-element spacing. The synthesis procedure is then step by step:

1. The interim difference array is kept as simple as possible, a four element array.
2. The interim sum array is a 19 by 19 element rectangular array.
3. The interim sum array excitations are obtained using the odd case of transformation based synthesis method with $I=J=1$, with no coefficient set to zero.
4. The archetypal linear array, consisting of 20 elements, is synthesised using the Zolotarev polynomial method with the maximum sidelobe ratio set at 20dB.
5. Since the interim difference array consists of only four elements, its collapsed distribution along the $\phi=0°$ cut will have only one zero, at $\psi=0$. With this zero removed from the archetypal linear array zeros, the prototype linear array excitations are computed. The number of elements in the prototype linear array are 19.
6. The transformation coefficients are selected for near-circular contours: $t_{00}=-\frac{1}{2}$ and $t_{01}=t_{10}=t_{11}=\frac{1}{2}$.
7. Compute the interim sum array distribution.

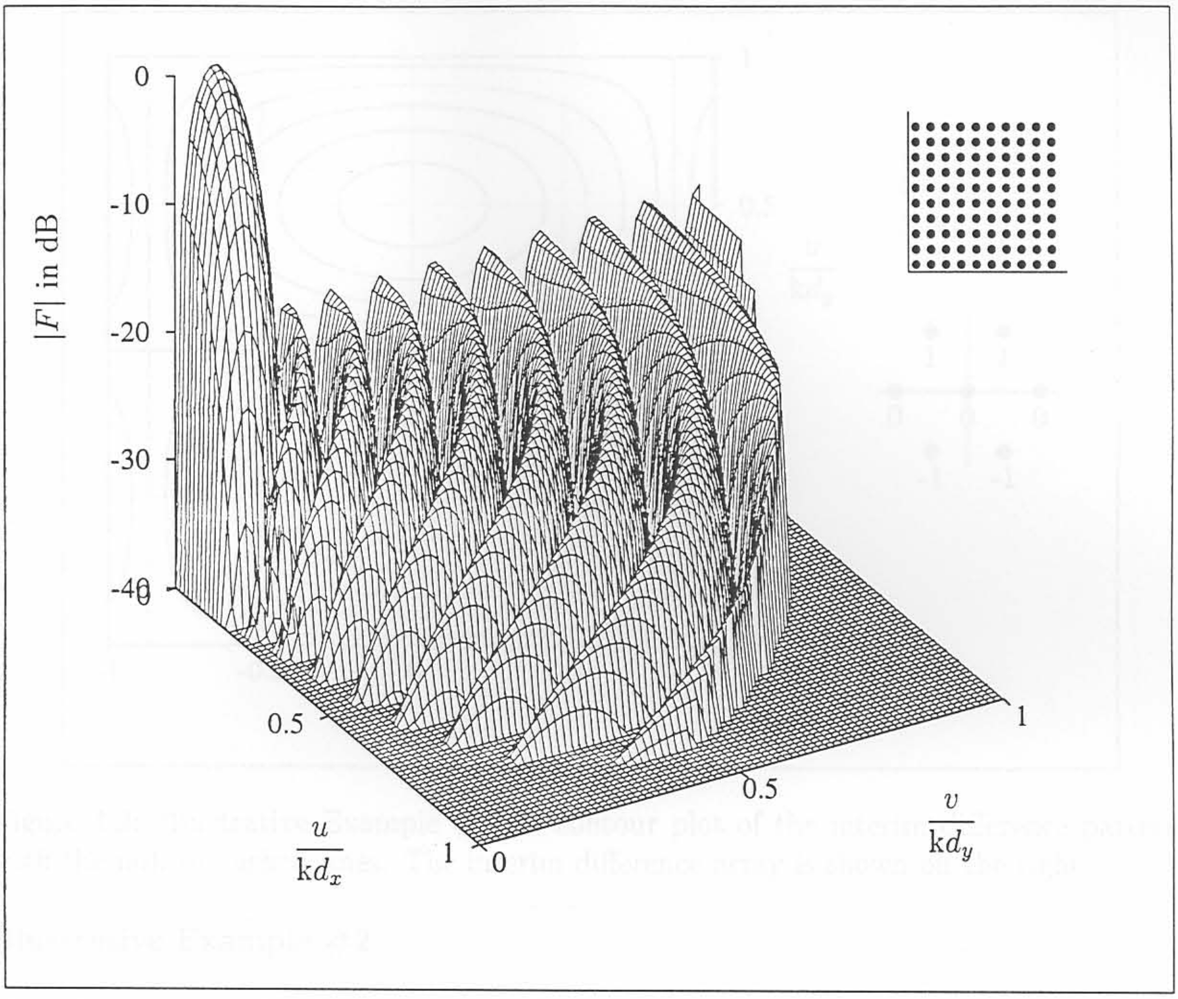

Figure 4.1: Illustrative Example #1: Surface plot of the planar array factor of the difference distribution.

8. The interim difference planar array is a four element array. Its excitations are required to be antisymmetric (180° out of phase) about the $\phi = 0°$ principal plane, and in-phase about the $\phi = 90°$ principal plane. The result is a factored difference pattern $f_d = \cos(\frac{1}{2}u)\sin(\frac{1}{2}v)$.

9. The final difference distribution is then computed by the convolution of the interim sum and interim difference array excitations.

The difference pattern is displayed in Figure 4.1. The sidelobe and all other pattern behaviour in the principal difference plane of the planar array are the same as that designed for the archetypal linear array. Furthermore, if the excitations of the archetypal linear array are real, then so are those of the final planar array.

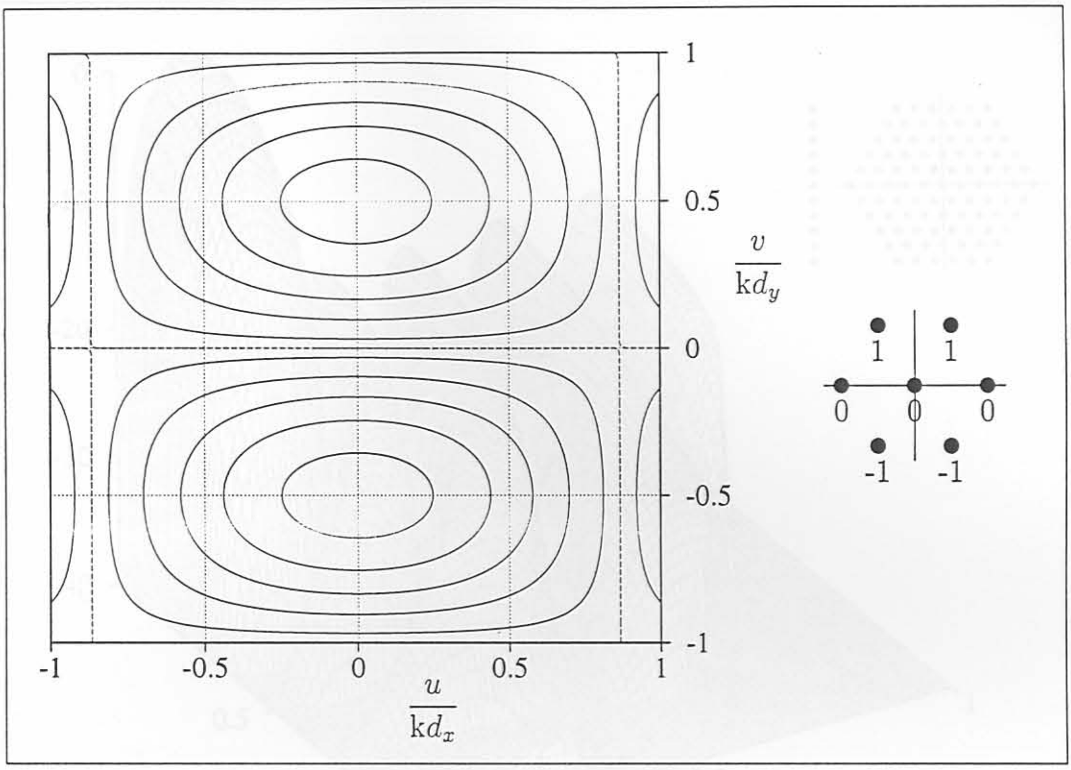

Figure 4.2: Illustrative Example #2: A contour plot of the interim difference pattern, with the null in dashed lines. The interim difference array is shown on the right.

Illustrative Example #2

Consider an example of a hexagonal planar array with five rings. Difference patterns about both principal planes are required. Assume the inter-element spacing (the distance between an element and its six closest neighbours) is $d = 0.577\lambda$. As the collapsed distributions along the principal planes differ, the synthesis procedure must be repeated for the two different cases. Let us look at the two cases separately, first with the difference null along $\phi = 0°$, and then with the difference null along $\phi = 90°$. The synthesis procedure will again be followed step by step.

1. The interim difference array for both cases is the smallest unit cell of a hexagonal array, a seven element (or one ring) hexagonal array.
2. From the first step, it is clear that the interim sum array will be a four ring hexagonal array.
3. The transformation coefficients are chosen to represent the unit cell of the interim sum array, as mentioned in Section 3.4.6), thus $I=2$ and $J=1$, with $t_{01} = t_{10} = t_{21} = 0$.
4. As the interim sum array consists of 4 hexagonal rings, the collapsed distribution along the x-axis will have 17 elements. Since $I=2$, the prototype linear array will

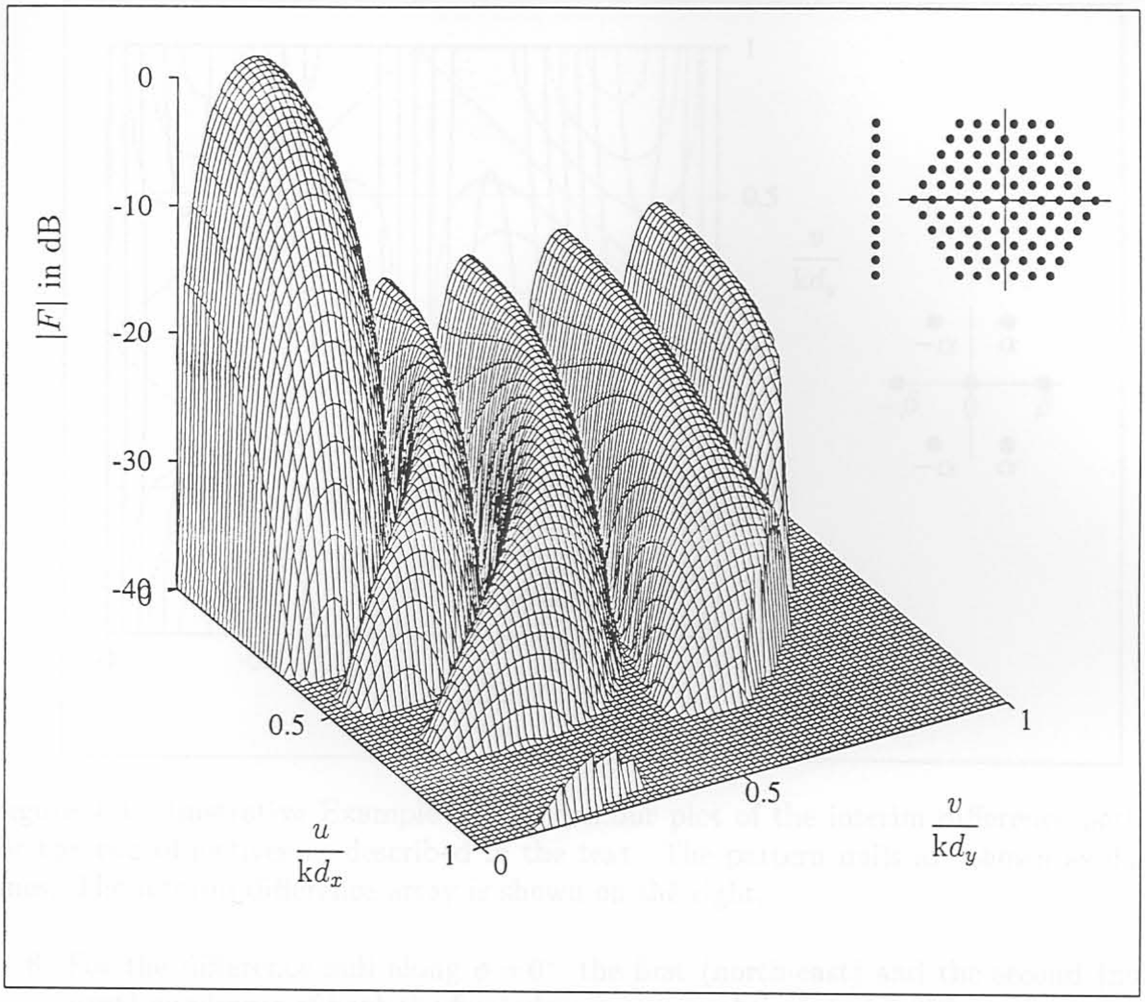

Figure 4.3: Illustrative Example #2: Surface plot of the planar array factor, with the array geometry (and the collapsed distribution) drawn on the right.

have 9 elements, or $Q=4$. Thus for this case the archetypal linear array is an 11 element array, with an inter-element spacing of a half wavelength (to avoid super directivity). The synthesis problem is solved using the Zolotarev method.

5. Since we know the interim difference array geometry, we can determine which zeros must be removed from the archetypal linear array factor, namely the zeros at $\psi=0°$ and $\psi=90°$. The remaining 8 zeros are used to compute the prototype linear array excitations.

6. The transformation function coefficients are chosen in order to obtain the minimum beamwidth in all cuts ($t_{00}=-\frac{1}{4}$, $t_{11}=1$, and $t_{20}=\frac{1}{4}$, as discussed in Section 3.5.2). This will also result in the maximum (non-super-directive) bore-sight slope for this particular geometry and case.

7. With the information obtained in steps 5 and 6, compute the interim sum array excitation.

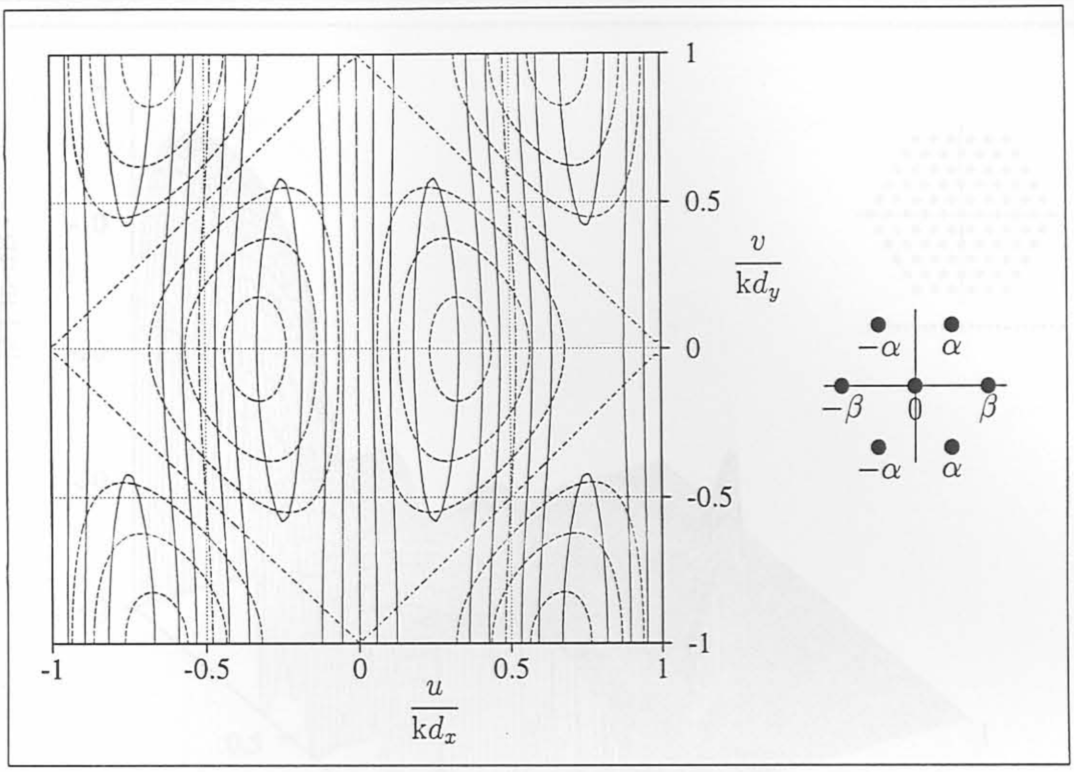

Figure 4.4: Illustrative Example #2: A contour plot of the interim difference patterns for the two objectives as described in the text. The pattern nulls are shown as dotted lines. The interim difference array is shown on the right.

8. For the difference null along $\phi = 0°$ the first (north-east) and the second (north-west) quadrants of both the final planar array and the interim difference array must be excited 180° out of phase with the third (south-west) and fourth (south-east) quadrants. All the elements at $y = 0$ must be switched off. Thus unit excitation, for the 4 non-zero elements, is the only solution for the interim difference array excitations. The factored difference pattern, drawn in Figure 4.2, is $f_d = \cos(u)\sin(v)$. The interim difference array is depicted on the right hand side of Figure 4.2.
9. The final planar array excitation is calculated from the interim sum array and the interim difference array excitations, using a two dimensional discrete convolution.

Figure 4.3 depicts the final planar array factor. The maximum sidelobe level is -20dB and the difference null is along the $\phi = 0°$ cut. In total only 80 non-zero elements are needed. The array geometry is drawn in the upper right hand corner of the figure.

Next we consider the case with the difference null along $\phi = 90°$. The first three steps are exactly the same as for the previous case, and will not be repeated. Starting from Step 4:

4. The collapsed distribution of the interim sum array in the $\phi = 90°$ plane is a 9 element linear array. The prototype linear array will have 9 elements, since $J = 1$.

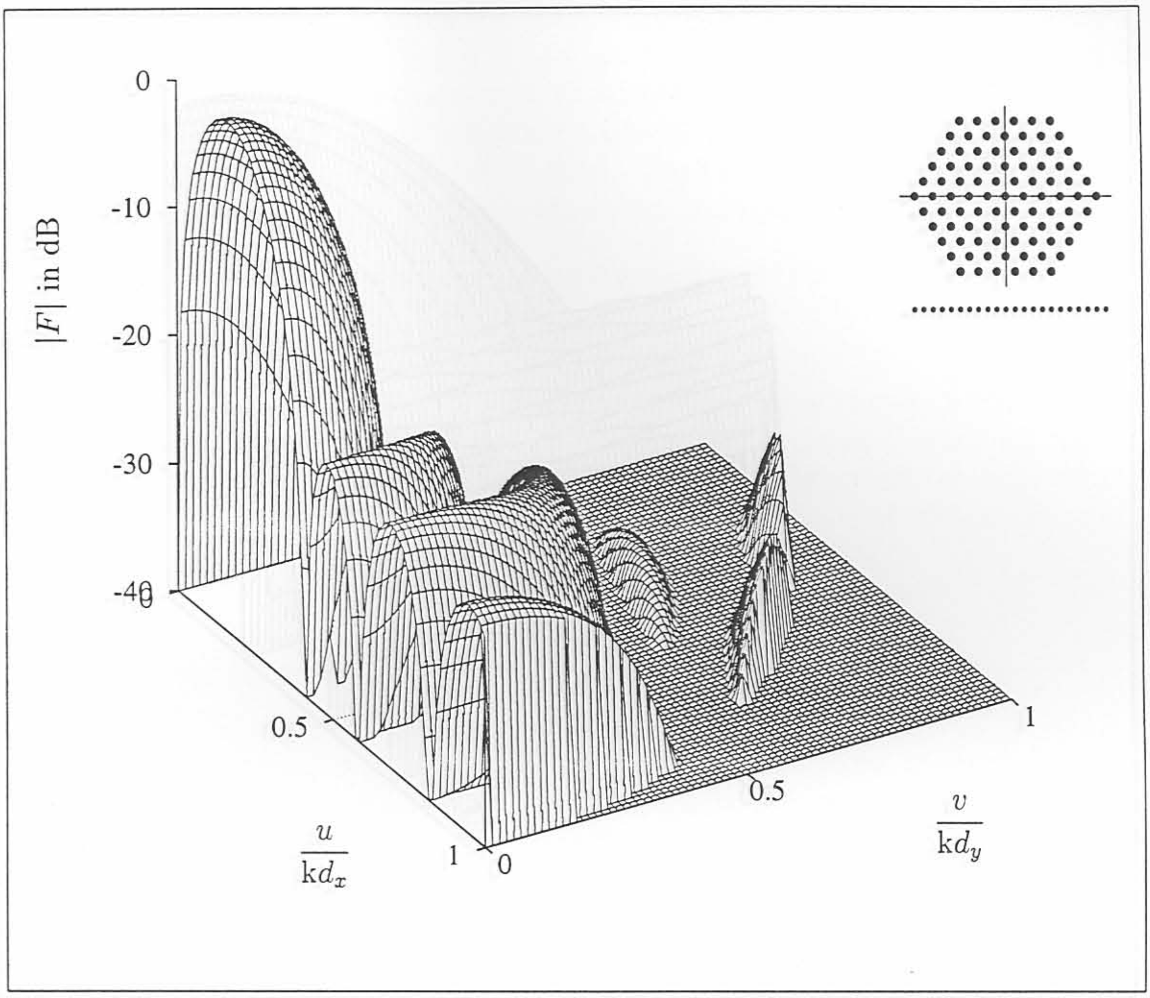

Figure 4.5: Illustrative Example #2: Surface plot of the planar array factor, with minimum beamwidths in all cuts as the design objective. The array geometry (and the collapsed distribution) is drawn on the right.

The interim difference removes two nulls from the planar pattern in the$=90^\circ$ plane, thus the archetypal linear array must have 11 elements (with half wavelength inter-element spacing as before). The archetypal linear array and the interim arrays do not differ for this example, but in general this may not be the case.

5. The zeros at $\psi=0^\circ$ and $\psi=90^\circ$ are factored from the archetypal linear array factor, and the remainder are used to calculate the prototype linear array excitations.

6. As in the previous case, the transformation function coefficients are chosen to minimise the beamwidth in all cuts. However, this is not the only sensible objective, maximisation of the bore-sight slope will result in a different set of transformation coefficients for this geometry and case ($t_{00} = -0.1051$, $t_{11} = 0.2168$ and $t_{20} = 0.8883$). A number of possibilities exist between these two extreme objectives, requiring engineering judgement to obtain the best solution for the application at

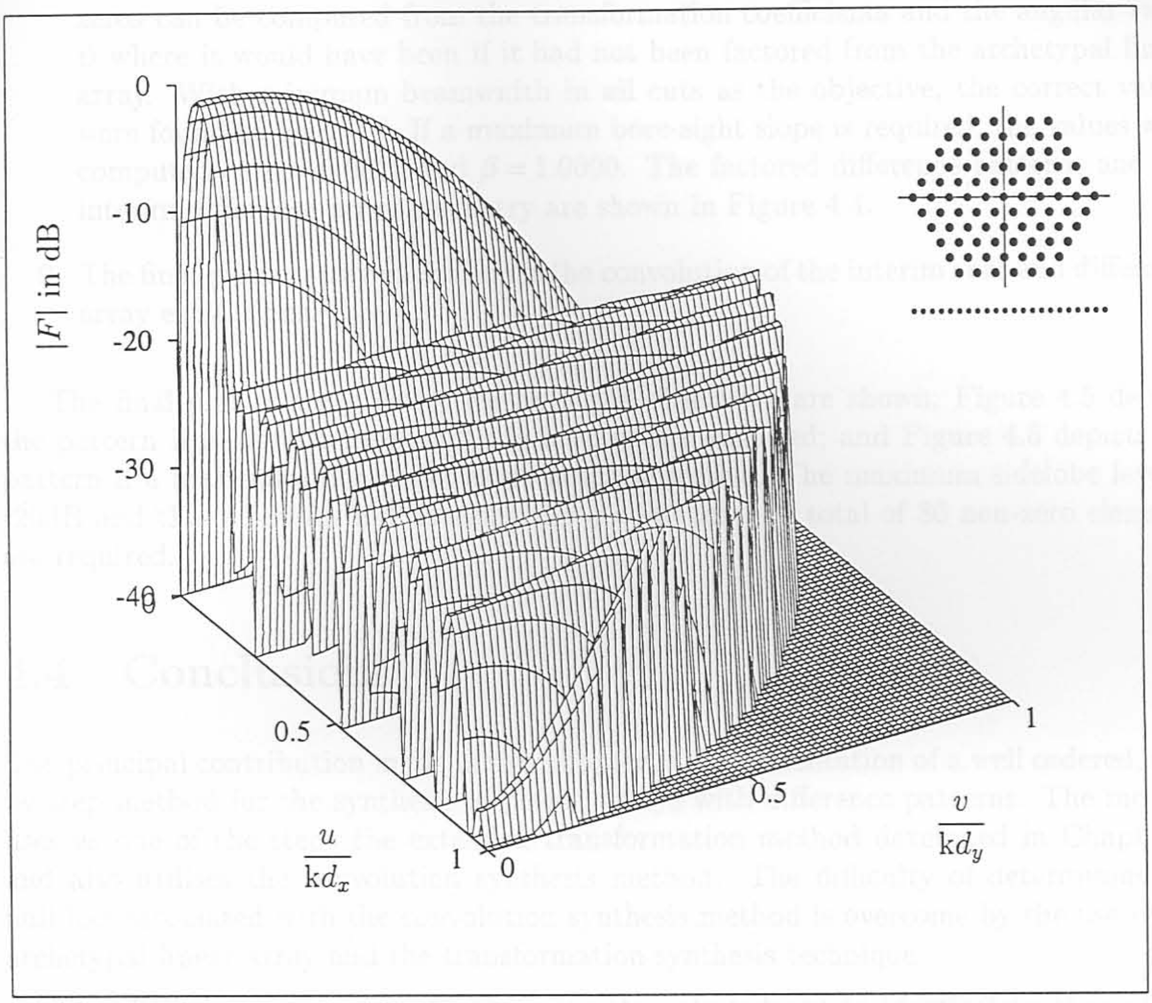

Figure 4.6: Illustrative Example #2: Surface plot of the planar array factor, designed for maximum bore-sight slope; with the array geometry (and the collapsed distribution) drawn on the right.

hand. To illustrate, the results of both these objectives will be shown.

7. The transformation based technique is used to compute the interim sum array excitation.

8. The first and fourth quadrants and the second and third quadrants of the final planar array and the interim difference array must be excited 180° out of phase to produce a difference null along $\phi = 90°$. All the elements at $y = 0$ must be switched off. This leaves 6 non-zero excitations to control the factored difference pattern

$$f_d = \alpha \sin(u)\cos(v) + \beta \sin(2u)$$

The values of α and δ must be chosen in such a manner that the zeros factored out of the archetypal linear array will be placed at the correct positions to ensure that the sidelobes will not be higher than the specification. The position of these zeros can be computed from the transformation coefficients and the angular value ψ where it would have been if it had not been factored from the archetypal linear array. With minimum beamwidth in all cuts as the objective, the correct values were found to be $\alpha = \beta$. If a maximum bore-sight slope is required, the values were computed as $\alpha = 0.0610$ and $\beta = 1.0000$. The factored difference patterns and the interim difference array geometry are shown in Figure 4.4.

9. The final planar array excitation is the convolution of the interim sum and difference array excitations.

The final planar array factor for both the objectives are shown; Figure 4.5 depicts the pattern if minimum beamwidth in all cuts are required; and Figure 4.6 depicts the pattern if a maximum bore-sight slope is the objective. The maximum sidelobe level is -20dB and the difference null is along the $\phi = 90°$ cut. A total of 86 non-zero elements are required.

4.4 Conclusions

The principal contribution made in this chapter is the presentation of a well ordered, step by step method for the synthesis of planar arrays with difference patterns. The method uses as one of the steps the extended transformation method developed in Chapter 3, and also utilises the convolution synthesis method. The difficulty of determining the null loci associated with the convolution synthesis method is overcome by the use of the archetypal linear array and the transformation synthesis technique.

The difference pattern performance in the selected cut is identical to that of the archetypal linear array used, and will thus be optimum if the latter is optimum. In the other pattern cuts the sidelobes are below those of the archetypal linear array, but not unnecessarily low. The technique in effect provides a structured procedure for spreading out the linear array excitations, thereby eliminating any guesswork that may otherwise be required. Due to the simplicity of the method the synthesis of even very large arrays is rapid, making it feasible to conduct parametric studies of array performance and to perform design trade-off studies.

Preliminary work has been published by the author [128]