

# Contributions to the Synthesis of Planar and Conformal Arrays

Acknowledgements

I would like to thank the following people without whom this thesis would not have been possible:

by

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Submitted in partial fulfillment of the requirements for the degree

Ph.D (Electronic Engineering)

in the

Faculty of Engineering

UNIVERSITY OF PRETORIA

August 2000



## Synopsis

Title: Contributions to the Synthesis of Planar and Conformal Arrays  
Name: E. Botha  
Promotor: Prof. J. Joubert  
Department: Electrical, Electronic and Computer Engineering  
Degree: Philosophiae Doctor (PhD) (Engineering)

## Acknowledgements

I would like to thank the following people without whom this thesis would not have been possible:

1. Derek McNamara, who interested me in antennas in general and array synthesis in particular; and who was my mentor throughout my studies and promotor for the first part of this thesis.
2. Johan Joubert, my promotor for the latter part of my thesis.
3. My wife, Jo-Anne, for her continued support and understanding.
4. My parents for their support and encouragement.
5. Friends and fellow students at Tukkies, Wimpe, Pieter, Martin, Thys, Louis, Marcus, Danie and Maggie for stimulating discussions.

Lastly, our Father in Heaven for providing every opportunity.

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The transformation technique, used for the synthesis of centro-symmetrical contoured beams for rectangular planar arrays, is extended to enable the synthesis of planar arrays with arbitrary contoured footprint patterns; planar arrays with non rectangular boundaries and planar arrays with triangular lattices. The transformation based synthesis technique utilises a transformation that divides the problem into two decoupled sub-problems. One sub-problem involves the determination of certain coefficients in order to achieve the required footprint contours, while the other sub-problem consists of a linear array synthesis.

A well ordered, step by step procedure for the synthesis of planar arrays with difference patterns is presented. The method utilises the convolution synthesis method and uses the extended transformation method as one of the steps. The technique in effect provides a structured procedure for spreading out the linear array excitations. The result is near-optimum difference patterns for planar arrays with rectangular or hexagonal lattices. The difference pattern performance in the selected cut is identical to that of the archetypal linear array used, and will thus be optimum if the latter is optimum. In the other pattern cuts the sidelobes are below those of the archetypal linear array, but not unnecessarily low.

Conformal array synthesis is an ill conditioned inverse problem with a multitude of local minima. Due to the non linear nature of conformal array synthesis an effective conformal array synthesis method must have a rapid rate of convergence and some measure of confidence that the result will be close to the optimal solution. The synthesis of arrays of arbitrary geometry and elements was stated as the search for the intersection of properly defined sets. Effective relaxation was implemented in the excitation space as well. A number of possible ways of calculating starting points were investigated, and a novel method to obtain a set of initial values as close to the global minimum as possible is proposed for shaped and contoured beam synthesis. The phase variation in the shaped beam region is slow and may be written as a function of few variables. Genetic algorithm is used to optimise these phase function variables. The importance of practical element patterns in the analysis and synthesis of conformal arrays is also shown.

## Abstract

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The transformation technique, used for the synthesis of centro-symmetrical contoured beams for rectangular planar arrays, is extended to enable the synthesis of planar arrays with arbitrary contoured footprint patterns, planar arrays with non rectangular boundaries and planar arrays with triangular lattices. The transformation based synthesis technique utilises a transformation that divides the problem into two decoupled sub-problems. One sub-problem involves the determination of certain coefficients in order to achieve the required footprint contours. The other sub-problem consists of a linear array synthesis, for which powerful methods for determining appropriate element excitations, already exist. The final planar array size is linked to the number of contour transformation coefficients and the prototype linear array size. The biggest advantage of the transformation based synthesis technique is its computational efficiency, making it feasible to conduct parametric studies of array performance design tradeoff studies even for very large arrays.

A well ordered, step by step procedure for the synthesis of planar arrays with difference patterns is presented. The method utilises the convolution synthesis method and uses the extended transformation method as one of the steps. The technique in effect provides a structured procedure for spreading out the linear array excitations, thereby eliminating any guesswork that may otherwise be required. The result is near-optimum difference patterns for planar arrays with rectangular or hexagonal lattices. The difference pattern performance in the selected cut is identical to that of the archetypal linear array used, and will thus be optimum if the latter is optimum. In the other pattern cuts the sidelobes are below those of the archetypal linear array, but not unnecessarily low. The synthesis procedure is very rapid for even very large arrays, making it feasible to conduct design trade-off and parametric studies.

Conformal array synthesis is an ill conditioned inverse problem with a multitude of local minima. Due to the non linear nature of conformal array synthesis an effective conformal array synthesis method must have a rapid rate of convergence and some measure of confidence that the result will be close to the optimal solution. The synthesis of arrays of arbitrary geometry and elements can be stated as the search for the intersection of properly defined sets. Proper sets were defined in both the radiation pattern space and excitation space; along with the necessary projector between these sets. Effective relaxation can be implemented in the excitation space as well. A number of possible ways of calculating starting points were investigated, and a novel method to obtain a set of initial values as close to the global minimum as possible is proposed for shaped and contoured beam synthesis. The phase variation in the shaped beam region is slow and may be written as a function of few variables. The starting pattern is a summation of component beams, each weighted by the proper value of the shaping function and phase shifted by the proper value of the phase function. Genetic algorithm is used to optimise these phase function variables. The importance of practical element patterns in the analysis and synthesis of conformal arrays is also demonstrated.

Keywords: Antenna theory, Antenna arrays, Antenna synthesis, Array synthesis, Planar arrays, Conformal arrays, Antenna pattern synthesis, Power synthesis, Conformal array synthesis, Planar array synthesis.

## Samevatting

Titel: Bydraes tot die sintese van Vlak- en Konforme-samestellings  
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Die transformasie tegniek, wat vir die sintese van sentro-simmetriese gekontoerde patrone van reghoekige vlaksamestellings gebruik word, is uitgebrei vir arbitrêre gekontoerde patroon sintese; die sintese van vlaksamestellings met nie reghoekige rande en die sintese van vlaksamestellings met driehoekige roosters. Die transformasie gebaseerde tegniek verdeel die probleem in twee subprobleme. Een subprobleem (kontoer transformasie probleem) behels die berekening van sekere koëffisiente om die verlangde kontoer te omskryf. Die ander subprobleem bestaan uit 'n prototipe liniêresamestelling sintese; waarvoor daar reeds kragtige metodes bestaan om die element aandrywings te bereken. Die grootte van die finale vlaksamestelling is afhanklik van die hoeveelheid kontoer transformasie koëffisiente en die aantal prototipe liniêresamestelling elemente. Die grootste voordeel van die metode is die rekenaar effektiwiteit daarvan, selfs vir groot samestellings. Dus is dit moontlik om parametriese en ontwerpskreterea studies te doen selfs vir baie groot samestellings.

'n Goed ge-ordende stap-vir-stap prosedure vir die sintese van verskil patrone van vlaksamestellings is voorgestel. Die metode behels die konvolusie sintese metode so wel as die uitgebreide transformasie gebaseerde tegniek as van die stappe. Die tegniek bied 'n gestruktureerde uitsprei van die samestelling aandrywings. Die prosedure lewer byna-optimum verskilpartone vir vlaksamestelling met reghoekige of heksagonale elementroosters. Die verskilpatroon in die hoofsnit is identies aan die patroon van die prototipe samestelling; en sal dus optimaal wees as die prototipe samestelling patroon optimaal is. Die sylobbe in ander patroon snitte is net so laag, of laer, as die van die prototipe samestelling; maar nie onnodig laag nie. Die prosedure is rekenaar effektief, wat parametriese studies vir baie groot samestellings is moontlik maak.

Konforme-samestelling sintese is 'n swak gekondisioneerde inverse probleem met meer as een lokale minimum. As gevolg van die nie-liniariteit van konforme-samestelling sintese moet enige toepaslike sintese metode vinnig konvergeer en 'n mate van vertrouwe bied dat die resultaat naby aan optimum is. Die sintese van samestellings met 'n arbitrêre geometrie en arbitrêre elemente is beskryf as die soek na die snyding van korrek gedefinieerde stelle. Stelle, met meegaande projektors, is in die aandrywingsvlak en die patroonvlak gedefinieer. 'n Begin punt so naby as moontlik aan die optimum punt is belangrik om konvergensie na die optimale punt se verseker. Die fase van die verveld is stadig variërend in die hoofgebied, en kan met behulp van net 'n paar veranderlikes beskryf word. Die beginpatroon is die sommering van komponent bundels, elk geweg met die korrekte waarde van die gespesifiseerde hoofbundel vorm en in fase geskuif met die waarde van 'n fasefunksie. Die fasefunksie veranderlikes word dan geoptimeer met Genetiese Algoritme. Die belangrikheid van praktiese elementpatrone in die analise sowel as sintese van konforme-samestellings is aangetoon.

Slutelwoorde: Antenneteorie, Antennesamestellings, Antennesintese, Samestellingsintese, Vlaksamestellings, Konformesamestellings, Antennepatroon sinteses, Drywingsintese, Konformesamestelling sintese, Vlaksamestelling sintese.

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