CHAPTER FOUR: Modeling Approach of the Study and Structure of the regional SADC maize Model

4.1 Introduction

In this chapter, the modeling approach followed and the structure of the SADC maize model are discussed. The theoretical foundations of the modeling approach are reviewed and the SADC regional maize model is specified and presented together with a discussion of estimation procedures used.

The first sub-section of this chapter reviews the basic elements of the economic theory of the firm and consumer behavior employed in this analysis. The aim of this section is to provide the theoretical foundations for the specification and empirical estimation of the supply and demand segments of the model presented later in this study. Good understanding of the underlying assumptions behind the specified supply and demand systems is important for explaining market behavior in the maize sub-sector (how maize producers and consumers make their production decisions and consumption choices). It also provides the basis for predicting demand and supply responses to changes in policies in SADC countries.

4.2 Supply System

Farm supply is the schedule of amounts farmers would be willing to produce at different *expected* price levels, with all other factors remaining constant (Ferris, 1998)

The analysis of agricultural producers' behavior presented here is based on the neo-classical theory of the firm. The theory assumes that producers maximize profit or net returns subject to some technical and institutional constraints. The technical constraints refer to the firm's production function, which define the physical relationship between factor inputs and the maximum output level for the given technology per unit of time (Varian, 1984). Institutional constraints relate to market structure, which determine the economic environment in which firms operate.

Consider a farm that uses land -A; labor -L, and other inputs (chemicals and capital) -K, in the production of maize. The expected output or the production function, which is simply a conceptualization of the technical relationship between inputs being used and output, can be written as $Q = F(A, L, K)$, where $\frac{\partial F}{\partial A} \geq 0, \frac{\partial F}{\partial L} \geq 0, \frac{\partial F}{\partial K} \geq 0$. (Factor inputs have non-negative marginal contribution to output, i.e. quasi-rents are not negative). Admissible production technology structures also require that the second derivatives be greater than zero. This imposes the law of diminishing marginal return, which also ensures that the production function is concave to the origin. Let p denote the expected output price, r the rental cost for land A, w the cost of labor L, and k the cost of other inputs K. Theory also assumes that output level and output prices are independently distributed random variables and that the farmer is risk neutral. The farmer's decision problem is to maximize expected profit: $Max \Pi(p, r, k, w, TFC) = Max\{pQ - rA - kK - wL - TFC\}$, where pQ is the expected revenue, rA is the cost of land, kK is the cost of capital and other inputs, wL is the cost of labor and TFC is total fixed cost. The first order conditions of profit maximization imply that,

$$p\frac{\partial Q}{\partial A} - r = 0, p\frac{\partial Q}{\partial K} - k = 0, p\frac{\partial Q}{\partial L} - w = 0.$$

These three equalities state that the farmer will maximize profit by producing output levels where the expected value of the marginal product of each input is equal to the input's cost. The second order conditions require concavity of the production function, which ensures convexity of the profit function to input and output prices. Assuming that the production function is invertible then optimum input demand can be expressed as a function of input and output prices, $K^{\alpha}(p, r, k, w), A^{\alpha}(p.r, k, w),$ and $L^{\alpha}(p, r, k, w)$ which are obtained from solving the first order condition equations. The input demand functions are homogenous of degree zero in input and output prices. Substituting input demand into the production function yields the output supply function, $Q^{\alpha} = f(K^{\alpha}, A^{\alpha}, L^{\alpha})$, which is also homogenous of degree zero in output and input prices.

From the convexity property of the profit function it follows that the expected supply function is upward sloping and that the input demand functions are downward sloping. It follows that the marginal effects of an increase in output price p on input demands are equal in absolute magnitude, but are of opposite sign, to the marginal effect of an increase in the corresponding input price on output supply.

Alternatively, the above derivation can be achieved by using the cost minimization paradigm, which give the same results. Another, commonly used method is the duality theory to solve for the input demand. The advantages of duality, as described by Lopez (1982), are that the simultaneous equation bias is avoided since profit and input demand functions are expressed as functions of exogenous variables and that the duality method can be used to compute the *mutatis mutandis* elasticities associated with supply and demand. The foundations of duality theory are the indirect profit and cost functions, which are obtained from the profit maximization and cost minimization specifications of the firms' supply decision. Following the same procedure as in profit maximization, input factor demand functions are obtained. The input factor demands can be written as a function of output and input prices $K^{\alpha}(p,r,k,w), A^{\alpha}(p,r,k,w)$ and $L^{\alpha}(p,r,k,w)$. Substituting the factor inputs into production function yields the profit maximizing output level $Q^{\alpha} = f(K^{\alpha}, A^{\alpha}, L^{\alpha})$. The indirect profit function is then $\Pi(p,r,k,w) = pf(A^{\alpha}, K^{\alpha}, L^{\alpha}) - rA - kK - wL$, which is a function of output and input prices. Using the Envelope Theorem, taking partial derivatives of the indirect profit functions with respect to output and input prices, yields the output supply

$$\frac{\partial \Pi}{\partial P} = f(A^{\alpha}, K^{\alpha}, L^{\alpha}), \frac{\partial \Pi}{\partial A} = -A^{\alpha}(p,r,k,w), \text{ and input demand functions}$$

$$\frac{\partial \Pi}{\partial K} = -K^{\alpha}(p,r,k,w) \text{ and } \frac{\partial \Pi}{\partial L} = -L^{\alpha}(p,r,k,w).$$

4.2.1   Elasticities of Supply and Input Demand

Economists have devised a convenient way of expressing relationships within the supply system using elasticities. Elasticity measures the degree of responsiveness of

the independent variable to a percentage change in the dependant variables. Generally, four types of elasticities are important in econometric studies, namely: own price elasticity, cross price elasticity, input price elasticity, and income elasticities. For simplicity, suppose the agricultural producer can produce only two commodities, wheat (w) and maize (m). The own price elasticity of supply measures the percentage change in output in response to a 1 % change in output price p_m of maize, *ceteris paribus*.

Mathematically, it is defined as $\varepsilon_{mm} = \dfrac{\dfrac{\Delta Q_m}{Q_m}}{\dfrac{\Delta P_m}{P_m}} = \dfrac{\Delta Q_m}{\Delta P_m} * \dfrac{P_m}{Q_m} \succ 0$. The cross price supply

elasticity measures the percentage change in output caused by a 1 % change in the price of another output, in this case the wheat price p_w, and the elasticity is defined

as $\varepsilon_{mw} = \dfrac{\partial Q_m}{\partial P_w} * \dfrac{P_w}{Q_m} \prec 0$. Similarly, the input price elasticities for other inputs -K, labour-L and land-A measure the percentage change in output given a 1 % change in one of these input prices. Input price elasticity are expressed as:

$\varepsilon_{mk} = \dfrac{\partial Q_m}{\partial k} * \dfrac{k}{Q_m} \prec 0, \ \varepsilon_{ml} = \dfrac{\partial Q_m}{\partial w} * \dfrac{w}{Q_m} \prec 0, \ \text{and} \ \varepsilon_{sA} = \dfrac{\partial Q_m}{\partial r} * \dfrac{r}{Q_m} \prec 0.$

There are many relationships among elasticities the most important, however is that of "no money illusion". This means that if all prices increase by the same percentage output quantity should not be affected, as the negative effects of input price increases will be offset by the positive effects of own price increases. This implies that a marginal change in the level of output Q_m, due to change in the price of an input is equal to negative value of the marginal change in input use following a marginal change in output price p. This can be mathematically represented as:

$\dfrac{\partial Q_s}{\partial k} = -\dfrac{\partial K}{\partial P_s}, \ \dfrac{\partial Q_s}{\partial w} = -\dfrac{\partial L}{\partial P_s}, \ \dfrac{\partial Q_s}{\partial r} = -\dfrac{\partial A}{\partial P_s}$. The symmetry relationship can therefore

be established as $\dfrac{\partial k}{\partial r} = \dfrac{\partial A}{\partial K}$, and similarly for all other inputs.

4.2.2 The Dynamics of Supply and Expectations Models

Heady et al (1961) postulated that farmers operate in a dynamic world where prices and input-output relationships are not known with certainty, and are subject to change through time. Producers are not in a position to fully adjust their input use or output in one period of time. It is for this reason that a dynamic approach may be more appropriate when modeling agricultural production. Since agricultural production occurs under less than perfect certainty, due to its biological nature, and because input use decisions for are sequential, time plays a crucial role. In the short-run, some inputs are fixed and in long run all inputs vary, which suggest that input use is a function of time. Thus, when taking time in consideration, there are various possible ways that the firm can adjust its input use and hence the movement from one equilibrium point to another. This adjustment is not instantaneous due to price uncertainty, fixity, and non-divisibility of factor inputs.

Various methods have been used to represent dynamic output supply response. The most commonly used method is the Koyck's distributed lag model (Koyck, 1954) and Nerlove's partial adjustment model (Nerlove, 1958) with various functional forms to represent producers' expectations. Since farmers do not know what will be the price level at harvest time or at what price they will sell their output, their cropping decisions are based on certain expectations. The most cited types of expectations in the literature are naive, extrapolative, adaptive, rational and quasi-rational. The most widely used is the adaptive expectations assumption. According to Nerlove's (1958) seminal paper, the partial adjustment model assumes that the change in price expectations in the current period is some proportion, ∂, of the error made in formulating expectations in the previous period. Producers adjust toward the optimum level of output Q_t according to the following equation $P_t^e - P_{t-1}^e = \partial(P_{t-1} - P_{t-1}^e)$ where P_t is the normalized price of output; "e" is the expectations operator in respective periods, and ∂ is the coefficient of expectation with $0 < \partial < 1$. Rewriting the above equation one gets $P_t^e = \partial P_{t-1} + (1-\partial)P_{t-1}^e$. This equation means that the expected price at time t is the sum of last period's expected price plus some adjustment factor and last period's actual price. In support of his

argument Nerlove (1956) stated the following hypothesis "...each period people revise their notion of 'normal' price in the proportion to the difference between the current price and the previous ideal 'normal' price." The above equation can be express as a first-order difference equation $P_t^e = \sum_{i=0}^{\infty} \partial (1-\partial)^i P_{t-1-i}$, which can be solved for P_t^e and is a weighted moving average of past actual prices where the weights decline with time. In this case, a highly simplified version of output supply function is used where output is a function of the expected price (normalized) and Z is an exogenous variable, $Q_t = \beta_0 + \beta_1 P_t^e + \beta_2 Z_t + \upsilon_t$ where β_1 is the long-term response, and υ_t is an error term. An algebraic manipulation of the price and output function yields the following output supply function,

$$Q_t = \beta_0 \partial + \beta_1 \partial P_{t-1} + (1-\partial)Q_{t-1} + \beta_2 [Z_t - (1-\partial)Z_{t-1}] + \upsilon_t - (1-\partial)\upsilon_{t-1}$$

Rewriting the above $Q_t = \Pi_0 + \Pi_1 P_{t-1} + \Pi_2 Q_{t-1} + \Pi_3 Q + \varepsilon_t$, where

$$\Pi_0 = \beta_0 \partial,\ \Pi_1 = \beta_1 \partial,\ \Pi_2 = 1-\partial,\ \Pi_3 = \beta_2,\ Q = Z_t - (1-\partial)Z_{t-1}, \varepsilon_t = \upsilon_t - (1-\partial)\upsilon_{t-1}.$$

The above is a output supply function of known variables only, i.e., lagged price, lagged output, and some exogenous variables. OLS estimation of the above equation will not yield reliable estimates because of the presence of serial correlation created by the lagged values of dependent variables. However, Maximum likelihood (MLE) or instrumental variable estimation methods will yield estimates with the desired properties.

Once the supply equation has been estimated, the short and long run multipliers can be computed. The short-run is a period of time during which some input factors of production are fixed and some inputs may vary in response to price, whereas in the long run all input factors can vary. The short and long run multipliers are obtained as follows:

$$\frac{\partial Q_{t+1}}{\partial P_{t+1}} = \Pi_1 (1 + \Pi_1 + \Pi_2 + \Pi_1^2 + ...) = \frac{\Pi_1}{1 - \Pi_2}$$

The short-run multiplier is the estimated value of the coefficient $\beta_1\partial$ and the long - run multiplier or long run response ∂ is the short- run multiplier divided by one minus the coefficient of lagged output. In terms of elasticities, the short-run elasticity is $E^s = \dfrac{\partial Q_t}{\partial P_{t-1}} * \dfrac{P_{t-1}}{Q_t} = \Pi_1 * \dfrac{\overline{P}}{\overline{Q}}$. And the long run elasticity is $E^L = \dfrac{\Pi_1}{1-\Pi_2} * \dfrac{\overline{P}}{\overline{Q}}$, where $\overline{P}, \overline{Q}$ are mean values of price and output. So far, the value of β has been assumed to be between zero and one. But when the value of β is one, it gives rise to the "cobweb" model, which shows how a higher price leads to a higher level of output in next period and given a downward sloping demand curve causes a lower price in the next period, which in turn leads to a low level of output and a high price in the next period, and so on. The other two commonly used expectation methods are the naive price expectation and rational expectations. For rational expectations it is presumed that producers do not make systematic errors. For the purpose of this study the adaptive expectation is adopted for the empirical estimation models used.

4.3 The Demand System
Aggregate demand for almost all agricultural crops can be divided into demand for direct use, i.e. primary demand, demand for intermediate use (a derived demand), and inventory demand. Consumer demand, or the retail demand, is defined as a schedule of quantities of a particular commodity that an individual consumer is willing and able to buy as the price of that commodity varies, all other factors held constant per unit of time. Derived demand is the demand for intermediate goods. For example, demand for maize for maize manufacturing and demand for maize by the beverage industry and other food manufacturing industries represent derived demand. The demand for stock, i.e. inventory demand, is due to the price expectations for precautionary reasons.

4.3.1   Consumer Demand
Retail demand for a commodity is the demand for a commodity in its final form. In our case, this is maize meal. The consumer demand relationship can be defined as quantity of maize meal demanded as a function of prices and income. The law of

demand states that there is an inverse relationship between quantity demanded and its own price. In other words, the demand curve is downward sloping.

The theoretical specification for food use is based upon consumer theory of utility maximization subject to budget constraints, i.e., consumer maximizes his or her utility function subject to a given level of income. Mathematically, the optimization problem is $\text{Max } U(\underline{X})$ subject to $I = \sum_{1}^{N} P_i X_i$, where $U(\underline{X})$ is the consumer's utility function, which is assumed to satisfy some regularity conditions, $\underline{X}$ is a vector representing the bundle of n goods and p_i is the price of good i. The utility function is a strictly quasi-concave and twice differentiable function. Consumer preferences are assumed to satisfy certain properties (reflexive, transitive, completeness, continuous and weakly monotonic). The optimization problem is solved using the Lagrange multiplier. Assuming that the second order conditions are satisfied for a global maximum, i.e., all income is spent. Solving the (n+1) first order equations, the demand functions of Xi's are obtained, which are implicit functions of prices and income. These implicit functions are homogenous of degree zero in prices and income. The indirect utility function is then obtained by substituting the solved values of Xi's into the direct utility function, i.e., $U^*(P_1,...P_N, I) = U^*[X_1^*(P_1,...P_N, I)...X_N^*(P_1,...P_N, I)$ The indirect utility function approach allows us to derive the uncompensated (Marshallian) demand function by differentiating the indirect utility function with respect to prices and income. The Marshallian demand function is obtained as follows

$$\frac{\dfrac{\partial U^*(P_1,...P_N, I)}{\partial P_i}}{\dfrac{\partial U^*(P_1,...P_N, I)}{\partial I}} = X_i^*(P_1,...P_N, I)$$

By applying the Hotelling-Wold identity to the indirect utility function the inverse uncompensated demand function is obtained (Johnson *et al*, 1984).

Alternatively, the problem can be approached as an expenditure minimization

problem. Inverting the indirect utility function and solving for I in terms of U and p gives the expenditure function. The problem can be stated as the minimum cost of attaining a given U^0 at a given price vector $\underline{P}$.

From the above derivation, the consumer demand for a commodity can be expressed as a function of prices and income (I) in its simplest form $Q^D = F(P_m, P_s, I)$ where Pm is the price of the commodity in question, and Ps is prices of competing and complementary goods. Summing the individual consumer demand yields the total consumer demand.

As in the case of supply it is useful to have measures of the responsiveness of quantity demanded due to changes in price and income (demand elasticities). The own price elasticity of demand is defined as the proportionate change in quantity demanded of X_i (maize in our case) due to a proportionate change in the price of X_i, ceteris *paribus*. Mathematically, the own price elasticity of demand is expressed as

$e_{ii} = \dfrac{\partial X_i}{\partial P_i} \dfrac{\overline{P_i}}{X_i} < 0$. The cross price elasticities of demand measure the proportionate

change in the quantity demanded of good X_i relative to the proportionate change in

the price of good j, *ceteris paribus*. Mathematically, if $e_{ij} = \dfrac{\partial X_i}{\partial P_j} \dfrac{\overline{P_j}}{X_i} > 0$, then the

good "i" is a substitute for good j and $e_{ij} < 0$ if good i is complementary. Finally the

income elasticity is defined as $\eta_j = \dfrac{\partial X_i}{\partial I} \dfrac{\overline{I}}{X_i}$. This measures the proportionate change

in the demand quantity of good "i" due to a proportionate change in income I.

As in the case of supply elasticities, certain interrelationships among the elasticity's can be established. Demand functions have four fundamental properties: homogeneity of degree zero, Engel aggregation, Cournot aggregation, and the Slutsky condition. For simplicity, a trend term (t) may be included in demand equations i.e., $Q^D = F(T, P_m, P_s, I)$ to capture the process of habit formation.

4.3.2 Inventory Demand

Many agricultural products are produced at one point of time during a crop year whereas consumption occurs through out the whole period. According to Bressler and King (1970), demand for stocks can be decomposed into a transaction demand, precautionary demand and speculative demand. Hence changes in stocks in the short-run can exert a considerable influence on supply.

For the purpose of this study the stock behavior specification is expressed as follows:

$$SC_t = S_t(SC_{t-1}, P_t, Q_t, Q_{t-1})$$

where SC is the change in stocks, P is the price, and Q is the quantity produced.

The above theoretical concepts contribute significantly to the understanding of supply and demand of agricultural commodities. Furthermore, the economic theory developed will help in portraying the reality of the SADC maize industry later in the empirical chapter.

### 4.4 Model Structure and Model Specification

In this sub section, the structure of the SADC maize industry is described using flow charts and price-quantity (P-Q) space. The model is a nonspatial partial equilibrium model—nonspatial because it does not identify trade flows between specific regions and partial equilibrium because only one commodity is modeled.

The flow chart is a causal ordering of the supply-utilization-price structure. It facilitates the understanding of the nature of the economic and statistical relationships among variables that influence production and consumption. It also explains how policy and other relevant variables influence production and the consumption of the commodity under consideration.

It has been common in crop modeling to start with area planted but unavailability of data on areas planted to maize in the SADC countries made it impossible to start modeling the structure of the SADC maize sector from the initial stage of area
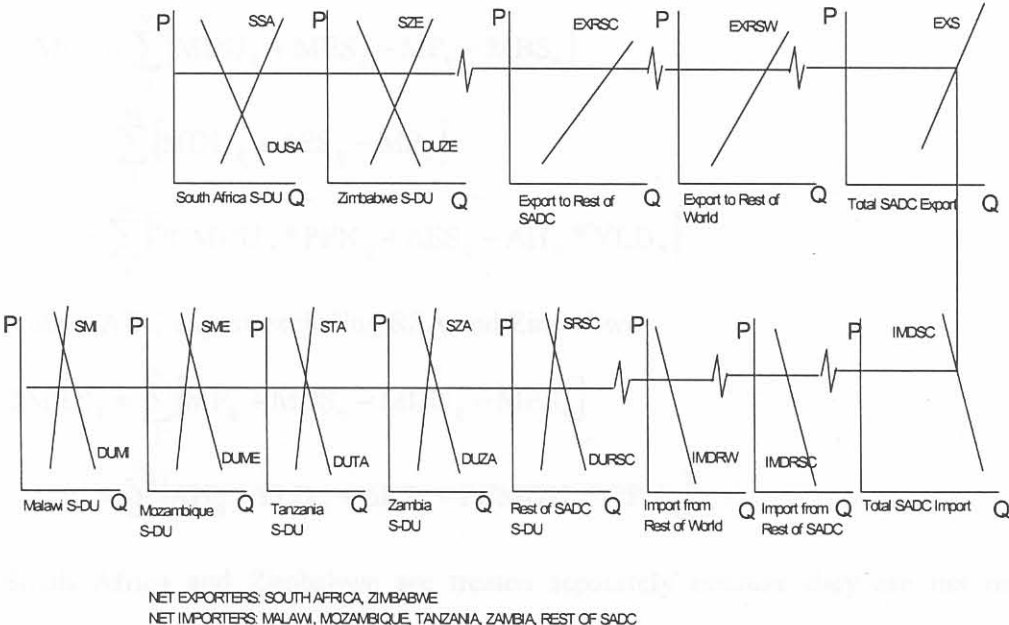
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planted. Area harvested is a good proxy for the area planted and is a reliable indicator of planned production. To capture the regional diversity in maize production, the SADC maize production block has been disaggregated to the country level so as to reflect the production characteristics in each member state. For an individual country, area harvested times yield per hectare gives the maize production. Total maize supply for an individual country is an identity and includes total production plus beginning stocks. Similarly total demand for maize for an individual country is also an identity and is equal to the sum of ending stocks, food demand and feed demand. Figure 4.1 depicts the structural components of the SADC maize model. The left hand side of figure 4.1 is the supply side and the right hand side represents the demand side. Summing across the individual countries the total SADC maize supply and total demand is computed. Equilibrium quantities and net trade are determined by equating excess demands and supplies across the region and explicitly linking price in each region to a world price. The price-linkage equation defines the degree of price transmission of external market conditions into the internal system. Trade in maize in SADC countries occurred whether or not price transmission is allowed.

Figure 4.1: Structure of the SADC Maize Model
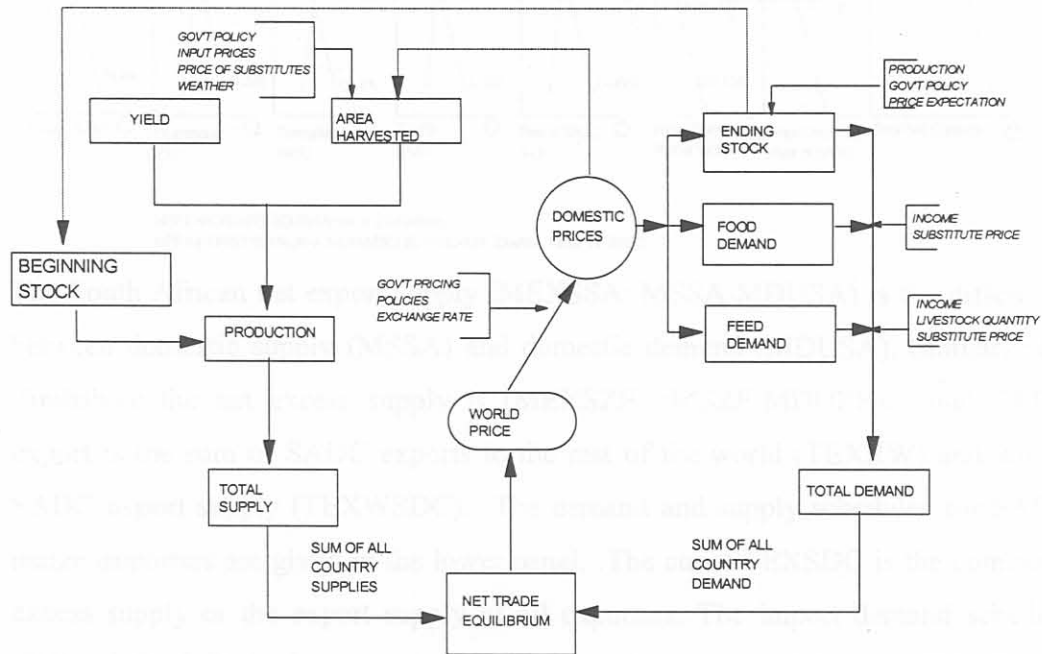
The above flow chart elaborates on the structure of the SADC maize sector discussed above and provide guidance towards the empirical estimation of the SADC maize model. The basic elements of a nonspatial equilibrium supply and demand model are illustrated in figure 4.2. The P - Q space depicts   the market at a specific point of time holding non-price factors constant. The P - Q space depicts the economic relationships, from the initial point of production to the final use of the product. The P - Q space is a convenient way of relating supply and utilization by means of price. Hence the flow chart and the P- Q space are closely linked.

Figure 4.2: Determination of Equilibrium Price and Quantities in the SADC Maize Model

NET EXPORTERS: SOUTH AFRICA, ZIMBABWE
NET IMPORTERS: MALAWI, MOZAMBIQUE, TANZANIA, ZAMBIA, REST OF SADC

The South African net export supply (MEXSSA=MSSA-MDUSA) is the difference between domestic supply (MSSA) and domestic demand (MDUSA). Similarly for Zimbabwe the net excess supply is (MEXSZE =MSZE-MDUZE). Total SADC export is the sum of SADC exports to the rest of the world (TEXRW) and within SADC export supply (TEXWSDC). The demand and supply schedules for SADC maize importers are given in the lower panel. The curve TEXSDC is the combined excess supply or the export supply of all exporters. The import demand schedule TIMDSDC of all the importers is the difference between total demand (TDSC) and

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total supply (TSSC), which includes import from the rest of the world, (IMPDW) and imports from other SADC countries (IMPDSC). The export demand schedule facing South Africa and Zimbabwe is the difference between import demand of all importers and export supply of all competitors in the region. The kinked price equilibrium is due to certain countries restrictive trade policies and pricing mechanisms, which are transmitted to domestic price from the world price variability. Trade equilibrium is achieved by the clearing of excess demands and supplies generated within each country and imports from the rest of the world and export to the rest of the world. The necessary components of
the model are given by the following equation:

Total SADC import including imports from the rest of the world

$$
\begin{aligned}
TIMD_t &= \sum_j^M \left[MDU_{tj} + MES_{tj} - MP_{tj} - MBS_{tj} \right] \\
&= \sum_j^M \left[MDU_{tj} + \Delta ES_{tj} - MP_{jt} \right] \\
&= \sum_j^M \left[PCMDU_{tj} * PPN_{tj} + \Delta ES_{tj} - AH_{jt} * YLD_{tj} \right]
\end{aligned}
$$

Total SADC export excluding RSA and Zimbabwe

$$
\begin{aligned}
TMEX_t &= \sum_j^N \left[MP_{tj} + MBS_{tj} - MDU_{tj} - MES_{tj} \right] \\
&= \sum_j^N \left[AH_{tj} * YLD_{tj} + \Delta ES_{tj} - PCMDU_{tj} * PPN_{tj} \right]
\end{aligned}
$$

South Africa and Zimbabwe are treated separately because they are net maize exporters.

South Africa excess supply

$$
\begin{aligned}
MEXSSA_t &= MSSA_t - DUSA_t \\
&= MAHSA_t * MYLDSA_t + \Delta ESSA_t - PCMDUSA_t * PPNSA_t
\end{aligned}
$$

Zimbabwe excess supply

$$
\begin{aligned}
MEXZE &= MSZE - DUZE \\
&= MAHZE_t * MYLDZE_t + \Delta ESZE_t - PCMDUZE_t * PPNZE_t
\end{aligned}
$$

Total SADC export

$$TEXSDC_t = MEXSSA_t + MEXZE_t + TMEX_t$$

Total SADC imports (SADC demand minus domestic supply)

$$TIMDSDC_t = TDSC_t - TSSC_t$$

SADC maize market equilibrium

$$NTMSDC_t = TEXSDC_t - TIMDSDC_t$$

Figure 4.3 is the P - Q space for a specific SADC maize producing country. The first block represents the farm level. The arrows indicate the directional influence of the variables, i.e. the expected sign of the parameter associated with the variable in the Total supply is equal to beginning stocks plus imports and total maize production.

The supply decision responds to the expected price of maize, a viable substitute crop, an index of input prices, and competing crop prices. The area harvested is influenced by the producer price of maize, the price of production inputs, the price of competing crops and some other exogenous variables, $AHt = F(AH_{t-1}, RPM_t, RCP_{t-1}, X_t,)$. Competing crops may differ from one country to another. Area harvested responds directly to the lagged price of maize, and consequently in the very short- run is perfectly inelastic, as represented by a vertical line. Competing crop prices, such as wheat prices, influence resource allocation, and are known as shifters. Area harvested times yield (AH_t*YLD_t), gives maize production. For most countries, the production block consists of the following: one behavioral equation, area harvested and one production identity, (maize area harvested times maize yield per hectare). The supply block consists of total domestic production, imports and beginning stocks.

Figure 4.3: P-Q Space for the Individual Country
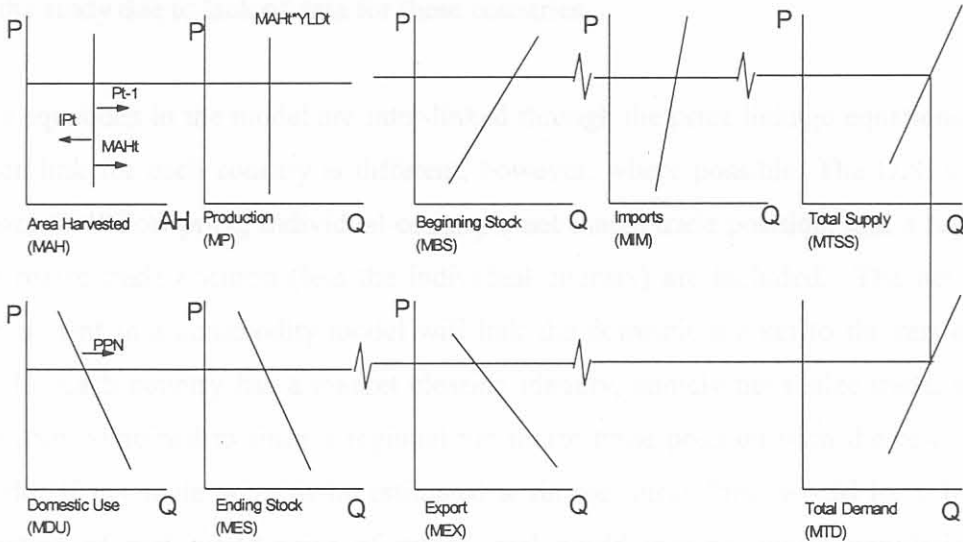
The demand block consists of the total domestic use, exports and ending stocks. The consumption demand for maize responds to the current domestic retail price of maize, with population and income as demand shifters. In general, human consumption demand for maize (in maize meal form) is influenced by the domestic price of maize and income and has a downward sloping curve. Total human consumption of maize is specified as demand per capita multiplied by total population. Net exports are used as the market clearing identity, which is implicitly influenced by the world price and vice versa. The domestic total supply block of the ith country (exporting or importing) is the cumulative sum of maize production, maize imports and beginning stocks. Assuming maize in any form is consumed by humans, maize utilization is expressed as per capita consumption, and is a function of consumer price of maize deflated by the consumer price index, and per capita GDP deflated by the GDP deflator and a time trend to account for habit formation. Ending stocks in period t are the beginning stocks for period t+1. However due to unavailability of data for ending stock, change in stocks is modeled for the each country.

The model contains seven country sub-models. The countries included in the study are Malawi, Mozambique, South Africa, Tanzania, Zambia, Zimbabwe, and the rest of the SADC, which includes Botswana, Lesotho, Mauritius, and Swaziland. Angola,

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the Democratic Republic of Congo, Namibia, and the Seychelles were not included in the study due to lack of data for these countries.

The equations in the model are inter-linked through the price linkage equations. The price link for each country is different, however, where possible. The U.S. yellow maize Gulf Port price, individual country's net maize trade position, and a regional net maize trade position (less the individual country) are included. The net trade component in a commodity model will link the domestic market to the rest of the world. Each country has a market clearing identity, namely net maize trade, which are then combined to form a regional net maize trade position with the rest of the world. If net trade were to be estimated a simple linear form would have been a function of real world price of maize, real world income, world population. A reduced from of the world price equation is derived from FAPRI's world grain model. This world price equation can be thought of as an inverted total net trade demand equation, i.e., SADC net exports and world price are negatively related.

4.5 Estimation and Model Solution Procedures

In the following subsections, the modeling approach, the estimation procedures and method for evaluating econometric models are discussed. The main issues involved in constructing a structural econometric commodity model are briefly discussed. An econometric model can be a single equation or set of equations that establish certain relationships among the institutional, definitional and behavioral variables. Broadly speaking, a forecasting model can be classified as a structural model. Econometric commodity models provide a powerful analytical tool for examining the complexities associated with agricultural commodity markets. The recent developments in statistical and economic theory and computational technologies have improved formulation and estimation methods. Statistical estimation techniques are used to estimate the equations to ascertain the relationship between the endogenous and the exogenous variables. For example, in the supply function, a positive sign is expected for output price and a negative sign for input price. At same time, the sum of the output price elasticities is expected to be equal to the sum

of the input price elasticities in absolute terms due to the fact that production functions are homogeneous of degree zero in prices.

The modeling exercise in this study starts with model specification, consisting of a set of estimable equations, which are linear in variables. The rational for this simple specification is that there is no *a priori* information as to the functional form. Second, the statistical estimation procedures are best developed for linear models, which help in computing the desirable analytical characteristics of the equations. For example the reduced form of linear models are easily estimable, and dynamic properties of the model can be evaluated readily. Reliability statistics and other test statistics are easily available to test the forecasts in the case of a linear reduced form. With these functions, the problems of structural change and updating the model can be handled easily.

The equations in the structural econometric model can be classified as either behavioral equations or as identities. Behavioral equations are based on economic theory and are estimated from historical data using statistical estimation tools. Identities are equations that hold true by definition. The relationships among different variables and the causal effects of these variables are explained by these equations. The sign of estimated parameters associated with the variables in the behavioral equations indicate the directional influence of the variables. For example, a negative sign is associated with the estimated parameters for the competing crops prices variable in an area-harvested equation.

The behavioral equations contain both endogenous and exogenous variables. Endogenous variables are explained by behavioral equations and/or identities. For example, in our case area harvested and maize production are all endogenous variables. Exogenous variables are variables that are not explained within the model and are considered to be known. For example, in this case, yield, GDP, the exchange rate and policy variables are considered as exogenous variables.

As stated earlier the structural econometric model of SADC maize sector consists of the maize production block, a demand block, a market clearing identity- net trade and the price linkage. All in all, there are 64 equations in the model. Out of these, 27 are behavioral equations. The behavioral equations consist of seven area harvested equations, seven change in stock equations, a per capita consumption equation for each of the seven countries, and a price linkage equation for all the countries except the rest of SADC which is directly linked to the U.S. Gulf port price. The remaining 37 equations are identities. The identities include production, yield, total domestic consumption, local net trade, regional net trade for each country, and a market clearing identity with the rest of the world. The total SADC supply is the sum of total SADC maize production, imports, and stock change.

All equations are estimated using the classical ordinary least squares (OLS) method. This step is based on *a priori* knowledge and economic theory, which helps in the identification of variables to be used in the behavioral equation. The OLS estimation technique chooses the line that minimizes the sum of squared deviation of the observation from the line. Let $Y = X\beta + \xi$ be linear model where Y is (nx1) column vector of n endogenous variable, X is a (n x m) matrix of exogenous variables, β is (mx1) vector of parameters to be estimated and ξ is (nx1) vector of normally, identically distributed error ($\xi \cup N(0, \sigma^2 I)$ where I is a (nxn) identity matrix. The OLS estimation technique is to minimize $\xi'\xi = Y'Y - 2\beta'X'Y + \beta'X'X\beta$. Solving for $\hat{\beta}' = (X'X)^{-1}X'Y$, which is unbiased estimator of the vector of unknown parameters β. In summary, under the above assumptions the parameters estimates of β are the best linear unbiased estimators (Greene, 2000). Once the coefficients of the equations are estimated individually, the equations and variables, which will forms part of the system and which satisfactorily explain inter-linkages among sectors are retained.

The econometric model of the SADC maize sector expresses interdependence of variables that influence the supply and utilization of maize in the SADC through a

system of simultaneous equations. Each equation in such a system describes a different relationship among a different set of the variables in the system. However all of these relationships are assumed to hold simultaneously. The OLS method of estimation is inadequate. The use of OLS may yield biased and inconsistent estimates unless the model is exactly identified. Various estimation procedures such as the two least square (2SLS), three stage least square (3SLS), instrumental variable methods (IV), full information maximum likelihood (FIML), and indirect least square method (ILS) are used to eliminate the simultaneous bias. Among these, the most common estimation technique for a simultaneous model is the 2SLS method. The 2SLS estimates is a useful estimation procedure for an over identified model. This estimation procedure uses information available from the specification of each equation to obtain unique estimates of each of the parameters in the system. The 2SLS estimates are both consistent and efficient. For the purpose of this study, the 2SLS estimation procedure is used. The results of the 2SLS estimation are reported and discussed in Chapter five.

The next step is to solve or simulate the model. The Gauss-Seidel solution algorithm is used to solve the model's simultaneous system of equations. For an in depth discussion of this algorithm, the reader is directed to Fair (1984). The underlying assumption for the Gauss-Seidel Algorithm is that the error term in each behavioral equation is zero. Since the model in this study is linear, the expected value of the error term is zero by the classical assumption. Hence, solving the model results in the predicted values of endogenous variable being equal to their expected values. The Gauss-Seidel technique requires that the equations in the model be rewritten with each endogenous variable on the left hand side of the equations.

4.6 Validation Tests

Once the model is solved, then the model performance is tested. The model is validated to verify its ability to replicate historical characteristics. Two simulation methods have been proposed for the validation of simultaneous equation systems. They are static and dynamic simulations over the historical period. A static

simulation is one where the historical values of the lagged endogenous variables are used each year the model is solved. In contrast, in the dynamic simulation, the lagged endogenous variables are assigned their estimated values in the initial period for which the model is solved. In successive periods, the previous-period solution values are used for the lagged endogenous variables. Thus, the model feeds off itself in generating estimates over the validation period. For the static solution (or simulation), the actual values of the predetermined variables are substituted in the equation. For dynamic solution the solved value from the previous period is used for the lagged endogenous variable (Greene 2000).

A naive method of evaluation is the visual inspection of a graphical plot of the actual values and the simulation replicates the actual values. This will give an indication of whether the historical simulation replicated the actual values. This is performed, by checking how well the simulation captures the turning points. Appendix A gives a graphical representation of the historical simulation.

Various statistical measures have been used to test the model. The most commonly used is the root-mean-square (RMSE) simulation error. The RMSE of the simulation for an endogenous variable for example MAH_t (Area Harvested) is defined

as $RMSE = \sqrt{\dfrac{1}{N}\sum_{1}^{N}\left[MAH_t^s - MAH_t^A\right]^2}$ where MAH_t^S is the simulated value of

maize area harvested (MAH) in period t and $MAH_t{}^A$ is the actual value of MAH and N is the number of observations. The RMSE measures the deviation of the simulated endogenous variable from the actual values. When the RMSE is compared with the average size of the variable BAH, the RMSE percentage error (RMSE%) is used

where $RMSE\% = \sqrt{\dfrac{1}{N}\sum_{1}^{N}\left[\dfrac{MAH_t^s - MAH_t^A}{MAH_t^A}\right]^2}$. Two other measures used are the

mean simulation error (MSE), defined as $MSE = \dfrac{1}{N}\sum_{1}^{N}\left[MAH_t^s - MAH_t^A\right]^2$ and the

associated means simulation percentage error (MSE%) MSE%

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$$\text{MSE\%} = \frac{1}{N} \sum_{1}^{N} \left[\frac{\text{MAH}_t^s - \text{MAH}_t^A}{\text{MAH}_t^A} \right]^2$$. Another statistic useful in evaluating the

performance of the historical simulations or ex post forecasts is the mean absolute

error (MARE), which is defined as follows, $\text{MARE} = \frac{1}{N} \sum_{i}^{N} \left| \frac{\text{MAH}_t^s - \text{MAH}_t^A}{\text{MAH}_t^A} \right|$. This

statistic is an increasing function of the absolute value of the model's prediction error

and is independent of the units of measurements. If MARE equals 0, the model fits

the historical data perfectly. Thus, MARE is bounded from zero but not from the

above and increases as the absolute value of the estimation error increases. Theil

(1971) proposed the widely used Theil's inequality coefficient,

$$U = \frac{\sqrt{\frac{1}{N} \sum_{1}^{N} \left[\text{MAH}_t^s - \text{MAH}_t^A \right]^2}}{\sqrt{\frac{1}{N} \sum_{1}^{N} \left(\text{MAH}_t^A \right)^2} \sqrt{\frac{1}{N} \left(\text{MAH}_t^s \right)^2}}$$

where U lies between zero and 1. If U=0, then there is a perfect fit. Whereas if U =1,

the model has a bad fit. Furthermore, Thiel's U statistics can be decomposed into

three components: the bias proportion U^M, the variance proportion U^S and the

covariance proportion U^C. It can be shown that $U^M + U^S + U^C = 1$. U^M measures the

extent to which the average values of simulated and actual values of the variable

differ. A value of U^M close to zero is desirable, and if U^M is not close to zero, this

indicates the presence of a systematic error. U^S indicates the model's capability to

replicate the degree of variability in the variable. A large U^S implies that the actual

series has fluctuated considerably whereas the simulated variable has not or vice

versa. U^C measures the unsystematic error. A values of $U^M = U^S = 0$ and $U^C = 1$ is

ideal. (Pindyck and Rubinfeld, 1998)

Apart from the above statistical measures, a method commonly used to evaluate the

properties of a model is the computation of impact multipliers from a deterministic

simulation. Multiplier analysis is concerned with the evaluation of changes in

endogenous variables caused by changes in an exogenous variable. Both static and

dynamic multipliers are considered here. Short-run multipliers explain changes in

endogenous variable over a single or specific period of time and dynamic multipliers explain changes that have a cumulative effect over a period of time. The results of this test are discussed in Chapter five. In this study, a further step is involved which is to formulate a baseline. A baseline is a prediction of the endogenous variables based on the expected changes in exogenous variables in the future. Finally, the model is used to evaluate the consequences of policy changes on the SADC maize sector.