Chapter 9

Study and Results

9.1 Aim

It is generally assumed that the current implied volatility level is the best proxy for the future level of implied volatility and hence the future price of an option. In this chapter, the GARCH option pricing method is applied to the implied volatility history of a warrant. The method is as described in section 8.2.2, where I propose calibrating the GARCH process over the "historical" implied volatility of the underlying financial instrument, in this case stock.

In this study, the current implied volatility level is compared to the GARCH level or equivalently, the predicted future price of the option compared to the predicted price of under the GARCH option pricing method

The predicted future price of a European option, to avoid arbitrage, is its current value adjusted for the relevant risk-free interest rate.

9.2 Methodology and Data

JSE warrants are generally short dated, that is of maturity less than one year. An option pricing model must thus be able to price a warrant, with as little calibration to historical data as possible. Here, a 30-day period of calibration to implied volatility was decided on in each case.

Although the warrants market is more liquid than the options market of SAFEX, there are still days where no new trade takes place in a specific warrant. The result, is that after a sharp drop in the price in the underlying equity, the intrinsic value of the replicating portfolio may be greater than the market price of the an untraded put warrant. The implied volatility of that warrant is thus undefined at such a date.

In this study, the chosen warrants where priced in a rolling window of one day (with a thirty day history each), from approximately thirty days after they where first traded, up to a date where either the implied volatility

is undefined or zero, the warrant reaches maturity or $2002/11/27^1$.

The 11 to 20 day ahead values of both the forecasts due to the actual warrant prices and the GARCH option prices are compared to the actual warrant prices of 11 to 20 days ahead. The measurement over a 10 day period gives a better indication of the forecasting power of the two methods.

The following put warrants were selected:

Result no:	Warrant	Underlying	Issuer	Date From	To
1	3ASAIB	ABSA	Investec Bank	2002/05/15	2002/11/06
2	3ASAUB	ABSA	UBS	2002/01/02	2002/11/06
3	5ASAIB	ABSA	Investec Bank	2002/01/08	2002/11/06
4	2AGLUB	Anglo American	UBS	2001/06/08	2002/04/04
5	3AGLIB	Anglo American	Investec Bank	2001/09/25	2002/02/27
6	7AGLIB	Anglo American	Investec Bank	2002/02/22	2002/07/25
7	BAGLIB	Anglo American	Investec Bank	2002/07/29	2002/09/17
8	3NEDUB	Nedcor	UBS	2002/01/02	2002/08/19
9	6NEDIB	Nedcor	Investec Bank	2002/07/08	2002/10/03
10	6NEDSG	Nedcor	Societe General	2002/08/23	2002/09/20
11	3OMLUB	Old Mutual	UBS	2002/01/02	2002/06/24
12	4OMLSG	Old Mutual	Societe General	2002/08/16	2002/09/16
13	5OMLIB	Old Mutual	Investec Bank	2002/05/15	2002/06/13
14	3SAPIB	Sappi	Investec Bank	2002/04/16	2002/10/07
15	3SAPUB	Sappi	UBS	2002/01/02	2002/11/06

Each warrant can be categorized in terms of time to maturity and moneyness:

- Time to maturity of a warrant is the amount of days left in the life of the warrant. A warrant's implied volatility tends to increase dramatically 70 to 60 days and closer, to maturity. It seems sensible to categorize results in terms of the time to maturity of the warrant. The two categories are maturity of less than 70 days and maturity of 70 days and more.

- Moneyness is defined as the stock price divided by the exercise price of a warrant. A put warrant is defined to be "out of the money" when the moneyness ratio is more that 1.1, "at the money" if the ratio is between 0.9 and 1.1 and "in the money" if the ration is less than 0.9.

9.3 Measures of Results

The accuracy of the implied volatility method and the GARCH option pricing method is measured in the following way:

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[1]The last date on which data was captured.

1. The current market implied volatility, $I(t)$ and the GARCH option price, $\sigma^2(t)$ at time t are adjusted to the (annual) risk-free rate r for each day of the 10 day period starting in 11 days,

$$
\begin{aligned}
I(t,i) &\equiv e^{r \times (10+i)/252} I(t) \\
\sigma^2(t,i) &\equiv e^{r \times (10+i)/252} \sigma^2(t)
\end{aligned}
$$

for $i = 1, \ldots, 10$.

2. The absolute percentage deviations between the two forecasts,

$$I(t,i) \text{ and } \sigma^2(t,i)$$

and the actual observed implied volatility in the market,

$$I(t+i+10,0)$$

is taken for each day of the 10 day period and weighed as follow

$$
\begin{aligned}
\Delta^i_{Actual} &= \frac{|I(t,i) - I(t+i+10,0)|}{I(t+i+10,0)} \\
\Delta^i_{GARCH} &= \frac{|\sigma^2(t,i) - \sigma^2(t+i+10,0)|}{I(t+i+10,0)}
\end{aligned}
$$

3. The following risk-measures are determined

$$
\begin{aligned}
\Delta^{below}_{Actual} &= \sum_{i=1}^{10} \Delta^i_{Actual} \mathbf{1}_{I(t,i)<I(t+i+10,0)} \\
\Delta^{above}_{Actual} &= \sum_{i=1}^{10} \Delta^i_{Actual} \mathbf{1}_{I(t,i)>I(t+i+10,0)} \\
\Delta^{below}_{GARCH} &= \sum_{i=1}^{10} \Delta^i_{GARCH} \mathbf{1}_{\sigma^2(t,i)<I(t+i+10,0)} \\
\Delta^{above}_{GARCH} &= \sum_{i=1}^{10} \Delta^i_{GARCH} \mathbf{1}_{\sigma^2(t,i)>I(t+i+10,0)}
\end{aligned}
$$

where and $\mathbf{1}$ is an indicator function.

The measure $\Delta^{below}_{\bullet}$ ($\Delta^{above}_{\bullet}$) is the sum of the absolute percentage deviations below (above) the actual implied volatilities. These measures don't only measure the absolute deviation, but also measures if the forecasts are above or below the actual implied volatilities. The sum of the measure $\Delta^{below}_{\bullet}$ and the measure $\Delta^{above}_{\bullet}$ give the absolute deviation.

9.4 Results

The results are given for the 15 mentioned warrants

- The columns denoted by time to maturity and moneyness are as explained in section 9.2.

- The column named "Observations" indicates the amount of separate tests done in each category of the specific warrant.

- The columns marked less and more are as explained in section 9.3.

- The following abbreviations are used:

    - ITM: In the money
    - ATM: At the money
    - OTM: Out of the money
    - CTM: Close to maturity
    - FFM: Far from maturity

9.4.1 The Results:

Warrant Name: 3ASAIB						
Time to maturity (days)	Observations	Moneyness	GARCH		Actual	
			Below	Above	Below	Above
Less than 70	36	In	2.47	0.05	0.19	0.20
	0	At	-	-	-	-
	0	Out	-	-	-	-
70 and Above	83	In	0.59	1.00	0.86	0.85
	0	At	-	-	-	-
	0	Out	-	-	-	-
Description:	The GARCH model predicts ITM, CTM warrants worse than the Actual model does. The GARCH models underpredicts ITM, CTM and overpredicts ITM FFM warrants. The GARCH model predicts ITM, FFM warrants slightly better than the Actual model does.					

Warrant Name: 3ASAUB						
Time to maturity (days)	**Observations**	**Moneyness**	**GARCH**		**Actual**	
			Below	**Above**	**Below**	**Above**
Less than 70	49	In	0.17	1.86	0	1.61
	0	At	-	-	-	-
	0	Out	-	-	-	-
70 and Above	160	In	0.89	0.98	0.92	0.93
	0	At	-	-	-	-
	0	Out	-	-	-	-
Description:	The GARCH model predicts ITM, CTM warrants worse than the Actual model does. Both the GARCH and Actual models overpredicts ATM and OTM CTM warrants. The GARCH model predicts ITM, FFM warrants slightly worse than the Actual model does.					

Warrant Name: 5ASAIB						
Time to maturity (days)	Observations	Moneyness	GARCH		Actual	
			Below	Above	Below	Above
Less than 70	37	In	0.03	5.53	0.01	5.54
	0	At	-	-	-	-
	0	Out	-	-	-	-
70 and Above	45	In	0.63	0.70	0.79	0.62
	0	At	-	-	-	-
	0	Out	-	-	-	-
Description:	The GARCH model predicts ITM, CTM warrants slightly worse than the Actual model does. The GARCH model predicts ITM, FFM warrants slightly better than the Actual model does.					

Warrant Name: 2AGLUB						
Time to maturity (days)	Observations	Moneyness	GARCH		Actual	
			Below	Above	Below	Above
Less than 70	0	In	-	-	-	-
	0	At	-	-	-	-
	0	Out	-	-	-	-
70 and Above	8	In	1.31	1.05	0	2.52
	90	At	1.26	8.71	0.01	10.36
	5	Out	1.49	7.03	0	6.92
Description:		The GARCH model predicts ITM, FFM warrants worse than the Actual model does.				
		The GARCH model predicts ATM, FFM warrants better than the Actual model does.				
		Both models overpredicts ATM and OTM FFM warrants.				
		The GARCH model predicts OTM, FFM warrants better than the Actual model does.				

Warrant Name: 3AGLIB						
Time to maturity (days)	**Observations**	**Moneyness**	**GARCH**		**Actual**	
			Below	**Above**	**Below**	**Above**
Less than 70	0	In	-	-	-	-
	0	At	-	-	-	-
	50	Out	2.61	2.90	0.03	6.13
70 and Above	0	In	-	-	-	-
	10	At	0	5.28	0	5.68
	43	Out	0.02	14.63	0	14.51
Description:	The GARCH model predicts OTM, CTM warrants better than the Actual model does. The GARCH model predicts ATM, FFM warrants slightly better than the Actual model does. Both models overpredicts ATM and OTM FFM warrants. The GARCH model predicts OTM, FFM warrants slightly worse than the Actual model does.					

Warrant Name: 7AGLIB						
Time to maturity (days)	Observations	Moneyness	GARCH		Actual	
			Below	Above	Below	Above
Less than 70	7	In	3.72	0	3.44	0
	24	At	2.76	0.03	3.04	0.01
	0	Out	-	-	-	-
70 and Above	0	In	-	-	-	-
	68	At	0.31	1.32	0.49	0.98
	6	Out	0.53	0.19	0.65	0.15

Description:

The GARCH model predicts ITM, CTM warrants better than the Actual model does.

The GARCH model predicts ATM, CTM warrants slightly better than the Actual model does.

Both the GARCH and Actual models underpredicts ATM and OTM CTM warrants.

The GARCH model predicts ATM, FFM warrants slightly worse than the Actual model does.

The GARCH model predicts OTM, FFM warrants slightly better than the Actual model does.

Warrant Name: BAGLSG						
Time to maturity (days)	**Observations**	**Moneyness**	**GARCH**		**Actual**	
			Below	**Above**	**Below**	**Above**
	7	In	3.72	0	3.44	0
Less than 70	24	At	2.76	0.03	3.04	0.01
	0	Out	-	-	-	-
	0	In	-	-	-	-
70 and Above	68	At	0.31	1.32	0.49	0.98
	6	Out	0.53	0.19	0.65	0.15
Description:	The GARCH model predicts ITM, CTM warrants worse than the Actual model does. The GARCH model predicts ATM, CTM warrants better than the Actual model does. Both the GARCH and Actual models underpredicts ATM and OTM CTM warrants. The GARCH model predicts ATM, FFM warrants worse than the Actual model does. The GARCH model predicts OTM, FFM warrants slightly better than the Actual model does.					

Warrant Name: 3NEDUB						
Time to maturity (days)	**Observations**	**Moneyness**	**GARCH**		**Actual**	
			Below	**Above**	**Below**	**Above**
Less than 70	0	In	-	-	-	-
	0	At	-	-	-	-
	0	Out	-	-	-	-
70 and Above	157	In	0.84	0.88	0.65	0.68
	0	At	-	-	-	-
	0	Out	-	-	-	-
Description:	The GARCH model predicts ITM, FFM warrants worse than the Actual model does.					

Warrant Name: 6NEDIB						
Time to maturity (days)	Observations	Moneyness	GARCH		Actual	
			Below	Above	Below	Above
Less than 70	17	In	0.35	0.33	0	0.83
	0	At	-	-	-	-
	0	Out	-	-	-	-
70 and Above	45	In	1.50	0.09	1.45	0.13
	0	At	-	-	-	-
	0	Out	-	-	-	-
Description:	The GARCH model predicts ITM, CTM warrants better than the Actual model does. The GARCH models underpredicts overpredicts ITM, FFM warrants. The GARCH model predicts ITM, FFM warrants slightly worse than the Actual model does.					

Warrant Name: 6NEDSG						
Time to maturity (days)	Observations	Moneyness	GARCH		Actual	
			Below	Above	Below	Above
Less than 70	7	In	0.73	0.11	0.76	0.13
	0	At	-	-	-	-
	0	Out	-	-	-	-
70 and Above	14	In	0.47	0.46	0.66	0.31
	0	At	-	-	-	-
	0	Out	-	-	-	-
Description:	The GARCH model predicts ITM, CTM warrants worse than the Actual model does. The GARCH model predicts ITM, FFM warrants slightly better than the Actual model does.					

Warrant Name: 3OMLUB						
Time to maturity (days)	**Observations**	**Moneyness**	**GARCH**		**Actual**	
			Below	**Above**	**Below**	**Above**
Less than 70	0	In	-	-	-	-
	0	At	-	-	-	-
	0	Out	-	-	-	-
70 and Above	118	In	0.18	1.99	0.42	1.28
	0	At	-	-	-	-
	0	Out	-	-	-	-
Description:	The GARCH model predicts ITM, FFM warrants worse than the Actual model does. Both the GARCH and Actual models overpredicts ITM, FFM warrants.					

Warrant Name: 4OMLSG						
Time to maturity (days)	**Observations**	**Moneyness**	**GARCH**		**Actual**	
			Below	**Above**	**Below**	**Above**
Less than 70	3	In	0.73	0.07	1.09	0
	0	At	-	-	-	-
	0	Out	-	-	-	-
70 and Above	19	In	2.56	0	2.43	0
	0	At	-	-	-	-
	0	Out	-	-	-	-
Description:	The GARCH model predicts ITM, CTM warrants better than the Actual model does.					
	The GARCH model predicts ITM, FFM warrants worse than the Actual model does.					
	Both the GARCH and Actual models overpredicts ITM, FFM and CTM warrants.					

Warrant Name: 5OMLIB						
Time to maturity (days)	**Observations**	**Moneyness**	**GARCH**		**Actual**	
			Below	**Above**	**Below**	**Above**
Less than 70	0	In	-	-	-	-
	0	At	-	-	-	-
	0	Out	-	-	-	-
70 and Above	41	In	2.13	0.00	2.43	0
	0	At	-	-	-	-
	0	Out	-	-	-	-
Description:	The GARCH model predicts ITM, FFM warrants better than the Actual model does. Both the GARCH and Actual models overpredicts ITM, FFM warrants.					

Warrant Name: 3SAPIB						
Time to maturity (days)	Observations	Moneyness	GARCH		Actual	
			Below	Above	Below	Above
Less than 70	18	In	0.23	0.15	0	0.03
	0	At	-	-	-	-
	0	Out	-	-	-	-
70 and Above	103	In	1.31	0.21	0.72	0.35
	0	At	-	-	-	-
	0	Out	-	-	-	-
Description:	The GARCH model predicts ITM, CTM warrants worse than the Actual model does. The GARCH model predicts ITM, FFM warrants worse than the Actual model does. Both the GARCH and Actual models overpredicts ITM, FFM warrants.					

Warrant Name: 3SAPUB						
Time to maturity (days)	Observations	Moneyness	GARCH		Actual	
			Below	Above	Below	Above
Less than 70	47	In	2.46	3.26	0	6.21
	0	At	-	-	-	-
	0	Out	-	-	-	-
70 and Above	162	In	3.12	1.87	1.09	2.73
	0	At	-	-	-	-
	0	Out	-	-	-	-
Description:	The GARCH model predicts ITM, CTM warrants better than the Actual model does. The GARCH model predicts ITM, FFM warrants worse than the Actual model does. Both the GARCH and Actual models predicts ITM warrants poorly.					

9.4.2 Conclusion to Results

In this study, the results due to implied volatility or actual observed market prices performed marginally better than the GARCH prices in the forecasting of market prices of 11 to 20 days in the future.

The forecast due to the actual observed market prices performed marginally better in both time to maturity classes for "in the money" warrants.

The GARCH option pricing forecasts where marginally better for "at the money" warrants with less than 70 days to maturity and "out of the money" warrants with more than 70 days to maturity.

9.4.3 Comments on Study and Results

No specific GARCH or ARMA process can ever be used to fully explain market dynamics. A GARCH process can for instance be useful only in forecasting options on certain assets, in certain market conditions, with a certain range of maturities. Thus plainly put, if an (implied) volatility process follows an approximate GARCH process, then use the GARCH process or option pricing methodology to forecast option prices, if not don't.

A general study, as done here defeats the purpose of GARCH processes to a certain extent, since a GARCH process must be tailor made to the specific market instrument and conditions.

This study does however show that GARCH series can be fitted to implied volatility with some success.