Pricing options under stochastic volatility

by

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Submitted in partial fulfillment of the requirements for the degree

Magister Scientiae in Mathematics of Finance

in the Faculty of Natural & Agricultural Sciences

University of Pretoria

Pretoria

January 2003

Summary

In this dissertation some of the real world deviations from the assumptions made in the Black-Scholes option pricing framework is investigated. Special attention is paid to volatility, the standard deviation of stock price returns. Unlike the assumption of constant volatility of increments in Brownian motion, volatility in the market is stochastic. Market models allowing for stochastic volatility are no longer complete as in the Black-Scholes framework. Options in incomplete markets are harder to price since investors demand higher returns for taking additional risk.

Duan (1995) proposed an option pricing measure for incomplete markets, due to stochastic volatility, called the Local Risk-Neutral Valuation Relationship (LRNVR). Under the LRNVR, the local risk neutral measure (Q) is equivalent to the real world measure (P), the conditional expected return under the Q measure equals the risk-free rate and the conditional one period ahead variances under both measures are equal, P almost surely. The LRNVR holds for consumers with familiar utility functions.

Stock returns are assumed to follow a Generalized Autoregressive Conditional Heteroscedastic (GARCH) process. This process is a discrete time statistical time series that is calibrated over stock returns. In this dissertation the LRNVR and related option pricing methodology is comprehensively investigated.

Warrants traded on the JSE Securities Exchange violates the Black-Scholes assumptions in two additional ways, short selling is restricted and the market is somewhat illiquid. One of the results of these violations is that the standard deviation and the implied volatility, volatility implied by the market price of the option, are out of sync. The implied volatility tends to be higher than the volatility of stock market returns.

In this dissertation the GARCH option pricing process is applied to the implied volatility of the warrant instead of the stock price process, as done by Duan. This method compares well with the use of implied volatility to price warrants.

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Acknowledgements

I would like to thank my supervisors, Professor Frik van Niekerk of the University of Pretoria and Johan Verwey of Gensec Bank for their support and guidance. My thanks to Professor Dawie de Jongh for advice and references to literature is also due. I am also grateful to my family for their motivation and support.

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