Chapter 3

THE MODEL

3.1. Introduction

Löfgren et al (2001) developed a standard CGE model to be used for policy analysis by the International Food Policy Research Institute (IFPRI). IFPRI, the World Bank and other researchers extensively use this model to analyze a wide range of policy issues. The model follows the neoclassical-structuralist approach and specifically focuses on developing countries. The data requirement of this model is a disaggregated SAM, as well as a set of trade elasticities. The model follows the SAM disaggregation of activities, commodities, factors, households and other institutions. The equations of the model simulate the actions of the institutions in this economy. The model is solved through a set of linear and non-linear equations using GAMS. Economical behavior is captured by parameters, some parameters are calibrated within the model, while the others are estimated using econometric techniques. Production and consumption decisions are simulated through optimizing first-order conditions subject to a set of constraints. A set of equations, called the system constraints, balance the goods and factor markets as well as the macroeconomic aggregates savings-investment, the government and the current account from the rest of the world.

The discussion in this chapter is mainly from the paper published by Löfgren et al (2001) on the standardized model. The mathematical model is included in the appendix for readers interested in CGE modeling.

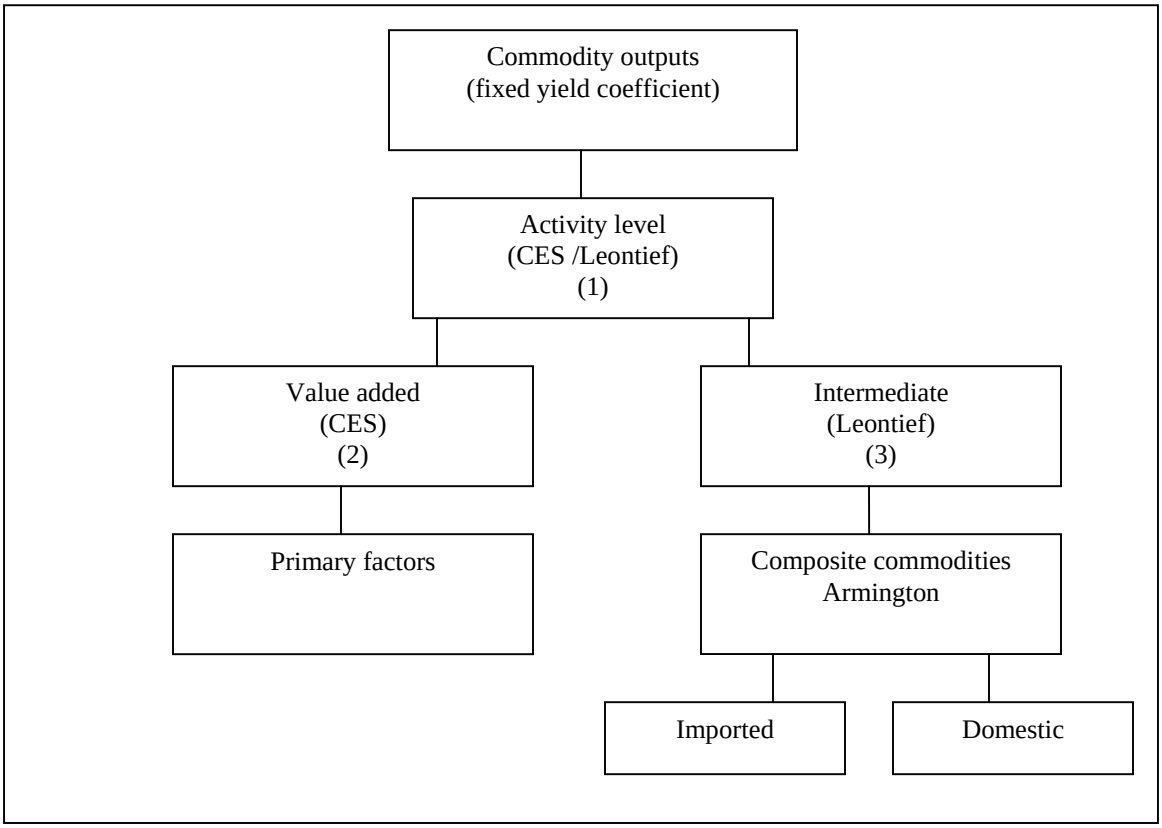
3.2. Activities, Production and Factor Markets

Each producer, represented by an activity, assumes to maximize profits. Profits are defined as the difference between revenue, factor cost and intermediate inputs. The level

of activity, yields, and commodity prices at the producer level, in turn, defines revenue. Profits are maximized subject to a production function.

This model allows for a separation between production activities and commodities. The model permits any activity to produce multiple commodities and any commodity to be produced by multiple activities. Each activity produces one or more commodities (and any commodity may be produced by more than one activity) according to a fixed yield coefficient. A nested hierarchy is used to illustrate the production decisions. Figure 3.1 illustrates the different production decisions by the producer:

Figure 3.1: Production Technology from (Löfgren et al, 2001)



(1) The producer must decide on the combination of intermediate inputs and factors to be used in the production process, according to their substitutability. This is known as the top-level activity function. Usually a Leontief function is applied to

this problem, but a CES function may also be used in identified sectors if there is sufficient evidence that suggests that the available techniques permit the aggregate mix between factors and intermediate inputs. For this application a Leontief function will be used for all the sectors. This is in line with a model constructed by Lewis (2001) for South Africa to look at the implications of HIV/Aids in South Africa.

(2) The second decision involves the combination of factors to be used. Each activity uses a set of factors up to the point where the marginal revenue product of each factor is equal to its wage. Wages may differ across activities. The model also allows for wage distortions across activities due to exogenous causes, for example discrimination, status, and health.

(3) The aggregate intermediate input decision is a Leontief function.

The modeler should choose appropriate factor market closures. The standard model provides three alternative factor market closures. The alternatives are discussed in the section on closures.

3.3. Institutions

The institutions in this model include households, enterprises, the government, and the rest of the world. Households are disaggregated in terms of income, as it is important to be able to determine the welfare impact on the different income groups of a change in VAT. It is, however, questionable whether or not it is appropriate to disaggregate households on the basis of income when one is concerned with income distribution. This analysis wants to determine what the effect of a change in VAT on household income is, but by classifying households into income deciles defeats the purpose. Other possible classifications to consider are classification by gender, race, and location.

Households receive income from the factors of production as well as transfers from the other institutions. Transfers from the rest of the world are fixed in foreign currency. Households use their income to consume, save, pay direct taxes, and for transfers to other institutions. Enterprises' income is also generated from factors of production (mostly in the form of returns on capital) as well as transfers from other institutions. Enterprises pay direct taxes, save, and may also transfer income to other institutions. Enterprises transfer surpluses to households. Enterprises do not consume. The income of the government is mainly from taxes. Taxes are received from other institutions (direct tax) as well as from indirect taxes. The government uses its income for government consumption, subsidies (producers' and exports) and for transfers to other institutions. The government's expenditure exceeds its income resulting in a budget deficit (or negative savings). The foreign sector's transaction with this economy is in terms of imports and exports and transfers to and from the rest of the world. Foreign savings (or the current account deficit) is the difference between foreign currency spending and receipts.

A set of closures balances the government, savings and investment, and the rest of the world. Again the different alternatives will be discussed in the section on closures.

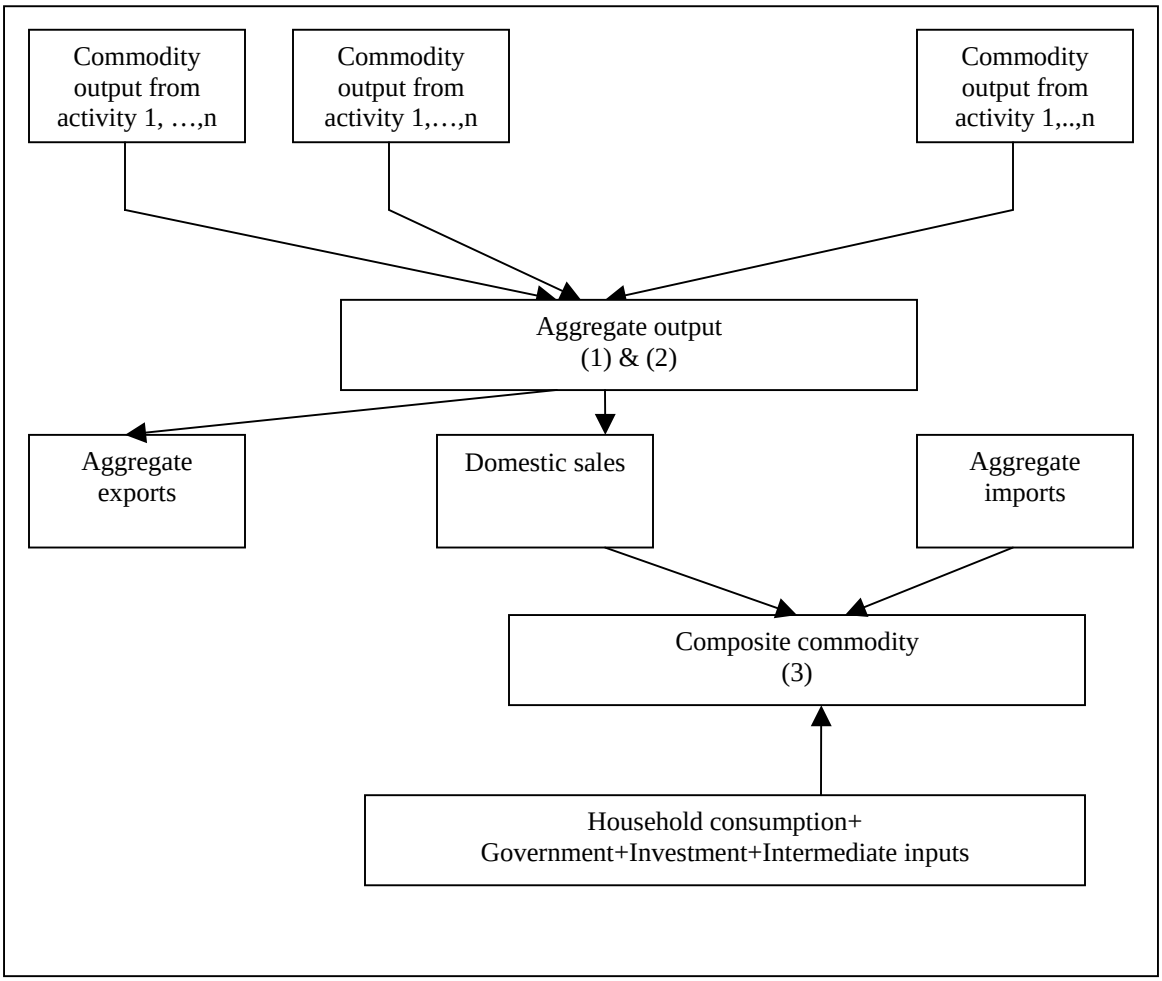
3.4. Commodity Markets

In this model all commodities enter the market. The producers' aim is to minimize the cost of supply subject to the substitutability between the different commodities. Figure 3.2 illustrates the decision of which commodities to produce:

(1) The first decision is to decide on the commodity by choosing the relevant activity. A CES function is modeled as aggregation function. The demand for the output of each activity is modeled by minimizing the cost of supplying a given quantity of aggregate output subject to its CES function. Activity-specific commodity prices implicitly clear the market for each disaggregated commodity.

(2) The second decision involves the allocation of domestic output between the domestic market and exports. Suppliers want to maximize sales revenue for any aggregated output level, subject to the imperfect transformability between exports and domestic sales. A CET function is modeled. The price received by domestic suppliers for exports is expressed in domestic currency and includes export taxes. The supply price for domestic sales is equal to the price paid by domestic demanders. Some commodities are not exported and total supply is passed to the domestic market. Domestic demand consists of household consumption, government consumption, investment, and intermediate inputs. Exports demands are infinitely elastic at given world prices.

Figure 3.2: Commodity Flows from Adapted from (Löfgren et al, 2001)



(3) All demand is for a composite good made up from domestic supply and imports. This also is the third decision, whether to import, or consume domestic production. The assumption is that domestic demanders minimize cost subject to the imperfect substitutability between domestic supply and imports. This is known as the Armington (1969) function and is a CES type function. Where domestic supply is zero, total demand comes from imports and where imports are zero total demand is supplied by domestic production. International markets showing infinitely elastic supply at world prices demand imported commodities. The import prices paid includes import tariffs.

There is no assumption of perfect substitutability between exports and domestic supply or between imports and domestic supply in this model. A set of elasticities of substitution specifies the substitutability of commodities.

3.5. Closures

3.5.1 Factor Market

The standard CGE model provides different alternative closures for the factor market, savings-investment, the government and the rest of the world. The most applicable closure may then be used for this application.

Factor market closures used in this CGE model firstly assumes capital and high skilled labor are fully employed and activity specific. The model forces each activity to hire the base-year quantity of labor. In this case the quantity of factors demanded is fixed as well as the economy-wide wage. The activity-specific wage terms and the supply variable are flexible. This closure is preferred for short-run analysis, or/and when the quality of factors differs significantly over sectors. Therefore this closure assumes full employment and that the factors are activity-specific.

Secondly this CGE model assumes that semi-skilled and low-skilled labor is unemployed and mobile. This forces the model to take the quantity supplied of each factor as fixed. The economy-wide wage variable is then allowed to vary so that the quantity of factors demanded is equal to the quantity of factors supplied. Each activity pays an activity specific wage that is determined by the economy-wide wage and the (fixed) wage distortion variable. This closure therefore assumes unemployment with factor mobility.

The impact of these closures is that for capital and high-skilled labor total employment will not change. The only change here would be the activity specific rental price of capital and the wage of high skilled labor. Capital and high-skilled labor may therefore not move between activities. For semi- and unskilled labor, wages will remain constant as these factors experience high levels of unemployment. The only factor that would change for semi- and unskilled labor is employment. The wage rate of semi- and unskilled labor is fixed at real wage level. The real wage is included in the model as the initial wage level multiplied by the consumer price index relative to the initial CPI level.

3.5.2 Government

Different closures for the government are used for different simulations. The following closures are used:

(A) The first closure assumes flexible government savings, with fixed direct tax rates. Government savings are treated as a residual. This closure is typically used when the government uses accommodating policy to finance, for example, zero-rating food.

(B) The second closure assumes fixed government savings. Direct tax rates of domestic institutions are adjusted endogenously to generate a fixed level of government savings. The same number of percentage points adjusts the base-year tax rates of selected domestic non-government institutions endogenously. This closure is more progressive, as an increase of taxes with the same number of

percentage points will tax the already high taxpayers at a higher rate than low taxpayers. This closure is typically used when government is using non-accommodating policy to finance, for example, zero-rating food. In this case an increase in direct taxes will be used to finance the cost of zero-rating food.

In all the closures government consumption is fixed either in real terms or as a share of nominal absorption.

An additional closure for the government balance was added. This closure allows the government deficit to remain fixed, while the statutory VAT rate change. The statutory VAT rate of either one commodity or all commodities can change, and this is the reason why the statutory VAT rate is defined over commodities.[1] This closure was used when food was zero-rated and the VAT on either business services or financial services was increased to eliminate the impact of the loss in VAT revenue. As the government closure used differs from simulation to simulation it will be discussed in more detail in Chapter 5, where the simulations are discussed.

3.5.3 Foreign Sector

The closure for the rest of the world assumes a flexible exchange rate with fixed foreign savings. The adjustment rule follows from observations made by Davies and Van Seventer (2003) who noted that foreign savings as defined by the national accounts behaved relatively constant over the last 10 years.

3.5.4 Saving-Investment

The savings-driven investment closure is used for most of the simulations. All non-government savings are now fixed by fixing the marginal propensity to save for all non-government institutions. Capital formation (investment) is flexible. The quantity of each commodity in the investment bundle is multiplied by a flexible scalar. This will ensure

[1] This can be seen in Chapter 4.

that the investment cost is equal to the savings value. The level of savings determines investment. Nell, in a paper read by him (2002) found, in the long-run, exogeneity between saving and investment in South Africa; private savings were strongly exogenous to private investment in the period 1977 to 2001. This implies that the savings level will determine investment. (Nell,2002:26).

3.6. Changes to the Standard IFPRI Model

The standardized model developed by Löfgren et al (2001) does not include more than one commodity tax, and for the purpose of this analysis it is necessary to do so as there are more than one category of commodity taxes in the South African tax system. The following commodity taxes are included:

Table 3.1: Additional Commodity Taxes Included in the CGE Model

Commodity Tax Category	Description	Parameter in Model
VAT	Value Added Tax	$tvat(c)$
Fuel Levy	Government levy on fuel	$tfuel\ (c)$
Excise	Excise taxes on goods	$texcise(c)$
Net other commodity taxes	Other taxes on production, excluding subsidies	$tproducts(c)$

The standard CGE model also does not include a statutory VAT rate, but rather an actual (or effective) value-added tax rate. To do the simulation in this study it was necessary to include a statutory vat rate variable, as well as a suitable equation linking the statutory vat rate with the actual vat rate. The following equation was added to the model:

$$TVAT(C) = STATVAT(C) * leakage(c)$$

where

$TVAT(C)$ is the actual VAT rate

$STATVAT(C)$ is the statutory VAT rate

leakage(*c*) is the ratio of the actual VAT rate to the statutory VAT rate

The VAT function in this model, however, does not model input and output taxes explicitly as it is not required for any of the simulations in this study. When the commodity is zero-rated the producers are still allowed to receive a credit for inputs, and VAT is still equivalent to a consumption tax (Gottfried and Wiegard,1990:2).

The complete model with all the adaptations for the South African specification is listed below in the Appendix.

3.7. Summary

The standard model developed by Löfgren et al (2001) will be used for the purpose of analyzing changes in the VAT structure on the South African economy. The standard model consists of a set of equations in the neoclassical-structuralist tradition. The activities, commodities, production factors, households and other institutions follow the SAM disaggregation. For this application the standard model is adapted to be representative of the South African economy. This is achieved by using a SAM as well as South African trade elasticities.

Appendix Chapter 3

APPENDIX CHAPTER 3

The Mathematical Model

Notational Principles

Endogenous variables	upper-case Latin letters without a bar
Exogenous variables	upper-case Latin letters with a bar
Parameters	lower-case Latin letters (with or without a bar) or lower-case Greek letters (with or without superscripts)
Set indices	lower-case Latin letters as subscripts to variables and parameters

Sets

A	Set of activities
ACES (contained in A)	Set of activities with a CES function at the top of the technology nest
C	Set of commodities
CD (contained in C)	Set of commodities with domestic sales of domestic output
CDN (contained in C)	Set of commodities without domestic market sales of domestic output (complement of CD)
CE (contained in C)	Set of exported commodities (with domestic production)
CEN (contained in C)	Set of non-exported commodities (complement of CE)
CM (contained in C)	Set of imported commodities
CMN (contained in C)	Set of non-imported commodities
CX (contained in C)	Set of commodities with domestic output
F	Set of factors
FLAB	Set of labor categories
H	Set of households
INS	Set of institutions (domestic and the rest of world)

INSD	Set of domestic institutions
INSDNG	Set of domestic non-government institutions

Parameters

α_a^a	efficiency parameter in the CES activity function
α_c^{ac}	shift parameter for domestic commodity aggregation function
α_c^q	CES function shift parameter
α_c^t	CET function shift parameter
α_a^{va}	efficiency parameter in the CES value-added function
β_{ch}^m	marginal share of consumption spending on marketed commodity *c* for household *h*
δ_a^a	CES activity function share parameter
δ_{ac}^{ac}	share parameter for domestic commodity aggregation function
δ_c^q	CES function share parameter
δ_c^t	CET function share parameter
δ_{fa}^{va}	CES value-added function share parameter for factor *f* in activity *a*
ρ_a^a	CES activity function exponent
ρ_c^{ac}	domestic commodity aggregation function exponent
ρ_c^q	CES function exponent
ρ_c^t	CET function exponent
ρ_a^{va}	CES value added function exponent
γ_{ch}^m	subsistence consumption of marketed commodity *c* for household *h*
γ_{ach}^h	subsistence consumption of home commodity *c* from activity *a* for household *h*
ϑ_{ac}	yield of output *c* per unit of activity *a*

$cwts_c$	weight of commodity *c* in the consumer price index
$dwts_c$	weight of commodity c in the producer price index
ica_{ca}	quantity of *c* per unit of aggregate intermediate input *a*
mps_i	base savings rate for domestic institutions *i*
$mps01_i$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates
pwe_c	f.o.b export price in foreign currency units
pwm_c	c.i.f import price in foreign currency units
$qdst_c$	quantity of stock change
$\overline{qg_c}$	base-year quantity of government demand
$\overline{qinv_c}$	base-year quantity of fixed investment demand
$shif_{if}$	share for domestic institution *i* in income of factor *f*
$shii_{ii'}$	share of net income of *i'* to *i* (*i' Є INSDNG'* and *i Є INSDNG*)
ta_a	tax rate for activity
te_c	export tax rate
tf_f	direct tax rate for factor *f*
$\overline{tins_i}$	exogenous direct tax rate for domestic institution *I*
$tins01_i$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates
tm_c	import tariff rate
tq_c	rate of sales tax (as share of composite price inclusive of sales tax)
$trnsfr_{if}$	transfer from factor f to institution *i*
tva_a	rate of value-added tax for activity *a*

Variables

DMPS	change in domestic institution savings rates (=0 for base; exogenous variable)

DPI	producer price index for domestically marketed output
EG	government expenditures
EH_h	household consumption expenditure
EXR	exchange rate (local currency unit per foreign currency unit)
$GSAV$	government savings
$MPS_{i'}$	marginal propensity to save for domestic non-government institutions (exogenous variable)
PA_a	activity price (gross revenue per activity unit)
PDD_c	demand price for commodities produced and sold domestically
PDS_c	supply price for commodities produced and sold domestically
PE_c	export price in local currency units
$PINTA_a$	aggregate intermediate input price for activity a
PM_c	import price in local currency units
PQ_c	composite commodity price (market price)
PVA_a	price of (aggregate) value-added
PX_c	aggregate producer price for commodity
$PXAC_{ac}$	producer price of commodity c for activity a
QA_a	quantity (level) of activity
QD_c	quantity sold domestically of domestic output
QE_c	quantity of exports
QF_{fa}	quantity demanded of factor f for activity a
QG_c	government consumption demand for commodity c
QH_{ch}	quantity of consumption of marketed commodity c for household h
$QINT_{ca}$	quantity of commodity c as intermediate input to activity a
$QINTA_a$	quantity of aggregate intermediate input
$QINV_c$	quantity of fixed investment demand for commodity c
QM_c	quantity of imports of commodity
QQ_c	quantity of goods supplied to domestic market (composite supply)

QVA_a	quantity of (aggregate) value-added
QX_c	aggregate quantity of domestic output of commodity
$QXAC_{ac}$	marketed output quantity of commodity *c* from activity *a*
$TINS_i$	rate of direct tax on domestic institutions *i*
$TABS$	total nominal absorption
$TRII_{ii'}$	transfers from institution *i'* to *i* (both in the set *INSDNG*)
WF_f	average factor price
YG	government revenue
YI_i	income of institution *i* (in the set *INSDNG*)
YIF_{if}	income to domestic institution from factor *f*

Exogenous Variables

$\overline{CPI}$	consumer price index (exogenous variable)
$\overline{DTINS}$	change in domestic institution tax share (=0 for base; exogenous variable)
$\overline{FSAV}$	foreign savings in foreign currency unit (exogenous variable)
$\overline{GADJ}$	government consumption adjustment factor (exogenous variable)
$\overline{IADJ}$	investment adjustment factor (exogenous variable)
$\overline{MPSADJ}$	savings rate scaling factor (=0 for base)
$\overline{QFS_f}$	quantity supplied of factor *f* (exogenous variable)
$\overline{TINSADJ}$	direct tax scaling factor (=0 for base; exogenous variable)
$\overline{WFDIST_{fa}}$	wage distortion factor for factor *f* in activity *a* (exogenous variable)

The Model

The model specification is according to Löfgren et al (2001). The adjustments for South Africa incorporated into the Löfgren et al (2001) model is also shown. The model allows

for three types of commodities, namely domestic goods consumed domestically, imports and exports. Some prices are determined outside the model, the rest of the prices are determined within the model.

1. Price Block

Import Price

$$PM_c = pwm_c \cdot (1 + tm_c) \cdot EXR \qquad c \in CM \qquad \textbf{---1.1}$$

where

$c \in C$	set of commodities (also referred to as c' and C')
$c \in CM (\subset C)$	set of imported commodities
$c \in CT (\subset C)$	set of domestic trade inputs (distribution commodities)

The import price is the price paid by domestic users for imported commodities exclusive of sales tax. The import price is expressed in the local currency unit (rand). The import price equation states that import price is equal to the world price of imports converted to domestic currency by multiplying with the exchange rate inclusive of import tariffs.

Export Price

$$PE_c = pwe_c \cdot (1 - te_c) \cdot EXR \qquad c \in CE \qquad \textbf{---1.2}$$

where

$c \in CE (\subset C)$	set of exported commodities (with domestic production)

The export price is the price received by domestic producers when they sell their output on world markets. The export price is also expressed in domestic currency (rand).

Export price is equal to the world export price converted to the domestic currency by multiplying with the exchange rate exclusive of export subsidies.

Demand Price of Domestic Non-Traded Goods

$$PDD_c = PDS_c \qquad c \in CD \qquad \textbf{---1.3}$$

where

$c \in CD (\subset C)$ set of commodities with domestic sales of domestic output

The price received by demanders and suppliers for domestic non-traded goods are equal. This is a deviation from the standard model. The standard model models transaction cost explicitly. In this application transaction costs are already included in the supply and use tables as intermediate inputs where required, and it is therefore not necessary to include transaction cost explicitly.

Absorption

$$PQ_c \cdot (1 - tq_c) \cdot QQ_c = PDD_c \cdot QD_c + PM_c \cdot QM_c \qquad c \in (CD \cup CM) \qquad \textbf{---1.4}$$

where

$$tq_c = leakage_c * STATVAT_c + tfeul_c + texcise_c + tproducts_c$$

Absorption (on the left-hand side) is the total domestic spending on a commodity at the domestic demander price, which in this model, will equal the domestic supplier prices, exclusive of sales tax. Absorption (on the right-hand side) is the sum of the value of domestic output and imports.

Marketed Output Value

$$PX_c \cdot QX_c = PDS_c \cdot QD_c + PE_c \cdot QE_c \qquad c \in CX \qquad \textbf{---1.5}$$

where

$c \in CX (\subset C)$ set of commodities with domestic output

The marketed output value is the sum of the value of domestic output sold domestically and the value of exports.

Activity Price

$$PA_c = \sum_{c \in C} PXAC_{ac} \cdot \vartheta_{ac} \qquad a \in A \qquad \textbf{---1.6}$$

where

$a \in A$ set of activities

The activity price is the revenue per activity unit and is the return from selling the output of the activity. The activity price is calculated by multiplying the yields per activity of output by the activity-specific commodity prices. This is then summed over all commodities because one activity might produce more than one commodity.

Aggregate Intermediate Input Price

$$PINTA_a = \sum_{c \in C} PQ_c \cdot ica_{ca} \qquad a \in A \qquad \textbf{---1.7}$$

The aggregate intermediate input price is the cost of the disaggregated intermediate inputs per unit of aggregate intermediate input. The aggregate intermediate input price is activity-specific and is the sum of the value of the quantity of a commodity per unit of the aggregate intermediate input. The quantity of the commodity per unit of the aggregate intermediate input is known as the intermediate input coefficient.

Activity Revenue and Costs

$$PA_a \cdot (1 - ta_a) \cdot QA_a = PVA_a \cdot QVA_a + PINTA_a \cdot QINTA_a \qquad a \in A \qquad \textbf{---1.8}$$

The value of activity revenue (net of activity taxes) is equal to the payments for value-added and intermediate inputs.

Consumer Price Index

$$\overline{CPI} = \sum_{c \in C} PQ_c \cdot cwts_c \qquad \textbf{---1.9}$$

Consumer prices change. The DPI functions as the numéraire.

Producer Price Index for Non-Traded Market Output

$$DPI = \sum_{c \in C} PDS_c \cdot dwts_c \qquad \textbf{---1.10}$$

The producer price index is fixed and functions as a numéraire in the model. This specification makes the model homogeneous to the degree zero. Price changes are then relative to the numéraire.

2. PRODUCTION AND TRADE BLOCK

CES technology: Activity Production Function

$$QA_a = \alpha_a^a \cdot (\delta_a^a \cdot QVA_a^{-\rho_a^a} + (1-\delta_a^a) \cdot QINTA_a^{-\rho_a^a})^{-\frac{1}{\rho_a^a}} \qquad a \in ACES \qquad \textbf{---2.1}$$

where

$a \in ACES(\subset A)$ set of activities with a CES function at the top of the technology nest

Production is carried out by activities that are assumed to maximize profits subject to the technology available. This equation states that the top level activity production function is a function of value-added and intermediate inputs. The function exponent, ρ , is a transformation of the elasticity of substitution between value-added and aggregate intermediate input.

CES technology: Value-Added Intermediate-Input Ratio

$$\frac{QVA_a}{QINTA_a} = \left(\frac{PINTA_a}{PVA_a} \cdot \frac{\delta_a^a}{1-\delta_a^a}\right)^{\frac{1}{1+\rho_a^a}} \qquad a \in ACES \qquad \textbf{---2.2}$$

The optimal mix of intermediate inputs and value-added is a function of the relative prices of intermediate inputs and value-added.

Value-Added and Factor Demands

$$QVA_a = \alpha_a^{va} \cdot \left(\sum_{f \in F} \delta_{fa}^{va} \cdot QF_{fa}^{-\rho_a^{va}}\right)^{-\frac{1}{\rho_a^{va}}} \qquad a \in A \quad f \in F \qquad \textbf{---2.3}$$

where

$f \in F$ set of factors (also referred to as F')

The quantity of value-added is a CES function of disaggregated factor quantities. The function ρ is a transformation of the elasticity of factor substitution.

Factor Demand

$$WF_f \cdot \overline{WFDIST_{fa}} = PVA_a \cdot (1 - tva_a) \cdot QVA_a \cdot \left(\sum_{f \in F'} \delta_{fa}^{va} \cdot QF_{fa}^{-\rho_a^{va}}\right)^{-1} \cdot \delta_{fa}^{va} \cdot QF_{fa}^{-\rho_a^{va}-1}$$

$$a \in A \quad f \in F \qquad \textbf{---2.4}$$

Activities demand factors at the point where the marginal cost of each factor is equal to the marginal revenue product of the factor. The marginal cost is defined on the left-hand side as the activity-specific factor price. The marginal revenue product is defined on the right-hand side net of intermediate input cost. The average factor price is an endogenous variable, while the wage-distortion variables are exogenous.

Disaggregated Intermediate Input Demand

$$QINT_{ca} = ica_{ca} \cdot QINTA_a \qquad a \in A \quad c \in C \qquad \textbf{---2.5}$$

For each activity, the demand for disaggregated intermediate inputs is determined via a standard Leontief type function. The demand for disaggregated intermediate inputs is equal to the level of intermediate input use multiplied with a fixed intermediate input coefficient.

Commodity Production and Allocation

$$QXAC_{ac} = \vartheta_{ac} \cdot QA_a \qquad a \in A,\ c \in CX,\ h \in H \qquad \textbf{---2.6}$$

where

$h \in H$ set of households

On the right-hand side, production quantities are defined as yields multiplied by the activity levels. On the left-hand side, the quantities are allocated to market sales.

Output Aggregation Function

$$QX_c = \alpha_c^{ac} \left(\sum_{a \in A} \delta_{ac}^{ac} \cdot QXAC_{ac}^{-\rho_c^{ac}} \right)^{-\frac{1}{\rho_c^{ac}-1}} \qquad c \in CX \qquad \textbf{---2.7}$$

Aggregated production of any commodity is defined as a CES aggregate of the different activities producing the commodity. The function exponent ρ specifies the degree of substitutability between different products. The values of ρ must be specified to ensure that the isoquant is convex to the origin, and therefore exhibits a diminishing technical rate of substitution.

First-Order Condition for Output Aggregation Function

$$PXAC_{ac} = PX_c \cdot QX_c \left(\sum_{a \in A'} \delta_{ac}^{ac} \cdot QXAC_{ac}^{-\rho_c^{ac}}\right)^{-1} \cdot \delta_{ac}^{ac} \cdot QXAC_{ac}^{-\rho_c^{ac}-1}$$

$$a \in A$$

$$c \in CX \qquad \textbf{---2.8}$$

This equation is derived from maximizing aggregate output QX at the price PX, subject to the aggregation function and the disaggregated commodity prices PXAC. The optimal quantity of the commodity is inversely related to the activity-specific price: a decline in the price of PXAC of one activity relative to the others would shift demand in that direction. The output QX is sold at a price PX and is produced with inputs QXAC at prices PXAC.

Output Transformation (CET) Function

$$QX_c = \alpha_c^t \cdot \left(\delta_c^t \cdot QE_c^{\rho_c^t} + (1-\delta_c^t) \cdot QD_c^{\rho_c^t}\right)^{\frac{1}{\rho_c^t}} \qquad c \in (CE \cap CD) \qquad \textbf{---2.9}$$

Domestic production is allocated to two destinations, namely domestic sales and exports. The CET function is determined by both exports and domestic sales. The CET function is identical to a CES function except for negative elasticities of substitution. The exponent ρ is a transformation of the elasticity of substitution between exports and domestic sales. This reflects the assumption of imperfect substitution between the two destinations. The value of ρ is determined to specify an isoquant that is concave to the origin.

Export-Domestic Supply Ratio

$$\frac{QE_c}{QD_c} = \left(\frac{PE_c}{PDS_c} \cdot \frac{1-\delta_c^t}{\delta_c^t}\right)^{\frac{1}{\rho_c^t - 1}} \qquad c \in (CE \cap CD) \qquad \textbf{---2.10}$$

This equation is determined by maximizing producer revenues subject to the CET function and a fixed quantity of output. This equation then defines the optimal mix of exports and domestic sales. The exports-domestic price ratio will determine the export-domestic demand ratio.

Output Transformation for Domestically Sold Outputs without Exports and for Exports without Domestic Sales

$$QX_c = QD_c + QE_c \qquad c \in (CD \cap CEN) \cup (CE \cap CDN) \qquad \textbf{---2.11}$$

where

$c \in CEN (\subset C)$ non-exported commodities

$c \in CDN (\subset C)$ commodities without domestic market sales of domestic output

This function replaces the CET function for produced commodities that are either sold domestically or exported, but not both. This function allocates the entire volume of output to either exports or domestic sales.

Composite Supply (Armington) Function

$$QQ_c = \alpha_c^q \cdot (\delta_c^q \cdot QM_c^{-\rho_c^q} + (1-\delta_c^q) \cdot QD_c^{-\rho_c^q})^{-\frac{1}{\rho_c^q}} \qquad c \in (CE \cap CD) \qquad \textbf{---2.12}$$

This function is called the Armington function and is a CES function. This function specifies the composite supply as a function of imports and domestic supply. Imperfect substitutability between imports and domestic supply is assumed. The elasticity of substitution is specified by ρ .

Import-Domestic Demand Ratio

$$\frac{QM_c}{QD_c} = \left(\frac{PDD_c}{PM_c} \cdot \frac{\delta_c^q}{1-\delta_c^q} \right)^{\frac{1}{1+\rho_c^q}} \qquad c \in (CM \cap CD) \qquad \textbf{---2.13}$$

This equation is obtained by minimizing cost subject to the CES function and a fixed quantity of composite supply. This is the optimal mix between imports and domestic supply. The domestic-import price ratio determines the import-domestic demand ratio. The elasticity of substitution is specified by ρ .

Composite Supply for Non-Imported Outputs and Non-produced Imports

$$QQ_c = QD_c + QM_c \qquad c \in [(CD \cap CMN) \cup (CM \cap CDN)] \qquad \textbf{---2.14}$$

where

$c \in CMN (\subset C)$ set of non-imported commodities

This function replaces the Armington function for commodities that are either imported or produced domestically, but not both.

3. Institution Block

Factor Income

$$YF_f = \sum_{a \in A} WF_f \cdot \overline{WFDIST_{fa}} \cdot QF_{fa} \qquad f \in F \qquad \textbf{---3.1}$$

where

YF_f income of factor f

This equation defines the total income of each factor and is equal to the sum of activity payments. Activity payments, in turn, are equal to activity-specific wages multiplied with employment levels.

Institutional Factor Incomes

$$YIF_{if} = shif_{if} \cdot [(1 - tf_f) \cdot YF_f - trnsfr_{rowf} \cdot EXR] \qquad i \in INSD \quad f \in F \qquad \textbf{---3.2}$$

where

$i \in INS$ set of institutions (domestic and rest of world)

$i \in INSD (\subset INS)$ set of domestic institutions

Total factor income is divided between domestic institutions in fixed shares after payment of direct factor taxes and transfers to the rest of the world. Transfers are converted to domestic currency by multiplying with the exchange rate.

Income of Domestic, Non-Government Institutions

$$YI_i = \sum_{f \in F} YIF_{if} + \sum_{i' \in INSDNG'} TRII_{ii'} + trnsfr_{igov} \cdot \overline{CPI} + trnsfr_{irow} \cdot EXR \qquad i \in INSDNG \quad \textbf{---3.3}$$

where

$i \in INSDNG (\subset INSD)$ set of domestic non-government institutions (also *INSDNG')*

The total income of domestic non-government institutions is equal to factor incomes, transfers from other institutions, which include other domestic non-government

institutions, the government and the rest of the world. Transfers from the rest of the world are converted to domestic currency by multiplying with the exchange rate.

Intra-Institutional Transfers

$$TRII_{ii'} = shii_{ii'} \cdot (1 - MPS_{i'}) \cdot (1 - TINS_{i'}) \cdot YI_{i'} \qquad i \in INSDNG,\ i' \in INSDNG' \qquad \textbf{---3.4}$$

Transfers between domestic non-government institutions are paid as fixed shares of the total institutional income net of direct taxes and savings.

Household Consumption Expenditures

$$EH_h = (1 - \sum_{i \in INSDNG} shii_{ih}) \cdot (1 - MPS_h) \cdot (1 - TINS_h) \cdot YI_h \qquad h \in H \qquad \textbf{---3.5}$$

where

$h \in H (\subset INSDNG)$ set of households

The total value of consumption spending by households is equal to the income net of taxes, savings and transfers to other domestic non-government institutions. Households are the only institution among domestic non-government institutions that consumes commodities.

Household Consumption Spending on Marketed Commodities

$$PQ_c \cdot QH_{ch} = PQ_c \cdot \gamma^{m}_{ch} + \beta^{m}_{ch} \cdot \left(EH_h - \sum_{c' \in C} PQ_{c'} \cdot \gamma^{m}_{c'h} - \sum_{a \in A} \sum_{c' \in C} PXAC_{ac'} \cdot \gamma^{h}_{ac'h} \right) \qquad c \in C,\ h \in H \qquad \textbf{---3.6}$$

This equation is determined by maximizing a utility function subject to a consumption expenditure constraint. This function is a LES function since spending on a commodity is a linear function of total consumption spending.

Investment Demand

$$QINV_c = \overline{IADJ} \cdot \overline{qinv_c} \qquad c \in C \qquad \textbf{---3.7}$$

Fixed-investment demand is defined as the base-year quantity multiplied by an adjustment factor. The adjustment factor is exogenous, and therefore also makes the investment quantity exogenous.

Government Consumption Demand

$$QG_c = \overline{GADJ} \cdot \overline{qg_c} \qquad c \in C \qquad \textbf{---3.8}$$

Government consumption demand is defined as the base-year quantity multiplied by an adjustment factor. This factor is also exogenous and therefore the quantity of government consumption is fixed.

Government Revenue

$$YG = \sum_{i \in INSDNG} TINS_i \cdot YI_i + \sum_{f \in F} tf_f \cdot YF_f + \sum_{a \in A} tva_a \cdot PVA_a \cdot QVA_a$$
$$+ \sum_{a \in A} PA_a \cdot QA_a + \sum_{c \in CM} tm_c \cdot pwm_c \cdot QM_c \cdot EXR + \sum_{c \in CE} te_c \cdot pwe_c \cdot QE_c \cdot EXR$$
$$+ \sum_{c \in C} tq_c \cdot PQ_c \cdot QQ_c + \sum_{f \in F} YF_{govf} + trnsfr_{govrow} \cdot EXR \qquad \textbf{---3.9}$$

where

$$tq_c = leakage_c * STATVAT_c + tfeul_c + texcise_c + tproducts_c$$

Total government revenue is the sum of revenues from taxes, factors and transfers from the rest of the world. Taxes include direct taxes from institutions, taxes on commodities, taxes on activities, import tariffs, but less exports subsidies. Taxes on commodities include VAT, fuel levies, excise duties and other taxes on products. Transfers are again converted to domestic currency by multiplying by the exchange rate.

Government Expenditures

$$EG = \sum_{c \in C} PQ_c \cdot QG_c + \sum_{i \in INSDNG} trnsfr_{igov} \cdot \overline{CPI} \qquad c \in C \qquad \textbf{---3.10}$$

Government spending is the sum of government spending on consumption goods and transfers.

4. SYSTEM CONSTRAINTS BLOCK

Factor Markets

$$\sum_{a \in A} QF_{fa} = \overline{QFS_f} \qquad f \in F \qquad \textbf{---4.1}$$

This equation equates the total factor quantity demanded and the total quantity supplied for each factor.

Composite Commodity Markets

$$QQ_c = \sum_{a \in A} QINT_{ca} + \sum_{h \in H} QH_{ch} + QG_c + QINV_c + qdst_c \qquad c \in C \qquad \textbf{---4.2}$$

This equation equates the quantity supplied and demanded of the composite commodity. The demand side includes demand for intermediate goods, household consumption demand, government consumption, investment, and stock changes. Government consumption, investment demand and stock changes are determined exogenously. Composite supply determines the demand for domestic sales and imports.

Current Account Balance for Rest of World (in Foreign Currency)

$$\sum_{c \in CM} pwm_c \cdot QM_c + \sum_{f \in F} trnsfr_{rowf} = \sum_{c \in CE} pwe_c \cdot QE_c + \sum_{i \in INSD} trnsfr_{irow} + \overline{FSAV} \qquad \textbf{---4.3}$$

The balance on the current account equates the spending and earning of foreign exchange. Two basic market-clearing closures may be used for the rest of the world. Foreign savings may be fixed, while the exchange rate equilibrates the current account. Alternatively, the exchange rate may be fixed while foreign savings fluctuate to determine equilibrium. This application will model a flexible exchange rate, with fixed foreign savings.

Government Balance

$$YG = EG + GSAV \qquad \textbf{---4.4}$$

The government sets current government revenue equal to the sum of current government expenditures and savings. Government investment is excluded from this balance.

Direct Institutional Tax Rates

$$TINS_i = \overline{tins_i} \cdot (1 + \overline{TINSADJ} \cdot tins01_i) + \overline{DTINS} \cdot tins01_i \qquad i \in INSDNG \qquad \textbf{---4.5}$$

This equation defines the direct tax rate of domestic non-government institutions. Three alternative closure rules may be modeled. The first closure models all variables on the

right-hand side as exogenous variables. Government savings is the endogenous variable that clears the government balance. The second closure fixes government savings. DTINS is now the flexible variable that clears the government balance by scaling the base-year tax rates of each tax-paying institution. Rates will change by a uniform number of percentage points for all tax-paying institutions. The third closure also fixes government savings. TINSADJ is now the flexible variable. Rates will now change by adding a uniform number of percentage points to the base-year tax rate for all taxpaying institutions. This will, however result in relatively large increases in the tax rate for relatively large base-year rates.

Institutional Savings Rates

$$MPS_i = \overline{mps_i} \cdot (1 + \overline{MPSADJ} \cdot mps01_i) + \overline{DMPS} \cdot mps01_i \qquad i \in INSDNG \qquad \textbf{---4.6}$$

The closure rule for the savings-investment balance will determine whether the right-hand side is considered fixed or whether either MPSAJD or DMPS are fixed. Fixing the right-hand side will in effect fix savings. Investment will adjust to equate the savings-investment balance.

Savings-Investment Balance

$$\sum_{i \in INSDNG} MPS_i \cdot (1 - TINS_i) \cdot YI_i + GSAV + EXR \cdot \overline{FSAV} = \sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c$$

---4.7

This equation states that total savings and total investment have to be equal. Total savings consists of savings by domestic non-government institutions, government savings and foreign savings. Foreign savings is converted to domestic currency by multiplying with the exchange rate. Total investment is the sum of the value of investment demand and stock changes. Closures may be either investment-driven or savings-driven. A

variable called Walras is added to the savings-investment. The value of this variable should be equal to zero. If it is not equal to zero, one or more equations are not satisfied and a general equilibrium solution has not been found.

Total Absorption

$$TABS = \sum_{h \in H} \sum_{c \in C} PQ_c \cdot QH_{ch} + \sum_{a \in A} \sum_{c \in C} \sum_{h \in H} PXAC_{ac} \cdot QHA_{ach}$$

$$+ \sum_{c \in C} PQ_c \cdot QG_c + \sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c \qquad \textbf{---4.8}$$

Total absorption is the total value of domestic final demand. This will equal GDP at market prices plus imports minus exports.

Ratio of Investment to Absorption

$$INVSHR \cdot TABS = \sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c \qquad \textbf{---4.9}$$

The right-hand side is the total investment value. Total investment value is calculated as a share in nominal absorption multiplied with total absorption. Total investment, in turn, is equal to the value of investment demand and the value of changes in stock.

Ratio of Government Consumption to Absorption

$$GOVSHR \cdot TABS = \sum_{c \in C} PQ_c \cdot QG_c \qquad \textbf{---4.10}$$

The right-hand side is the value of government consumption. The value of government consumption is calculated as a share in nominal absorption multiplied with total absorption.