Appendix A

Technical fields

Description	Field name
Vehicle's description	$vehicle(k).description$
Vehicle's volumetric capacity	$vehicle(k).cv$
Vehicle's weight capacity	$vehicle(k).cw$
Vehicle's fixed cost	$vehicle(k).f$
Vehicle's availability	$vehicle(k).avail$
Customer's name	$customer(i).name$
Customer's geographical position	$customer(i).x$
	$customer(i).y$
Customer's volumetric demand	$customer(i).dv$
Customer's weight demand	$customer(i).dw$
Service time at customer	$customer(i).st$
Maximum lateness allowed at customer	$customer(i).Lmax$
Lateness penalty factor for customer	$customer(i).alpha$
Number of time windows for customer	$customer(i).tw$
Artificial customer index	$customer(i).index$
Artificial customer reference	$customer(i).ref$
Earliest allowed time of arrival at artificial customer	$customer(i).earliest$
Latest allowed time of arrival at artificial customer	$customer(i).latest$
Actual time of arrival at artificial customer	$customer(i).actual$

Appendix B

Complete algorithm

```
Capture input information
    Capture vehicle information in VEHICLES
        for each vehicle k
            Capture vehicle registration number as vehicle(k).description
            Capture vehicle's volumetric capacity as vehicle(k).cv
            Capture vehicle's weight capacity as vehicle(k).cw
            Capture vehicle's fixed cost as vehicle(k).f
            Capture vehicle availability as vehicle(k).avail

    Set average speed as 55 km/h
    Sort available vehicles
        Clear and set VEHAVAIL as an available vehicle matrix
        for all available vehicles in VEHICLES
            Add vehicle to VEHAVAIL
            Sort VEHAVAIL in ascending order on <volumetric capacity>

    Capture general CUSTOMER information
        Capture customer information in CUSTOMER
        for each customer i
            Capture customer's name as customer(i).name
            Capture customer's position as customer(i).x and customer(i).y
            Capture customer's volumetric demand as customer(i).dv
            Capture customer's weight demand as customer(i).dw
            Capture service time at customer as customer(i).st
            Capture maximum lateness at customer as customer(i).Lmax
            Capture lateness penalty factor for customer as customer(i).alpha
            Capture number of time windows for customer as customer(i).tw
```

```
        for each entry, i, in CUSTOMER
            if CUSTOMER has multiple time windows
                Split customer into customer(i).tw artificial customers
                Add artificial customer to ARTIF
                Capture the time window information for each ARTIFicial customer
            else
                Add the CUSTOMER as a single ARTIFicial customer
                Capture the time window information for the single ARTIFicial customer
        Calculate the DISTance matrix between all the ARTIFicial nodes

Initialise algorithm
    Set the ROUTED matrix as empty
    for all the ARTIFicial nodes, except the depot (node 1)
        Add the ARTIFicial node to the UNROUTED matrix

Initialise TOUR
    Set the TOUR index (t) to 1
    Establish the starting time for the TOUR
        Starting time for the current TOUR is e_0 + s_0
        (It is assumed that vehicles are not loaded at the beginning of the depot's time window)
    Assign vehicle to TOUR
        Set the first vehicle in VEHAVAIL as the current vehicle for the TOUR
        Update vehicle availability
            Locate the current vehicle in VEHICLE
            Set vehicle(k).availability = 0
            Recalculate VEHAVAIL

Build TOUR
    While UNROUTED is not empty
        Initialise ROUTE with seed customer
            Set ROUTE index (r) to 1
            Assign current ROUTE to current TOUR
            Establish the starting time for the TOUR
            Set ROUTE load to zero

            Assign the depot as starting and ending node for the current ROUTE
            Select a seed customer from the UNROUTED nodes
                Calculate the time window compatibility matrix (TWCM) for all UNROUTED nodes
                    for each node combination (a,b) where node b is serviced after node a
                        Calculate the earliest possible arrival at b as arrival_earliest
                            e_a - the earliest allowed arrival at node u
                            s_a - the service time at node a
                            t_ab - the travel time between node a and node b
                            arrival_earliest = e_a + s_a + t_ab
                        Calculate the latest possible arrival at b as arrival_latest
                            l_a - the latest allowed arrival at node a
```

```
                s_a - the service time at node a
                t_ab - the travel time between node a and node b
                arrival_latest = l_a + s_a + t_ab
            if the earliest possible arrival at b is before the latest allowed arrival at b
                Calculate time window compatibility (TWC)
                    TWC_ab = min {arrival_latest, l_b} - max {arrival_earliest, e_b}
            else
                TWC is negative infinity

        Calculate the number of infeasible time windows for each UNROUTED node
            for each UNROUTED node (i)
                Determine how many times in row i of TWCM is TWC negative infinity
                Determine how many times in column i of TWCM is TWC negative infinity
                Calculate the total number of infeasibilities by adding row and column count

        if there are infeasible time windows for any UNROUTED node
            The seed customer is the node with the most number of infeasible time windows
        else
            Calculate the COMPATIBILITY vector
                for each UNROUTED node (a) in the TWCM
                    row = TWCM(a,:)
                    column = TWCM(:,a)
                    compatibility(a) = sum(row) + sum(column) - TWCM(a,a)
            The seed customer is the node with the lowest COMPATIBILITY

    Insert seed customer
        Insert seed customer on current ROUTE
        Update UNROUTED customers
            Remove seed customer from UNROUTED
            Remove any other artificial nodes related to seed customer from UNROUTED
            Update ROUTE load

Expand partial ROUTE
    while UNROUTED is not empty and there are customers that fit into the current ROUTE
        Clear the node selection matrix C2
        for each UNROUTED node (u)
            Clear the node insertion matrix C1
            Select the best position to insert node u on the current ROUTE
                for each edge (i,j) on the current ROUTE
                Determine feasibility to add node u
                    Infeasible if either TWC_iu or TWC_uj is unfeasible
                    Infeasible if TOUR capacity is exceeded by u
                if it is feasibile to evaluate node u between i and j
                    Update the C1 vector for the insertion positions
                        Calculate c_1(i,u,j)
                            Calculate c_11(i,u,j)
```

d_{ij} - the distance between nodes i and j
$c_{11}(i,u,j) = d_{iu} + d_{uj} - \mu d_{ij}$, $\mu >= 0$ (in Dullaert, $\mu = 1$)
Calculate $c_{12}(i,u,j)$
e_i - earliest allowed arrival at node i
w_i - the waiting time before node i
a_i - the actual start time before node node i
s_i - the service time at node i
t_{ij} - the travel time between nodes i and j
b_i - the original start of service at node i
b_i^{new} - the start of service at node i after node u has been inserted

$b_u = max\{e_u, a_i + s_i + t_{iu}\}$
$b_j = a_j$
$b_i^{new} = max\{e_i, b_u + s_u + t_{uj}\}$
Calculate $c_{12}(i,u,j) = b_j^{new} - b_j$
Calculate $c_{13}(i,u,j)$
Select to use either *ACS*, *AOOS*, or *AROS*
ACS
Q - the load of the current vehicle before node u
$\underline{Q}$ - the max capacity of the current vehicle before node u
Q^{new} - the new load of the vehicle after node u
$\underline{Q}^{new}$ - the new max capacity of the vehicle after node u
$F(C)$ - the fixed cost of the smallest available vehicle that can service a demand of size C for a subtour
$ACS = F(Q^{new}) - F(Q)$
if new vehicle is indicated
flag new vehicle number

AOOS
$AOOS = ACS - F(\underline{Q}^{new} - Q^{new})$
if new vehicle is indicated
flag new vehicle number

AROS
$F'(C)$ - the fixed cost of the largest available vehicle whose capacity is less than or equal to C
$P(z)$ - the capacity of the smallest available vehicle that can service a demand of z
L $\omega = P(z_i + z_j) - P(max\{z_i, z_j\})$
$\delta(\omega) = 1$ *if* $\omega > 0$, *otherwise* 0
$\delta(\omega) = 1$ *if* $Q^{new} > \underline{Q}$, *otherwise* 0
$AROS = ACS - \delta(\omega)F'(\underline{Q}^{new} - Q^{new})$
$c_{13}(i,u,j)$ = *any one of ACS, AOOS, or AROS*
if new vehicle is indicated
flag new vehicle number

> a_i - weight factors. The weight need not add up to 1
> $c_1(i,u,j) = a_1 c_{11}(i,u,j) + a_2 c_{12}(i,u,j) + a_3 c_{13}(i,u,j)$

Add the $c_1(i,u,j)$ value to *C1(m).value*

else

Check next edge on current ***ROUTE***

Select the best edge (i^*,j^*) based on the lowest ***C1*** matrix value

Update the ***C2*** matrix for the insertion position

Calculate $c_2(i^*,u,j^*)$

> d_{ou} - the distance between the depot (node o) and node u
> t_{ou} - the travel time between the depot (node o) and node u
> s_u - the service time at node u
> λ - in Solomon combinations of 1 and 2 are used
> $F(q_u)$ - the fixed cost of the smallest available vehicle that can service the demand of node u
> $c_2(i^*,u,j^*) = \lambda(d_{ou} + t_{ou}) + s_u + F(q_u) - c_1^{best}(i,u,j)$

Add the $c2(i^*,u,j^*)$ value to the ***C2*** matrix

Sort ***C2*** in ascending order

Find first time-feasible node (u^*), starting at the beginning of ***C2***

While no u^* has been found, and end of ***C2*** has not been reached

Check for time feasibility

> Check node u's time feasibility
> $a_u = \max\{e_u, a_i + s_i + t_{iu}\}$
> if $a_u \le l_u + L_u^{max}$
> Check node j's time feasibility
> $a_j^{new} = \max\{a_j, a_u + s_u + t_{uj}\}$
> if $a_j^{new} \le l_j + L_j^{max}$
> Check rest of ***ROUTE***'s r nodes for time feasibility
> While feasible
> $a_r^{new} = \max\{a_r, a_{r-1} + s_{r-1} + t_{r-1,r}\}$
> if $a_r^{new} \le l_r + L_r^{max}$
> then feasible
> else
> infeasible
> else
> infeasible
> else
> infeasible

if feasible

Identify applicable node as u^*

else
Check next element of ***C2***

if a unique u^* node has been identified
Insert node u^*
Update ***UNROUTED*** customers
Remove u^* from ***UNROUTED***
Remove any other artificial nodes related to u^* from ***UNROUTED***
Update ***ROUTE***
Update ***ROUTE*** load
if new vehicle has been indicated
if $Q^{new} > Q$
Find the smallest available vehicle to service Q^{new}
Update ***VEHAVAIL***
Change the availability status of the current vehicle to available
Change the availability status of the new vehicle to unavailable
Assign new vehicle to current ***TOUR***
Recalculate ***VEHAVAIL***

Recalculate ***ROUTE*** schedule for nodes
Actual start-time at origin (a_o) is the start-time indicated for the current route
for each node (i) on the current ***ROUTE***, except the depot at both ends
$a_i = max\{e_i , a_{i-1} + s_{i-1} + t_{i-1,i}\}$
$w_i = max\{0 , e_{i+1} - (a_i + s_i + t_{i,i+1})\}$
Calculate actual arrival at the depot (n^{th} node) at the end of the current ***ROUTE***
$a_n = a_{n-1} + s_{n-1} + t_{n-1,n}\}$
else
Initialize new ***ROUTE***

Expand TOUR
Determine multi-route feasibility
Check the actual arrival time at the depot of the previous ***ROUTE*** of the current ***TOUR*** (a_n)
if $a_n + s_o + 1\ hour < l_o^{max}$
then feasible
else
infeasible
if feasible
Initialize new ***ROUTE***
else
if the last ***ROUTE*** of the ***TOUR*** has no nodes other than the depot
Eliminate ***ROUTE*** from ***TOUR***
Initialize new ***TOUR***

```
Define ORPHANs
    if UNROUTED is not empty
        Assign all elements in UNROUTED to ORPHANs
        Clear UNROUTED

Report initial solution
    Calculate the OBJective function value for the initial solution
    Report initial solution
        for each TOUR
        Report all TOUR and ROUTE information
```

Appendix C

Output

The output presented in this appendix are representations of the actual text files (*.*txt*) generated by *MATLAB 6.5*. Consider the following excerpt from the output for the $R1$ problem class.

```
 :        :
Tour:    2
Vehicle: v2
Route:   1
Customer  Actual time
   c11        0.00
   c881      13.00
   c511      86.00
   c272     347.00
   c11      358.31
Route:   2
Customer  Actual time
   c11      358.31
   c412     412.00
   :         :
```

The last digit (either 1 or 2) represents the customer's specific time window during which it will be serviced. For example, the second tour is assigned vehicle 2. The first and the last nodes on every route is $c11$. The $c1$ denotes the *depot*, while the last digit, 1, denotes the first time window (and for the depot, the only time window).

The third customer (the fourth node) on the first route of the second tour is $c272$. The $c27$ denotes customer 27, while the last digit, 2, indicates that customer 27 is serviced during its second specified time window, and to be specific, at 347 minutes after the vehicle left the depot.

At the end of each output file, any orphans are indicated. Orphans are defined to be customers that could not be inserted into any routes. The total scheduling distances is also indicated.

C.1 Problem class $R1$

The following output was generated my *MATLAB* for the problem class $R1$: randomly distributed customers with a short scheduling horizon.

```
Tour:     1
Vehicle: v1
Route:    1
Customer  Actual time
   c11        0.00
   c561      17.00
   c681      62.00
   c671     161.00
   c11      172.89
Route:    2
Customer  Actual time
   c11      172.89
   c401     179.00
   c231     267.00
   c482     411.00
   c11      422.44
Route:    3
Customer  Actual time
   c11      422.44
   c912     492.00
   c992     512.00
   c11      523.65
Route:    4
Customer  Actual time
   c11      523.65
   c112     559.00
   c11      570.33

Tour:     2
Vehicle: v2
Route:    1
Customer  Actual time
   c11        0.00
   c881      13.00
   c511      86.00
```

```
   c272    347.00
   c11     358.31
Route:   2
Customer  Actual time
   c11     358.31
   c412    412.00
   c432    422.67
   c712    448.00
   c11     459.20
Route:   3
Customer  Actual time
   c11     459.20
   c842    459.95
   c782    503.00
   c11     514.84
Route:   4
Customer  Actual time
   c11     514.84
   c282    516.67
   c982    531.00
   c11     541.40
Route:   5
Customer  Actual time
   c11     541.40
   c92     560.00
   c11     571.91

Tour:    3
Vehicle: v3
Route:   1
Customer  Actual time
   c11       0.00
   c451     60.00
   c121    345.00
   c11     356.91
Route:   2
Customer  Actual time
   c11     356.91
   c802    420.00
   c921    430.67
   c11     441.49
Route:   3
Customer  Actual time
   c11     441.49
```

```
   c32     443.45
   c752    456.00
   c11     467.00
Route:   4
Customer  Actual time
   c11     467.00
   c622    535.00
   c11     546.80

Tour:    4
Vehicle: v4
Route:   1
Customer  Actual time
   c11       0.00
   c861     68.00
   c651    216.00
   c261    240.00
   c11     251.49
Route:   2
Customer  Actual time
   c11     251.49
   c571    291.00
   c812    309.00
   c11     320.27
Route:   3
Customer  Actual time
   c11     320.27
   c182    346.00
   c11     357.87
Route:   4
Customer  Actual time
   c11     357.87
   c792    413.00
   c11     424.87
Route:   5
Customer  Actual time
   c11     424.87
   c502    428.00
   c202    490.00
   c11     501.49
Route:   6
Customer  Actual time
   c11     501.49
   c602    502.02
```

```
   c11      512.55

Tour:     5
Vehicle: v5
Route:    1
Customer  Actual time
   c11        0.00
   c331      10.00
   c211      64.00
   c851      74.38
   c11       85.58
Route:    2
Customer  Actual time
   c11       85.58
   c351     216.00
   c722     232.00
   c11      244.20
Route:    3
Customer  Actual time
   c11      244.20
   c612     269.00
   c732     384.00
   c11      394.96
Route:    4
Customer  Actual time
   c11      394.96
   c972     401.00
   c11      411.95
Route:    5
Customer  Actual time
   c11      411.95
   c662     413.07
   c832     423.62
   c11      435.00
Route:    6
Customer  Actual time
   c11      435.00
   c462     492.00
   c11      502.44
Route:    7
Customer  Actual time
   c11      502.44
   c252     511.00
   c11      521.29
```

```
Tour:    6
Vehicle: v6
Route:   1
Customer  Actual time
   c11       0.00
   c641     42.00
   c191     63.00
   c11      73.91
Route:   2
Customer  Actual time
   c11      73.91
   c311     75.15
   c541    105.00
   c11     116.49
Route:   3
Customer  Actual time
   c11     116.49
   c162    117.64
   c341    188.00
   c11     199.62
Route:   4
Customer  Actual time
   c11     199.62
   c901    206.00
   c11     216.91
Route:   5
Customer  Actual time
   c11     216.91
   c592    334.00
   c11     345.71
Route:   6
Customer  Actual time
   c11     345.71
   c932    378.00
   c11     389.49
Route:   7
Customer  Actual time
   c11     389.49
   c492    426.00
   c11     437.47
Route:   8
Customer  Actual time
   c11     437.47
```

```
   c82      483.00
   c11      493.84

Tour:     7
Vehicle: v7
Route:    1
Customer  Actual time
   c11        0.00
   c951     170.00
   c11      181.22
Route:    2
Customer  Actual time
   c11      181.22
   c372     278.00
   c11      289.42
Route:    3
Customer  Actual time
   c11      289.42
   c892     354.00
   c11      364.85
Route:    4
Customer  Actual time
   c11      364.85
   c301     365.78
   c442     430.00
   c11      441.44

Tour:     8
Vehicle: v8
Route:    1
Customer  Actual time
   c11        0.00
   c361     114.00
   c11      124.76
Route:    2
Customer  Actual time
   c11      124.76
   c321     190.00
   c11      201.04
Route:    3
Customer  Actual time
   c11      201.04
   c152     287.00
   c11      297.91
```

```
Route:    4
Customer  Actual time
   c11     297.91
   c702    313.00
   c11     323.67
Route:    5
Customer  Actual time
   c11     323.67
   c422    397.00
   c11     407.82

Tour:     9
Vehicle: v9
Route:    1
Customer  Actual time
   c11       0.00
   c171    150.00
   c11     162.27
Route:    2
Customer  Actual time
   c11     162.27
   c52     244.00
   c11     254.82
Route:    3
Customer  Actual time
   c11     254.82
   c962    271.00
   c72     285.00
   c11     297.29
Route:    4
Customer  Actual time
   c11     297.29
   c392    325.00
   c11     336.13
Route:    5
Customer  Actual time
   c11     336.13
   c242    347.00
   c11     358.56

Tour:     10
Vehicle: v10
Route:    1
Customer  Actual time
```

```
   c11      0.00
   c1002    225.00
   c11      235.84
Route:    2
Customer  Actual time
   c11      235.84
   c772     237.65
   c11      249.47
Route:    3
Customer  Actual time
   c11      249.47
   c632     268.00
   c11      279.31

Tour:     11
Vehicle: v11
Route:    1
Customer  Actual time
   c11        0.00
   c942     182.00
   c11      193.36
Route:    2
Customer  Actual time
   c11      193.36
   c522     219.00
   c11      230.29

Tour:     12
Vehicle: v12
Route:    1
Customer  Actual time
   c11        0.00
   c472     229.00
   c11      239.69

Tour:     13
Vehicle: v13
Route:    1
Customer  Actual time
   c11        0.00
   c142     189.00
   c11      200.31
Route:    2
Customer  Actual time
```

```
    c11      200.31
    c132     209.00
    c11      220.31

Tour:     14
Vehicle: v14

Tour:     15
Vehicle: v15

Tour:     16
Vehicle: v16

Tour:     17
Vehicle: v17

Tour:     18
Vehicle: v18

Tour:     19
Vehicle: v19

Tour:     20
Vehicle: v20

Tour:     21
Vehicle: v21

Tour:     22
Vehicle: v22

Tour:     23
Vehicle: v23

Tour:     24
Vehicle: v24

Tour:     25
Vehicle: v25

Tour:     26
Vehicle: v26

Tour:     27
```

```
Vehicle: v27

Tour:    28
Vehicle: v28

Tour:    29
Vehicle: v29

Tour:    30
Vehicle: v30

Tour:    31
Vehicle: v31

Tour:    32
Vehicle: v32

Tour:    33
Vehicle: v33

Tour:    34
Vehicle: v34

Tour:    35
Vehicle: v35

Tour:    36
Vehicle: v36

Tour:    37
Vehicle: v37

Tour:    38
Vehicle: v38

Tour:    39
Vehicle: v39

Tour:    40
Vehicle: v40

Tour:    41
Vehicle: v41
```

```
Tour:    42
Vehicle: v42

Tour:    43
Vehicle: v43

Tour:    44
Vehicle: v44

Tour:    45
Vehicle: v45

Tour:    46
Vehicle: v46

Tour:    47
Vehicle: v47

Tour:    48
Vehicle: v48

Tour:    49
Vehicle: v49

Tour:    50
Vehicle: v50

Tour:    51
Vehicle: v51

Tour:    52
Vehicle: v52

Tour:    53
Vehicle: v53

Tour:    54
Vehicle: v54

Tour:    55
Vehicle: v55

Tour:    56
Vehicle: v56
```

```
Tour:    57
Vehicle: v57

Tour:    58
Vehicle: v58

Tour:    59
Vehicle: v59

Tour:    60
Vehicle: v60

Tour:    61
Vehicle: v61

Tour:    62
Vehicle: v62

Tour:    63
Vehicle: v63

Tour:    64
Vehicle: v64

Tour:    65
Vehicle: v65

Tour:    66
Vehicle: v66

Tour:    67
Vehicle: v67

Tour:    68
Vehicle: v68

Tour:    69
Vehicle: v69

Tour:    70
Vehicle: v70

Tour:    71
```

```
Vehicle: v71
Route:    1
Customer  Actual time
   c11      0.00
   c1012   411.00
   c11     422.60
Route:    2
Customer  Actual time
   c11     422.60
   c532    495.00
   c11     505.67

Tour:      72
Vehicle: v72
Route:    1
Customer  Actual time
   c11        0.00
   c872    410.00
   c11     422.36
Route:    2
Customer  Actual time
   c11     422.36
   c292    474.00
   c11     485.38
Route:    3
Customer  Actual time
   c11     485.38
   c62     495.00
   c11     506.71

Tour:    73
Vehicle: v73
Route:    1
Customer  Actual time
   c11      0.00
   c822    309.00
   c11     319.36
Route:    2
Customer  Actual time
   c11     319.36
   c552    384.00
   c11     395.02

Tour:      74
```

```
Vehicle: v74
Route:    1
Customer  Actual time
   c11        0.00
   c742     104.00
   c11      114.44
Route:    2
Customer  Actual time
   c11      114.44
   c692     154.00
   c11      165.47
Route:    3
Customer  Actual time
   c11      165.47
   c382     346.00
   c11      356.65
Route:    4
Customer  Actual time
   c11      356.65
   c102     385.00
   c11      395.40

Tour:     75
Vehicle: v75
Route:    1
Customer  Actual time
   c11        0.00
   c582      76.00
   c11       87.11
Route:    2
Customer  Actual time
   c11       87.11
   c222     263.00
   c11      274.38
Route:    3
Customer  Actual time
   c11      274.38
   c42      336.00
   c11      347.78
Route:    4
Customer  Actual time
   c11      347.78
   c22      366.00
   c11      376.80
```

```
Orphans: none

Total distance: 11260
```

C.2 Problem class $R2$

The following output was generated my *MATLAB* for the problem class $R2$: randomly distributed customers with a long scheduling horizon.

```
Tour:     1
Vehicle: v1
Route:    1
Customer  Actual time
   c11        0.00
   c821      25.00
   c651      37.89
   c571      80.00
   c321      91.02
   c611     102.25
   c691     113.27
   c681     124.73
   c191     134.89
   c91      145.62
   c201     216.00
   c481     240.00
   c521     411.00
   c601     891.00
   c711     903.53
   c501     1226.00
   c861     1313.00
   c732     2254.00
   c11      2265.07

Tour:     2
Vehicle: v2
Route:    1
Customer  Actual time
   c11        0.00
   c841      43.00
   c241      54.65
   c421      64.82
   c51      138.00
```