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APPENDICES

APPENDIX A

The least squares approximation technique

Imprecisions in the financial market may result in prices and/or rates that do not always form a smooth curve. When determining an approximate fit to the term structure of interest rates, it is evident that all data points are not in perfect relation to each other. The data set can be approximated in several ways. However, the approximated curve should:

- be a smooth and continuous function of time; and
- have a smooth and continuous first derivative.

An approximation method that is commonly used to find the maximum likelihood estimate of the model parameters, is least squares approximation (Burden & Faires, 1989:425-451). The set of data points is fitted to a model which is a linear combination of specified functions of the term, t. The general form for this model is

$$y(t) = \sum_{k=1}^{M} a_k \, \xi_k(t)$$

where $\xi_k(t)$ = arbitrary fixed functions of t.

a_k = M adjustable coefficients, M < number of data points

The coefficients a are determined by minimizing the function:

$$\chi^2 = \sum_{i=1}^{N} \left[\frac{y_i - \sum_{k=1}^{M} a_k \, \xi_k(t_i)}{\sigma_i} \right]^2$$

where y_i = discrete data points, each with a term to maturity of t_i years,

σ_i = standard deviation of data point i.

Since the standard deviation serves as a weighting factor, it can be replaced by a weighting factor in order to give a bigger weighting to more tradable bonds. Different functions for ξ can be used in order to accommodate the particular shape of the curve being fitted. The least squares approximation technique works sufficiently well for many curve shapes, especially when more than one function is used.

APPENDIX B

The pricing of swap options

The price of a coupon bond is a non-linear function of the yield-to-maturity, given by

$$P_k = \sum_{i=1}^{n-1} \gamma_k e^{-t_i^{(k)}\eta_k} + (1 + \gamma_k) e^{-t_n^{(k)}\eta_k}.$$

In order to price a European option on a bond yield, it is usually treated as an option on the bond price, using price-volatility in the Black model. The price of swaps, on the other hand, is a *linear* function of the fixed rate. Options on swaps (swaptions) are also valued by using the Black model, but using the yield-volatility.

European swaptions are an example of options that can be priced by an exact solution, since the price is a linear function of the fixed rate (Jamshidian, 1996). The value of an n-year swap paying a fixed rate of R%, making m payments per year, is given by

$$S(R) = \frac{R \cdot N}{m}\sum_{i=1}^{mn} e^{-r_i t_i} + Ne^{-r_{mn} t_{mn}} - N$$

which is a linear function of the swap rate R. The rates $\{r_i\}$ are the zero-coupon rates for each payment period. At expiry, the payoff, h, of a swaption is therefore a linear function of the difference in two interest payments:

$$h = \max\{S(R) - S(R_X), 0\}$$

$$= \sum_{i=1}^{mn} \frac{N}{m} e^{-r_i t_i} \max\{(R - R_X), 0\}$$

where R_X is the strike rate. The coefficient of $\max\{(R - R_X),0\}$ is therefore the value of the payoff per 1% gain in the swap rate. The price of an option to receive a fixed rate can therefore be calculated by using the Black exact solution and calculating the expected payoff in percentage terms, and multiplying with the payoff per 1%:

$$\sum_{i=1}^{mn} \frac{N}{m} e^{-r_i t_i} \left(R_F N(d_1) - R_X N(d_2)\right)$$

where R_F is the forward swap rate and

$$d_1 = \frac{\ln\left(R_F / R_X\right) + \sigma^2 T/2}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

This is the same as using the swap price and price volatility to calculate the option price.

In order to compare the above with a bond, it is necessary to explain the fundamental difference between a bond and a swap. For a *swap*, the change in price for a 1 point change in the swap rate is constant. Due to the convexity of the *bond* price, the change in price for a 1 point move in the yield, changes, depending on the particular base yield. The difference in value per point for an out-the-money strike can be very different from the at-the-money value per point. It is clear that the described method to price a swap option cannot be directly applied to an option on a coupon bond.

Opsomming

'n Analise van die
termynstruktuur van rentekoerse
en opsies op effekte in die
Suid-Afrikaanse kapitaalmark

deur

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Afgeleide instrumente vorm 'n integrerende deel van handel in die finansiële wêreld en maak die gebruik van akkurate waarderingsmodelle en risiko-modelle noodsaaklik. Hierdie studie fokus op die Suid-Afrikaanse vasterentedraende mark, en evalueer bestaande modelle en prosedures. Alle waarderingsmodelle maak staat op sekere aannames en het gevolglik beperkings. Tekortkominge in die Suid-Afrikaanse vasterentedraende mark word derhalwe aangespreek, eerstens die termynstruktuur van rentekoerse en tweedens, die waardering van opsies op effekte.

Aangesien die vasterentedraende mark ten volle afhanklik is van die termynstruktuur van rentekoerse, is dit die belangrikste faktor in die prysberekening van enige vasterentedraende

afgeleide instrument. Die Suid-Afrikaanse effektemark verhandel hoofsaaklik in koepondraende effekte en bykans geen inligting is beskikbaar vir nulkoeponeffekte (wat die termynstruktuur bepaal) nie. 'n Verbeterde weergawe van die optrek-metode (bootstrap-method) vir die bepaling van 'n nulkoepon opbrengskurwe word derhalwe voorgestel. Die nulkoepon opbrengskurwe vorm die fondament vir die prysberekening van enige vanieljeproduk in die vasterentedraende mark en dien as inset vir die prysberekening van opsies op effekte wanneer 'n geen-arbitrage model gebruik word.

Die studie poog vervolgens om te verbeter op bestaande metodes om die waarde van Suid-Afrikaanse opsies op effekte te bepaal. 'n Studie na die eienskappe van die Hull-White model (1990) het gedien as motivering om die model toe te pas op Suid-Afrikaanse opsies, wat Amerikaans van aard is. Die Hull-White model moes egter aangepas word alvorens dit toegepas kon word op Suid-Afrikaanse opsies, omdat laasgenoemde in plaas van die prys van die effek, die opbrengskoers as trefprys gebruik. Aangesien die numeriese oplossing van die model die huidige termynstruktuur van rentekoerse as inset gebruik, is die nulkoepon opbrengskurwe weereens hier aangewend. Optimale omstandighede waaronder opsies op effekte vroeg uitgeoefen word, is bespreek.

Die gekompliseerde aard van die Hull-White model het die ontwikkeling van 'n vereenvoudigde model vir beurs-verhandelde opsies op die South African Futures Exchange (SAFEX), geregverdig. 'n Beurs-verhandelde opsie word nie beïnvloed deur die korttermyn risikovrye rentekoers-veranderlike nie, aangesien die onderliggende instrument die effek se termynkontrak-koers is. Daar kan dus aanvaar word dat, in plaas van die korttermynrentekoers of die prys, die effek se opbrengskoers vir die ooreenstemmende termynkontrak gebruik kan word as die stogastiese veranderlike. 'n Model wat hierdie proses as uitgangspunt gebruik, is soortgelyk aan die Black-model (1976), maar spreek egter meeste van laasgenoemde se nadele aan.

Summary

An analysis of the
term structure of interest rates
and bond options in the
South African capital market

by

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The enormous impact of derivatives in the financial world necessitates the use of accurate valuation and risk-forecast models. This study focuses on the South African fixed income market and evaluates current models and procedures. All valuation models depend on certain assumptions and therefore have limitations. Certain inefficiencies experienced in the South African fixed income market are addressed, firstly, term structure analysis, and, secondly, bond option valuation.

Since the fixed income market is entirely based on the term structure of interest rates, it remains the most important input in the pricing of any fixed income derivative security. The South African bond market trades mainly in coupon bonds, and little or no data is available

for zero-coupon instruments. (The term structure of interest rates is determined by the zero coupon rates.) An improved bootstrap method for the derivation of a zero-coupon yield curve is proposed. The zero-coupon yield curve is the basis for pricing all vanilla products in the fixed income market and serves as an important input in pricing bond options using a no-arbitrage model.

The study hence attempts to improve on existing methods to value South African bond options. An analysis of the characteristics of the Hull-White model (1990) served as motivation to apply the model to South African over-the-counter bond options, which are American options. The Hull-White model has had to be adjusted for its application to South African bond options, as these options are traded on the yield-to-maturity of the bond, rather than the price. Since the numerical solution to the Hull-White model uses the current term structure of interest rates as an input, the zero-coupon curve is used. Optimum conditions for the early exercise of over the counter bond options are discussed.

The complexity of the Hull-White model encouraged the development of a simplified model for exchange-traded options on the South African Futures Exchange (SAFEX). An exchange-traded bond option has no short-term risk-free rate component, as the underlying instrument is the bond future and the only payment is being made to a margin account where interest is earned. Therefore, instead of using the risk-free rate as the stochastic variable, it is possible to assume that the yield-to-maturity of the bond, and not the price, follows a Brownian motion. A pricing model for options on the future yield of a bond is in many ways similar to the Black model (1976). However, the yield-based model addresses most of the disadvantages of the Black model.