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CHAPTER 8

CONCLUSIONS

Although a large body of research already exists in the area of derivative securities, the characteristics of the South African fixed income market pose particular challenges for researchers in this market. This study has set out to add value to a specific area where little work has been done up till now, building on the theoretical work of other practitioners and scientists.

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8.1 Summary

The aim of this study was to add value to the South African fixed income market by analysing the existing procedures and models that are being used and, where necessary, to make a contribution by recommending alternatives.

Chapter 2 introduced the basic theory of pricing derivative securities. If a unique probability measure could be found so that the relative prices in an economy become martingales, then a continuous economy would be complete and free of arbitrage opportunities. The no-arbitrage assumption plays an important role in many pricing models. The concepts of no-arbitrage, martingales and partial differential equations serve as the basis for the valuation of derivative securities.

Chapters 3 and 4 were concerned with the term structure of interest rates. The short-term risk-free rate of interest (or spot rate) is the cornerstone of the fixed income market. The short-term risk-free rate and the market consensus on the future change in this rate form the term structure of interest rates. The term structure of interest rates determines the price of fixed deposits, bonds, swaps and other derivative securities in the fixed income market.

The zero-coupon yield curve can serve as the basis for estimating all other fixed income instruments. The estimation of the zero-coupon yield curve is, therefore, fundamental in order to price all other derivatives accurately, including bond options. The standard bootstrap method is cumbersome and the procedure of estimating the zero-coupon rates causes discrepancies. For these reasons, Chapter 4 introduced an iterative bootstrap method. This method starts with a first guess for the zero curve and then uses an iterative procedure which

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converges to the actual zero-coupon curve. Convergence to the actual yield curve following this method is proved. This method generates a zero-coupon curve in a much smoother and manageable way, without having to use other time-consuming numerical methods such as the Newton Raphson technique.

Chapter 5 discussed relevant bond option pricing models and focussed on the most appropriate model, the Hull-White model. The Hull-White model is based on the stochastic behaviour of the short-term rate and prices European options using the exact solution for the partial differential equation obtained. Hull and White further introduced a trinomial tree numerical approach to obtain a fair value for an American option, as discussed in Chapter 6. In order to use the Hull-White model for South African OTC bond options, the model was adjusted to make provision for a yield-strike convention, rather than a price-strike. The influence of the different strike conventions was shown for both coupon and zero-coupon bonds. The influence of the shape of the term structure became clear when the results of both a sharply increasing and decreasing term structure were compared. The successful use of the Hull-White model in practice depends largely on the estimation of the volatility parameters σ_r and a. The calibration of these parameters was discussed for zero-coupon and coupon bonds.

Since the convention in South Africa is to use a European model for the pricing of American options, the difference between European and American options was determined according to the Hull and White model, in order to establish an error-factor. Empirical results for South African options show a significant difference between the European and American prices.

SAFEX-traded bond options are options on the future yield of a bond, with an initial margin and a margin account on which interest is earned. Since the short-term risk-free rate does not

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influence these options directly, a model based on the stochastic behaviour of the future yield-to-maturity is more suitable. Chapter 7 discussed an option pricing model for SAFEX-traded bond options. The simplicity of this model makes it comparable to the Black model, which prices bond options using the stochastic behaviour of the *price* of the bond. A specific benefit of the yield-based model is that it addresses the disadvantages of the Black model.

8.2 Conclusions

This study adds value to several areas in the fixed income market, with specific reference to the South African market. The impact of the results is significant. The areas of contribution can be divided into three categories:

- construction of a zero-coupon yield curve using a new method, called the iterative bootstrap method;
- pricing and calibrating longer dated American OTC bond options with the yield-strike convention, using a modified Hull-White model; and
- pricing SAFEX options on the future yield of a bond, using a new methodology.

The development of an iterative bootstrap technique benefits the estimation of a zero-coupon yield curve, first for trading purposes and, secondly, as input to obtain bond option prices using the Hull-White numerical solution.

The Hull-White numerical solution was modified in order to price options on the yield-to-maturity of a bond. The influence of the strike-convention, as well as the shape of the yield curve, is shown. The early-exercise value for American options becomes significant in some cases. The results illustrate the impact of using a European model to price American over-the-

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counter options. A convenient way of calibrating the Hull-White model to market data is suggested.

The characteristics of exchange-traded bond options made it feasible to develop a simplified model for SAFEX options. Empirical evidence of the correlation between price-volatility and the yield of the bond indicates a fundamental problem in applying the Black model to South African futures options. Since the yield-based method addresses all the major disadvantages of the Black model, it can be used with much more confidence to price future options, especially out-the-money options. More efficient hedging is also possible when one uses the delta of the yield-based model.

8.3 Recommendations

The fixed income market remains an area that requires further research and refinement, especially in South Africa. Historically, the South African yield curve has been one of the most interesting yield curves in emerging markets, because it has many different, and, sometimes unusual shapes. Generally this causes a problem, since it complicates the fit of data points. The success of the iterative bootstrap method is largely due to the approximation technique used to fit the data points. Although success has been achieved in this study in using different combinations of linear functions, these functions require further adjustments, particularly when there is a large change in the shape of the yield curve. The development of a fitting procedure that adjusts automatically to the shape of the yield curve would add more value to the iterative bootstrap technique.

Vanilla European and American options on coupon bonds, which simultaneously address

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options on swaps, have been considered in this study. Other options on fixed rate instruments, as well as exotic options are, however, areas of research which have drawn little attention so far. Future research in these areas is recommended.

Volatility remains the most important input parameter in any option pricing model. The longer the option term, the bigger the influence of the expected volatility on the option price. Accurate estimation of the volatility parameters used in the Hull-White model numerical method becomes more crucial when there is no benchmark. Intensive research in this area will benefit practical users of the Hull-White model for South African options and will make the model more accessible to practitioners at large.

The distribution of the yield-to-maturity of bonds, as shown in this study, remains an interesting problem. Empirical research in this area to find an improved fit to the distribution of data, and a model to approximate the valuation of an option that satisfies this distribution can add more value.