

**An analysis of the
term structure of interest rates
and bond options in the
South African capital market**

by

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