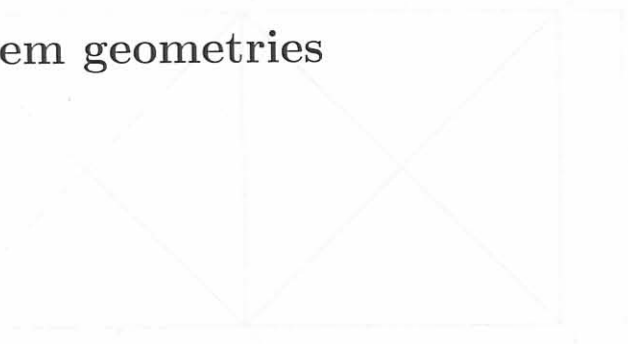


# Appendix B

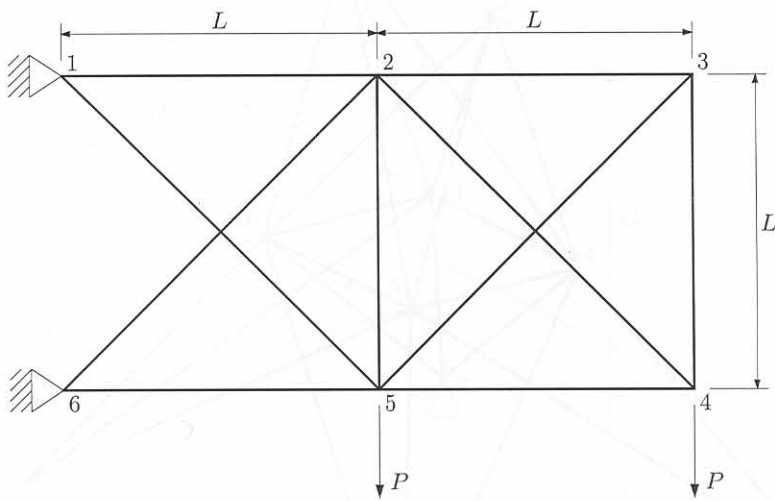
## Structural test problems

### B.1 Test problem geometries



Material	Aluminum
Young's Modulus	70 GPa
Yield Strength	270 MPa
Ultimate Tensile Strength	350 MPa
Area Moment of Inertia	1.0e-06 m <sup>4</sup>
Length	1.0 m
Volume	1.0e-06 m <sup>3</sup>

Figure B.1: 10-bar truss

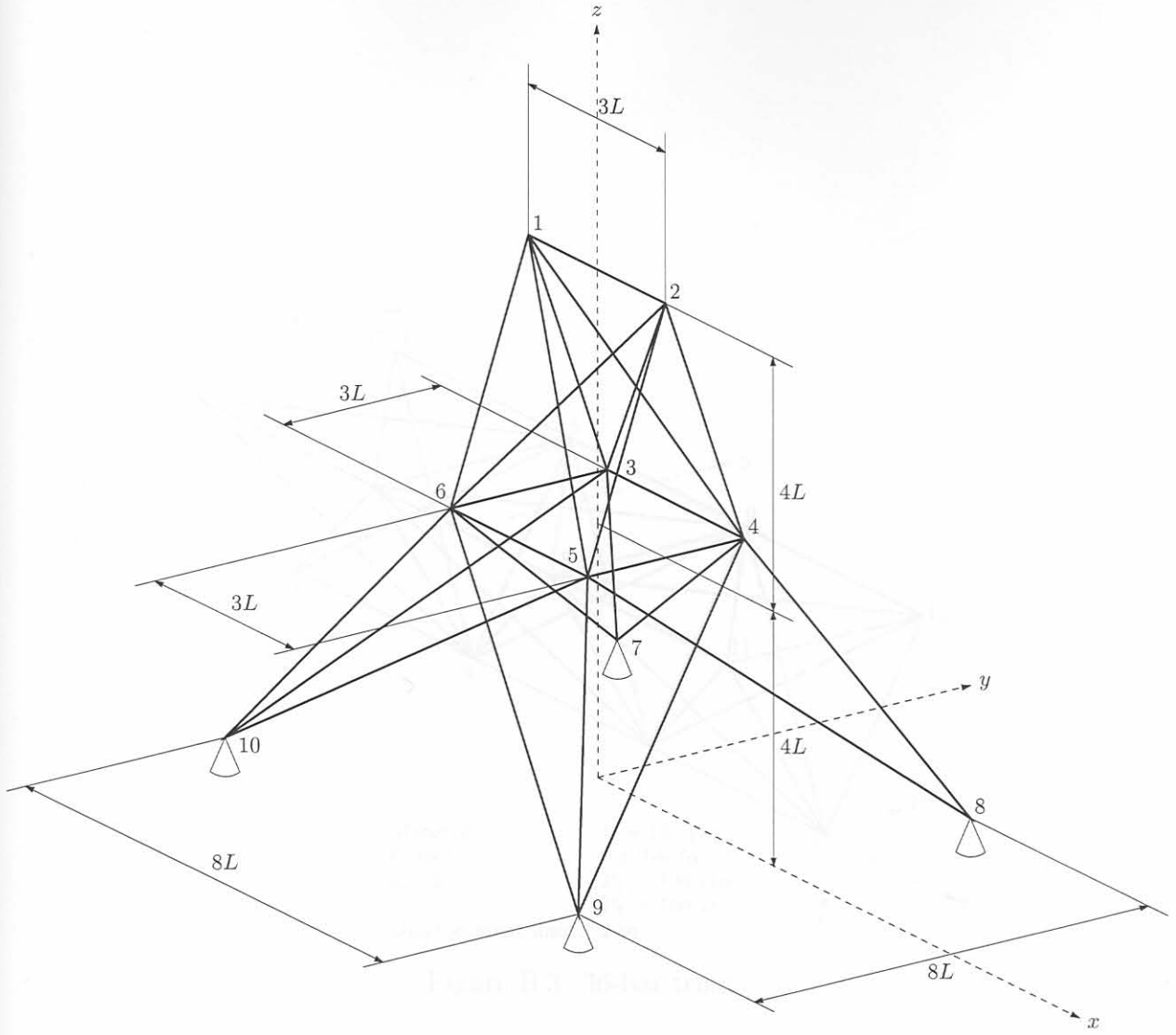


Material:  $E = 10^7$  psi  
 Density:  $0.1 \text{ lbm/in}^3$   
 Yield stress: 25000 psi  
 Displacement limit: 2 in  
 Length:  $L = 360$  in  
 Load:  $P = 100$  kip

Figure B.1: 10-bar truss

Material:  $E = 10^7$  psi  
 Density:  $0.1 \text{ lbm/in}^3$   
 Yield stress: 25000 psi  
 Displacement limit: 2 in  
 Length:  $L = 360$  in

Figure B.2: 10-bar truss



Material:	$E = 10^7$ psi
Density:	$0.1$ lbm/in <sup>3</sup>
Stress limit:	$40000$ psi
Displacement limit:	$0.35$ in
Length:	$L = 25$ in

Figure B.2: 25-bar truss

B.2 Numerical results

B.2.1 Penalty constraint method without social pressure

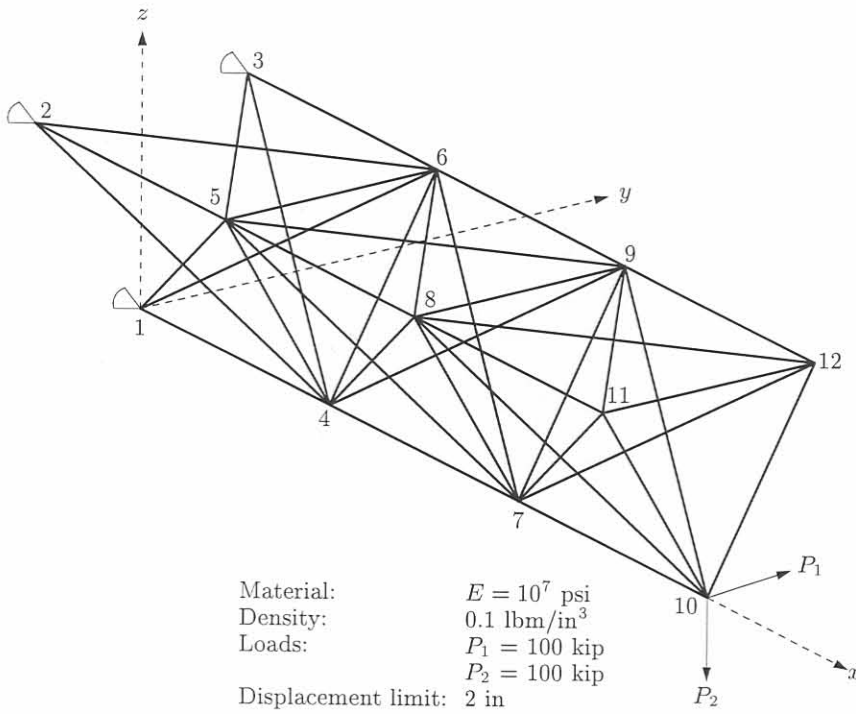


Figure B.3: 36-bar truss

## B.2 Numerical results

### B.2.1 Penalty constraint method without social pressure

$\lambda_{\text{max}}$ (lbs)	max. iter.	max. iter.	max. iter.	max. iter.
0	10	10	10	10
100	10	10	10	10
200	10	10	10	10
300	10	10	10	10
400	10	10	10	10
500	10	10	10	10
600	10	10	10	10
700	10	10	10	10
800	10	10	10	10
900	10	10	10	10
1000	10	10	10	10
1100	10	10	10	10
1200	10	10	10	10
1300	10	10	10	10
1400	10	10	10	10
1500	10	10	10	10
1600	10	10	10	10
1700	10	10	10	10
1800	10	10	10	10
1900	10	10	10	10
2000	10	10	10	10
2100	10	10	10	10
2200	10	10	10	10
2300	10	10	10	10
2400	10	10	10	10
2500	10	10	10	10
2600	10	10	10	10
2700	10	10	10	10
2800	10	10	10	10
2900	10	10	10	10
3000	10	10	10	10
3100	10	10	10	10
3200	10	10	10	10
3300	10	10	10	10
3400	10	10	10	10
3500	10	10	10	10
3600	10	10	10	10
3700	10	10	10	10
3800	10	10	10	10
3900	10	10	10	10
4000	10	10	10	10
4100	10	10	10	10
4200	10	10	10	10
4300	10	10	10	10
4400	10	10	10	10
4500	10	10	10	10
4600	10	10	10	10
4700	10	10	10	10
4800	10	10	10	10
4900	10	10	10	10
5000	10	10	10	10
5100	10	10	10	10
5200	10	10	10	10
5300	10	10	10	10
5400	10	10	10	10
5500	10	10	10	10
5600	10	10	10	10
5700	10	10	10	10
5800	10	10	10	10
5900	10	10	10	10
6000	10	10	10	10
6100	10	10	10	10
6200	10	10	10	10
6300	10	10	10	10
6400	10	10	10	10
6500	10	10	10	10
6600	10	10	10	10
6700	10	10	10	10
6800	10	10	10	10
6900	10	10	10	10
7000	10	10	10	10
7100	10	10	10	10
7200	10	10	10	10
7300	10	10	10	10
7400	10	10	10	10
7500	10	10	10	10
7600	10	10	10	10
7700	10	10	10	10
7800	10	10	10	10
7900	10	10	10	10
8000	10	10	10	10
8100	10	10	10	10
8200	10	10	10	10
8300	10	10	10	10
8400	10	10	10	10
8500	10	10	10	10
8600	10	10	10	10
8700	10	10	10	10
8800	10	10	10	10
8900	10	10	10	10
9000	10	10	10	10
9100	10	10	10	10
9200	10	10	10	10
9300	10	10	10	10
9400	10	10	10	10
9500	10	10	10	10
9600	10	10	10	10
9700	10	10	10	10
9800	10	10	10	10
9900	10	10	10	10
10000	10	10	10	10

Table B.2.1: Numerical results with a penalty factor of  $10^6$  and  $\epsilon = 10^{-6}$ .

	Optimum solution	PSO-DIV variant solution		
$\epsilon_s$ (%)		5	2	1
$\bar{f}_{best}^g$ (lbs)		4781.55	4682.07	4643.86
$\bar{\sigma}$		34.75	10.32	6.66
$N_{fe}$ (ave.)		781	949	1185
<i>Reliability</i>		10/10	9/10	8/10
Best found $f_{best}^g$ (lbs)	4607.1	4713.56	4665.98	4632.90
$NI$	0	0.1628	0.0426	0.0604
$x_1$	25.358	19.674	21.142	24.528
$x_2$	5.000	5.706	5.039	5.326
$x_3$	17.839	18.897	19.904	15.263
$x_4$	11.238	7.655	10.988	14.938
$x_5$	5.000	5.094	6.901	5.171
$x_6$	5.000	6.442	6.125	5.524
$x_7$	5.000	10.601	5.469	5.493
$x_8$	15.501	14.787	14.801	14.552
$x_9$	15.893	9.400	13.410	13.297
$x_{10}$	5.000	9.837	6.249	6.104
$N_{fe}$		492	1177	888

 Table B.1: Convex 10-bar truss results with *a priori* stopping criterion

	Optimum solution	PSO-DIV variant solution		
$\epsilon_s$ (%)		5	2	1
$\bar{f}_{best}^g$ (lbs)		5292.40	5153.62	5105.78
$\bar{\sigma}$		15.120	5.776	4.818
$N_{fe}$ (ave.)		1304	1609	2019
<i>Reliability</i>		10/10	10/10	10/10
Best found $f_{best}^g$ (lbs)	5060.85	5260.94	5144.22	5094.34
$NI$	0	0.0784	0.129	0.0886
$x_1$	30.522	29.208	27.461	29.148
$x_2$	0.100	0.462	0.787	0.355
$x_3$	23.200	22.853	24.337	23.354
$x_4$	15.223	11.717	12.868	13.999
$x_5$	0.100	0.344	0.336	0.130
$x_6$	0.551	0.833	0.971	0.327
$x_7$	7.457	10.230	7.614	9.086
$x_8$	21.036	26.118	23.568	19.466
$x_9$	21.528	18.619	20.730	21.847
$x_{10}$	0.100	1.135	0.105	0.650
$N_{fe}$		1034	1002	1541

 Table B.2: Non-convex 10-bar truss results with *a priori* stopping criterion

	Optimum solution	PSO-DIV variant solution		
		5	2	1
$\epsilon_s$ (%)				
$\bar{f}_{best}^g$ (lbs)		37487.34	36415.83	36378.53
$\bar{\sigma}$		41.812	37.836	4.740
$N_{fe}$ (ave.)		2970	2823	2433
<i>Reliability</i>				
	Optimum solution	PSO-DIV variant solution		
$\epsilon_s$ (%)		5	2	1
$\bar{f}_{best}^g$ (lbs)		569.22	555.08	550.29
$\bar{\sigma}$		3.639	1.144	3.639
$N_{fe}$ (ave.)		1324	1941	2528
<i>Reliability</i>		10/10	10/10	10/10
Best found $f_{best}^g$ (lbs)	545.04	559.55	552.32	549.30
$NI$	0	0.000	0.000	0.00996
$x_1$	0.010	0.268	0.525	0.085
$x_2$	2.042	1.741	2.134	1.960
$x_3$	3.002	3.341	2.705	3.087
$x_4$	0.010	0.226	0.019	0.063
$x_5$	0.010	0.154	0.021	0.073
$x_6$	0.683	0.679	0.672	0.756
$x_7$	1.623	1.881	1.737	1.675
$x_8$	2.671	2.486	2.740	2.533
$N_{fe}$		1322	2787	2083

 Table B.3: 25-bar truss results with *a priori* stopping criterion

 Table B.4: Cvxopt 36-bar truss results with *a priori* stopping criterion



	Optimum solution	PSO-DIV variant solution		
		5	2	1
$\epsilon_s$ (%)		5	2	1
$\bar{f}_{best}^g$ (lbs)		37457.34	36415.98	36078.53
$\bar{\sigma}$		44.012	37.836	4.740
$N_{fe}$ (ave.)		2970	5833	7438
Reliability		10/10	10/10	10/10
Best found $f_{best}^g$ (lbs)	35726	37375.39	36325.53	35602.03
$NI$	0	0.00836	0.000	0.0192
$x_1$	38.715	29.886	34.809	48.914
$x_2$	24.111	39.694	25.539	16.455
$x_3$	7.138	6.318	10.703	8.539
$x_4$	95.047	93.355	94.658	97.451
$x_5$	59.794	47.074	54.997	64.444
$x_6$	14.435	13.139	15.291	14.104
$x_7$	5.000	7.213	5.240	5.056
$x_8$	5.000	7.962	6.816	5.380
$x_9$	14.564	17.831	16.800	15.543
$x_{10}$	5.000	5.263	6.162	5.075
$x_{11}$	5.000	5.492	5.966	5.058
$x_{12}$	5.000	7.800	5.264	5.043
$x_{13}$	28.042	38.046	36.440	23.952
$x_{14}$	28.042	27.796	27.225	28.301
$x_{15}$	27.684	36.206	25.934	27.852
$x_{16}$	27.684	18.230	29.674	28.017
$x_{17}$	28.653	31.476	29.977	30.275
$x_{18}$	28.653	33.133	23.198	24.458
$x_{19}$	5.000	5.940	5.159	5.072
$x_{20}$	5.000	5.154	5.183	5.033
$x_{21}$	5.000	5.409	5.653	5.012
$N_{fe}$		2421	6546	10374

 Table B.4: Convex 36-bar truss results with *a priori* stopping criterion

	Optimum solution	PSO-DIV variant solution		
Stopping $N_{fe}$		2000	1000	500
$\epsilon_a$		0.01	0.01	0.01
$\bar{f}_{best}^g$ (lbs)		4606.26	4611.13	4606.56
$\bar{\sigma}$		0.2540	5.590	0.794
$N_{fe}$ (ave.)		26193	16298	7048
<i>Reliability</i>		10/10	10/10	10/10
Best found $f_{best}^g$ (lbs)	4607.1	4606.06	4607.19	4606.04
$NI$	0	0.001040	0.00433	0.00102
$x_1$	25.358	25.269	24.956	25.311
$x_2$	5.000	5.000	5.001	5.000
$x_3$	17.839	17.898	18.256	17.919
$x_4$	11.238	11.312	11.204	11.200
$x_5$	5.000	5.000	5.000	5.000
$x_6$	5.000	5.000	5.000	5.000
$x_7$	5.000	5.000	5.209	5.000
$x_8$	15.501	15.478	15.132	15.495
$x_9$	15.893	15.842	16.046	15.859
$x_{10}$	5.000	5.000	5.000	5.000
$N_{fe}$		18984	19131	8167

Table B.5: Convex 10-bar truss results with logical stopping criterion

	Optimum solution	PSO-DIV variant solution		
Stopping $N_{fe}$		2000	1000	500
$\epsilon_a$		0.01	0.01	0.01
$\bar{f}_{best}^g$ (lbs)		5064.05	5064.36	5064.70
$\bar{\sigma}$		7.306	5.119	7.986
$N_{fe}$ (ave.)		23242	13219	6688
<i>Reliability</i>		10/10	10/10	10/10
Best found $f_{best}^g$ (lbs)	5060.85	5059.89	5060.10	5060.06
$NI$	0	0.0175	0.000	0.00131
$x_1$	30.522	29.992	29.880	29.997
$x_2$	0.100	0.100	0.100	0.100
$x_3$	23.200	23.088	23.505	23.629
$x_4$	15.223	15.329	15.281	15.232
$x_5$	0.100	0.100	0.100	0.100
$x_6$	0.551	0.570	0.564	0.552
$x_7$	7.457	7.459	7.426	7.419
$x_8$	21.036	21.169	21.193	21.166
$x_9$	21.528	21.708	21.546	21.450
$x_{10}$	0.100	0.100	0.100	0.100
$N_{fe}$		21824	16991	8537

Table B.6: Non-convex 10-bar truss results with logical stopping criterion

	Optimum solution	PSO-DIV variant solution		
Stopping $N_{fe}$		2000	1000	500
$\epsilon_a$		0.01	0.01	0.01
$\bar{f}_{best}^g$ (lbs)		546.56	547.67	547.11
$\bar{\sigma}$		1.092	1.501	1.402
$N_{fe}$ (ave.)		9914	6520	3003
Reliability		10/10	10/10	10/10
Best found $f_{best}^g$ (lbs)	545.04	545.33	545.77	545.89
$NI$	0	0.000136	0.00007	0.0100
$x_1$	0.010	0.010	0.010	0.010
$x_2$	2.042	2.025	2.093	2.143
$x_3$	3.002	2.995	3.140	2.962
$x_4$	0.010	0.010	0.010	0.010
$x_5$	0.010	0.010	0.010	0.010
$x_6$	0.683	0.641	0.672	0.614
$x_7$	1.623	1.645	1.525	1.564
$x_8$	2.671	2.729	2.675	2.793
$N_{fe}$		6436	5659	4122

Table B.7: 25-bar truss results with logical stopping criterion

$f_{best}^g$	28.553	28.522	28.553	28.518
$NI$	3.000	5.000	3.000	3.000
$x_1$	5.000	5.000	5.000	5.000
$x_2$	5.000	5.000	5.001	5.000
$N_{fe}$		37506	22427	12329

Table B.8: Convex 35-bar truss results with logical stopping criterion

## B.2.2 Social pressure modification method

	Optimum solution	PSO-DIV variant solution		
Stopping $N_{fe}$		2000	1000	500
$\epsilon_a$		0.01	0.01	0.01
$\bar{f}_{best}^g$ (lbs)		35743.58	35726.16	35673.95
$\bar{\sigma}$		65.370	116.551	0.445
$N_{fe}$ (ave.)		39886	23577	14666
<i>Reliability</i>		9/10	10/10	10/10
Best found $f_{best}^g$ (lbs)	35726	35672.98	35673.23	35673.12
$NI$	0	0.167	0.028	0.008
$x_1$	38.715	38.577	38.691	39.184
$x_2$	24.111	24.273	24.310	23.735
$x_3$	7.138	7.226	7.048	7.154
$x_4$	95.047	94.594	94.604	94.536
$x_5$	59.794	59.825	59.597	59.624
$x_6$	14.435	14.318	14.478	14.410
$x_7$	5.000	5.000	5.000	5.000
$x_8$	5.000	5.000	5.000	5.000
$x_9$	14.564	14.453	14.559	14.501
$x_{10}$	5.000	5.000	5.000	5.000
$x_{11}$	5.000	5.000	5.000	5.000
$x_{12}$	5.000	5.000	5.001	5.000
$x_{13}$	28.042	28.039	28.118	27.880
$x_{14}$	28.042	27.969	27.691	28.055
$x_{15}$	27.684	27.525	27.460	27.667
$x_{16}$	27.684	27.550	27.790	27.497
$x_{17}$	28.653	28.533	28.515	28.501
$x_{18}$	28.653	28.522	28.555	28.618
$x_{19}$	5.000	5.000	5.000	5.000
$x_{20}$	5.000	5.000	5.000	5.000
$x_{21}$	5.000	5.000	5.001	5.000
$N_{fe}$		37506	22427	12329

Table B.8: Convex 36-bar truss results with logical stopping criterion

B.2.2 Social pressure modification method

	IV variant		
	solution	2	1
$\mu$ (%)	5	2	1
$f_{best}$ (kN)	4819.40	4094.14	4050.74
$\sigma$	14.042	5.206	1.382
$N_{FE}$ (ave.)	425	936	1392
Reliability	10/10	10/10	10/10
Best found $f_{best}$ (kN)	4597.2	4790.98	4682.30
$\Delta$	0.000	0.0000	0.0000
$\Delta$	21.877	21.761	25.865
$\Delta$	7.607	5.701	5.032
$\Delta$	17.649	19.879	21.067
$\Delta$	11.251	11.611	11.446
$\Delta$	6.131	5.280	5.071
$\Delta$	6.904	3.242	3.063
$\Delta$	7.096	7.290	7.832
$\Delta$	17.507	19.478	13.139
$\Delta$	17.861	16.221	15.198
$\Delta$	7.000	6.870	5.366
$\Delta$	5.0	9.6	3.8

Table 10: Comparison of results of PSOA Convex 16-bar truss results with a 2000 stopping criteria.

2

	Optimum solution	PSO-DIV variant solution		
$\epsilon_s$ (%)		5	2	1
$\bar{f}_{best}^g$ (lbs)		4819.40	4694.14	4650.74
$\bar{\sigma}$		14.042	5.206	1.882
$N_{fe}$ (ave.)		425	936	1302
<i>Reliability</i>		10/10	10/10	10/10
Best found $f_{best}^g$ (lbs)	4607.1	4790.98	4682.30	4647.62
$NI$	0	0.0000	0.0000	0.0000
$x_1$	25.358	24.761	25.665	24.181
$x_2$	5.000	5.701	5.122	5.032
$x_3$	17.839	19.879	21.799	21.096
$x_4$	11.238	11.817	8.908	11.449
$x_5$	5.000	6.231	5.286	5.071
$x_6$	5.000	6.904	5.242	5.063
$x_7$	5.000	7.290	7.832	6.155
$x_8$	15.501	10.476	13.130	12.456
$x_9$	15.893	16.221	13.696	15.765
$x_{10}$	5.000	6.876	6.365	5.913
$N_{fe}$	21500	610	954	1180

 Table B.9: Social pressure modified PSOA: Convex 10-bar truss results with *a priori* stopping criterion

 Table B.10: Social pressure modified PSOA: Non-convex 10-bar truss results with *a priori* stopping criterion

## APPENDIX B. STRUCTURAL TEST PROBLEMS

	Optimum solution	PSO-DIV variant solution		
$\epsilon_s$ (%)		5	2	1
$\bar{f}_{best}^g$ (lbs)		5293.93	5153.06	5108.54
$\bar{\sigma}$		11.401	8.142	3.451
$N_{fe}$ (ave.)		803	1083	1366
<i>Reliability</i>		10/10	10/10	10/10
Best found $f_{best}^g$ (lbs)	5060.85	5277.19	5134.86	5100.52
$NI$	0	0.0066	0.0000	0.0032
$x_1$	30.522	26.401	28.174	29.812
$x_2$	0.100	0.184	0.480	0.186
$x_3$	23.200	22.034	23.646	25.610
$x_4$	15.223	17.376	14.359	16.452
$x_5$	0.100	0.497	0.139	0.128
$x_6$	0.551	1.355	0.245	0.496
$x_7$	7.457	6.992	8.497	7.422
$x_8$	21.036	25.164	23.827	20.411
$x_9$	21.528	22.834	21.015	20.741
$x_{10}$	0.100	0.491	0.113	0.153
$N_{fe}$		897	969	1206

Table B.10: Social pressure modified PSOA: Non-convex 10-bar truss results with *a priori* stopping criterion



	Optimum solution	PSO-DIV variant solution		
		5	2	1
$\epsilon_s$ (%)		5	2	1
$f_{best}^g$ (lbs)		37450.83	36489.63	36074.39
$\bar{\sigma}$		59.918	67.874	11.962
$N_{fe}$ (ave.)		1237	1973	2431
<i>Reliability</i>		10/10	10/10	10/10
Best found $f_{best}^g$ (lbs)	545.04	561.42	550.38	549.82
$NI$	0	0.000	0.000	0.000
$x_1$	0.010	0.292	0.092	0.055
$x_2$	2.042	1.964	2.208	2.012
$x_3$	3.002	3.014	3.165	2.744
$x_4$	0.010	0.142	0.051	0.025
$x_5$	0.010	0.433	0.068	0.039
$x_6$	0.683	0.621	0.726	0.735
$x_7$	1.623	1.809	1.452	1.823
$x_8$	2.671	2.683	2.616	2.638
$N_{fe}$		1062	765	3069

 Table B.11: Social pressure modified PSOA: 25-bar truss results with *a priori* stopping criterion

 Table B.12: Social pressure modified PSOA: 25-bar truss results with *a priori* stopping criterion

	Optimum solution	PSO-DIV variant solution		
		5	2	1
$\epsilon_s$ (%)		5	2	1
$\bar{f}_{best}^g$ (lbs)		37450.93	36489.65	36074.39
$\bar{\sigma}$		59.918	67.874	11.952
$N_{fe}$ (ave.)		1232	1973	2431
<i>Reliability</i>		10/10	9/10	10/10
Best found $f_{best}^g$ (lbs)	35726	37340.09	36385.04	36044.45
$NI$	0	0.0144	0.012	0.011
$x_1$	38.715	31.449	41.804	35.538
$x_2$	24.111	27.103	21.082	22.024
$x_3$	7.138	16.533	6.752	18.632
$x_4$	95.047	86.386	97.298	90.770
$x_5$	59.794	51.480	64.502	60.527
$x_6$	14.435	18.869	15.385	14.742
$x_7$	5.000	5.674	5.510	5.214
$x_8$	5.000	6.386	7.810	5.518
$x_9$	14.564	19.458	15.522	17.323
$x_{10}$	5.000	16.851	5.298	5.191
$x_{11}$	5.000	16.223	5.944	5.450
$x_{12}$	5.000	5.160	5.316	5.646
$x_{13}$	28.042	33.090	35.448	27.734
$x_{14}$	28.042	25.307	15.944	30.645
$x_{15}$	27.684	27.160	26.140	27.238
$x_{16}$	27.684	30.415	29.242	27.214
$x_{17}$	28.653	29.223	29.592	30.169
$x_{18}$	28.653	31.422	27.427	25.223
$x_{19}$	5.000	5.342	5.249	5.147
$x_{20}$	5.000	5.778	5.803	5.065
$x_{21}$	5.000	5.505	5.110	5.136
$N_{fe}$		1091	1778	1490

 Table B.12: Social pressure modified PSOA: Convex 36-bar truss results with *a priori* stopping criterion

	Optimum solution	PSO-DIV variant solution		
Stopping $N_{fe}$		2000	1000	500
$\epsilon_a$		0.01	0.01	0.01
$\bar{f}_{best}^g$ (lbs)		4606.11	4606.12	4612.72
$\bar{\sigma}$		0.163	0.077	15.733
$N_{fe}$ (ave.)		9505	8552	6269
<i>Reliability</i>		10/10	10/10	10/10
Best found $f_{best}^g$ (lbs)	4607.1	4606.03	4606.04	4606.03
$NI$	0	0.000	0.000	0.000
$x_1$	25.358	25.342	25.337	25.313
$x_2$	5.000	5.000	5.000	5.000
$x_3$	17.839	17.843	17.817	17.854
$x_4$	11.238	11.244	11.234	11.245
$x_5$	5.000	5.000	5.000	5.000
$x_6$	5.000	5.000	5.000	5.000
$x_7$	5.000	5.000	5.000	5.000
$x_8$	15.501	15.467	15.541	15.470
$x_9$	15.893	15.886	15.842	15.893
$x_{10}$	5.000	5.000	5.000	5.000
$N_{fe}$		7392	6379	5866

Table B.13: Social pressure modified PSOA: Convex 10-bar truss results with logical stopping criterion

	Optimum	PSO-DIV variant		
	solution	solution		
Stopping $N_{fe}$		2000	1000	500
$\epsilon_a$		0.01	0.01	0.01
$\bar{f}_{best}^g$ (lbs)		5067.51	5062.33	5066.69
$\bar{\sigma}$		17.509	5.121	7.309
$N_{fe}$ (ave.)		10194	8011	4885
<i>Reliability</i>		10/10	10/10	10/10
Best found $f_{best}^g$ (lbs)	5060.85	5059.85	5059.89	5060.16
$NI$	0	0.00128	0.001	0.001
$x_1$	30.522	29.999	29.998	29.901
$x_2$	0.100	0.100	0.100	0.100
$x_3$	23.200	23.268	23.323	23.484
$x_4$	15.223	15.129	15.320	15.171
$x_5$	0.100	0.100	0.100	0.100
$x_6$	0.551	0.554	0.553	0.545
$x_7$	7.457	7.454	7.456	7.445
$x_8$	21.036	21.232	21.280	21.320
$x_9$	21.528	21.670	21.448	21.494
$x_{10}$	0.100	0.100	0.100	0.100
$N_{fe}$		10260	8212	6569

Table B.14: Social pressure modified PSOA: Non-convex 10-bar truss results with logical stopping criterion

	Optimum solution	PSO-DIV variant solution		
		10/10	10/10	10/10
Stopping $N_{fe}$		2000	1000	500
$\epsilon_a$		0.01	0.01	0.01
$\bar{f}_{best}^g$ (lbs)		546.84	547.95	550.34
$\bar{\sigma}$		1.478	2.371	3.223
$N_{fe}$ (ave.)		9596	6301	3277
Reliability		10/10	10/10	10/10
Best found $f_{best}^g$ (lbs)	545.04	545.21	545.55	545.58
$NI$	0	0.000	0.000	0.000
$x_1$	0.010	0.010	0.010	0.018
$x_2$	2.042	2.121	2.138	2.139
$x_3$	3.002	2.893	3.052	2.878
$x_4$	0.010	0.010	0.010	0.018
$x_5$	0.010	0.010	0.010	0.014
$x_6$	0.683	0.671	0.663	0.667
$x_7$	1.623	1.611	1.527	1.604
$x_8$	2.671	2.717	2.704	2.727
$N_{fe}$		7126	6774	3492

Table B.15: Social pressure modified PSOA: 25-bar truss results with logical stopping criterion

	Optimum solution	PSO-DIV variant solution		
Stopping $N_{fe}$		2000	1000	500
$\epsilon_a$		0.01	0.01	0.01
$\bar{f}_{best}^g$ (lbs)		38983.18	35742.00	35762.24
$\bar{\sigma}$		70.389	60.567	56.896
$N_{fe}$ (ave.)		9287	7844	5102
<i>Reliability</i>		10/10	10/10	10/10
Best found $f_{best}^g$ (lbs)	35726	35673.91	35673.90	35674.71
$NI$	0	0.00829	0.00826	0.00837
$x_1$	38.715	38.607	38.109	38.178
$x_2$	24.111	24.665	24.890	25.031
$x_3$	7.138	6.811	7.077	6.885
$x_4$	95.047	94.767	94.926	94.320
$x_5$	59.794	59.637	59.424	59.633
$x_6$	14.435	14.391	14.432	14.520
$x_7$	5.000	5.000	5.000	5.000
$x_8$	5.000	5.000	5.000	5.000
$x_9$	14.564	14.514	14.549	14.539
$x_{10}$	5.000	5.000	5.000	5.000
$x_{11}$	5.000	5.000	5.000	5.000
$x_{12}$	5.000	5.000	5.000	5.000
$x_{13}$	28.042	28.087	27.902	27.588
$x_{14}$	28.042	27.806	28.011	28.233
$x_{15}$	27.684	27.220	27.611	27.737
$x_{16}$	27.684	27.721	27.413	27.571
$x_{17}$	28.653	28.756	28.612	28.563
$x_{18}$	28.653	28.455	28.496	28.571
$x_{19}$	5.000	5.000	5.000	5.000
$x_{20}$	5.000	5.000	5.000	5.000
$x_{21}$	5.000	5.000	5.000	5.000
$N_{fe}$		18846	12802	10250

Table B.16: Social pressure modified PSOA: Convex 36-bar truss results with logical stopping criterion

## B.2.3 Summary of standard penalty method vs. penalty method with social pressure

	Optimum solution	Constriction factor	Dynamic inertia and velocity red.
$\epsilon_s$ (%)		1	1
$\bar{f}_{best}^g$		36054.13	36074.39
$\bar{\sigma}$		29.991	11.952
$N_{fe}$ (Ave.)		4442	2431
<i>Reliability</i>		10/10	10/10
$f_{best}$	35726	35975.82	36044.45
$NI$	0	0.00291	0.011
$x_1$	38.715	34.040	35.538
$x_2$	24.111	32.391	22.024
$x_3$	7.138	6.479	18.632
$x_4$	95.047	100.794	90.770
$x_5$	59.794	61.501	60.527
$x_6$	14.435	13.427	14.742
$x_7$	5.000	5.227	5.214
$x_8$	5.000	5.119	5.518
$x_9$	14.564	12.137	17.323
$x_{10}$	5.000	5.291	5.191
$x_{11}$	5.000	5.571	5.450
$x_{12}$	5.000	5.070	5.646
$x_{13}$	28.042	27.526	27.734
$x_{14}$	28.042	27.858	30.645
$x_{15}$	27.684	29.411	27.238
$x_{16}$	27.684	24.788	27.214
$x_{17}$	28.653	26.359	30.169
$x_{18}$	28.653	30.316	25.223
$x_{19}$	5.000	5.119	5.147
$x_{20}$	5.000	5.265	5.065
$x_{21}$	5.000	5.020	5.136
$N_{fe}$		4436	1490

Table B.17: 36-bar truss: Comparison between constriction factor and dynamic inertia / velocity reduction variants

**B.2.3 Summary of standard penalty method vs. penalty method with social pressure**

$\alpha = 5\%$

$\hat{f}_{max}$ (lbs)	4781.55	4819.40
$\hat{f}_{min}$ (lbs)	34.746	14.04
Number of iterations	760	425
Feasibility ratio	10/10	10/10
Optimal $\hat{f}_{max}$ (lbs)	4713.56	4790.93
Optimal infeasibility	0.1628	0.0000
Time (sec)	162	62

$\alpha = 1\%$

$\hat{f}_{max}$ (lbs)	4694.14	
$\hat{f}_{min}$ (lbs)	5.306	
Number of iterations	930	
Feasibility ratio	10/10	
Optimal $\hat{f}_{max}$ (lbs)	4692.5	
Optimal infeasibility	0.0000	
Time (sec)	114	

$\alpha = 1\%$

$\hat{f}_{max}$ (lbs)	4756.1	
$\hat{f}_{min}$ (lbs)	1.25	
Number of iterations	1015	
Feasibility ratio	10/10	
Optimal $\hat{f}_{max}$ (lbs)	4647.62	
Optimal infeasibility	0.0000	
Time (sec)	1180	

Table B.2.3. 10-bar problem: Comparison between concrete implementations



	Standard penalty method	Penalty with social pressure
$\epsilon_s = 5\%$		
$\bar{f}_{best}^g$ (lbs)	4781.55	4819.40
$\bar{\sigma}$	34.746	14.04
$N_{fe}$ (ave.)	780	425
Convergence ratio	10/10	10/10
Best found $f_{best}^g$ (lbs)	4713.56	4790.98
Normalized infeasability	0.1628	0.0000
$N_{fe}$	492	610
$\epsilon_s = 2\%$		
$\bar{f}_{best}^g$ (lbs)	4682.07	4694.14
$\bar{\sigma}$	10.322	5.206
$N_{fe}$ (ave.)	949	936
Convergence ratio	9/10	10/10
Best found $f_{best}^g$ (lbs)	4665.98	4682.30
Normalized infeasability	0.0426	0.0000
$N_{fe}$	1177	954
$\epsilon_s = 1\%$		
$\bar{f}_{best}^g$ (lbs)	4643.86	4650.74
$\bar{\sigma}$	6.659	1.882
$N_{fe}$ (ave.)	1185	1302
Convergence ratio	8/10	10/10
Best found $f_{best}^g$ (lbs)	4632.90	4647.62
Normalized infeasability	0.0604	0.0000
$N_{fe}$	888	1180

Table B.18: Convex 10-bar problem: Comparison between constraint implementations

	Standard penalty method	Penalty with social pressure
$\epsilon_s = 5\%$		
$\bar{f}_{best}^g$ (lbs)	5292.40	5293.94
$\bar{\sigma}$	15.120	11.401
$N_{fe}$ (ave.)	1304	803
Convergence ratio	10/10	10/10
Best found $f_{best}^g$ (lbs)	5260.94	5277.12
Normalized infeasability	0.0784	0.0066
$N_{fe}$	1034	897
$\epsilon_s = 2\%$		
$\bar{f}_{best}^g$ (lbs)	5153.62	5153.06
$\bar{\sigma}$	5.776	8.142
$N_{fe}$ (ave.)	1609	1083
Convergence ratio	10/10	10/10
Best found $f_{best}^g$ (lbs)	5144.22	5134.86
Normalized infeasability	0.129	0.000
$N_{fe}$	1002	969
$\epsilon_s = 1\%$		
$\bar{f}_{best}^g$ (lbs)	5105.78	5108.54
$\bar{\sigma}$	4.818	3.451
$N_{fe}$ (ave.)	2019	1366
Convergence ratio	10/10	10/10
Best found $f_{best}^g$ (lbs)	5094.34	5100.52
Normalized infeasability	0.0886	0.0032
$N_{fe}$	1541	1206

Table B.19: Non-convex 10-bar problem: Comparison between constraint implementations

	Standard penalty method	Penalty with social pressure
$\epsilon_s = 5\%$		
$\bar{f}_{best}^g$ (lbs)	569.22	569.24
$\bar{\sigma}$	3.639	3.034
$N_{fe}$ (ave.)	1324	1029
Convergence ratio	10/10	10/10
Best found $f_{best}^g$ (lbs)	559.55	561.42
Normalized infeasability	0.000	0.000
$N_{fe}$	1322	1062
$\epsilon_s = 2\%$		
$\bar{f}_{best}^g$ (lbs)	555.08	554.56
$\bar{\sigma}$	1.144	1.893
$N_{fe}$ (ave.)	1941	1329
Convergence ratio	10/10	10/10
Best found $f_{best}^g$ (lbs)	552.32	550.38
Normalized infeasability	0.000	0.000
$N_{fe}$	2787	765
$\epsilon_s = 1\%$		
$\bar{f}_{best}^g$ (lbs)	550.29	550.26
$\bar{\sigma}$	3.639	0.204
$N_{fe}$ (ave.)	2528	2616
Convergence ratio	10/10	10/10
Best found $f_{best}^g$ (lbs)	549.30	549.82
Normalized infeasability	0.0099	0.000
$N_{fe}$	2083	3069

Table B.20: Non-convex 25-bar problem: Comparison between constraint implementations

	Standard penalty method	Penalty with social pressure
$\epsilon_s = 5\%$		
$\bar{f}_{best}^g$ (lbs)	37457.34	37450.93
$\bar{\sigma}$	44.012	59.918
$N_{fe}$ (ave.)	2970	1232
Convergence ratio	10/10	10/10
Best found $f_{best}^g$ (lbs)	37375.39	37340.09
Normalized infeasibility	0.0083	0.0144
$N_{fe}$	2421	1091
$\epsilon_s = 2\%$		
$\bar{f}_{best}^g$ (lbs)	36415.98	36489.65
$\bar{\sigma}$	37.836	67.874
$N_{fe}$ (ave.)	5833	1973
Convergence ratio	10/10	9/10
Best found $f_{best}^g$ (lbs)	36325.53	36385.04
Normalized infeasibility	0.000	0.0120
$N_{fe}$	6546	1778
$\epsilon_s = 1\%$		
$\bar{f}_{best}^g$ (lbs)	36078.53	36074.39
$\bar{\sigma}$	4.740	11.952
$N_{fe}$ (ave.)	7438	2431
Convergence ratio	10/10	10/10
Best found $f_{best}^g$ (lbs)	35602.03	36044.45
Normalized infeasibility	0.0192	0.0114
$N_{fe}$	10374	1490

Table B.21: Convex 36-bar problem: Comparison between constraint implementations

Figure B.2: Non-convex 10-bar truss: Typical history plot for constraint violation and maximum velocity variance.

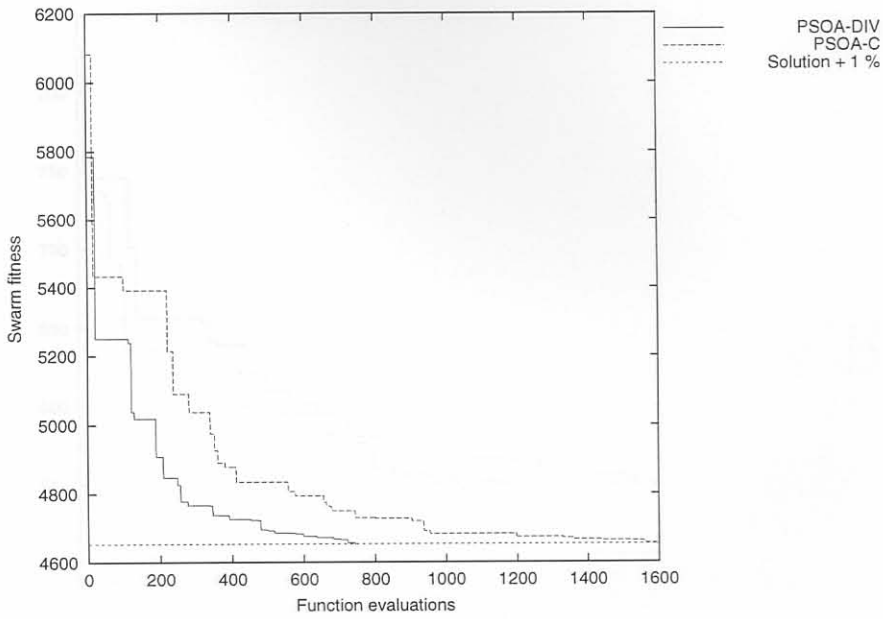


Figure B.4: Convex 10-bar truss: Typical history plot for contraction and dynamic inertia and maximum velocity variants

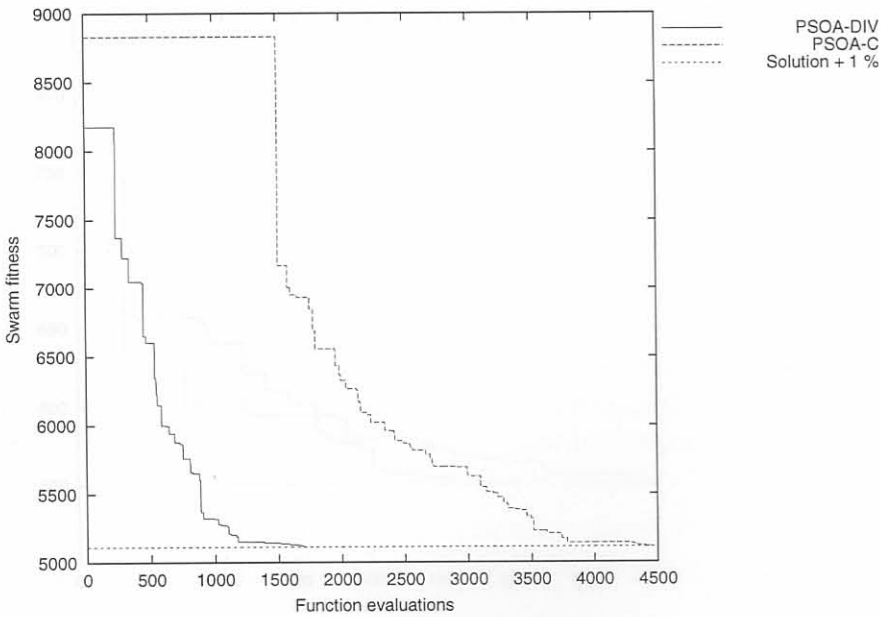


Figure B.5: Non-convex 10-bar truss: Typical history plot for contraction and dynamic inertia and maximum velocity variants

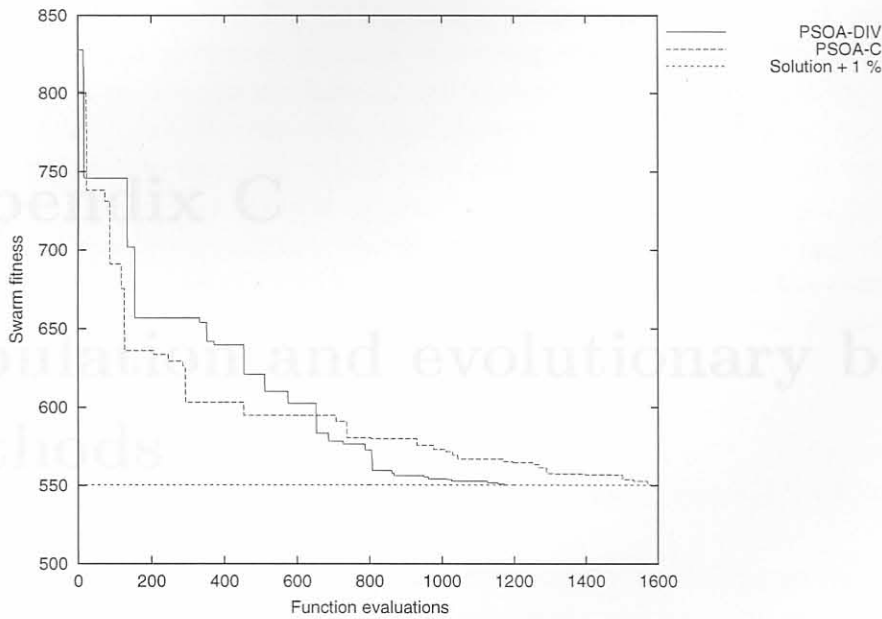


Figure B.6: Non-convex 25-bar truss: Typical history plot for contraction and dynamic inertia and maximum velocity variants

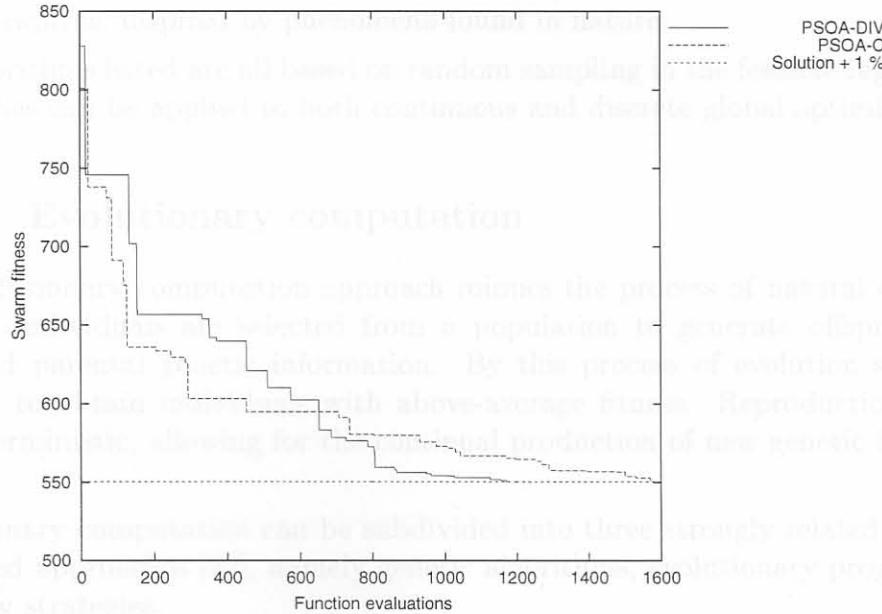


Figure B.7: Non-convex 36-bar truss: Typical history plot for contraction and dynamic inertia and maximum velocity variants

Genetic algorithms (GA's) were first introduced by Holland [53, 54, 55] and subsequently studied by De Jong [56, 57, 58], Goldberg [59, 60, 61, 62], and others.