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Chapter 6

Sizing Design of Truss Structures

6.1 Overview

In this chapter the application of the constriction and dynamic inertia and maximum velocity variants to the optimal sizing design of truss structures are studied. A simple methodology is proposed to accommodate the stress and displacement constraints during initial iterations, when a large number of particles may be infeasible. In this approach, increased social or peer pressure is exerted on infeasible particles to increase their rate of migration to feasible regions.

The development of this chapter is as follows: Firstly, the optimal size and shape design problem is formulated, whereafter the method for accommodating constraints into the PSOA is outlined. This is followed by the application of the PSOA to several well known problems in size optimization with dimensionality of up to 21.

6.2 Problem formulation

In the optimal sizing design of truss structures, the cross-sections of structural members are selected as the design variables $\boldsymbol{x}$. The minimum attainable structural weight is selected as the objective function, subject to allowable stress, displacement and linear buckling constraints. The optimal design problem detailed in Section 4.2 is then reformulated to include constraints as follows: Find the minimum weight f^* such that

$$f^* = f(\boldsymbol{x}^*) = \min f(\boldsymbol{x}) = \boldsymbol{a}^T\boldsymbol{x} \ , \tag{6.1}$$

subject to the general inequality constraints

$$g_j(\boldsymbol{x}^*) \leq 0, j = 1, 2, \ldots, m \ , \tag{6.2}$$

where $\boldsymbol{a}$ and $\boldsymbol{x}$ are column vectors in $\mathbb{R}^n$ and f and g_j are scalar functions of the design variables $\boldsymbol{x}$. The inequality constraints g_j represent the stress, strain, displacement or linear buckling constraints. The finite element method (FEM) may be used to approximate the objective function f and the constaint functions g_j.

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6.3 Accommodation of constraints

To facilitate the inclusion of the constraints (6.2) in the PSOA, (6.1) is modified to become

$$\tilde{f} = f(\boldsymbol{x}) + \sum_{j=1}^{m} \lambda_j [g_j(\boldsymbol{x})]^2 \mu_j(g_j) , \tag{6.3}$$

with

$$\mu_j(g_j) = \begin{cases} 0 & \text{if} \quad g_j(\boldsymbol{x}) \leq 0 \\ 1 & \text{if} \quad g_j(\boldsymbol{x}) > 0 \end{cases} , \tag{6.4}$$

and penalty parameters $\lambda_j > 0$, prescribed. In a typical search, the λ_j parameters are increased linearly with the number of function evaluations. This prevents undue enforcement of the constraints in early stages of the search, while ensuring that the final constraint violations are sufficiently small.

6.3.1 Social pressure

To increase the likelihood of particles migrating to feasible regions in the initial stages of the search, increased social or peer pressure is exerted, at the cost of cognitive learning. In this approach, the best particle value p^i_{best} and best swarm value g_{best} are only updated if the normalized infeasibility (NI^i) is smaller than a to be specified tolerance NI_{allow}. Hence cognitive learning of an initially infeasible bird, represented by c_1 and $\boldsymbol{p}^i_k$, is sacrificed. Only social pressure, represented by c_2 and $\boldsymbol{p}^g_k$, is retained.

This simple idea is implemented as follows: Steps 2(b) and 2(c) in the formal algorithm presented in Section 3.3 are replaced by:

2. (b) If $f^i_k < p^i_{best}$ and $NI^i < NI_{allow}$ then $p^i_{best} = f^i_k$, $\boldsymbol{p}^i_k = \boldsymbol{x}^i_k$, else $c_1 = 0$
   (c) If $f^i_k < g_{best}$ and $NI^i < NI_{allow}$ then $g_{best} = f^i_k$, $\boldsymbol{p}^g_k = \boldsymbol{x}^i_k$

Selecting $c_1 = 0$ is not necessarily optimal, and superior approaches will doubtless be suggested in future.

6.4 Stopping criteria

A number of logical stopping criteria may be specified for the PSOA. Amongst others, the algorithm can be terminated when the average swarm velocity or momentum reaches a prescribed fraction of the initial velocity and momentum. The algorithm can also be terminated when a specific number of iterations or function evaluations S occur without improvement in the best position $\boldsymbol{p}^g_k \leftrightarrow g_{best}$, within a prescribed tolerance ϵ_s.

In constructing stopping criteria, it is important to develop criteria which protect against over sampling of the objective function. Simultaneous, the competing objective of premature convergence should be prevented. While not necessarily optimal, two criteria here are

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considered here, namely the *a priory* condition, and the logical condition. They are selected here solely for their simplicity and illustrative powers.

A number of additional stopping criteria may be constructed. For instance, the algorithm can be terminated when the average momentum of the swarm reaches a prescribed fraction of the initial average momentum. A similar argument may off course be used for the average velocity of the swarm.

6.4.1 *A priori* stopping condition

The algorithm is terminated once it obtains the *a priori* known optimum within a prescribed tolerance. This stopping condition is commonly used in the training of neural networks [23], where the output error is minimized. In general, this is not a sensible stopping criterion for structural optimization. However, numerical results are included herein, since the criterion gives a good indication of how fast the algorithm converges to the region of the optimum, and presents a useful guide in estimating the required overhead in terms of number of function evaluations required by other stopping methods.

6.4.2 Logical stopping condition

In this condition, the swarm best value f^g_{best} is monitored as the search progresses. If there is no improvement for S specified function evaluations within a specified threshold tolerance ϵ_s, the search is stopped.

6.5 On swarm parameters

As shown by [15], the PSOA is sensitive to, in particular, the parameters w, c_1 and c_2, although dynamic inertia reduction reduces this sensitivity as opposed to constriction, as shown in Chapter 5.

In this study, unless otherwise stated, all results are generated using swarms consisting of 20 agents, with the cognitive and social scaling factors c_1 and c_2 both set to 2. In each case, the allowable normalized infeasibility NI_{allow} is set to 0.02, and the penalty parameters λ_j, $j = 1, 2, \ldots, m$, are linearly scaled from 10^3 to 10^6 in 4000 function evaluations, whereafter λ_j is constant.

6.6 Numerical results

The test set under consideration is tabulated in Table 6.1. The table gives the dimension n, the number of constraints m and the nature of the problems under consideration.

For the numerical results presented in Appendix B.2 each problem is analyzed 10 times, with the normalized infeasibility NI, the average fitness value f_{ave}, standard deviation $\bar{\sigma}$ and cost

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Problem Name	Problem Nature	n	m
10-Bar	Convex	10	32
10-Bar	Non-Convex	10	34
25-Bar	Non-Convex	8	84
36-Bar	Convex	21	95

Table 6.1: Structural test problems

N_{fe} being reported for each problem. For the sake of completeness, the best fitness value with its position and the associated number of function evaluations is also given.

In the following discussions, ϵ_a represents the tolerance in the *a priori* stopping condition, and '*Reliability*' the number of times the algorithm converged within 10000 function evaluations. S represents the number of function evaluations elapsed without improvement in best function value, within a tolerance ϵ_s, before termination.

6.6.1 Convex 10-bar truss

The structure is depicted in Figure B.1, and is described in, amongst others, [43]. For the *a priori* stopping condition, tabulated results are presented in Table B.1. (In the table, social pressure is not exerted on initially infeasible birds.) The table reveals that the function evaluations N_{fe} required for convergence is relatively low. In addition, the cost does not increase dramatically as ϵ_a is decreased from 0.05 to 0.01. However, the reliability decreases as ϵ_a becomes stricter, while the normalized infeasibility also decreases. Even so, the normalized infeasibility is some 6% for $\epsilon_a = 1\%$.

Upon the introduction of social pressure, (Table B.9), the reliability becomes 100%, while the decrease in constraint violations (from 16% to zero) is significant. Simultaneously, the computational effort N_{fe} decreases. (In Table B.1, the cost of unconverged searches is not reflected in the average cost.)

Finally, the high overhead of the logical stopping criterion is reflected by a comparison between Tables B.10 and B.6. In both cases, social pressure is exerted, while the latter table uses the logical stopping criterion, with $S = 2000$, 1000 and 500. Apparently, $S = 1000$ is adequate. Nevertheless, the the results indicate that the development of improved stopping criteria in future is of interest.

6.6.2 Non-convex 10-bar truss

This problem is also depicted in Figure B.1, and is amongst others, described in [43, 44]. The physical geometry for this problem is identical to the convex 10-bar truss, the only difference being a modified loading condition which induces multiple local minima in the fitness function [45, 46]

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For the sake of brevity, extensive results regarding the influence of the *a priori* stopping condition and social pressure are not given in tabulated form for this (and the following) problems. It suffices to state that the same trend as for the convex 10-bar truss is observed.

Numerical results for the logical stopping criterion and social pressure are presented in Table B.14. Once again, requiring $S = 1000$ is adequate. The normalized infeasibility upon convergence is acceptably small (approximately 0.1%).

6.6.3 Non-convex 25-bar truss

This structure is depicted in Figure B.2 [43, 47], and numerical results for the logical stopping criterion combined with social pressure are presented in Table B.7. For this problem, there is a more pronounced sensitivity to the value of S, and $S = 500$ would suffice.

6.6.4 Convex 36-bar truss

The final test problem is depicted in Figure B.3 [43, 45]. This convex problem is relatively difficult, with 21 design variables, and 95 constraints present. Numerical results using the *a priori* stopping criterion are presented in Table B.4, and using the logical stopping criterion in Table B.8.

The solutions obtained by using the *a priori* stopping condition (Table B.4) are all slightly infeasible, indicating that a higher value for λ_j would be beneficial. Nevertheless, in all cases the constraint violations are less than 1%.

In comparison with the *a priori* stopping criteria results, the logical stopping method performs very poorly, both in terms of cost and constraint violations. This poor performance was the main motivation for the introduction of the social pressure operator, for which numerical results are presented in the following section.

6.6.5 Effect of social pressure

The effect of social pressure is investigated for all of the structural test problems, and results with different *a priori* stopping values are summarized in Appendix B.2.3. Dramatic improvements in cost and normalized infeasability can be observed for all of the problems in this summary. When considering the logical stopping condition a similar trend is observed (compare Tables B.8 and B.16).

6.6.6 Comparison between PSOA-C and PSOA-DIV variants

Table B.17 illustrates the difference in performance between the constriction factor and dynamic inertia and velocity reduction. The average (f_{ave}) and best (f_{best}) fitness values obtained compare closely for the two methods, with the standard deviation $\bar{\sigma}$ for dynamic inertia and velocity reduction roughly 1/3 of the value for the constriction factor. The cost

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(N_{fe}) associated with the average and best fitness values for dynamic inertia and velocity reduction are roughly half the respective costs with constriction. (The same trend is observed for the other problems studied.)

6.6.7 Fitness history

Typical fitness histories, comparing the constriction and dynamic inertia and maximum velocity variants are presented in Figures B.4, B.5, B.6, and B.7. In terms of convergence rate, it is clear that the PSOA-DIV variant outperforms the PSOA-C variant.

6.7 Summary

The derivative free particle swarm optimization algorithm can effectively be used for the optimal sizing design of truss structures. While few results for constrained functions using the PSOA have previously been presented, social pressure is used herein to increase the likelihood of migration to feasible regions during the initial phases of the swarm search, thereby sacrificing the cognitive learning ability of initially infeasible particles. Use of the social operator leads to reduced cost with the logical stopping criteria, as well as limiting the constraint violations within a specified tolerance in all of the structural test problem cases.