MIMO Channel Modelling

for

Indoor Wireless Communications

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Sentech Chair in Broadband Wireless Multimedia Communications (BWMC)
Department of Electrical, Electronic and Computer Engineering

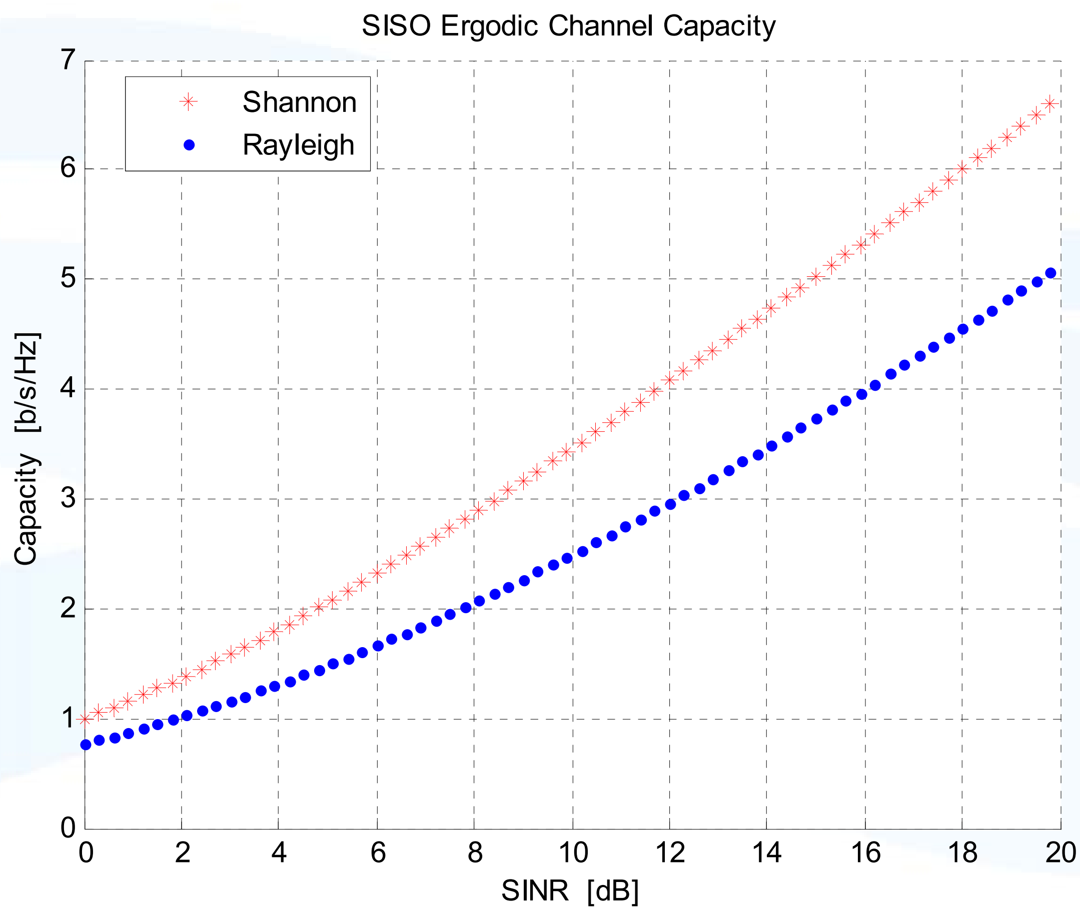
Presentation Outline

1. Introduction
2. Geometric Modelling
3. WB MIMO Measurement System
4. Model Assessment
   - Capacity
   - Spatial Correlation
   - Double Directional Channel
5. Maximum Entropy Approach to Channel Modelling
6. What has been Achieved?
7. Outputs

What is a MIMO System?

- Given an arbitrary wireless communication system, one considers a link for which the TX end and as well the RX end is equipped with multiple antenna elements.
- TX antenna signal and RX antennas at the other end are ‘combined’ in such a way that the BER or data rate(bps) of the communication for each MIMO user will be improved.

Traditional SISO System



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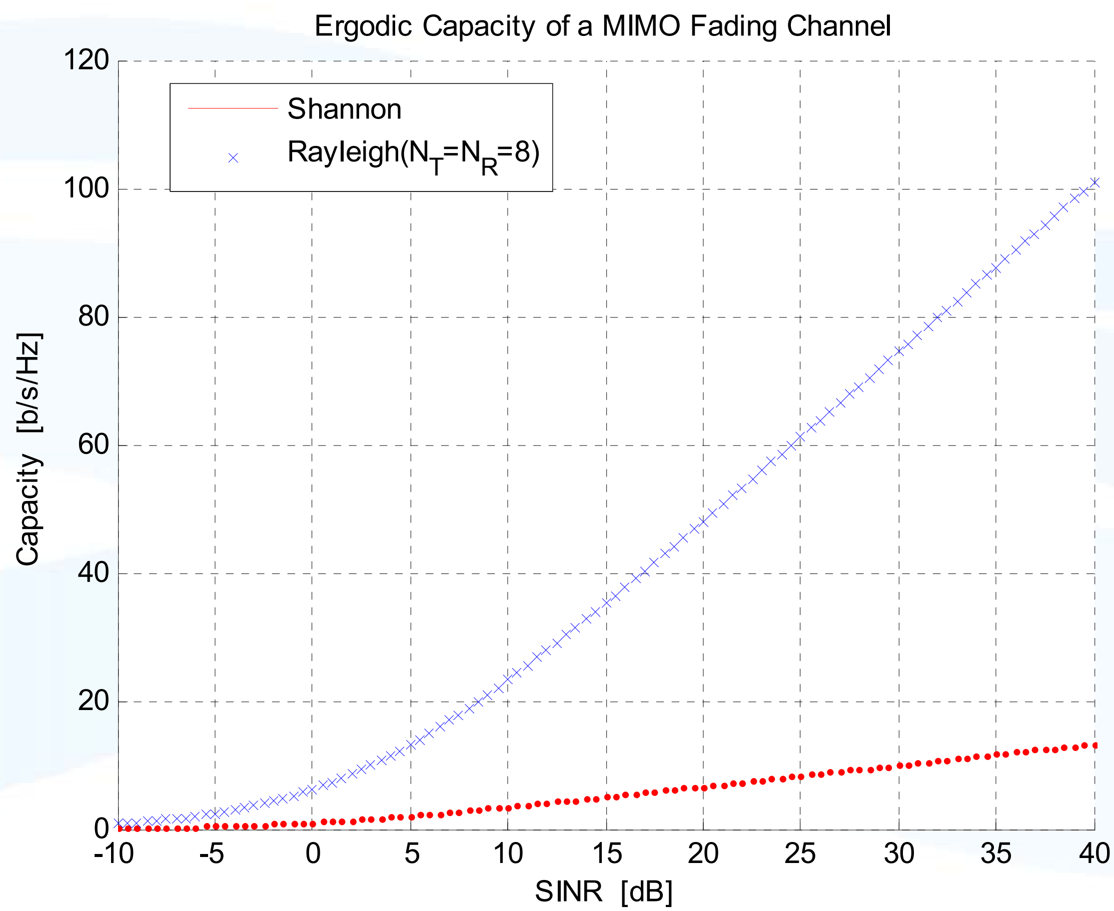
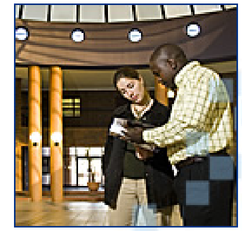
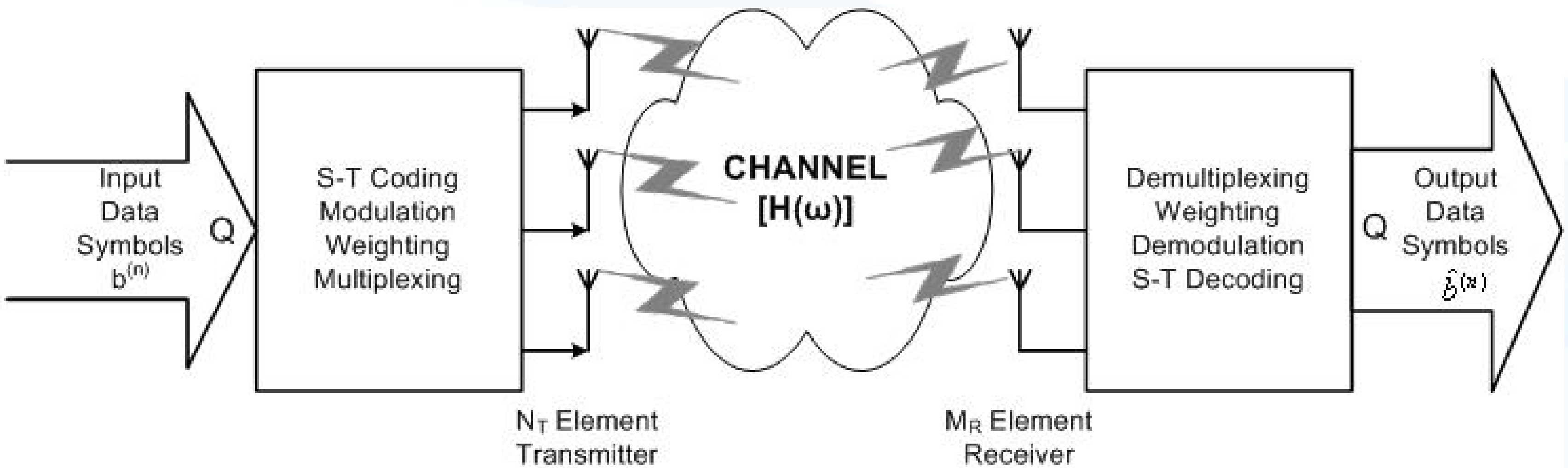
Opportunities for MIMO Technology – "Beyond the Shannon Bound"



Block Diagram of a MIMO Wireless System





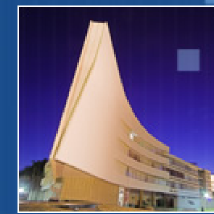
Benefits of MIMO Systems

- Spectral efficiency improvement
- Increases network's quality (QoS)
- Data rate increases substantially
- Operator's revenue increases
- Meet needs for future applications and services in 3G, 4G and NGN…

MIMO Channel Modelling

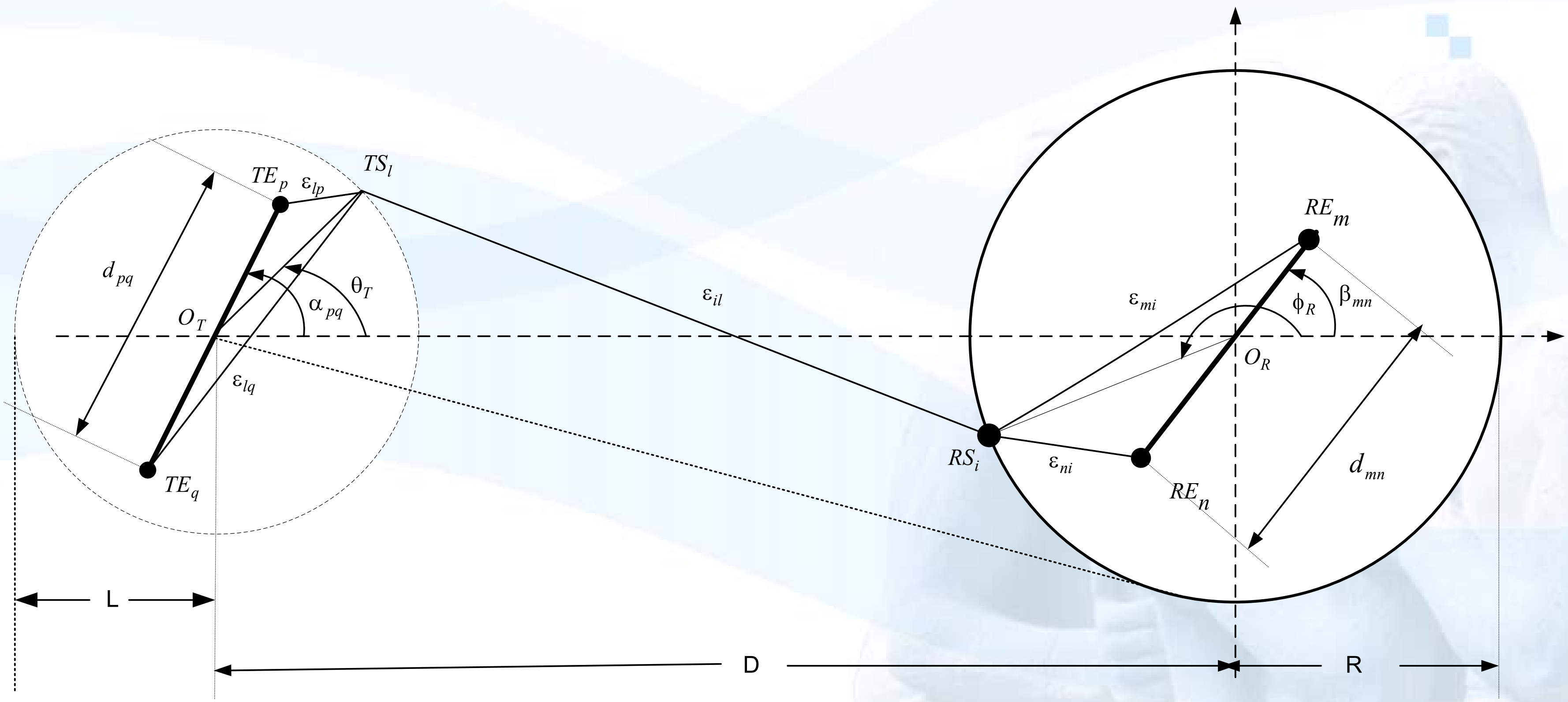
- MIMO systems increase capacity of wireless channel without increasing system BW in a rich scattering environment
- Space-time coding is informed by channel behaviour
- Various approaches to channel modelling:
  - Ray tracing
  - Geometric modelling
  - Channel sounding
- Channel sounding arguably most accurate representation of real world channels – ‘At a cost!!’

Geometric Modelling: System Description



- Fixed wireless scenario at 2.4 GHz
- Uniform scattering at TX
- Von Mises pdf of scatterers at RX with varying degrees of isotropic scattering
- Derive ST Model
- Present a ST correlation function with some key elements such as antenna element spacing, degree of scattering, AoA at user and antenna array configuration

Geometric Model for a 2x2 MIMO Channel



Mathematical Equation

This MIMO system can be written using the complex baseband notation as:

$$\bar{y}(t) = \bar{H}(t)\bar{x}(t) + \bar{n}(t)$$

$\bar{H}(t)$ is the channel matrix of complex path gains $h_{ij}(t)$ between TX_j and RX_i.

$\bar{n}(t)$ is the complex envelope of the AWGN with zero mean from each receive element,

$\bar{x}(t)$ is the transmit vector made up of the signal transmitted from each TX n_tx1 antenna element,

$\bar{y}(t)$ is the receive vector made up of the signal from each point RS_i.

The channel gain, $h_{mp}(t)$, for the link TE_p - RE_m as shown in Fig. 1, can be written as:

$$h_{mp}(t) = \sqrt{\Omega_{mp}} \lim_{L,N \to \infty} \frac{1}{\sqrt{LN}} \sum_{l=1}^{L} \sum_{i=1}^{N} g_{il} \times \exp\left\{ j\psi_{il} - \frac{2\pi j}{\lambda}\left[\varepsilon_{lp} + \varepsilon_{il} + \varepsilon_{mi}\right]\right\}$$

The Cross Correlation Function

The space-time correlation between two links, TE_p – RE_m and TE_q - RE_n as shown in Figure can be defined as:

$$\rho_{mp,nq}(\tau,t) = E[h_{mp}(t).h^*_{nq}(t+\tau)/\sqrt{\Omega_{mp}\Omega_{nq}}]$$

One can write:

$$h_{mp}(t) = \sqrt{\Omega_{mp}} \lim_{L,N\to\infty} \frac{1}{\sqrt{LN}} \sum_{l=1}^{L} \sum_{i=1}^{N} g_{il} \times \exp\left\{ j\psi_{il} - \frac{2\pi j}{\lambda}\left[\varepsilon_{lp} + \varepsilon_{il} + \varepsilon_{mi}\right]\right\}$$

$$h^*_{nq}(t) = \sqrt{\Omega_{nq}} \lim_{L,N\to\infty} \frac{1}{\sqrt{LN}} \sum_{l=1}^{L} \sum_{i=1}^{N} g_{il} \times \exp\left\{ -j\psi_{il} + \frac{2\pi j}{\lambda}\left[\varepsilon_{lq} + \varepsilon_{il} + \varepsilon_{ni}\right]\right\}$$

Making the respective substitutions gives:

$$\rho_{mp,nq} = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \exp\left\{\frac{-2\pi j}{\lambda}\left[\varepsilon_{\theta p} - \varepsilon_{\theta q} + \varepsilon_{m\phi} - \varepsilon_{n\phi}\right]\right\}.p(\theta_T)p(\theta_R)d\theta_T d\theta_R$$

Joint Antenna Correlation Function

One can write the JACF [7, 11, 13] as: $$\rho_{mp,nq} \cong \rho_{pq}^{TX} . \rho_{mn}^{RX}$$

If the pdf of scatterers at the TX is: $$p(\theta) = 1/2\pi$$

And at the RX the scattering distribution can be described by the von Mises PDF as

$$p(\phi) = \frac{\exp\left[k\cos(\phi - \mu)\right]}{2\pi I_0(k)}$$

where:

$$\mu \in [-\pi, \pi]$$

k is the isotropic scattering parameter

Φ is the mean direction of the AOA seen by the user

I_o is the zero order modified Bessel function

Antenna Correlation Functions

Simplifying the equations, one gets closed form expressions:

$$\rho_{pq}^{TX} = I_0[jc_{pq}] \qquad where; c_{pq} = \frac{2\pi d_{pq}}{\lambda}$$

$$\rho_{mn}^{RX} = \frac{1}{I_0(k)} I_0\left[k^2 - b_{mn}^2 + j2kb_{mn}\cos(\mu - \beta_{mn})\right]^{1/2} \qquad where; b_{mn} = \frac{2\pi d_{mn}}{\lambda}$$

Using:

$$\overline{X} = \overline{U}\sqrt{\overline{R_T}};$$

$\overline{R_T}$ is the $n_T \times n_T$ matrix of the TX antenna correlation

$$C = \log_2 \det\left(I_{n_R} + \frac{\rho}{n_T}\mathrm{HH}^H\right)$$

RESULTS

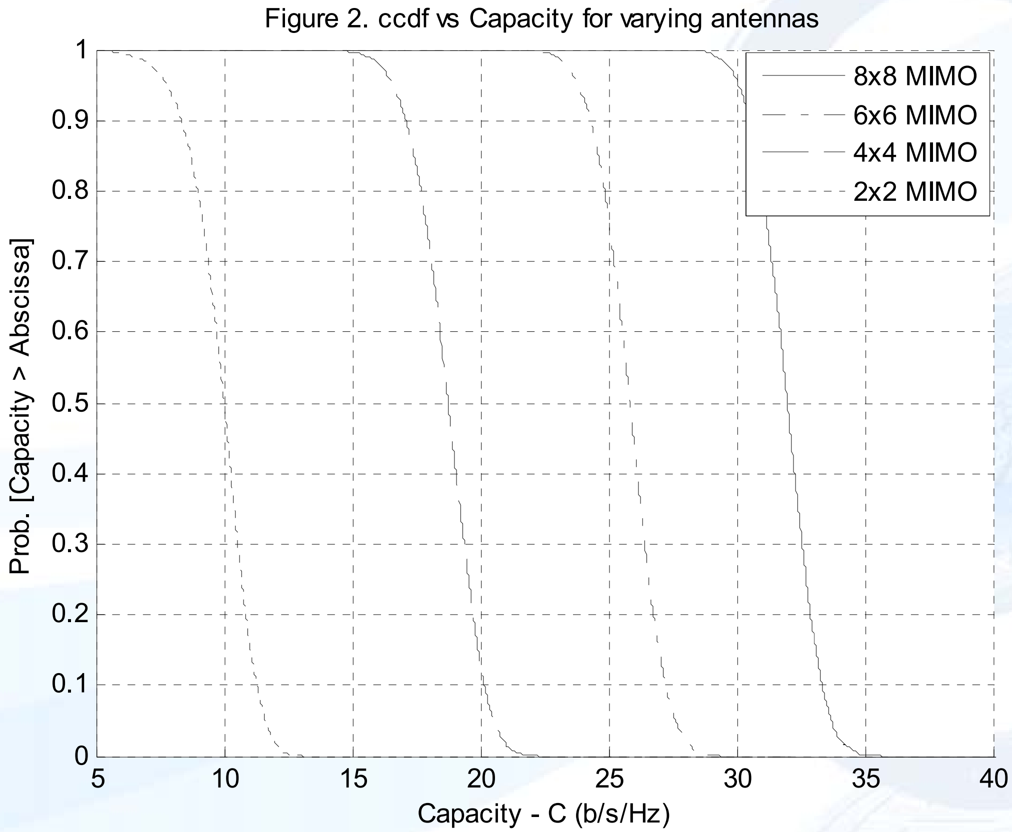


Figure 2. ccdf vs Capacity for varying antennas
8x8 MIMO
6x6 MIMO
4x4 MIMO
2x2 MIMO
Prob. [Capacity > Abscissa]
Capacity - C (b/s/Hz)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
5
10
15
20
25
30
35
40

RESULTS

Figure 3 ccdf vs Capacity for varying antennas element spacing at RX

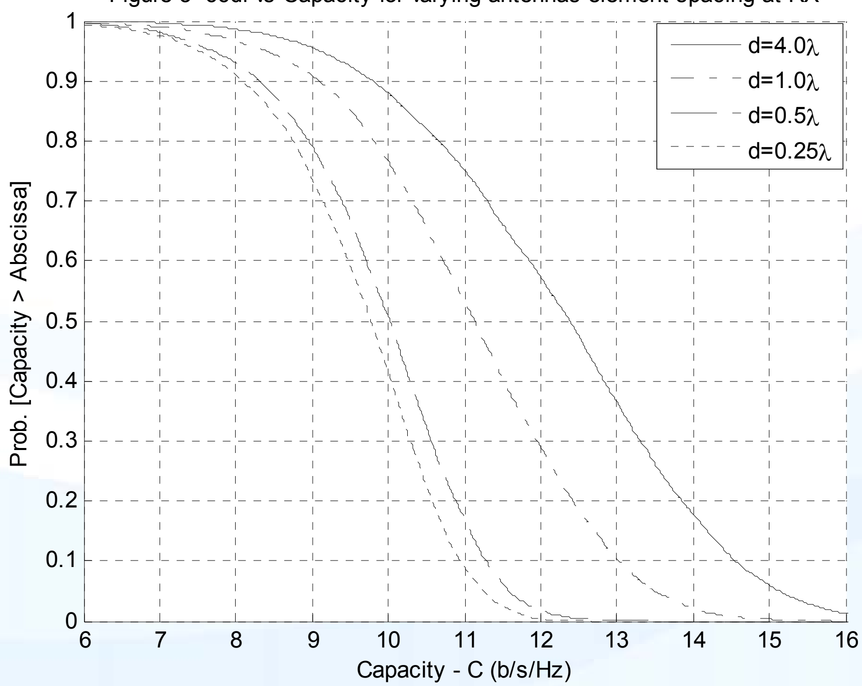
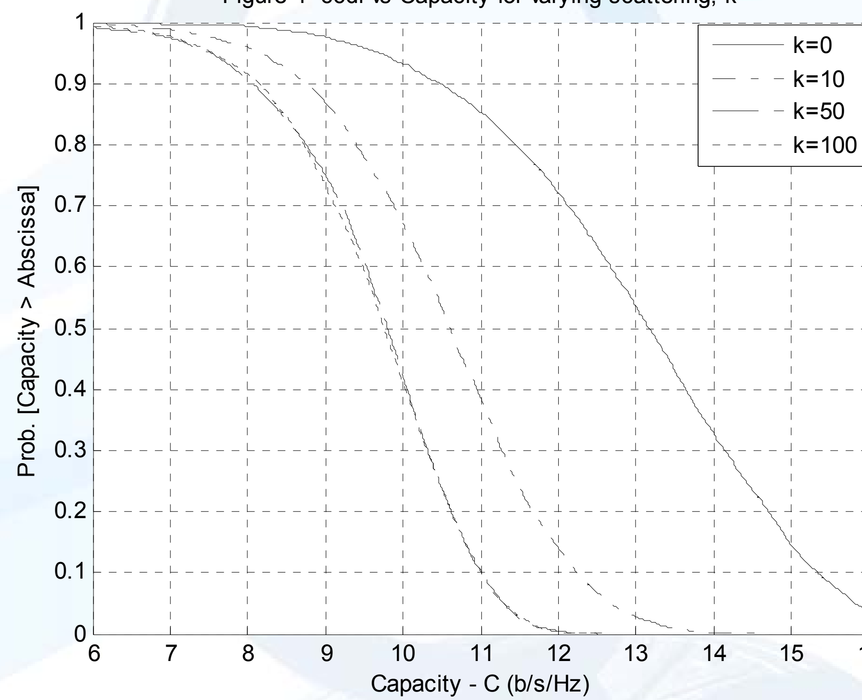
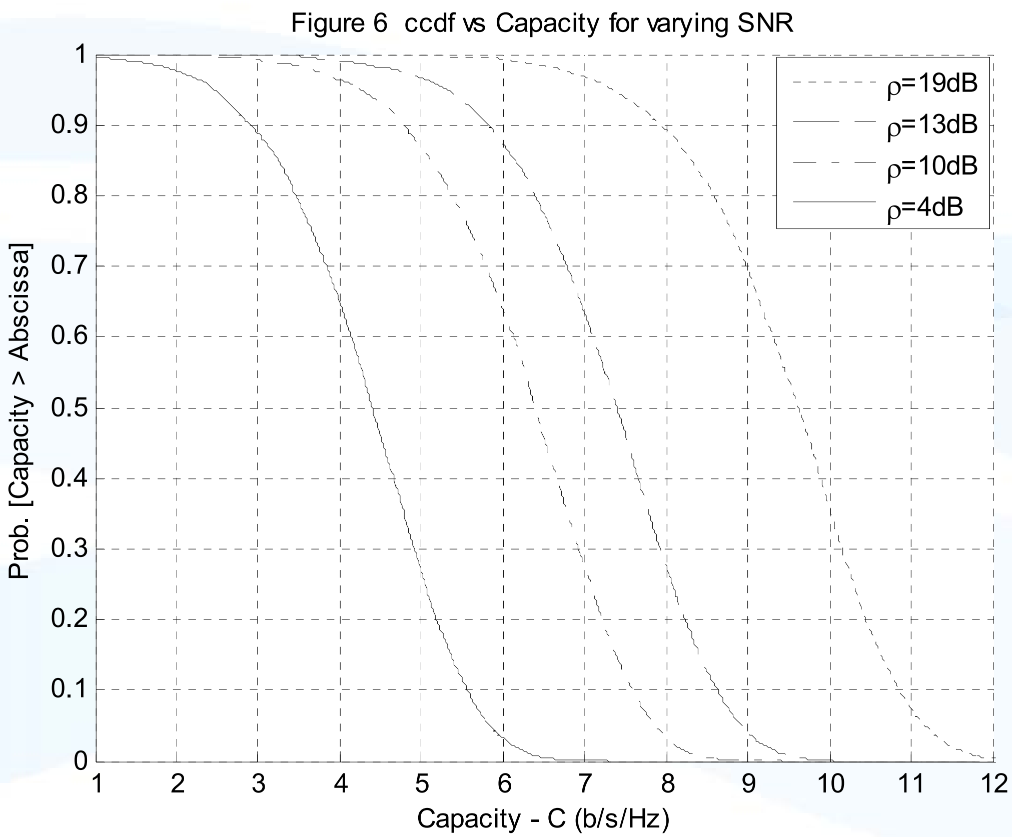


Figure 4 ccdf vs Capacity for varying scattering, k



RESULTS



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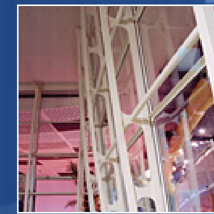
Geometric Modelling Conclusions



1. This model gives an indication of the theoretical performance gains of a MIMO system

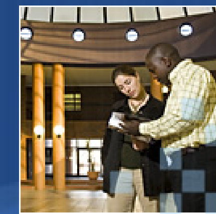
2. From a geometric based model a joint correlation function and TX and RX correlation functions were derived in a neat, compact and closed form

3. Model incorporates the key parameters such as configuration of antenna array, number of antenna elements, antenna spacing, antenna orientation and degree of scattering at RX.

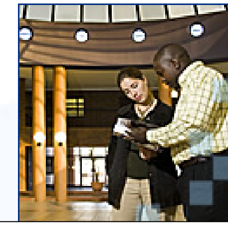
4. Shown that number of antenna elements has greatest impact on channel capacity.

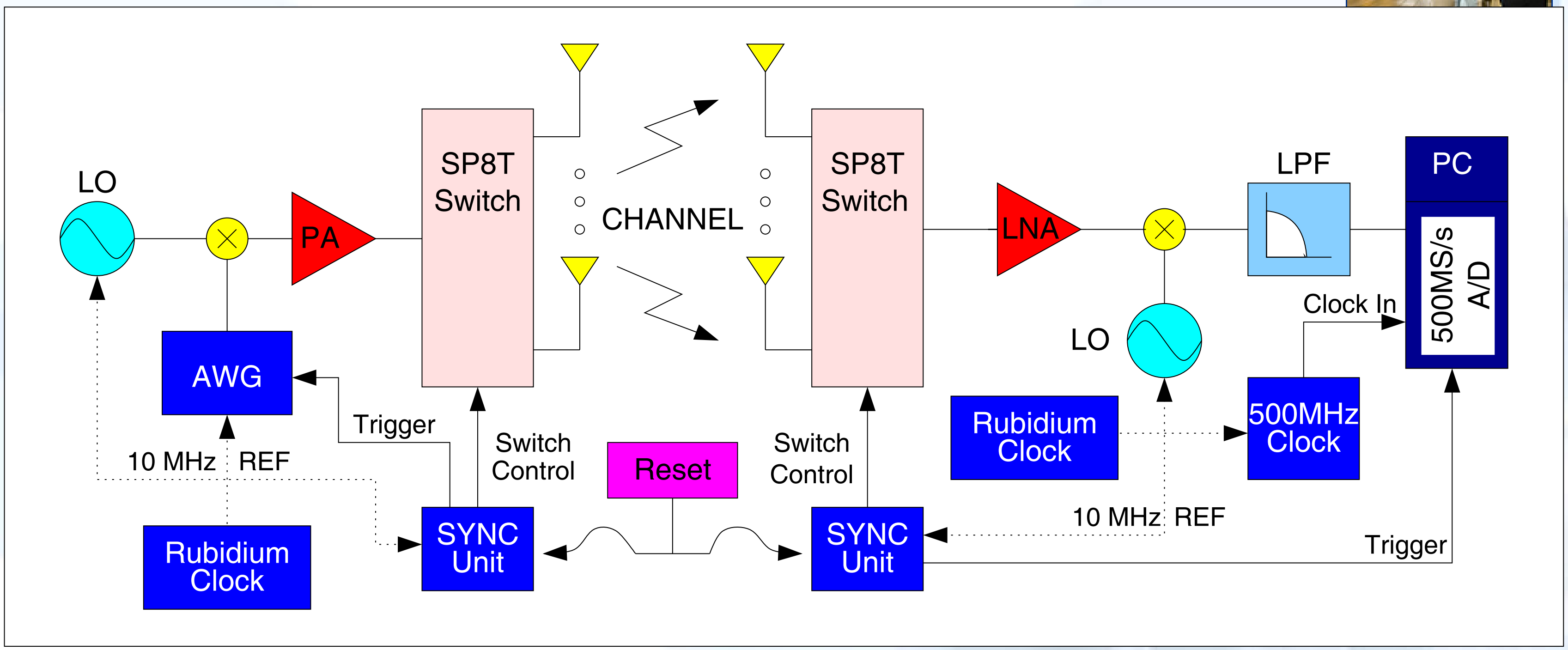
5. Could be a simplification of real environment!

WB MIMO Measurement System



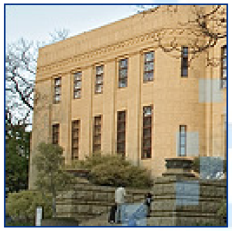
WB MIMO Channel Sounder

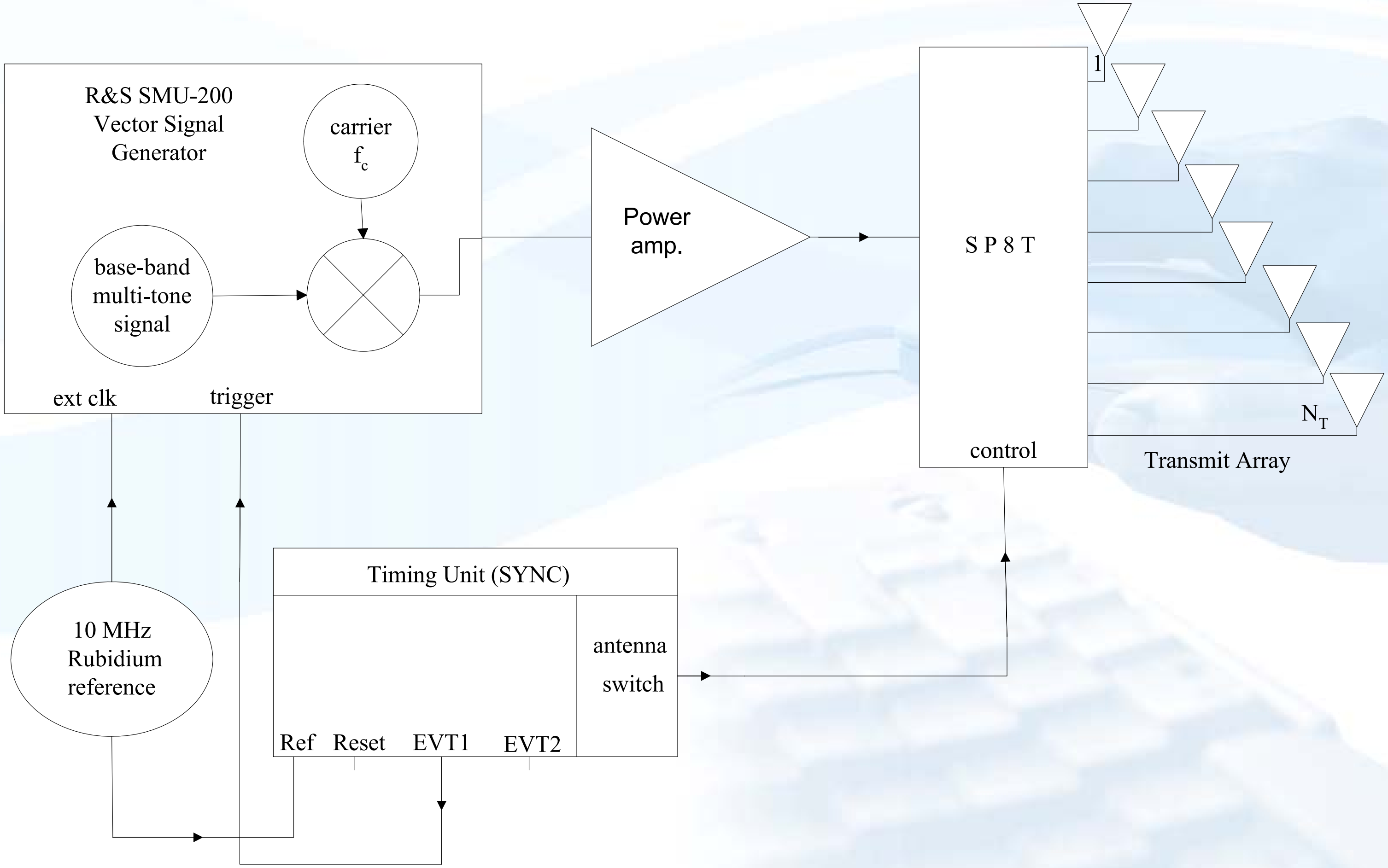




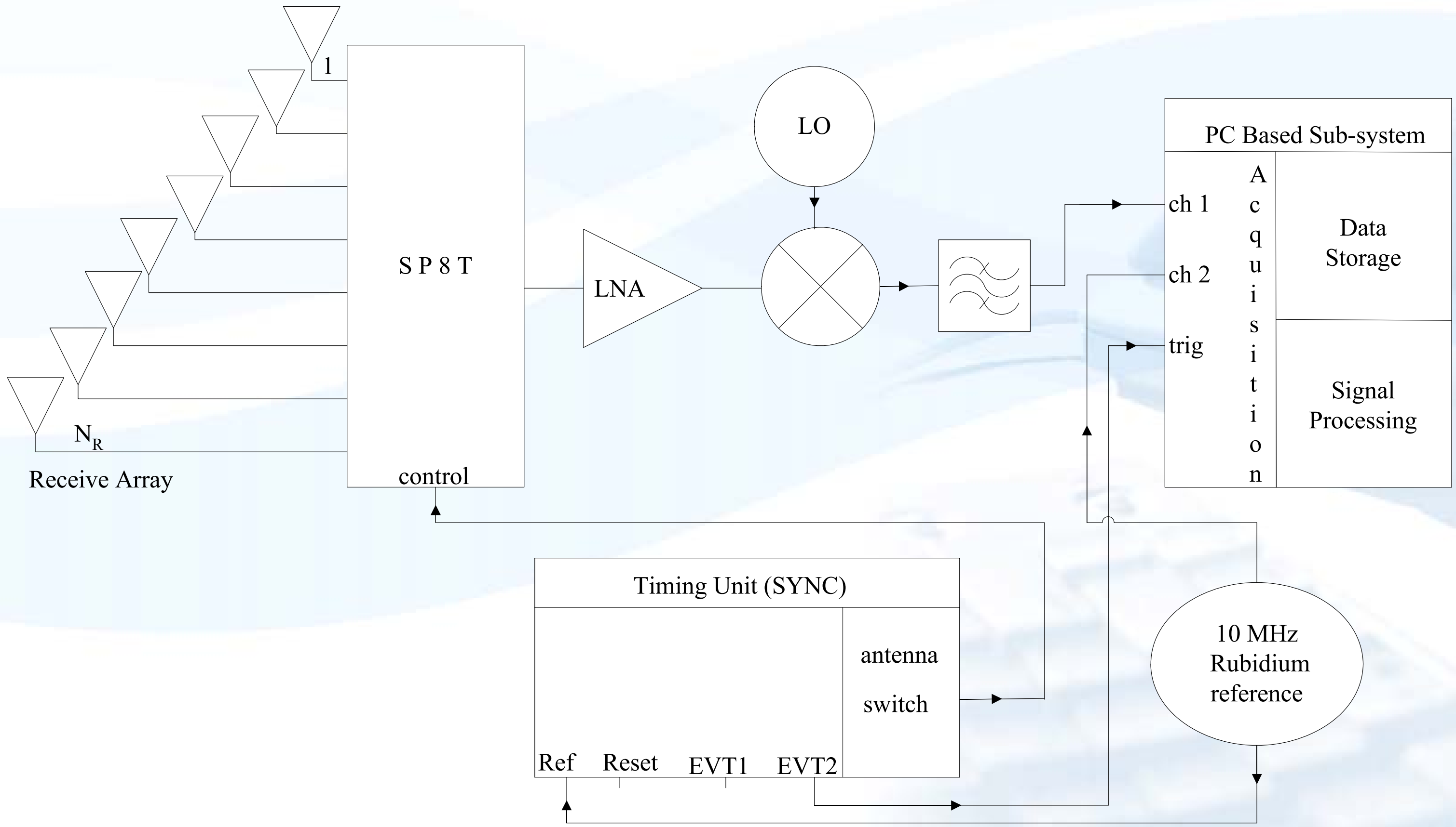
- **Low Cost 8x8 Architecture: switched array, COTS components/instruments**
- **PC-based A/D simplifies data processing (MATLAB)**
- **Up to 100 MHz instantaneous bandwidth**
- **2-6 GHz center frequency**

System Implementation - TX





System Implementation - RX



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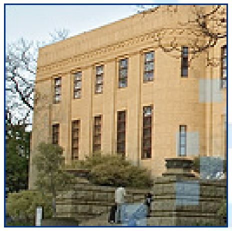
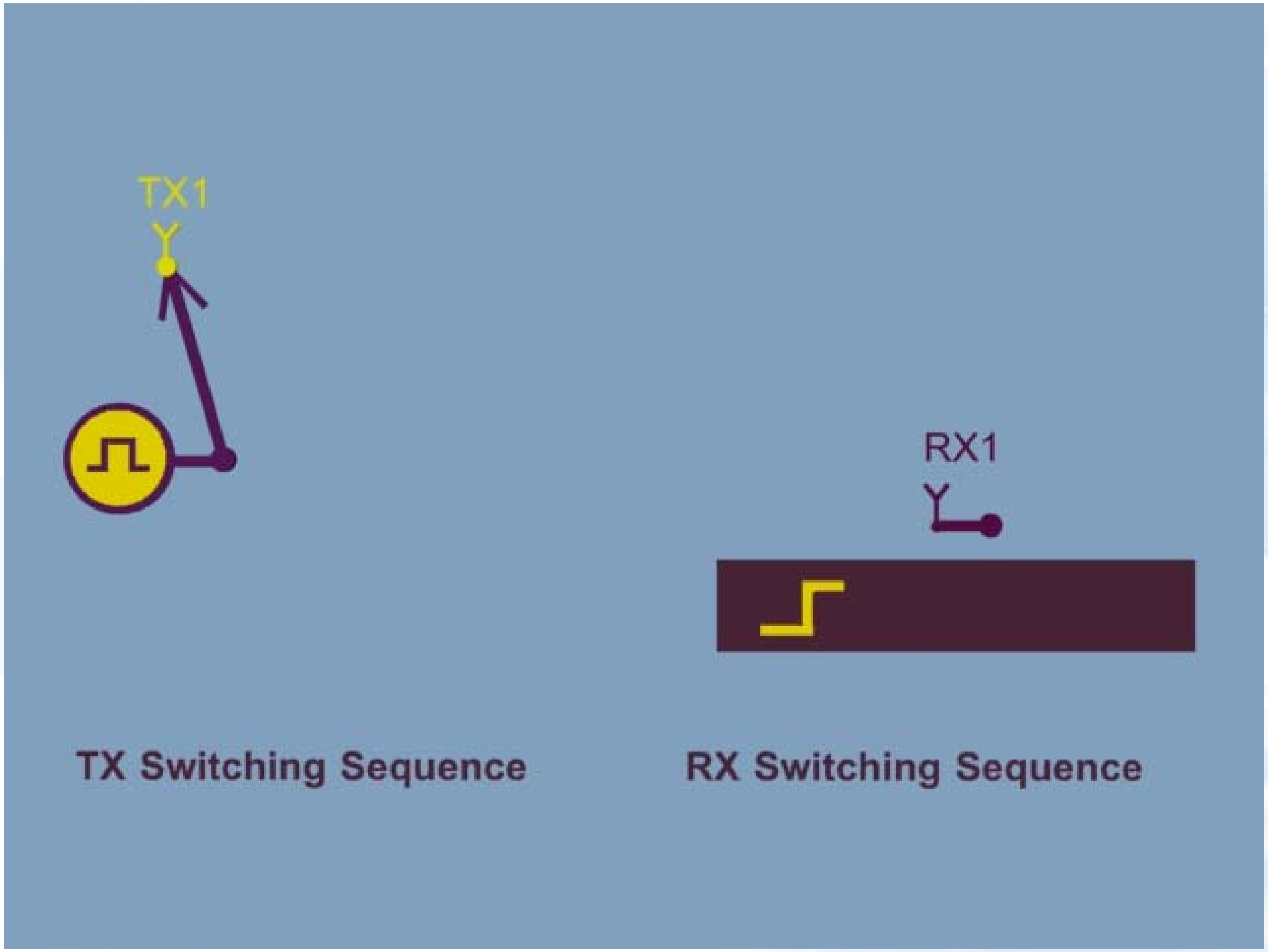
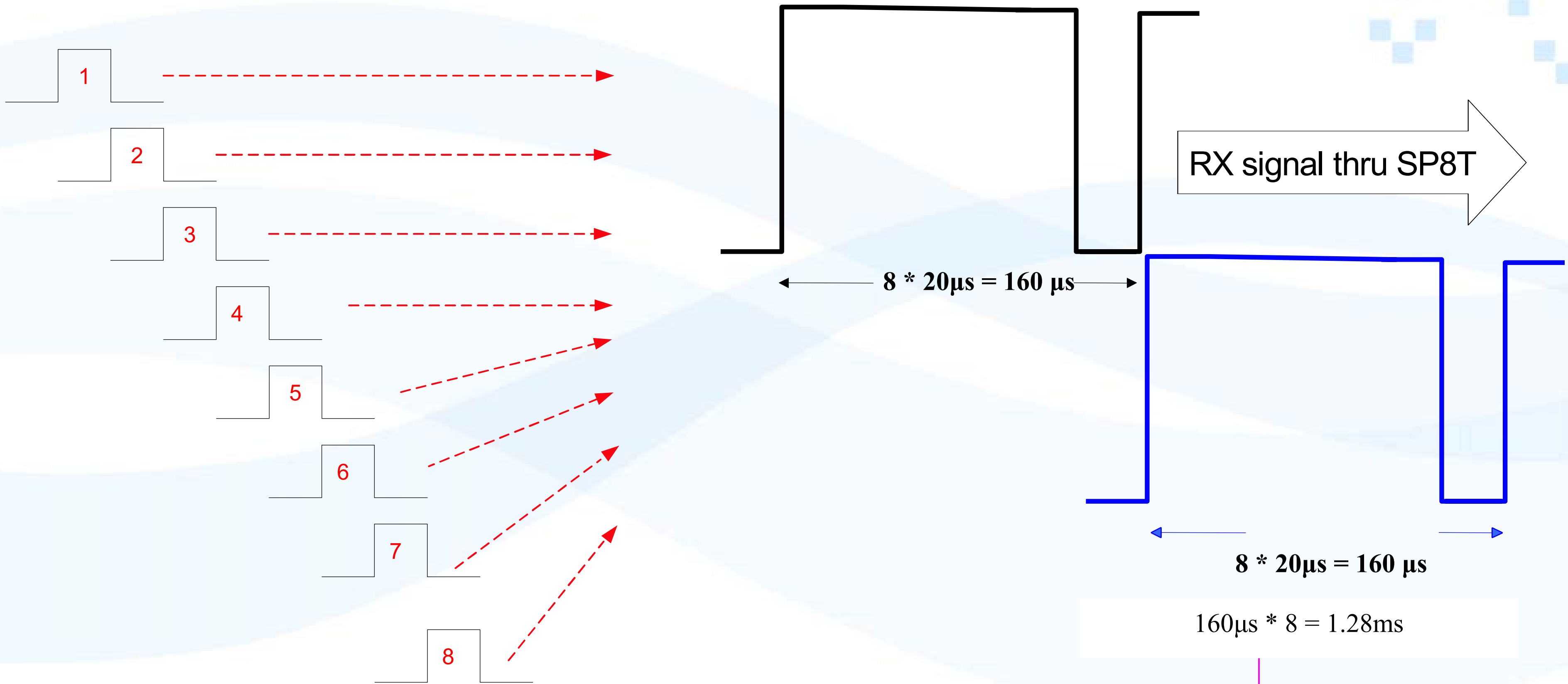
Measurement Method – UP System





Synchronization Sequences



TX signal from SP8T

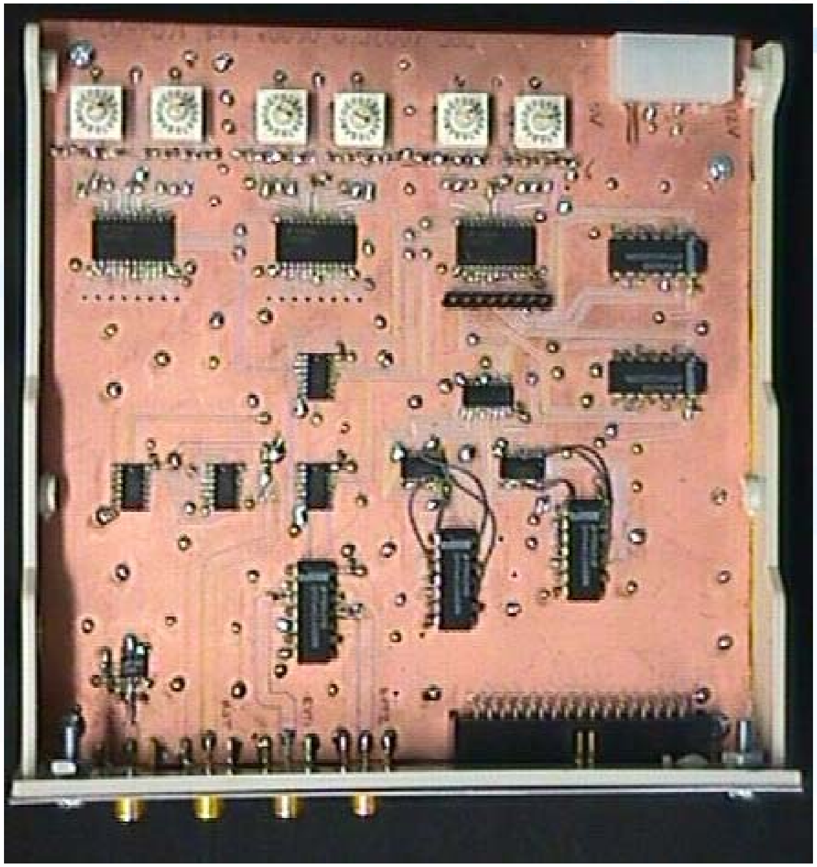
160µs * 8 = 1.28ms

1.28ms + 198.72ms = 200ms

200ms * 20snapshots = 4s

4s * 2sequences = 8s

Synchronization Unit (SYNC)



Measurement System

The multi-tone signal is of the form:

$$x(t) = \sum_{i=0}^{N} \cos(2\pi f_i t + \varphi_i)$$

$f_i = (0.5 + i)MHz$

$i = 0,1,...,39$

$\varphi_i = \{0, \pi\}$; is random (but fixed) phase shift for each tone that spreads the signal energy in time

To avoid artifacts associated with turning the signal on and off abruptly, the multitone signal of length *T* is multiplied by a Gaussian windowing function of the form

$$w(t) = \begin{cases} e^{-(T_1 - t)^2/2\sigma^2}, & 0 \le t < T_1 \\ e^{-(T_2 - t)^2/2\sigma^2}, & T_2 < t \le T \\ 1, & Otherwise \end{cases}$$

where:

T_1 and T_2 are the limits of the window

σ standard deviation controls the rise and fall time of the window

Hence:

$$Y(f) = X(f) * W(f)$$

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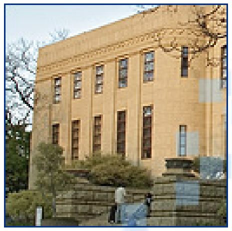
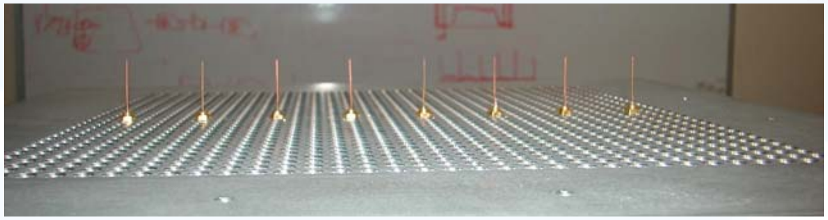
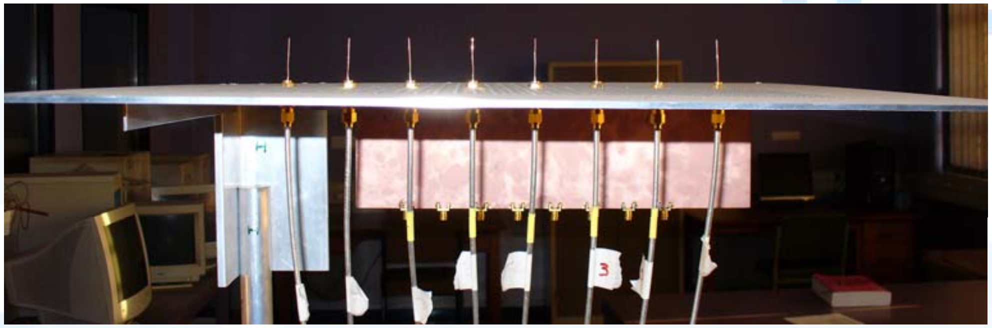
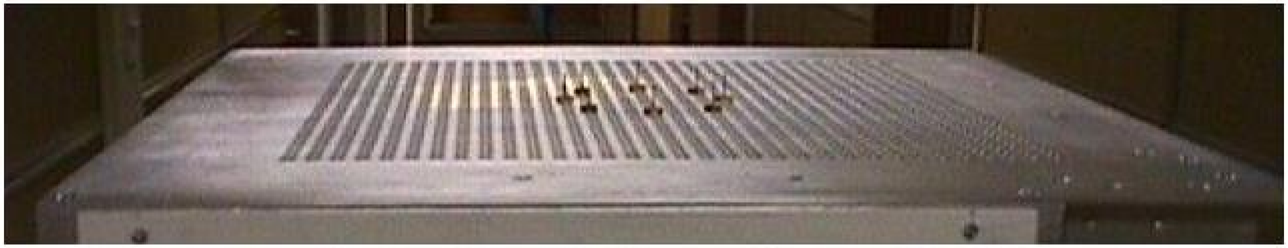
Monopole Antennas

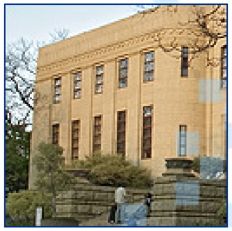


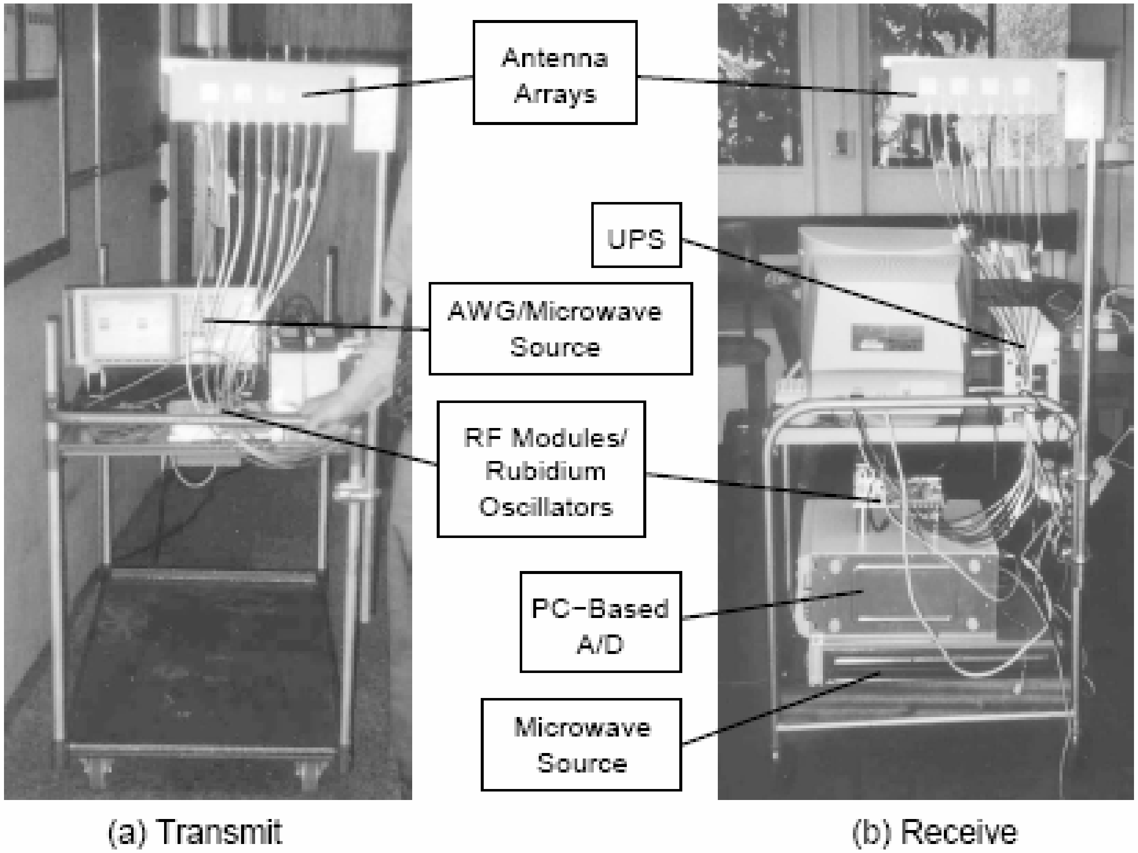




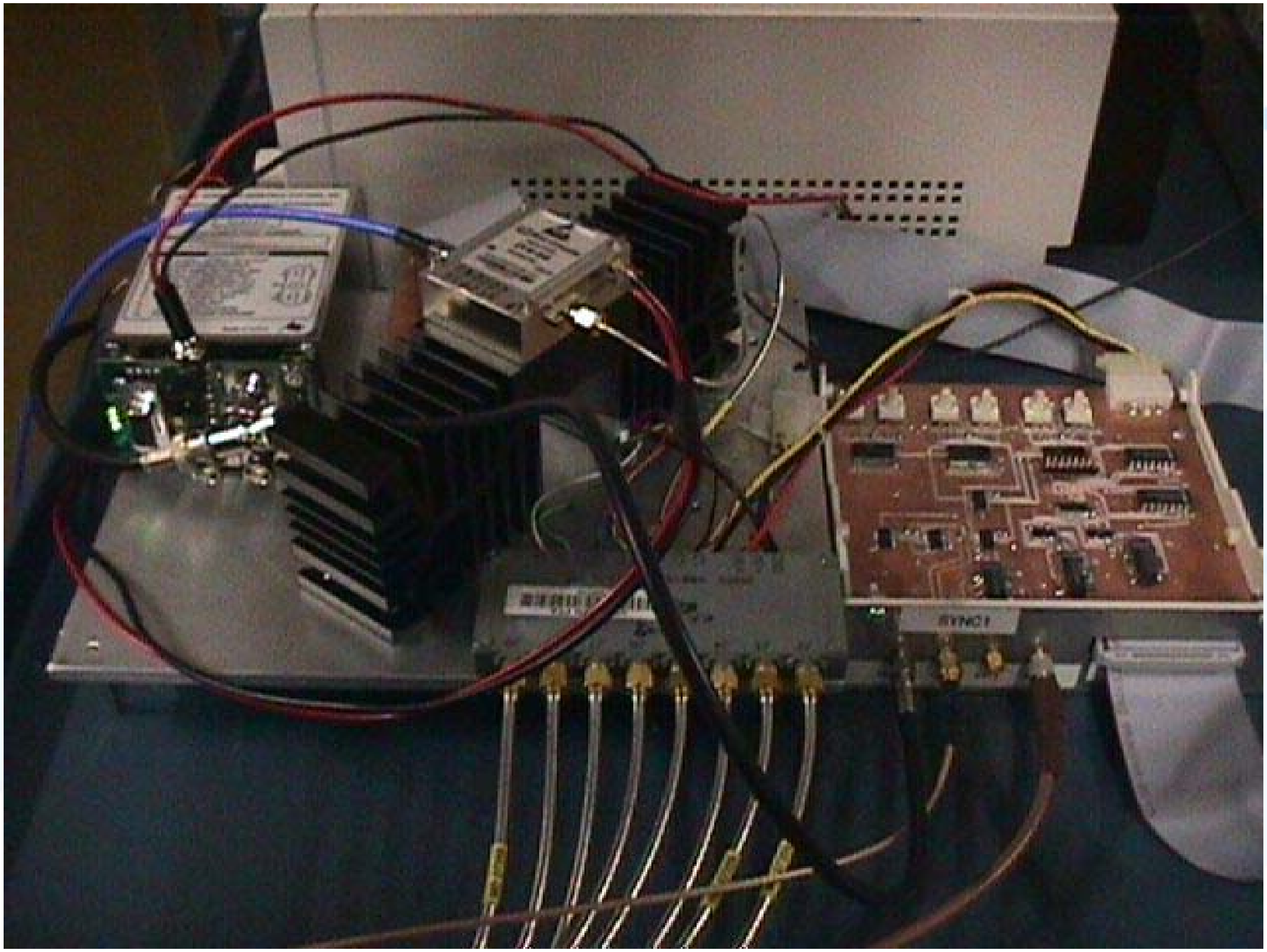


System developed and deployed at UP



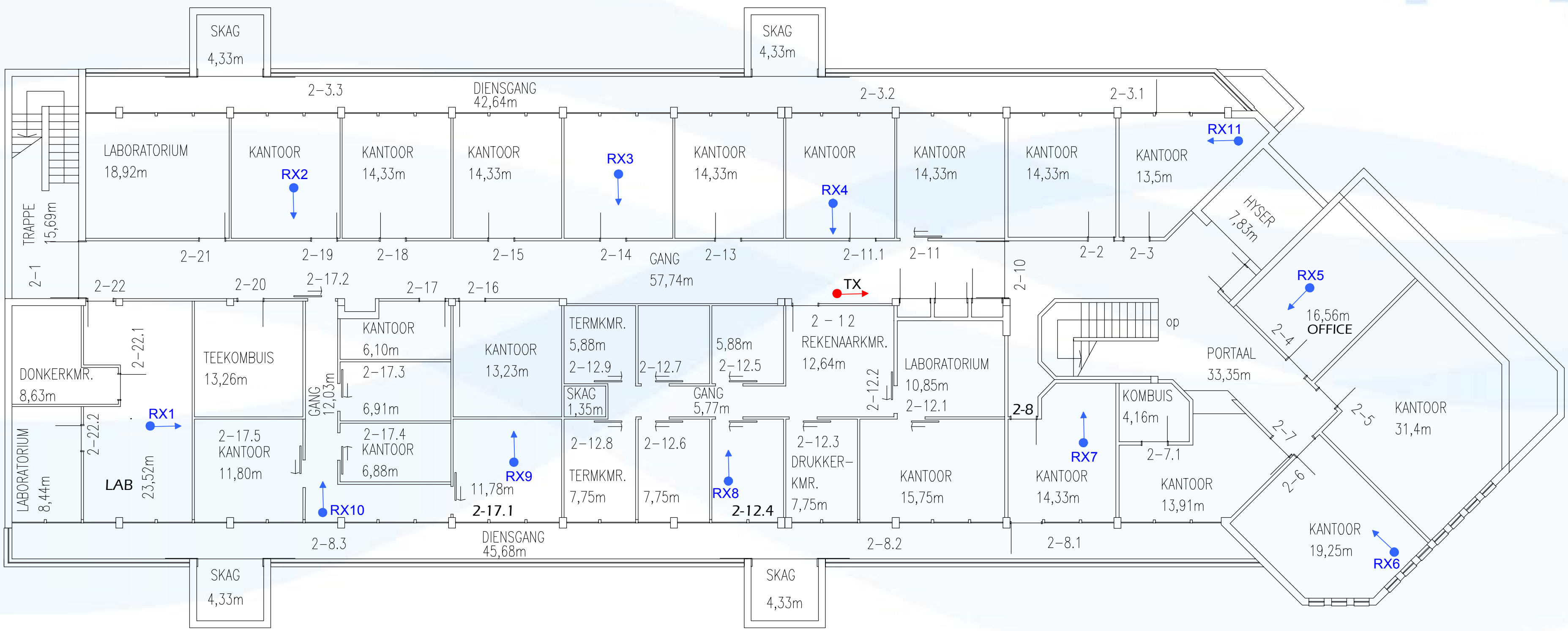


TX Hardware – 5.2 GHz



RX Indoor Measurement Locations

Measurement Environment – 2nd floor CEFIM



Data Processing

The Channel Matrix, H is represented in the ff form:

H(f,rx,tx,s,ss):

(Freq. bins,

RX antennas,

TX antennas,

sequence no,

snapshots)

Channel Matrix Normalization: remove effect of path loss

$$\widetilde{\mathbf{H}}^{(n)} = \left(\frac{1}{N_R N_T N_S} \sum_{m=1}^{N_S} \left\| \mathbf{H}^{(m)} \right\|_F^2 \right)^{-1/2} \mathbf{H}^{(n)}$$

$\widetilde{\mathbf{H}}^{(n)}$ = n^{th} normalized channel matrix

$\mathbf{H}^{(\mathbf{n})}$ = n^{th} non-normalized channel matrix

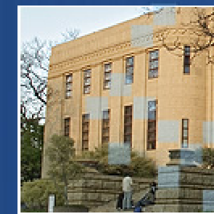
$\left\| \cdot \right\|_F$ = Frobenius norm

N_R = N_T = 8

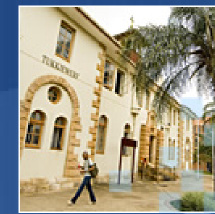
N_S = no. of channel measurements (snapshots and freq. bins)

(.)^H = the conjugate matrix transpose

WB MIMO Channel Sounder System Conclusion



- True channel behavior requires a system capable of direct channel measurement.
- Presented a successfully deployed ‘switched array’ system capable of probing from 2-6 GHz with a channel BW of 100MHz.
- Capable of having up to 8 TX and 8 RX antennas in an indoor environment.
- Reliable and repeatable measurements were taken in 3 different indoor environments at 2.4GHz and 5.2GHz.



Data Analysis and Model Assessment

- Capacity Modelling
- Spatial Correlation
- Double Directional Channel

Capacity for a 8x8 MIMO employing UCA

Channel capacity with no CSI

$$C = \log_2 \det(\mathbf{I} + \frac{\rho}{N_T} \widetilde{\mathbf{H}} \widetilde{\mathbf{H}}^H),$$

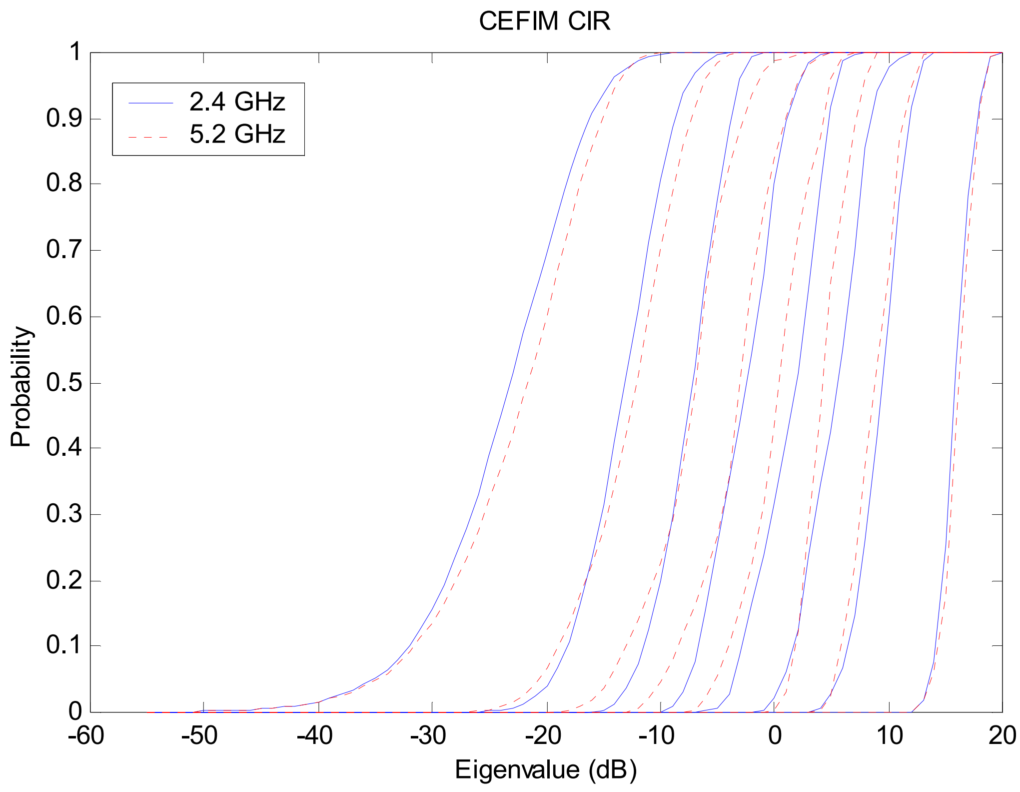
Average Channel Capacity cross BW

$$\overline{\mathrm{C}}_{\mathrm{loc}} = \left(\sum_{\mathrm{k}=1}^{\mathrm{K}} \mathrm{C}_{\mathrm{k}} \right) \Big/ \mathrm{K}$$

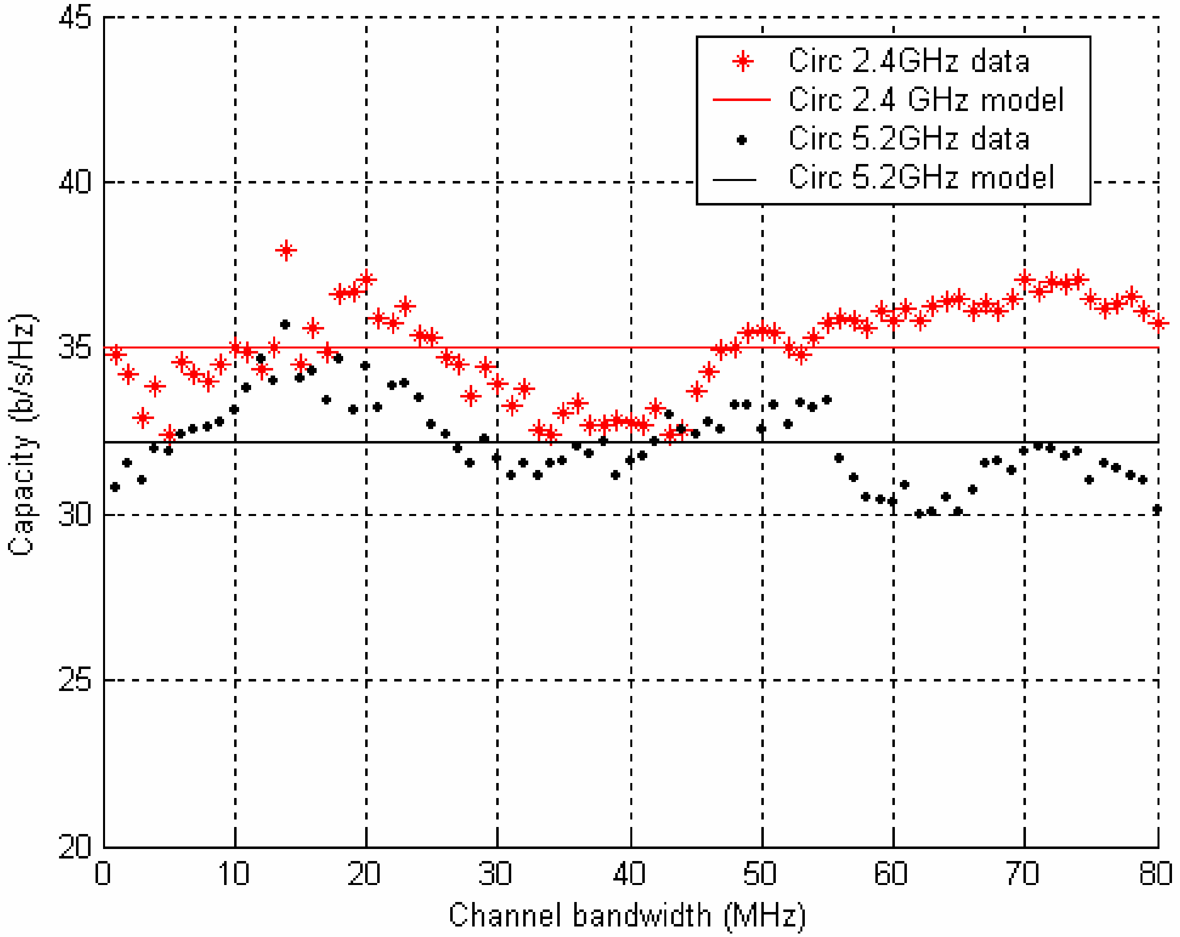
K = no. of frequency bins

RESULTS

Channel eigenvalue cdfs for circular arrays at 2.4GHz and 5.2GHz



Results
Capacity vs excitation BW at location 9



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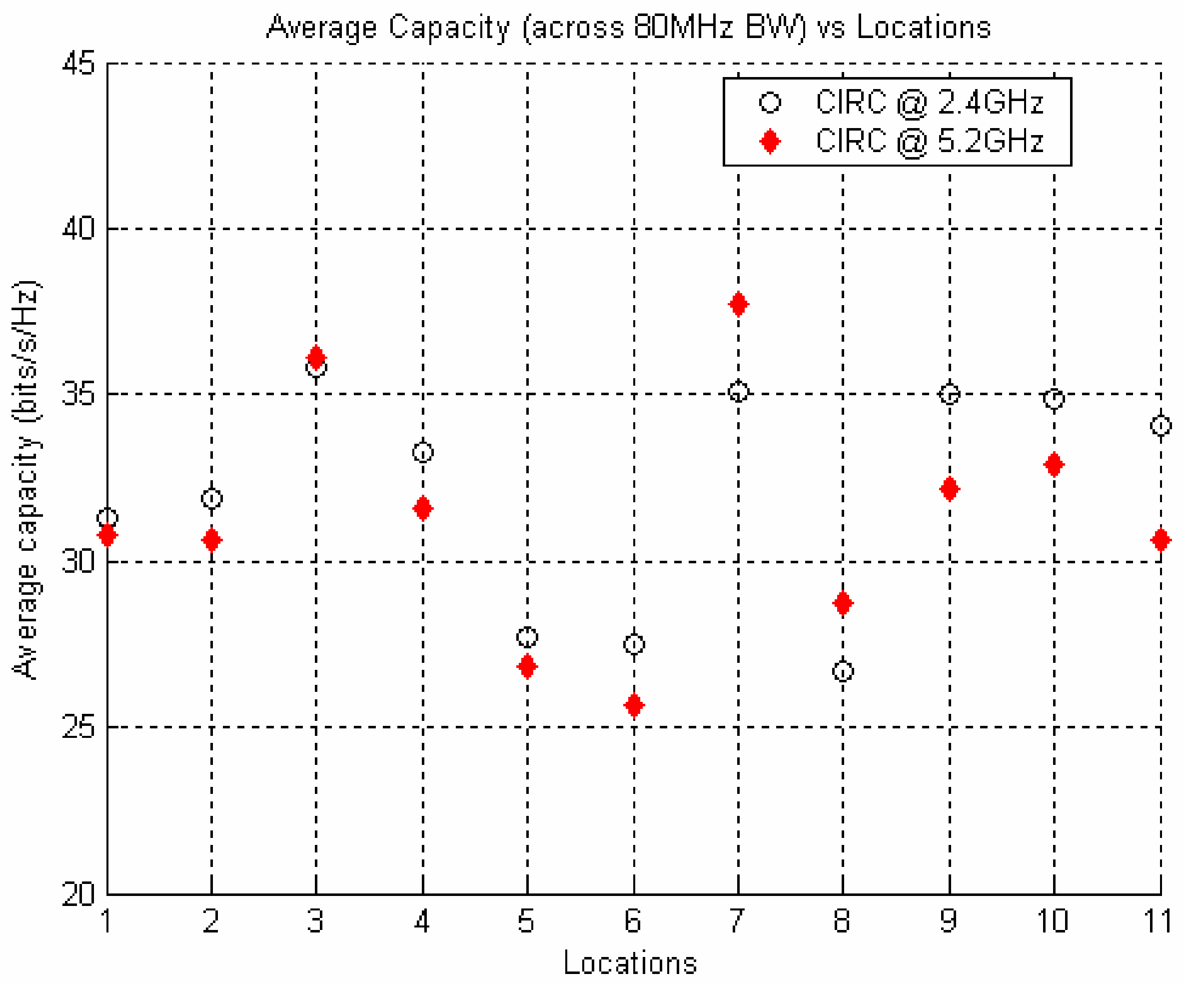
Circular Array Average Capacity



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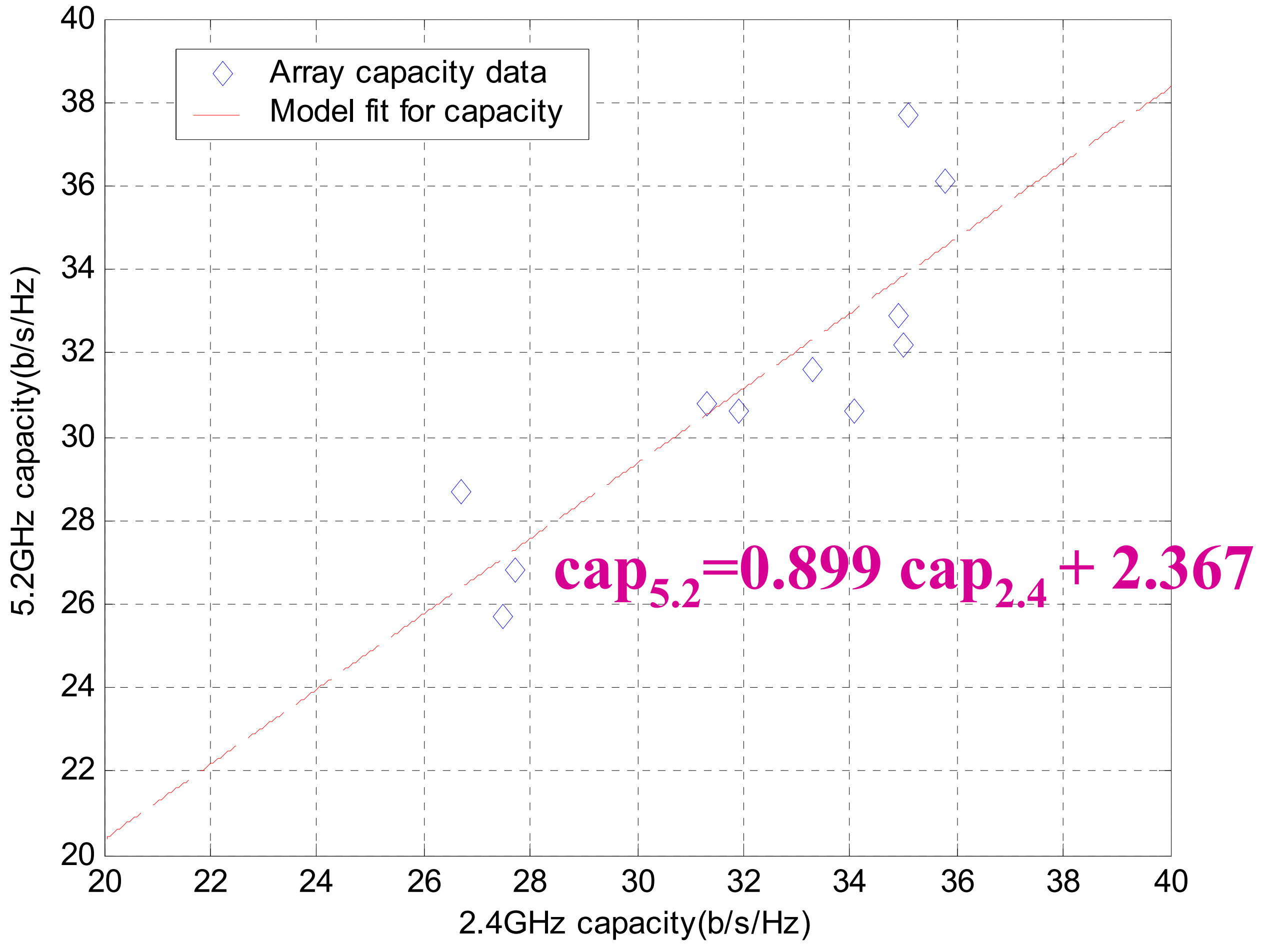
Results
Modelling freq scaling of capacity for UCA



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Spatial Correlation:
Shift Invariant Correlation - ULA

The correlation coefficient at the RX for element displacement ℓ:

$$\rho_{\ell} = \frac{\left[\sum_{k=1}^{N_S}\sum_{j=1}^{N_T}\sum_{i=1}^{N_R-\ell} H_{i,j}^{(k)} H_{i+\ell,j}^{(k)*}\right]}{\left[\left(\sum_{k=1}^{N_S}\sum_{j=1}^{N_T}\sum_{i=1}^{N_R-\ell} \left|H_{i,j}^{(k)}\right|^2\right)\left(\sum_{k=1}^{N_S}\sum_{j=1}^{N_T}\sum_{i=1}^{N_R-\ell} \left|H_{i+\ell,j}^{(k)}\right|^2\right)\right]^{1/2}}$$

$H_{i,j}^{(k)}$ is the k^{th} channel snapshot from the j^{th} TX to the i^{th} RX antenna

$\mathbf{N_s}$= 20*80; is the number of snapshots taken across all freq. bins and observations

$\mathbf{N_T} = \mathbf{N_R} = 8$; is number of TX and RX antennas respectively

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Modelling of Correlation

Magnitude of correlation coefficient, ρ_ℓ could be modelled as:

$$y_\ell = e^{-b\ell\Delta x}$$

where:

Δx is the element separation in wavelengths

b is the estimated decorrelation parameter

Average mean square error (MSE) at {TX,RX}:

$$\bar{d} = \frac{1}{\{N_T, N_R\}} \sum_{\ell=0}^{\{N_T, N_R\}-1} \left(|\rho_\ell| - y_\ell\right)^2$$

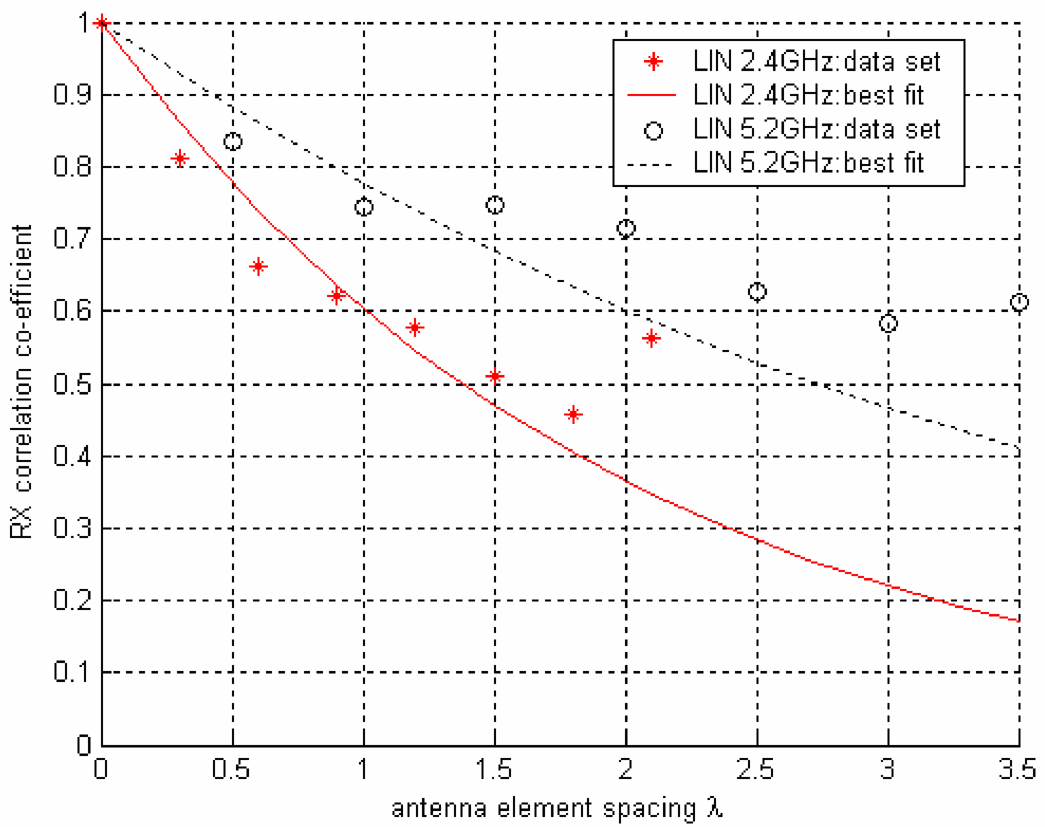
Frequency scaling analysis through linear regression of decorrelation or capacity by:

$$q_{5.2} = a_1 + a_2 q_{2.4}$$

where: $q_{\{5.2,2.4\}}$ is either the capacity or decorrelation at 5.2 GHz and 2.4 GHz, respectively, and a_1 and a_2 are obtained with a minimum MSE fit

Results:

Calculated relative correlation coefficients with curve fit for RX location 4



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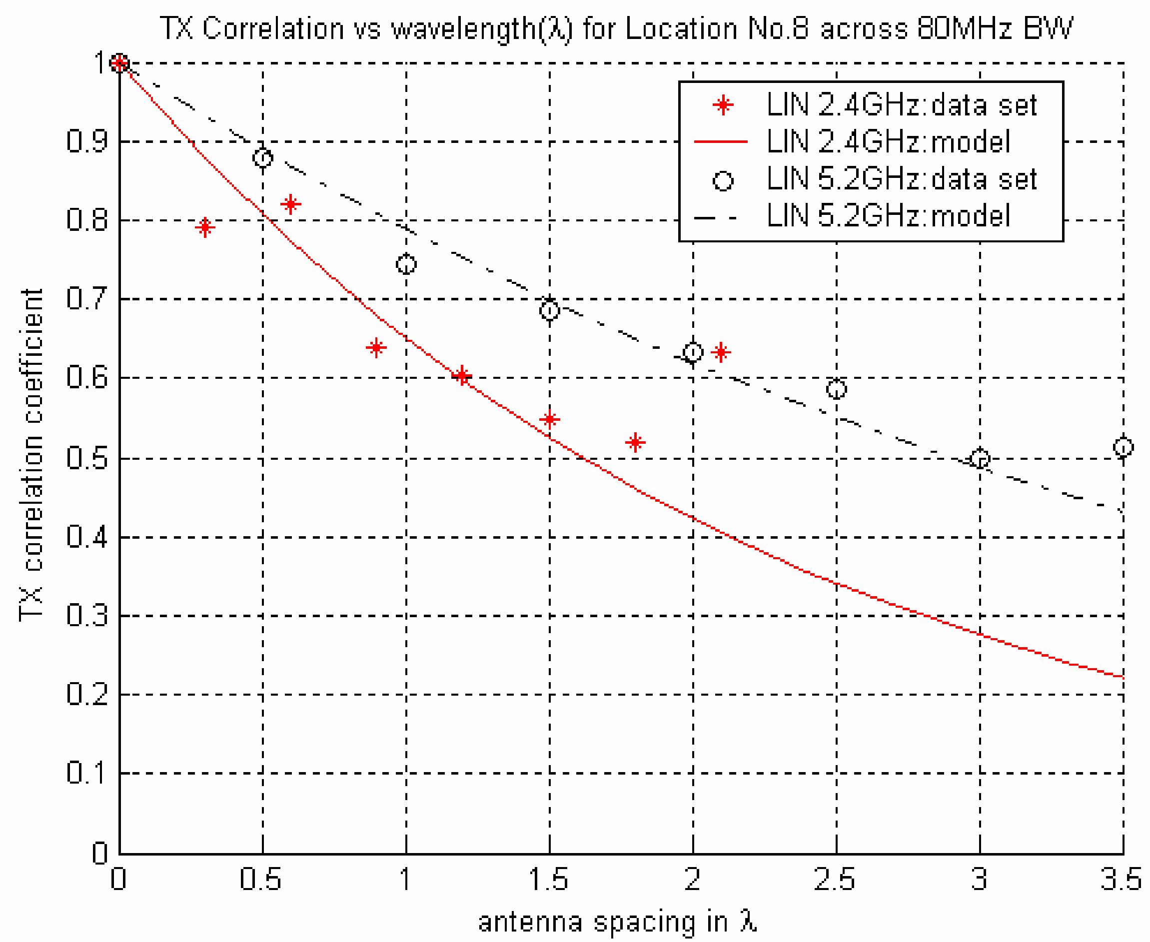
Results:

Calculated relative correlation coefficients with curve fit for TX location 8



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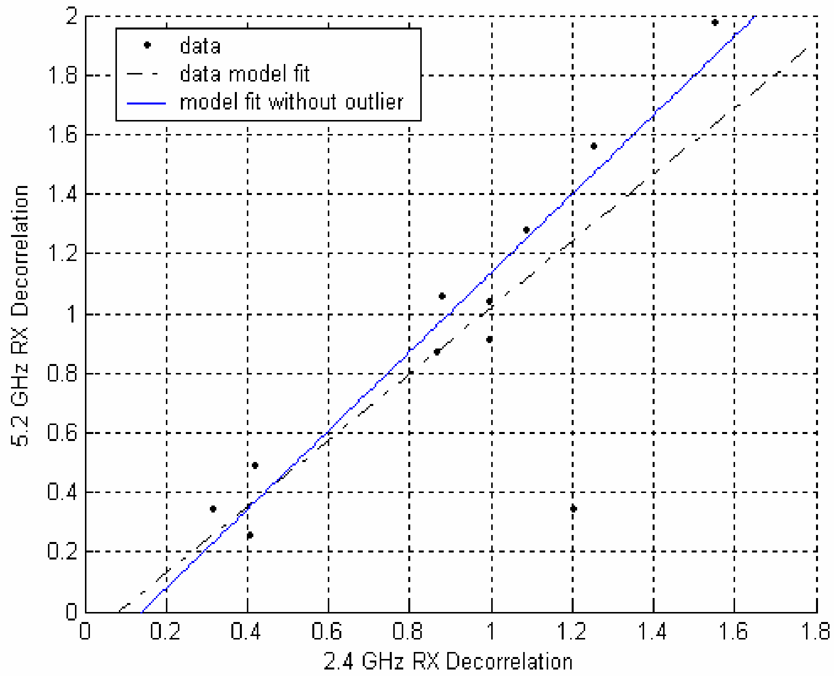
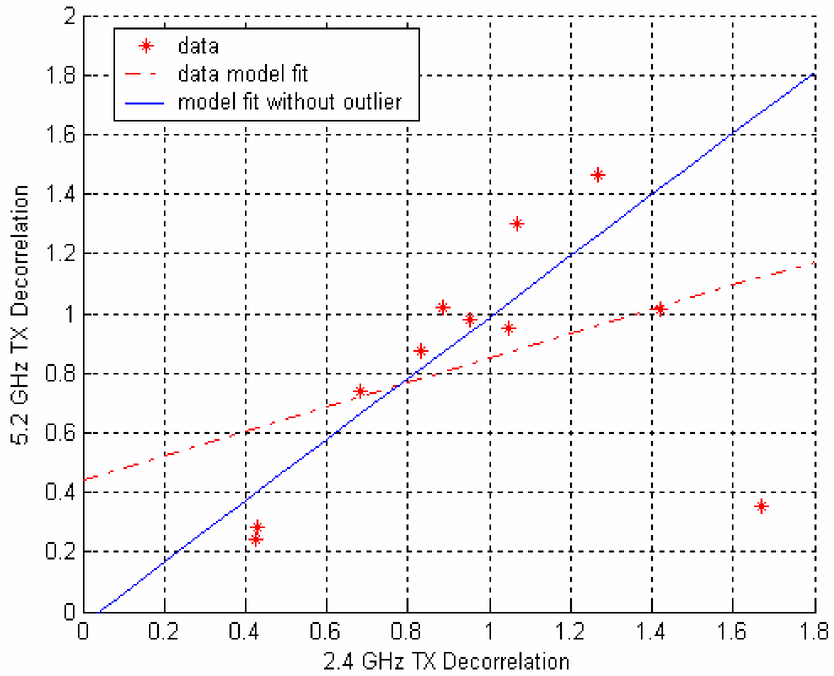
Results:

Relationship of decorrelation with respect to frequency scaling

RX



TX



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Results

TABLE I
DECORRELATION PARAMETER (b) AND ERROR wrt WAVELENGTH (λ) AT RX

Locations	5.2 GHz		2.4 GHz	
	b	error (%)	b	error (%)
1	0.8702	3.27	0.8690	4.76
2	1.2795	1.76	1.0903	4.48
3	1.5591	5.64	1.2546	2.00
4	0.2550	1.06	0.4080	0.75
5	1.0536	2.57	0.8799	3.49
6	0.3432	0.98	0.3182	1.15
7	0.9071	0.72	0.9978	0.17
8	0.4883	1.46	0.4190	0.35
9	0.3442	0.45	1.2042	1.18
10	1.0403	2.40	0.9980	2.90
11	1.9721	5.41	1.5548	3.65

Results

TABLE II

DECORRELATION PARAMETER (b) AND ERROR wrt WAVELENGTH (λ) AT TX

Locations	5.2 GHz		2.4 GHz	
	b	error (%)	b	error (%)
1	0.9769	1.54	0.9511	0.18
2	1.0189	2.66	0.8849	1.69
3	1.2988	1.68	1.0666	2.34
4	0.2400	2.96	0.4270	2.90
5	0.8746	2.08	0.8325	0.06
6	0.7402	5.84	0.6833	0.56
7	1.4634	1.13	1.2678	1.38
8	0.2845	0.51	0.4302	0.84
9	0.3530	1.03	1.6701	0.96
10	1.0134	0.75	1.4210	3.15
11	0.9514	0.49	1.0462	0.76

Data Analysis and Model Assessment: Conclusion

Capacity for UCA wrt frequency scaling:

- **Observed a linear model fit**
- **Variance of 1.91**
- **High degree of correlation of capacities**
- **Hence capacity at different centre freq can be reliably predicted**

Model for Spatial correlation of ULA at a location

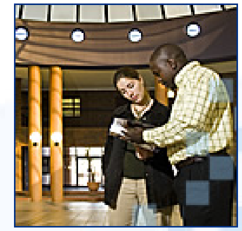
- **RX**
  - **exponential model fit gives typical MSE of 0.2% and 3.3% at two carrier freq’s**
  - **only 20% of locations gave MSE of 4%-5.5%(max)**
  - **average error at 5.2GHz = 2.34%**
  - **average error at 2.4 GHz = 2.26%**
- **TX**
  - **Similar to RX**
  - **average error at 5.2GHz = 1.9%**
  - **average error at 2.4 GHz = 1.4%**

Conclusion wrt Spatial Correlation

- Spatial correlation wrt frequency scaling
  - decorrelation parameter shows high dependence
  - linear model gives MSE of 0.012 at RX
  - linear model gives MSE of 0.034 at TX
  - strong dependence of correlation at two centre frequency's implies:
    - high correlation in directional signature of multi-path propagation
    - level of multi-path may be very similar
- Results useful for ST coding, MIMO system development, network planning, channel measurement campaigns

Double Directional Channel Modelling for indoor co-located WB MIMO measurements

Double Directional Channel



• Previous modelling efforts [Steinbauer, et al] have defined the double directional channel in terms of a paired discrete plane-wave departures and arrivals at the TX and RX

• Indoor environments have much more severe multipath, hence extracting individual plane-wave arrivals can be very difficult

• Hence the new approach we proposed is to define the double directional response in terms of spatial power spectra obtained from the joint TX/RX Bartlett or Capon beamformers

Double directional channel: Capon beamformer

$$P_{CAP}(\nu_T, \nu_R) = \frac{1}{\mathbf{a}(\nu_T, \nu_R)^H \hat{\mathbf{R}}^{-1} \mathbf{a}(\nu_T, \nu_R)}$$

$\{\cdot\}^H$ denotes the complex conjugate transpose

ν_T and ν_R are the azimuth angles at the TX and RX respectively

$\hat{\mathbf{R}}$ is the sample covariance matrix

$\mathbf{a}(\nu_T, \nu_R)$ Joint steering vector is defined as:

$$\mathbf{a}(\nu_T, \nu_R) = \mathbf{a}_T(\nu_T) \otimes \mathbf{a}_R(\nu_R)$$

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Double directional channel: Bartlett beamformer

Sample covariance matrix is computed as:

$$\hat{\mathbf{R}} = \frac{1}{K}\sum_{k}\mathbf{h}^{(k)}\mathbf{h}^{(k)H}$$

K is the total number of frequency bins

$\mathbf{h}^{(k)} = \mathrm{Vec}\{\mathbf{H}^{(k)}\}$ $\mathrm{Vec}\{\cdot\}$ - vector operation to stack a matrix into a vector

$\mathbf{a}_{\{T,R\}}$ are the separate array steering vectors for the TX and RX

Similary the joint Bartlett Beamformer is defined as:

$$P_{BAR}(\nu_T,\nu_R) = \frac{\mathbf{a}(\nu_T,\nu_R)^H \hat{\mathbf{R}} \mathbf{a}(\nu_T,\nu_R)}{\mathbf{a}(\nu_T,\nu_R)^H \mathbf{a}(\nu_T,\nu_R)}$$

Correlation coefficient (Metric) – DDC Spectra

Similarly of spectra is evaluated through correlation coefficient as

$$\rho = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N}(P_{2.4,ij} - \overline{P}_{2.4})(P_{5.2,ij} - \overline{P}_{5.2})}{\sqrt{\left[\sum_{i=1}^{N}\sum_{j=1}^{N}(P_{2.4,ij} - \overline{P}_{2.4})^2\right]\left[\sum_{i=1}^{N}\sum_{j=1}^{N}(P_{5.2,ij} - \overline{P}_{5.2})^2\right]}}$$

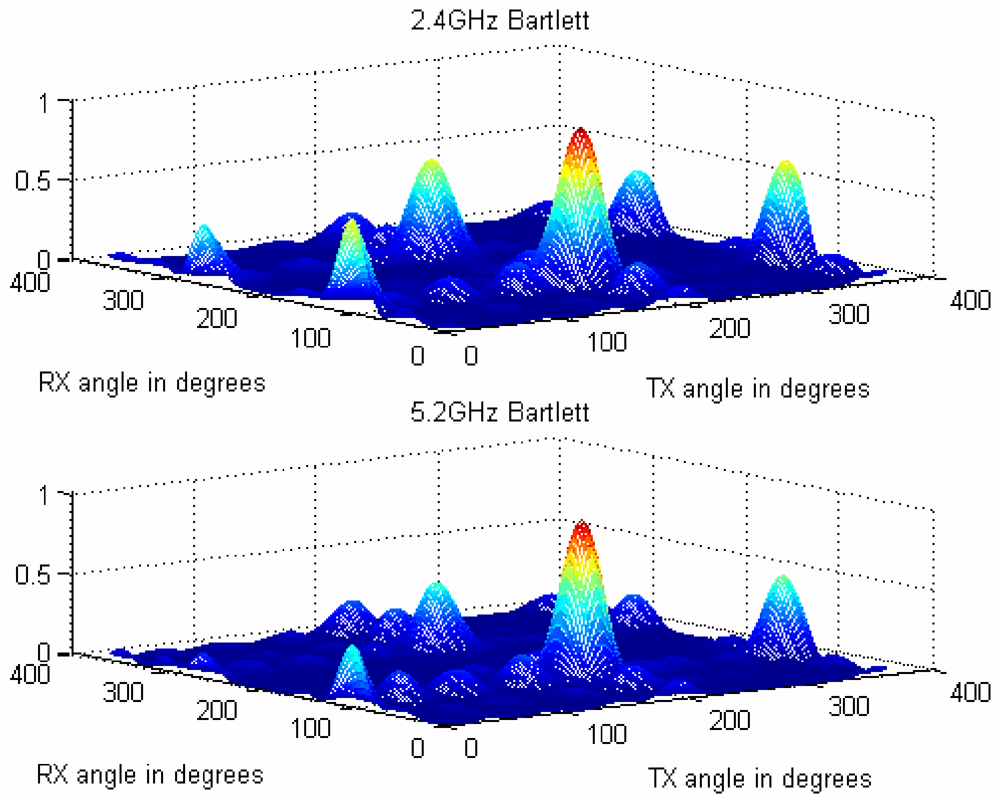
N no. of discretization points; f = 2.4 GHz or 5.2 GHz

$$P_{f,ij} = P_{\{CAP,BAR\}}(\nu_{T,i}, \nu_{R,j})$$

$$\nu_{T,i} = \nu_{R,i} = \frac{2\pi(i-1)}{N}$$

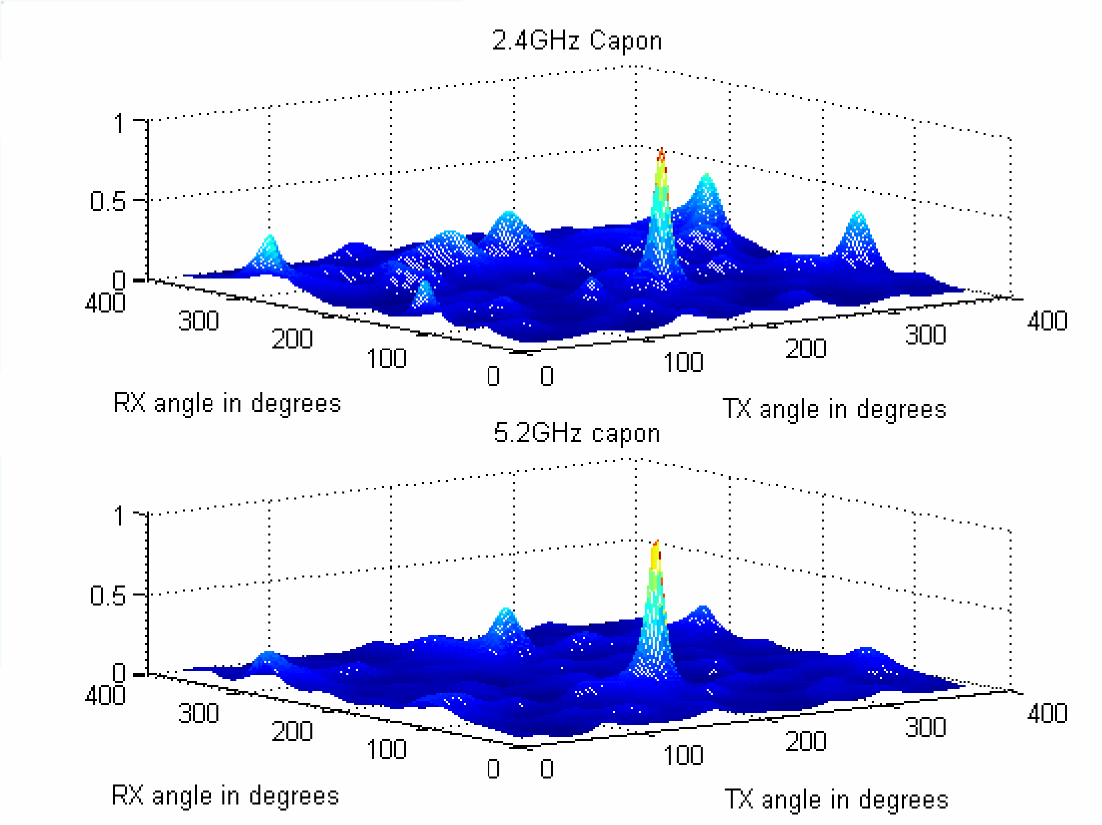
$$\overline{P}_f = \frac{1}{N^2}\sum_i\sum_j P_{f,ij}$$

Results
Measured spatial power spectra at RX location 4: Bartlett beamformer

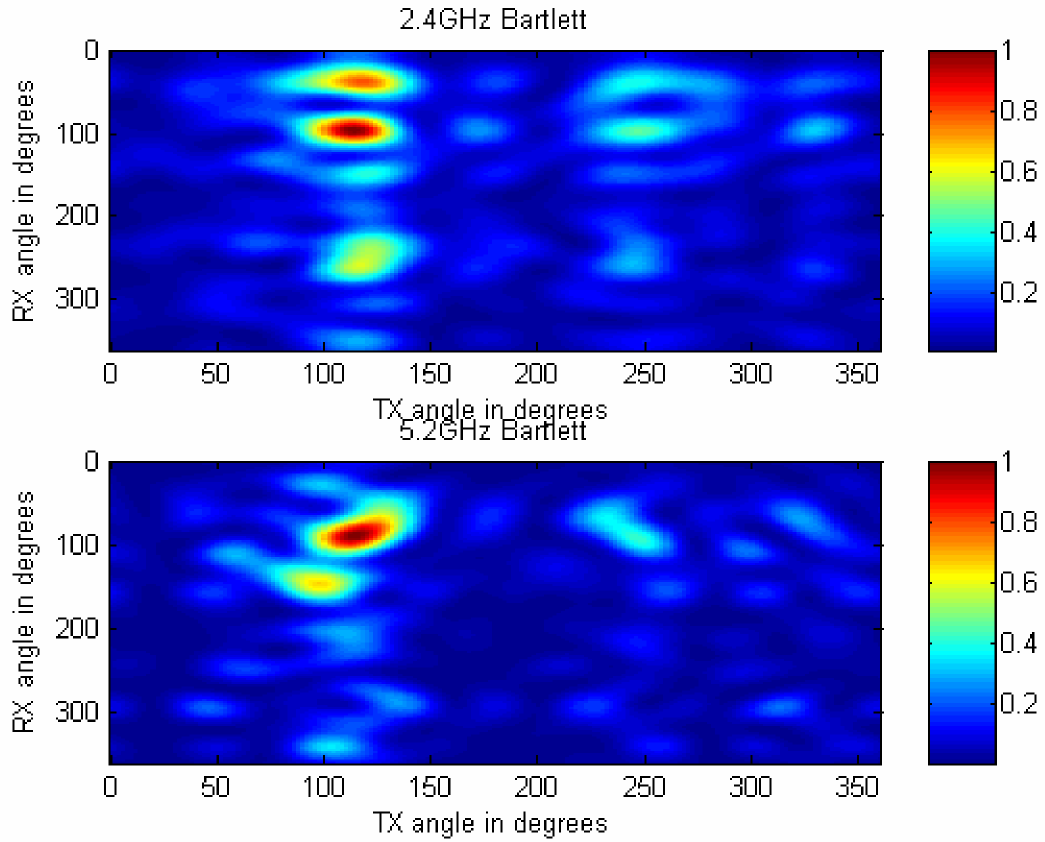


'Mesh'

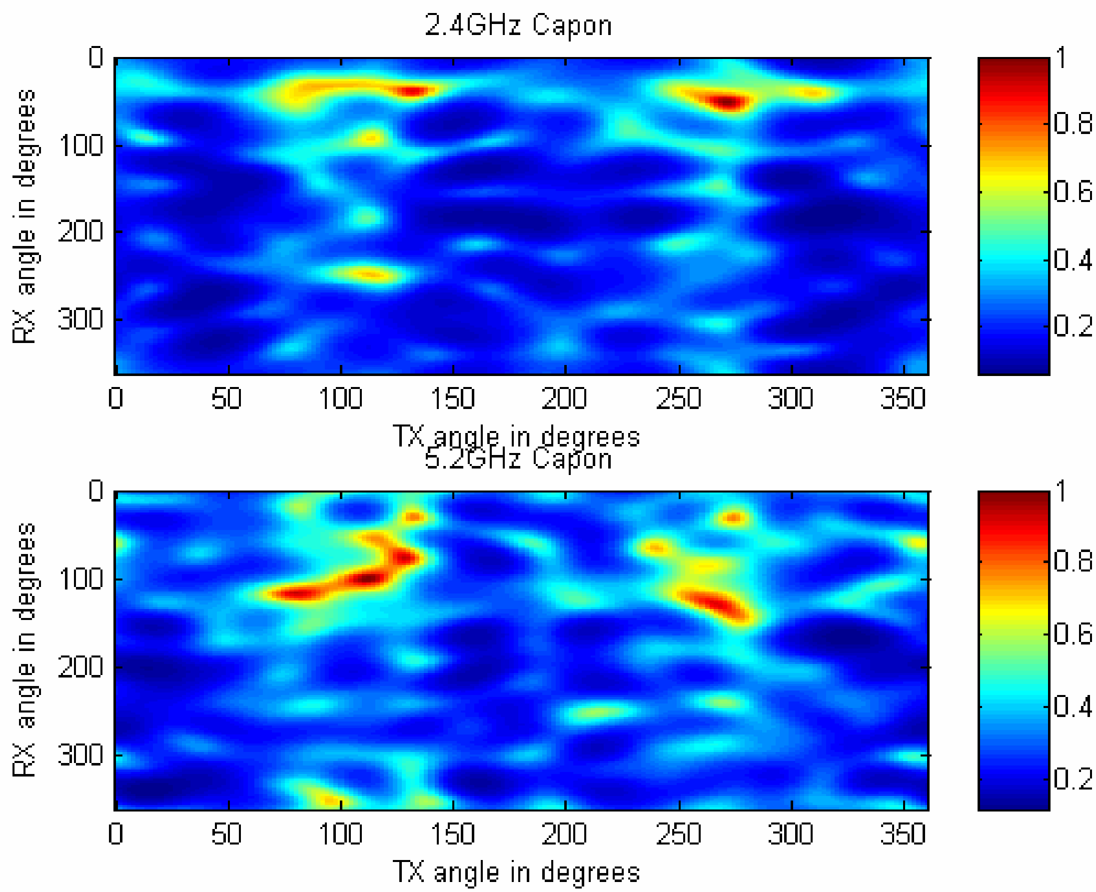
Results
Capon at Location 4



Results
Measured spatial power spectra at RX location 11



Results
Capon at Location 11



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Results
Spectral Correlation at 2.4- & 5.2 GHz

Physical Location	1	2	3	4	5	6	7	8	9	10	11
Bartlett Beamformer	0.37	0.56	0.43	0.56	0.62	0.59	0.35	0.51	0.33	0.25	0.41
Capon Beamformer	0.73	0.77	0.72	0.94	0.59	0.46	0.56	0.76	0.56	0.16	0.63

DDC: Conclusion

- Presented frequency scaling in DD Channel
- High degree of similarity
  - Implies MPP at two frequencies is mainly due to specular reflections
  - Good correlation of power spectra
- Suggests scaling of channel behaviour prediction
- Frequency scaling could save time and cost ito network planning if used properly

MIMO Channel Modelling:

Maximum Entropy Approach

Introduction

- Modeling using Kronecker Model – [Kai Yu, et. al.]

$$R_H = R_T \otimes R_R$$

  - allows one to compute the full joint covariance matrix from the separate Rx and Tx covariance matrices

- Discrepancies in results from above for capacity (eigenvalues) and joint spatial spectra – [Özcelik, et. al.]
  - especially for larger arrays with higher correlation

- Use of partial information from full covariance in separate TX / RX covariance information - [Weichselberger, et. al.]

Maximum Entropy (ME) Approach

- Information theoretic approach proposed by [Debbah and Müller, TIF'05]
  - By not imposing any artificial structure on channel, ME principle should provide more accurate and consistent modeling
  - This is based only on channel knowledge at hand

Our New Approach

- Applying ME to derive the full joint covariance based on knowledge of only the separate TX and RX covariance

- Investigate that this ME approach offers any channel modeling improvement

- Compare the spatial power spectra:
  - using the double directional channel
  - to indoor WB measurement campaign
  - Kronecker Model

Model Description (ME1)

Suppose we know the TX and RX covariance as $\mathbf{R}_T$ and $\mathbf{R}_R$, respectively and stats of channel constrained by:

$$\mathrm{E}\{\mathbf{H}\mathbf{H}^H\} = \mathbf{R}_R \quad \mathrm{E}\left\{\sum_k H_{ik}H_{jk}^*\right\} = R_{R,ij} \qquad (1)$$

$$\mathrm{E}\{\mathbf{H}^H\mathbf{H}\} = \mathbf{R}_T^T \quad \mathrm{E}\left\{\sum_k H_{ki}H_{kj}^*\right\} = R_{T,ij} \qquad (2)$$

$$p(\mathbf{H}) \geq 0 \qquad (3)$$

$$\int p(\mathbf{H})d\mathbf{H} = 1, \qquad (4)$$

To maximize the entropy wrt above constraints;

$$\int p(\mathbf{H}) \log p(\mathbf{H}) d\mathbf{H}$$

p(**H**) is the joint pdf of the elements of channel matrix **H**

ME Model (ME2)

We can write the Lagrangian L[p(**H**)], and set its derivate equal to zero (ie. *d*L/*d*p =0), giving

$$p(\mathbf{H}) = \exp\left[\lambda_0 + \sum_{ij} \mu_{R,ij} \sum_k H_{ik} H^*_{jk} + \sum_{ij} \mu_{T,ij} \sum_k H_{ki} H^*_{kj}\right]$$

$$= c_0 \exp(\sum_{ijk\ell} \mu_{R,ik} H_{ij} H^*_{k\ell} \delta_{j\ell} + \sum_{ijk\ell} \mu_{T,j\ell} H_{ij} H^*_{k\ell} \delta_{ik})$$

writing $c_0=\exp(\lambda_0)$, gives

$$p(\mathbf{H}) = c_0 \exp\left[\sum_{ijkl} H_{ij} H^*_{k\ell} \underbrace{(\mu_{R,ik}\delta_{j\ell} + \mu_{T,j\ell}\delta_{ik})}_{-R^{-1}_{ij,k\ell}}\right]$$

this is the form of standard MCN pdf

ME Model (3)

This can be expressed with the Kronecker Product as

$$\mathbf{R} = -(\underbrace{\mathbf{I}_T \otimes \boldsymbol{\mu}_R + \boldsymbol{\mu}_T \otimes \mathbf{I}_R}_{\mathbf{A}})^{-1}.$$

Which is different from the Kronecker Model [Yu, et. al.]

$$\mathbf{R} = \mathbf{R}_T \otimes \mathbf{R}_R.$$

The eigenvalue decomposition (EVD) of $\mu_{T,R}$:

$$\boldsymbol{\mu}_T = \boldsymbol{\xi}_T \boldsymbol{\Lambda}_T \boldsymbol{\xi}_T^H, \qquad \boldsymbol{\mu}_R = \boldsymbol{\xi}_R \boldsymbol{\Lambda}_R \boldsymbol{\xi}_R^H,$$

Hence we can write:

$$\mathbf{R}^{-1} = -\underbrace{(\boldsymbol{\xi}_T \otimes \boldsymbol{\xi}_R)}_{\boldsymbol{\xi}} \underbrace{(\mathbf{I}_T \otimes \boldsymbol{\Lambda}_R + \boldsymbol{\Lambda}_T \otimes \mathbf{I}_R)}_{\boldsymbol{\Lambda}} \underbrace{(\boldsymbol{\xi}_T \otimes \boldsymbol{\xi}_R)^H}_{\boldsymbol{\xi}^H},$$

ME (4)

- The eigenvectors are just the Kronecker product of the separate TX and RX eigenvectors
- The eigenvalues can be found by substituting **R** into the original constraints

Hence we can re-write the TX or RX covariance, eg

$$R_{R,ik} = \sum_j R_{ij,kj},$$

With the RX covariance constraint as:

$$\{-\mathbf{R}_R\}_{ik} = \sum_m \xi_{R,im}\xi^*_{R,km} \underbrace{\sum_n \Lambda^{-1}_{mn,mn}}_{-D_{R,mm}}.$$

simply the EVD of RX covariance

$$\mathbf{R}_R = \boldsymbol{\xi}_R \mathbf{D}_R \boldsymbol{\xi}_R^H.$$

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ME (5)

To solve for **R** (full) we need to solve

$$D_{R,mm} = d_{R,m} = -\sum_n \frac{1}{\lambda_{R,m} + \lambda_{T,n}}$$

$$D_{T,nn} = d_{T,n} = -\sum_m \frac{1}{\lambda_{R,m} + \lambda_{T,n}}$$

One method to solve is by indirect approach since **H** is a Gaussian process, ME maximizes det(**R**)

Need to find **Λ**, such that det(- $\mathbf{\Lambda}^{-1}$) is maximized, ie if:

$$f_{mn} = -\Lambda^{-1}_{mn,mn},$$

Maximize $\det(\mathbf{R}) = \prod_{ij} f_{ij}$ subject to ff constraints:

$$d_{T,j} = \sum_i f_{ij} \qquad d_{R,i} = \sum_j f_{ij} \qquad f_{ij} \geq 0.$$

ME (6)

Since constraints are linear and $-\Pi_{ij} f_{ij}$ is convex

⟹ can use linear programming to find initial guess for f_{ij}

⟹ use gradient descent method to find minimum of function

Data Processing

- Since multipath path scattering in indoors is severe, we define the double directional response in terms of spatial power spectra for joint RX/TX Bartlett beamformers as

$$P(\nu_T, \nu_R) = \frac{\mathbf{a}(\nu_T, \nu_R)^H \hat{\mathbf{R}} \mathbf{a}(\nu_T, \nu_R)}{\mathbf{a}(\nu_T, \nu_R)^H \mathbf{a}(\nu_T, \nu_R)}.$$

$\upsilon_{T,R}$ = azimuth angle at TX or RX

$\hat{R}$ = sample covariance matrix

The joint steering vector is defined as:

$$\mathbf{a}(\nu_T, \nu_R) = \mathbf{a}_T(\nu_T) \otimes \mathbf{a}_R(\nu_R)$$

Sample covariance matrix

$$\hat{\mathbf{R}} = \frac{1}{K} \sum_k \mathbf{h}^{(k)} \mathbf{h}^{(k)H}$$

$h^{(k)} = \mathrm{Vec}\{H^{(k)}\}$

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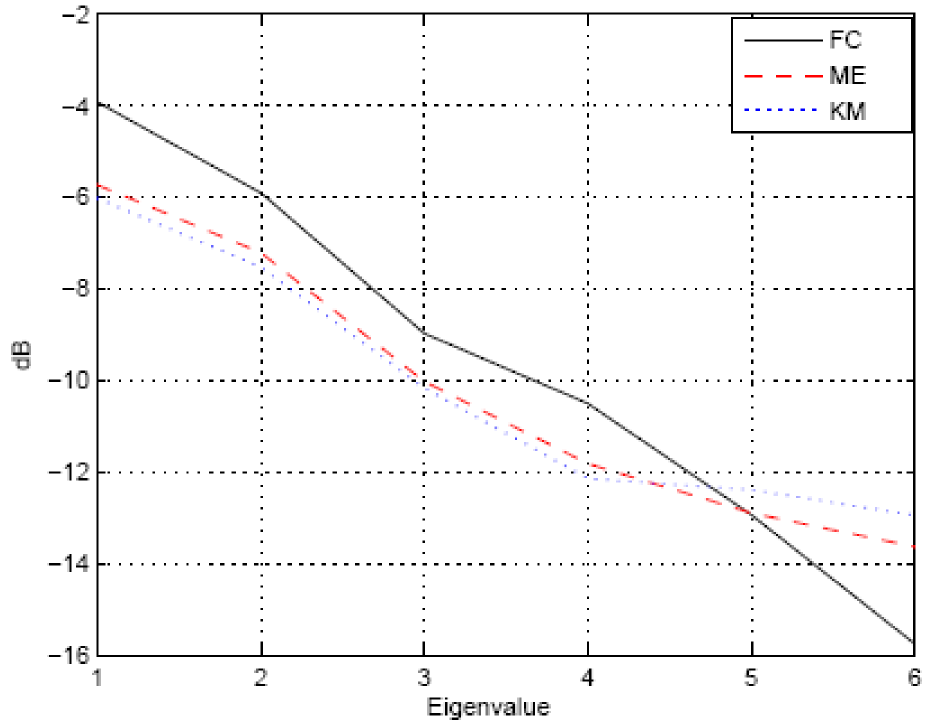
Results – Location 3



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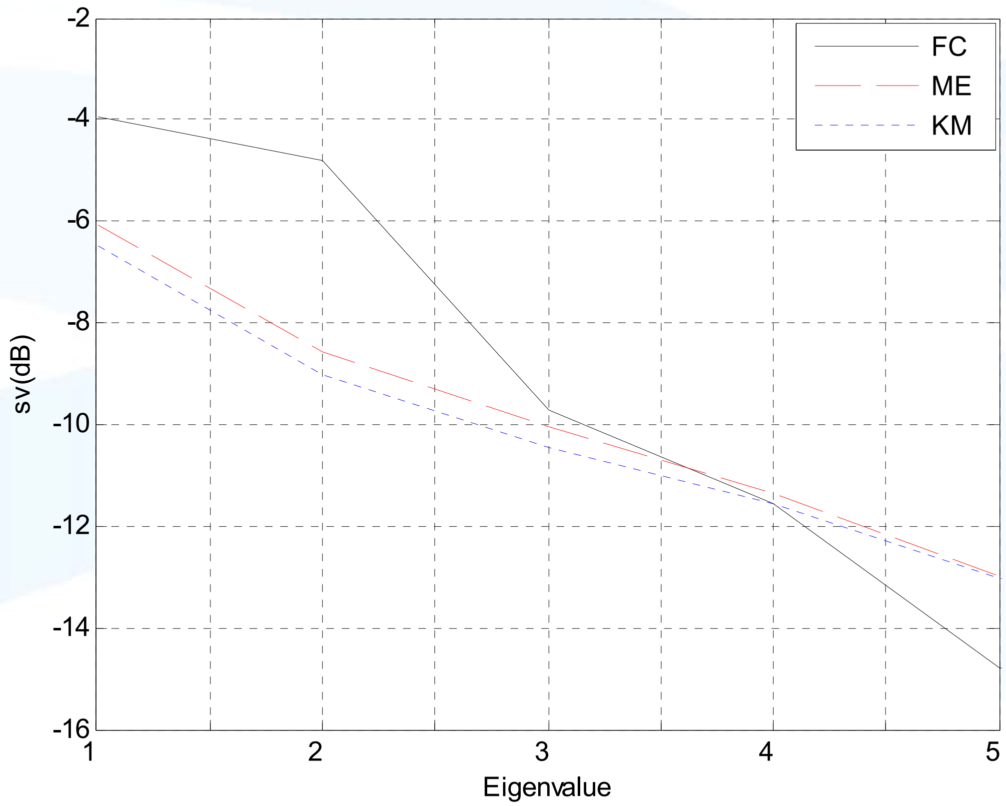
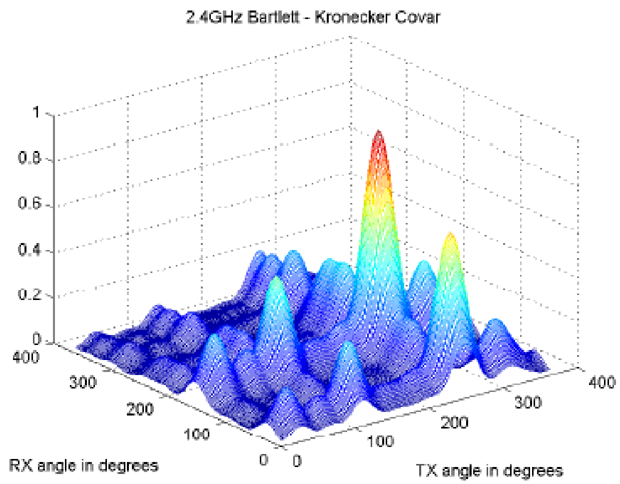
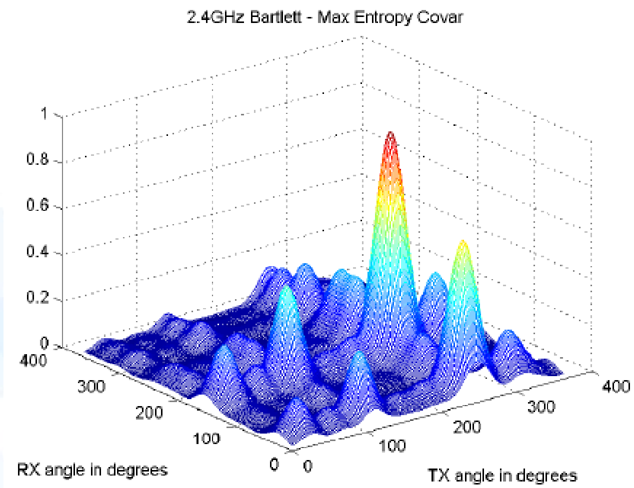
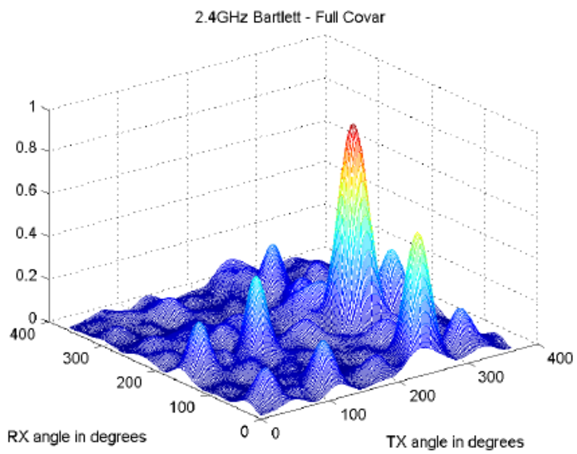
Results – Location 9

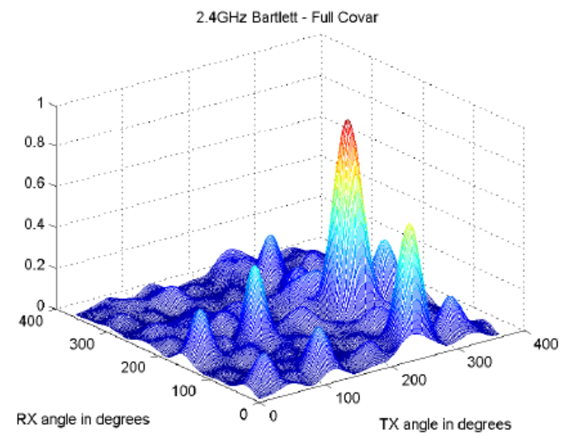


Results – Location3









Results
Joint correlation of spatial power spectra

CORRELATION COEFFICIENT OF SPATIAL POWER SPECTRA

Locations	Bartlett beamforming at 2.4 GHz	
	$\rho_{FC,ME}$	$\rho_{FC,KM}$
1	0.9387	0.9377
2	0.9258	0.9088
3	0.9470	0.99264
4	0.9927	0.9887
5	0.9716	0.9615
6	0.9818	0.9798
7	0.8850	0.8889
8	0.9872	0.9845
9	0.9028	0.8966
10	0.9007	0.8985
11	0.9261	0.9229

ME Approach to Channel Modelling: Conclusion

- Presented a ME approach for obtaining full covariance when only separate covariances are known
- The eigenvalues are different from the Kronecker Model
- For this indoor environment at 2.4GHz, KM and ME Model gave very similar double directional power spectra results but different metric
- This means that the modeled channels attain ME bound
- This suggests that the KM model represents a fundamental limit
- Opportunity for further testing of model

What has been achieved? (1)

1. Presented a Geometric Model for flat fading wireless indoor environment with scattering at both the TX and RX which have some of key components that affect the channel behavior.

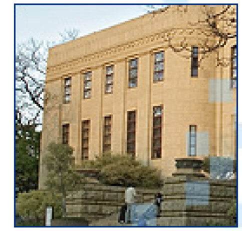
2. Developed a unique WB MIMO Channel Sounder capable of operating in the 2- 6 GHz range with an excitation BW of 100MHz.

3. Developed and presented the concept of ‘frequency scaling’ in describing MIMO channel behavior.

What has been achieved? (2)

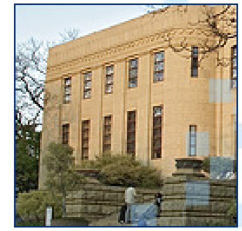
4. Modelled capacity for a UCA
5. Modelled spatial correlation for a ULA
6. Extended the Double Directional Channel Model to include spatial power spectra through joint beamforming.
7. Developed the new MIMO Channel Model (MWL Model) based on the maximum entropy approach.
8. Established an agenda for further research and MIMO channel characterization

Outputs



- Artifact (WB MIMO Channel Sounder)

- Currently have 10 publications
  - 4 Journal publications
    - 3 Published
    - 1 Accepted for Publication
  - 5 Peer reviewed International Conference Publications
  - 1 Invited Paper: International Conference Publication



Thank You

for

attending

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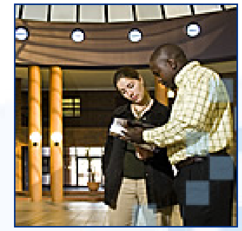

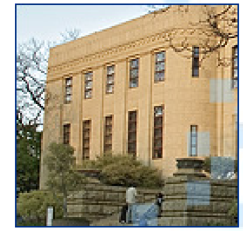

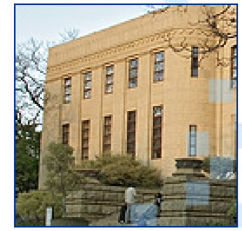

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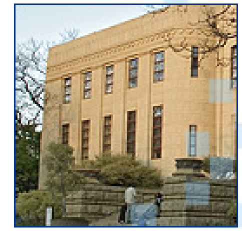

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