

# MUTUAL ADMITTANCE BETWEEN CPW-FED SLOTS ON CONDUCTOR-BACKED TWO-LAYER SUBSTRATES

by

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## SUMMARY

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### MUTUAL ADMITTANCE BETWEEN CPW-FED SLOTS ON CONDUCTOR-BACKED TWO-LAYER SUBSTRATES

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Slot dipole antennas fed by coplanar waveguide (CPW) on substrates consisting of a single dielectric layer exhibit various attractive qualities, including significantly wider impedance bandwidth than comparable microstrip patch antennas. For applications that call for unidirectional radiation, such as antennas on airframes, a conducting back plane is needed. A CPW on a conductor-backed single-dielectric-layer substrate will always experience power leakage into the TEM parallel-plate mode. On the other hand, it is possible to design CPW lines on conductor-backed two-layer substrates that are free from leakage into the substrate. However, once the CPW is used as feed line to a slot dipole, power leakage into the  $TM_0$  substrate mode caused by the transition between the CPW and the radiating slot, and by the radiating slot itself, may still severely compromise radiation efficiency.

This study has two main contributions to offer. First, a paucity of work on CPW-fed slot antennas on conductor-backed two-layer substrates is alleviated by providing a fuller characterization of single-slot behaviour on two-layer parallel-plate substrates than is currently available, and by systematically investigating a practically feasible minimum antenna configuration, namely broadside twin slots, that is not debilitated by the problem of substrate mode leakage. Results obtained with the moment-method-based electromagnetic simulator IE3D that emphasize the trade-off between radiation efficiency and impedance bandwidth are presented; they can be used for design purposes. For instance, with respect to single slots on a substrate with an electrically thin top dielectric layer and an air bottom layer, it is shown that radiation efficiency increases and bandwidth decreases

as height of the bottom substrate layer increases. For broadside twin slots, it is demonstrated that spacing close to half a wavelength of the two-layer parallel-plate  $TM_0$  mode apart can yield a large improvement in radiation efficiency over that of a single slot (a reduction in bandwidth however occurs).

The second main contribution is the development of an approach for finding the mutual admittance  $Y_{12}$  between CPW-fed slots on conductor-backed two-layer substrates that can be more readily incorporated in an iterative array design procedure than a moment-method-based technique, yet is of comparable accuracy; it is built on a standard reciprocity-based expression. As an initial step, the mutual admittance between CPW-fed slots on a conductor-backed two-layer substrate with an air bottom layer is characterized using IE3D. This involves presenting curves for  $Y_{12}$  between twin slots against slot separation  $d$  along standard paths for slot half-lengths in the vicinities of the first and second resonant half-lengths of the corresponding isolated slots (such data might be used towards a first-order array design), and a study of the effect of back plane distance (*i.e.*, bottom layer height) on mutual coupling. The bulk of the thesis however is devoted to the above reciprocity-expression approach. Simplifying assumptions are outlined that make it possible to determine  $Y_{12}$  against  $d$  by performing a once-only moment-method analysis of each slot in isolation, and then calculating external and internal reaction integrals at each value of  $d$ . This is significantly more economical than carrying out a full moment-method analysis of the whole twin-slot structure at every instance of  $d$ . Evaluation of the internal reaction integral requires the appropriate component of the spatial-domain Green's function for the substrate, which is derived in a form containing Sommerfeld-type integrals; treatment of singularities is discussed. The reciprocity-expression approach is verified by comparing  $Y_{12}$  against  $d$  curves for twin slots and non-identical slot pairs on a variety of conductor-backed two-layer substrates to IE3D simulations. A procedure that involves judicious selection of reference planes is introduced by which agreement between the methods for the special case of twin slots with the same half-length as the corresponding isolated second-resonant slot can be even further improved. A measurement is provided that validate theoretical calculations.



## OPSOMMING

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### **WEDERSYDSE ADMITTANSIE TUSSEN KOPLANÊRE GOLFLEIER-GEVOERDE GLEUWE OP SUBSTRATE MET TWEE DIËLEKTRIESE LAE EN 'N AGTERSTE GRONDVLAK**

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**Sleutelwoorde:** koplanêre golfleier-gevoerde gleuve, wedersydse admittansie, self-admittansie, parallel-plaat substrate met twee diëlektriese lae, antenne-samestellings

Gleuf-dipool antennes gevoer deur koplanêre golfleier (KPG) op substrate wat uit 'n enkele diëlektriese laag bestaan, het verskeie aantreklike eienskappe, onder meer beduidend beter impedansie-bandwydte as vergelykbare mikrostrook plakantennes. Vir toepassings wat unidireksionele straling benodig soos antennes wat op lugrame gemonteer is, word 'n geleidende agterste grondvlak benodig. 'n KPG op 'n substraat wat uit een diëlektriese laag bestaan met 'n grondvlak agter, sal altyd die TEM parallel-plaat-orde opwek en drywing afstaan aan hierdie orde. Aan die ander kant is dit moontlik om KPG-lyne te ontwerp op substrate met twee diëlektriese lae en 'n agterste grondvlak, wat nie drywing aan substraatordes sal verloor nie. Sodra hierdie KPG egter gebruik word om 'n gleuf-dipool te voer, kan drywingsverlies aan die  $TM_0$  substraat-orde wat veroorsaak word deur die oorgang tussen die transmissielyn en die gleuf, asook deur die gleuf self, die stralingeffektiwiteit van die antenne ernstig benadeel.

Die twee hoofbydraes van hierdie studie is die volgende. Eerstens word die relatief min beskikbare navorsing oor KPG-gevoerde antennes op twee-diëlektriese-laag-substrate met 'n agterste grondvlak aangevul deur 'n vollediger karakterisering van die gedrag van enkele gleuve op hierdie substrate. Ook word 'n realiseerbare gleufkonfigurasie, naamlik wye-sy tweeling-gleuve wat die probleem van drywingsverlies aan substraatordes kan oorbrug, stelselmatig ondersoek. Resultate verkry met behulp van die momente-metode-gebaseerde simulator IE3D wat die gee-en-neem-verwantskap tussen stralingeffektiwiteit en impedansie-bandwydte illustreer, word getoon; hierdie resultate kan



gebruik word vir ontwerpsdoeleindes. Dit word byvoorbeeld getoon dat die stralingseffektiwiteit van enkelgleuwe op 'n substraat met 'n elektries-dun boonste diëlektriese laag en 'n onderste laag wat uit lug bestaan, toeneem en die bandwydte afneem as die hoogte van die onderste laag toeneem. Ook word gewys dat, wanneer wye-sy tweeling-gleuwe ongeveer helfte van die golflengte van die  $T M_0$ -orde in die substraat uitmekaar gespasieer is, 'n groot toename in stralingseffektiwiteit verkry kan word ten opsigte van dié van 'n enkele gleuf ('n afname in bandwydte vind egter plaas).

Die tweede hoofbydrae is die ontwikkeling van 'n metode om die wedersydse admittansie  $Y_{12}$  tussen KPG-gevoerde gleuwe op twee-diëlektriese-laag-substrate met 'n agterste grondvlak te bepaal wat meer geredelik in 'n iteratiewe samestelling-ontwerpsprosedure geïntegreer kan word as 'n momente-metode-gebaseerde tegniek, maar wat vergelykbare akkuraatheid het; die metode is gebaseer op 'n bekende wederkerigheidsformule. As 'n eerste stap word die wedersydse admittansie tussen KPG-gevoerde gleuwe op 'n twee-diëlektriese-laag-substraat met 'n agterste grondvlak waarvan die onderste laag lug is, gekarakteriseer met behulp van IE3D. Dit behels krommes vir  $Y_{12}$  teenoor  $d$  (die afstand tussen die gleuwe) vir standaard konfigurasies van tweeling-gleuwe met halflengtes in die omgewing van die eerste- en tweede-resonante halflengtes van die ooreenstemmende geïsoleerde gleuwe, asook 'n studie van die invloed van die afstand van die agterste grondvlak (*i.e.*, hoogte van die onderste diëlektriese laag) op die wedersydse koppeling (hierdie data kan aangewend word vir eerste-orde samestellings-ontwerpsdoeleindes). Die grootste deel van die tesis word gewy aan die wederkerigheidsformule-benadering hierbo genoem. Aannames word uiteengesit wat dit moontlik maak om  $Y_{12}$  teenoor  $d$  te bepaal deur elk van die geïsoleerde gleuwe slegs eenmalig te analiseer met behulp van die momente-metode, en dan eksterne en interne reaksie-integrale by elke waarde van  $d$  te bereken. Dit is aansienlik meer ekonomies as om 'n volle momente-metode-analise van die hele twee-gleuf-struktuur by elke waarde van  $d$  uit te voer. Vir berekening van die interne reaksie-integrale word die tersaaklike komponent van die ruimtelike-domein Green-funksie van die substraat vereis; daar word getoon hoe laasgenoemde afgelei kan word in 'n vorm wat Sommerfeld-tipe integrale bevat, en die numeriese hantering van singulariteite word bespreek. Die wederkerigheidsformule-benadering word geverifieer deur krommes vir  $Y_{12}$  teenoor  $d$  vir tweeling-gleuwe en nie-identiese gleufpare op 'n verskeidenheid van twee-diëlektriese-laag-substrate met 'n agterste grondvlak, te vergelyk met IE3D-simulasies. 'n Prosedure wat 'n toepaslike keuse van verwysingsvlakke behels, word aan die hand gedoen om die ooreenkoms tussen die metodes vir die spesiale geval van tweeling-gleuwe met dieselfde halflengte as die ooreenstemmende geïsoleerde tweede-resonante gleuf selfs verder te verbeter. 'n Meting word



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getoon wat teoretiese berekeninge bevestig.



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