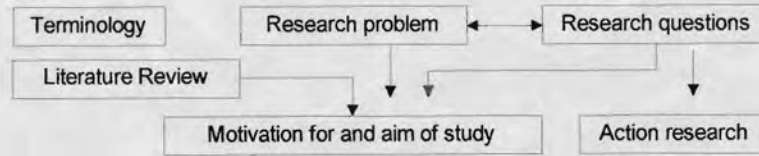
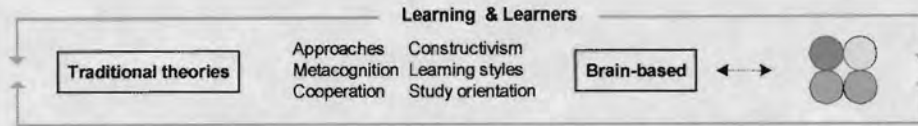


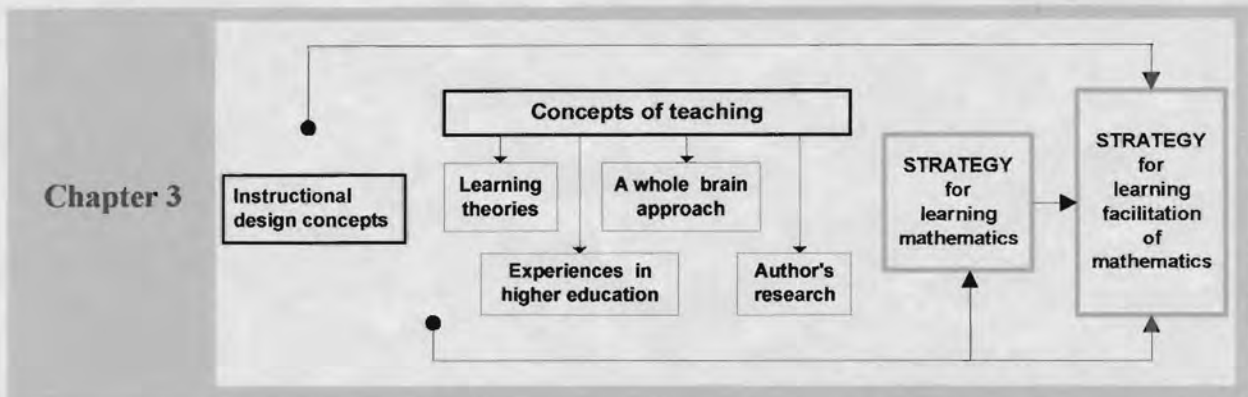
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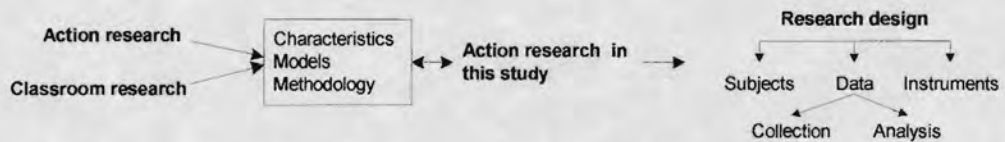
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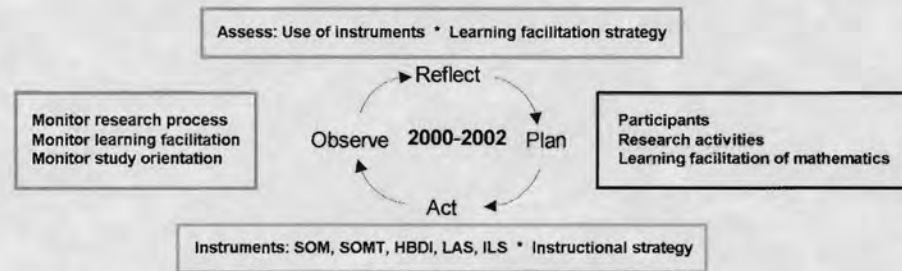
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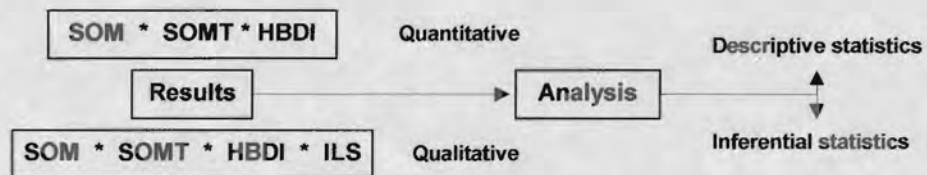
Chapter 4



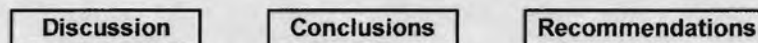
Chapter 5



Chapter 6

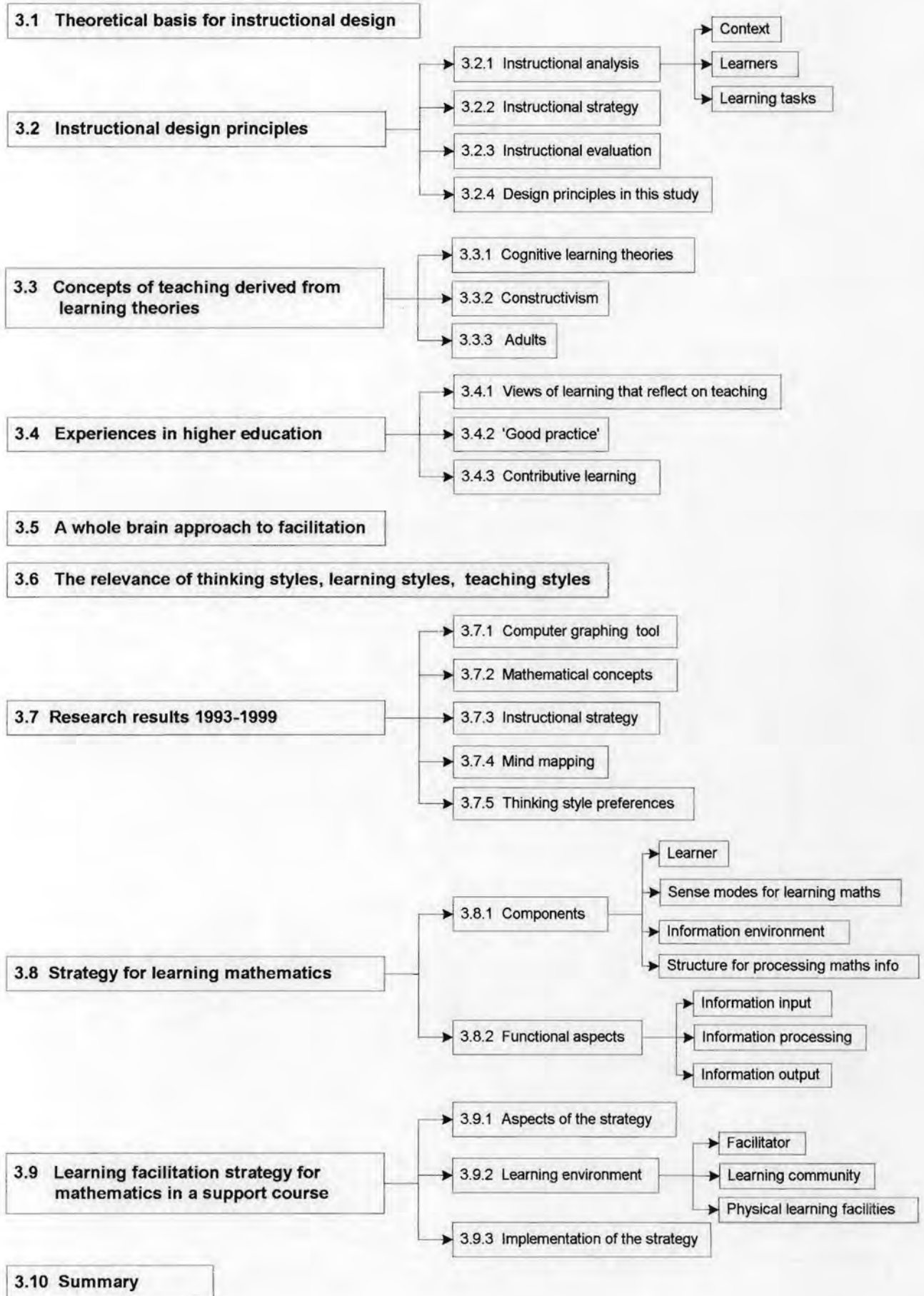


Chapter 7





A learning facilitation strategy for mathematics



Chapter 3

A learning facilitation strategy for mathematics

3. Introduction

The sheer magnitude of human knowledge renders its coverage by education an impossibility; rather, the goal of education is better conceived as helping students develop the intellectual tools and learning strategies needed to acquire the knowledge that allows people to think productively about Mathematics. (Bransford et al., 1999:5).

In this chapter a theoretical basis for instructional design principles that are applicable to the present study are identified. Examples of perspectives on instruction that contribute to a narrative for the learning facilitation of mathematics in a support course are discussed. A brain-based approach to learning and teaching and aspects of teaching resulting from experiences in higher education are addressed. The results of previous research by the author on the learning facilitation of mathematics in a support course are also summarised. The chapter concludes with the proposal of a strategy for the learning facilitation of mathematics in a support course.

3.1 A theoretical basis for instructional design

Gagné and Briggs (1974:4) point out that

The purpose of designed instruction is to activate and support the learning of the individual student. The purpose of planned instruction is to help each person develop as fully as possible, in his or her own individual directions.

Gagné and Briggs (1974:4-5) regard the following five characteristics as fundamental when instruction is designed, namely:

1. Instructional design must be aimed at **aiding an individual's own learning**.
2. Instructional design has **immediate and long-range phases**. Long-range aspects are more complex and varied and concerns designing a course or an entire instructional

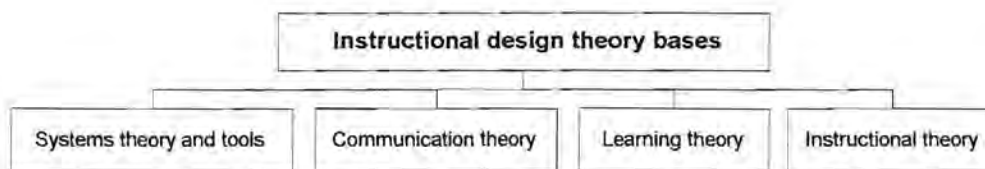
system. The immediate phase involves day-to-day planning and this phase can benefit from proper long-range design.

3. Systematically designed instruction can greatly **affect individual human development**. Providing only a nurturing environment is not sufficient — specific opportunities must be created for learners to develop fully.
4. Instructional design should be conducted by means of a **systems approach**.
5. Designed instruction must be based on knowledge of **how human beings learn**.

According to Smith and Ragan (1993) the most critical assumptions underlying instructional design are that the resulting instruction should be effective, efficient and appealing. Furthermore, students must participate actively and interact mentally and physically with material to be learnt. They also point out that evaluation of the instruction as well as of the learner's performance should be done and these insights ought to be used to revise the instruction. There should also be congruence among objectives, learning activities and assessment — the objectives should direct the activities and assessment.

Smith and Ragan (1993:14) regard the general systems theory, the communication theory, theories of learning and theories of instruction as the major theories that contribute to instructional design. This view is illustrated in Figure 3-1.

Figure 3-1 Theoretical bases for instructional design



Adapted from Smith and Ragan (1993:23)

According to Smith and Ragan's (1993) view, the **systems theory** highlights the interrelationships between the components in instruction and the **communication theory** is important for describing the way in which information is transferred from one person to another. The communication theory is also significant for the selection of the media that is used to support instruction.

The importance of **theories of learning** for instructional design is summarised by Ertmer and Newby (1993:50) when they remark that:

Learning theories provide instructional designers with verified instructional strategies and techniques for facilitating learning as well as a foundation for intelligent strategy selection.

On the other hand Cross (1998:7) points out that researchers of the learning phenomenon should not ignore *the experience of teachers [in higher education] who have spent lifetimes accumulating knowledge about learning*. She continues and adds that *faculty should not exchange views about student learning with no reference to what scholars know through study of the matter* (Cross, 1998:7).

The author of this thesis is of the opinion that the views of Cross stress the fact that **relevant theory** and **judicious experiences** from **practice** should be seen in congruence and should be interpreted for the design of tertiary mathematics instruction in a support course. The author also feels that all educators involved in tertiary mathematics instruction should actively and continually be involved in improving their practice through critical self-reflection and by taking cognisance of existing knowledge relevant to the learning and teaching of mathematics. A deep understanding of what "tertiary learning of mathematics" really entails may eventually not only foster our understanding of what constitutes a program of learning in mathematics but also what constitutes a program of **learning facilitation** in tertiary mathematics. Zuber-Skerritt (1992a; 1992b & 1997) argues that the implementation of action research²⁰ not only has the ability to improve teaching in higher education but can also contribute to research in tertiary (mathematics) education.

For the purposes of the instructional design underpinning the learning facilitation strategy proposed in this study, examples of perspectives on instruction that may have relevance for the focus of this study are discussed in the following section including:

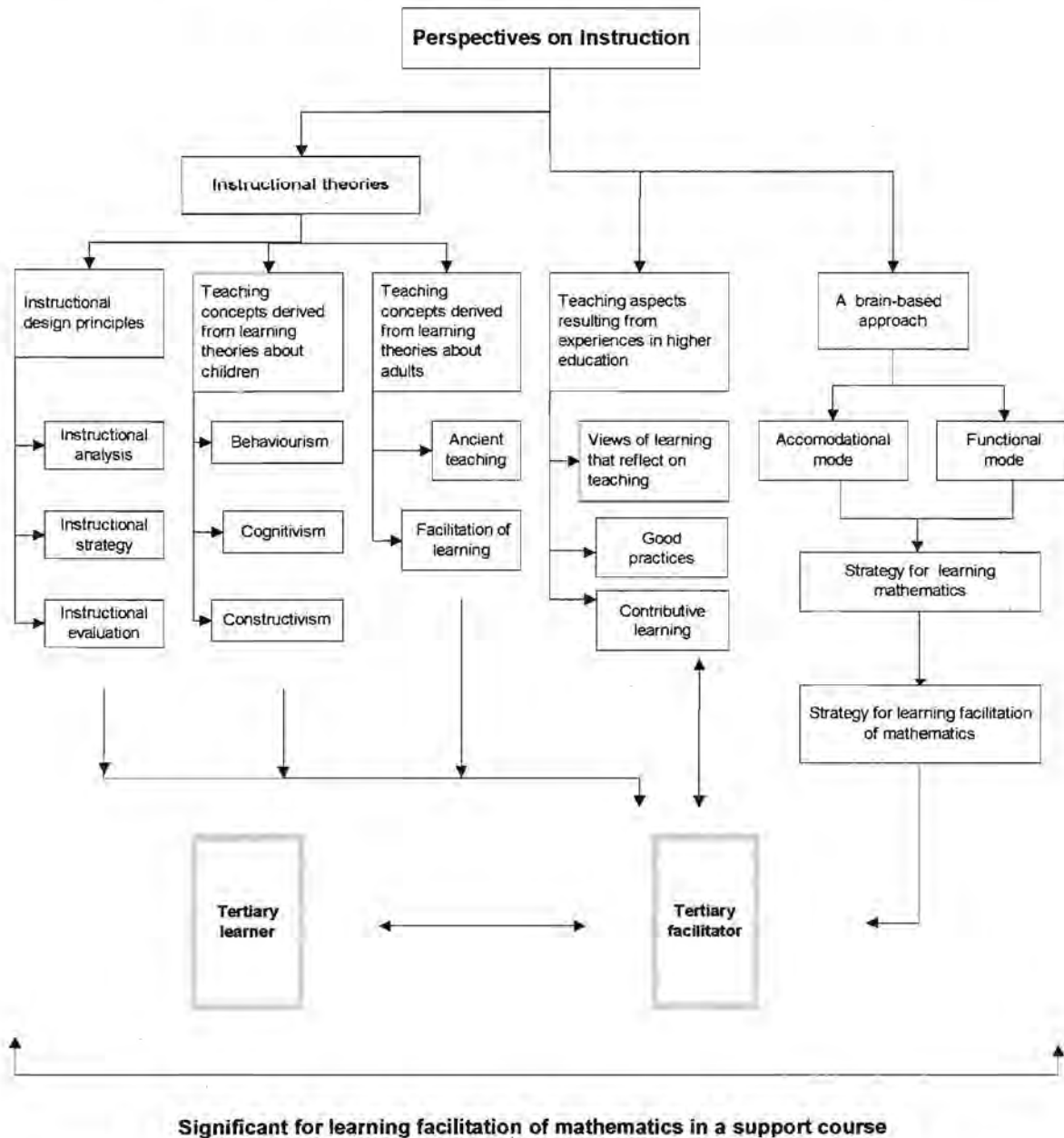
- Accepted instructional design principles.
- Concepts of teaching derived from learning theories about children.
- Concepts of teaching derived from learning theories about adults.

²⁰ Action research is discussed in Chapter 4 as the main methodology in this study.

- Concepts of teaching derived from experiences in undergraduate education.
- A brain-based integrated approach to teaching.

Figure 3-2 gives an overview of the focus as outlined above.

Figure 3-2 Examples of perspectives on instruction that contribute to a narrative for learning facilitation of mathematics in a support course



Proposed by the author of this thesis



3.2 Instructional design principles

According to Smith and Ragan (1993), an analysis of instruction, a strategy for instruction and the evaluation of the instruction constitute the main characteristics of instructional design principles. In a comparative analysis of models of instructional design Andrews and Goodson (1980) examined forty models of instructional design from various sources and identified 'common tasks' in model development. Gagné and Briggs (1974) present their views on instructional design principles as 'stages' in the design of instructional systems.

If the design principles of Smith and Ragan (1993), as mentioned above, are used as a frame of reference, the 'tasks' identified by Andrews and Goodson (1980) and the 'stages' of Gagné and Briggs²¹ (1974) can be categorised according to Smith and Ragan's principles. Table 3-1 summarises these tasks and stages according to the instructional design principles of an analysis of instruction, a strategy for instruction and the evaluation of the instruction. The last mentioned principles are discussed in the following sections.

Table 3-1 A summary of instructional design principles

Principle proposed by Smith and Ragan (1993):	Analysis of instruction
Corresponding tasks identified by Andrews and Goodman (1980):	<ul style="list-style-type: none"> Formulation of goals. Development of pretest and post test matching goals. Analysis of goals for skills/learning required. Characterisation of learner population. Assessment of need, problem identification, occupational analysis, competence or training requirements. Formulation of system and environmental descriptions and identification of constraints.
Corresponding stages identified by Gagné and Briggs (1974):	<ul style="list-style-type: none"> Analysis of needs, goals and priorities. Analysis of resources, constraints and alternative delivery systems. Analysis of course objectives. Define performance objectives.

²¹ It should be pointed out that Andrews and Goodman (1980) included Gagné and Briggs's (1974) 'stages' in all of the fourteen 'tasks' they identified.



Principle proposed by Smith and Ragan (1993):	A strategy for instruction
Corresponding tasks identified by Andrews and Goodman (1980):	Sequencing. Formulation of strategy to match subject matter and learner requirements. Selection of media to implement strategies. Development of courseware based on strategies. Development of materials and procedures for installing , maintaining and periodically repairing the instructional program.
Corresponding stages identified by Gagné and Briggs (1974):	Scope of delivery design. Sequence of delivery. Analysis of goals for skills/learning required. Prepare lesson plans. Selecting media and material. Teacher preparation. Installation and diffusion.
Principle proposed by Smith and Ragan (1993):	Evaluation of instruction
Corresponding tasks identified by Andrews and Goodman (1980):	Empirical tryout of courseware with learner population. Diagnosis of learning and courseware failures. Revision of courseware based on diagnosis. Consideration of alternative solutions. Costing instructional programs.
Corresponding stages identified by Gagné and Briggs (1974):	Assessing student performance. Formative evaluation. Field testing, revision. Summative evaluation.

Compiled from Smith and Ragan (1993); Andrews and Goodman (1980); Gagné and Briggs (1974)

For the purposes of this study, the instructional design principles as formulated by Smith and Ragan (1993) will be used as a frame of reference for the learning facilitation strategy defined in the study. In the opinion of the author these principles, on the one hand, incorporate conventional views on instructional design as postulated, for example, by Andrews and Goodman (1980), Gagné and Briggs (1974), Gagné and Driscoll (1988) and Reigeluth (1983). On the other hand, the principles as formulated by Smith and Ragan (1993) serve as contemporary and appropriate categories with regard to the learning facilitation strategy defined in this thesis.

In the following sections an overview of Smith and Ragan's (1993) design principles is given and their relevance for the present study is summarised in Table 3-2 on page 87.

3.2.1 Instructional analysis

According to Smith and Ragan (1993) any design and development of instruction should be preceded by an analysis in order to ensure that the instruction is relevant for the intended learners in their learning environment. They distinguish between the following three aspects of instructional analysis:

3.2.1.1 Analysis of instructional context

Analysis of instructional context involves a needs assessment and a description of the learning environment in which the instruction will take place.

A **needs assessment** can be indicated by a number of reasons, including the following:

- Learning goals not being met by given current instruction.
- Current instruction being inefficient.
- Current instruction being unappealing.
- A lack of instruction in a given area.
- New goals being added to the curriculum.
- Changes in learner population.

Analysis of the **learning environment** involves looking at existing conditions that surround and support instruction, namely:

- Characteristics of teachers that will be using the instructional material.
- Existing curricula into which the instruction must fit.
- Availability of instructional media equipment.
- Characteristics of the facilities and the organisation in which the planned instruction will take place.
- Factors in the larger community in which the instructional organisation exists.

3.2.1.2 Analysis of learners

Learner characteristics are regarded as an important aspect when designing instruction (Andrews & Goodman, 1980; Bransford *et al.*, 1999; Cross & Steadman, 1996; Felder &



Brent, 1999; Gagné & Briggs, 1974; Gagné & Driscoll, 1988; Jensen, 1996; McKeachie, 1999; Reigeluth, 1983).

Smith and Ragan (1993) identify four major categories of learner characteristics to be considered in instructional design. In the opinion of the author of this thesis these categories are also relevant for the strategies defined in the thesis and noteworthy mentioning.

- Cognitive (general and specific).
- Physiological.
- Affective.
- Social.

These categories can further be classified in terms of the following dimensions:

- Similarities among people or differences between people.
- Characteristics that change over time or those that remain relative stable over time.

Smith and Ragan (1993:55) use these categories to formulate an outline of learner characteristics by means of which learners can be analysed. The following categories from this outline can be included in a learner analysis for the purpose of this study:

Cognitive characteristics that include

- General and specific aptitudes.
- Language and reading level.
- Level of visual literacy, ability to gain information from graphics.
- Cognitive processing styles – preferred and most effective.
- Specific prior knowledge.

Physiological characteristics that include sensory perception.

Affective characteristics that include

- Interests.
- Motivation (to learn).
- Attitude toward subject matter.
- Academic self-concept.
- Anxiety level.
- Attribution of success (locus of control).

Social characteristics that include relationship with peers and tendencies toward cooperation or competition.



3.2.1.3 Analysis of learning tasks.

A needs assessment should reflect what learners are currently unable to do. The process of the task analysis is necessary in order reformulate the findings of the needs assessment so that it can be used to guide subsequent instructional design. The following steps summarises Smith and Ragan's (1993) views on performing a learning task analysis:

1. Writing instructional goals. These are statements of what learners should be able to do at the conclusion of instruction and can be lesson goals, unit goals or course goals.
2. Determining learning outcomes. Smith and Ragan use the system of Gagné's learning outcomes²² as foundation for a learning outcomes analysis.
3. Conducting an information-processing analysis of the instructional goal. This analysis describes the mental processes that the learner might go through in completing the goal.
4. Conducting a prerequisite analysis with regard to each step in the information-processing analysis. This means answering the question: "What must the learner know or be able to do to achieve each step?"
5. Writing performance objectives. These are statements of what learners should be able to do when they have completed a segment of instruction.

The outcomes of the instructional analysis phase of the learning facilitation strategy proposed in this thesis was reported by the author in an earlier study (Steyn, 1998). In Table 3-2 on page 87 a summary is given of the main aspects regarding instructional analysis as applied to the present study.

3.2.2 Instructional strategy

According to Reigeluth (1983) instructional strategies are composed of three aspects, namely, organisational strategy, delivering strategy and management strategy.

Instructional organisational strategy refers to the sequence of instruction, the content and the presentation of the content. At the lesson-level Smith and Ragan (1993) regard instructional events as comprising of an introduction, a body, conclusion and assessment. Figure 3-3 is a summary of the contents of each of these aspects.

²² Gagné's learning outcomes are discussed in Chapter 2, Section 2.1.2.

Figure 3-3 Instructional events at lesson-level

Introduction	Body	Conclusion	Assessment
<ul style="list-style-type: none">• Gain attention• Establish instructional purpose• Arouse interest and motivation• Preview lesson	<ul style="list-style-type: none">• Recall prior knowledge• Focus attention• Employ learning strategies• Practise• Evaluative feedback	<ul style="list-style-type: none">• Summarise and review• Transfer knowledge• Re-motivate and close	<ul style="list-style-type: none">• Assess performance• Evaluative feedback and remediate

Adapted from Smith and Ragan (1993:160)

Instructional delivery strategy characteristics deal with the type of instructional media that will be used and the grouping of the learners.

Instructional management strategies guide the arrangement of the organisational and delivery strategies, namely the scheduling of the instructional events and the mechanisms for delivering these events.

In Table 3-2 on page 87 a summary is given of the mentioned instructional strategies as applied to the present study.

3.2.3 Instructional evaluation

Smith and Ragan (1993:388) stress the point that there are two kinds of evaluation that are critical and essential to the design of instruction. On the one hand, the performances of students need to be evaluated to determine whether the objectives of the instruction have been reached. On the other, the instructional material has to be evaluated. According to them the evaluation of the instructional material occurs at two separate stages during the design process, namely, during the development of the instruction (formative evaluation) and after the instructional strategy have been implemented (summative evaluation).

When an action research approach to classroom practice is followed (as was done in the study reported in this thesis), formative evaluation forms an integral part of the reflection phase of the action research model. The research results reported in Chapters 6 and 7 can be regarded as a summative evaluation of the instructional strategy proposed in this thesis.

3.2.4 Instructional design principles in the present study

In Table 3-2 below a summary is given of the instructional design considerations regarding learning facilitation in the present study using the instructional design principles according to Smith and Ragan (1993) as a frame of reference.

Table 3-2 Instructional design considerations regarding the present study

Instructional design principles (Smith & Ragan, 1993)	Principles as applied to the present study
---	--

Instructional analysis: Context

Needs assessment	<p>How can the exploration of two-dimensional functions through their graphs enhance the conceptualisation of fundamental mathematical concepts that underpin a study in calculus?</p> <p>What are the requirements for graphing technology to ensure meaningful visualisation of two-dimensional functions to promote better understanding of the mathematical concepts involved?</p> <p>How should instruction be structured to foster learner cognition and conceptualisation incorporating the visualisation of two-dimensional functions?</p>
Learning environment	<p>The development of the mathematics potential of a learner on a support course needs to be done in addition to the course activities of a main-stream mathematics course. An innovative approach is necessary to keep instruction <i>effective, efficient</i> and <i>appealing</i>. Using computer graphing technology poses the possibility that learners <i>actively participate</i> and that they interact <i>mentally</i> and <i>physically</i> with the material to be learnt.</p>

Table 3-2 continues on the next page.

Instructional analysis: Analysis of learners – Learner characteristics

Cognitive	<p>Students who have the interest and ability to study engineering but who do not meet the admission requirements of the institution need support regarding the development of their mathematics potential. Mainstream courses in mathematics do not necessarily provide this support per se.</p> <p>A developmental approach is proposed addressing the following aspects: existing knowledge in mathematics; language and reading competency in mathematics; competency in graphical analysis and synthesis; thinking and learning styles; problem solving.</p> <p>A brain-based approach aimed at neural patterning and re-patterning²³ and whole brain utilization²⁴ is proposed.</p>
Physiological	<p>Sensory perception relevant for mathematics namely, visual, auditive and kinaesthetic perception, should be incorporated in the learning facilitation strategy.</p>
Affective	<p>Aspects regarding study attitude, study habits, mathematics anxiety, meta-cognition and knowledge pertaining to thinking and learning preferences should be integrated into the learning facilitation strategy. Learners should be made aware of their study orientation towards mathematics.</p>
Social	<p>Mathematical communication between facilitator and student and peer communication and learning regarding mathematics should be pursued and encouraged. These are essential steps in creating a learning community.</p>

Instructional analysis: Learning tasks

The primary aim with an analysis of the learning tasks is to determine whether the instructional strategy has contributed to the development of the learner's mathematical potential.

A quantitative analysis of performance in mathematics in the short term will give an indication of possible development of potential. However, eventual long term success in engineering mathematics is the ultimate goal.

Table 3-2 continues on the next page

²³ See Haile's view regarding the functions of the cerebral cortex and neural networks discussed in Chapter 2, section 2.3.1 on page 51.

²⁴ See Herrmann's four quadrant whole brain model discussed in Chapter 2, section 2.3.4.



Instructional strategy

Organisational	The instructional events at lesson level in the strategy proposed in the present study differ significantly from Smith and Ragan's (1993) model. Lesson level activities comprise of students doing structured worksheets using a computer graphing utility, analysing, interpreting, synthesizing, communicating and writing down mathematics. The instruction is individualised and at a learner's own pace. Feedback and remediation are given through communication with the lecturer, tutors and peer discussions.
Delivery	Mathematics activities are done in a computer laboratory setting with each student at his/her own computer. Explorative activities are structured by making use of a computer graphing utility, a paper based workbook and completion of answer sheets formatted to match a specific worksheet. ²⁵
Management	The use of technology for a group of learners necessitates prior planning and arrangement. Consideration has to be given to the fact that not all learners have been exposed to technology and that technology can be non-accessible at times.

Instructional Evaluation

Learner performance	Evaluation of learner performance is not restricted to the course content of the support course, but is in essence reflected in learner performance in the mainstream mathematics courses.
Instructional material	The instructional material being used has been evaluated formatively and summatively. ²⁶ However, due to the action research approach followed in the support course, the improvement of the instructional material is an ongoing process.

Compiled by the author of this thesis

3.3 Concepts of teaching derived from learning theories

It was pointed out in Chapter 2 that the "tertiary learner" of mathematics in a support course should be regarded as a developing learner. This view necessitates a diverse approach to understanding such a learner and also requires a diverse approach in discussing the "tertiary teaching" of mathematics in a support course. In this section a brief discussion of examples of teaching concepts derived from learning theories will be given.

²⁵ During this study the instructional material, *Fundamentals of 2-D Function Graphing - A practical workbook for precalculus and introductory calculus* (Greybe, Steyn & Carr, 1998) was used.

²⁶ Steyn (1998).



The focus in each case will be with relevance to the strategy proposed in the present study and according to the following two questions formulated by Ertmer and Newby (1993:53), namely:

What basic assumptions/principles of the theory are relevant to instructional design?

How should instruction be structured to facilitate learning?

3.3.1 Concepts of teaching resulting from cognitive learning theories

Cognitive learning theories emphasise that instruction should be aimed at making information meaningful to learners and helping learners relate new information to their existing knowledge.

Examples of the assumptions and principles of cognitive learning theories that are relevant to the instructional strategy in this study and the way that the instruction should be structured according to these principles are summarised as follows.

1. **Learner involvement**

The instructional design regarding mathematics for students on a support course should promote the active involvement of the learner. Astin (1985:36) points out that the vast and diverse literature on higher education and human learning indicates that *the key to an effective learning experience is student involvement*. According to Astin (1985:36) *student involvement refers to the amount of physical and psychological energy that the student devotes to the academic experience*. Although Astin (1985) sees involvement in a wider (macro) context than its mere relevance to a specific course, the author of this thesis is of opinion that the underlying principle of **activity** to promote academic well-being is also applicable on the micro level of a single course. In the mathematics component of the support course described in this study, the learner is in essence **personally involved** in the development of his/her mathematics potential through the facilitation of mathematical content and the development of skills as proposed in the strategy for learning facilitation defined in this thesis.

In the learning facilitation strategy for mathematics proposed in this study, learner involvement, for example on a micro level in a typical class activity, entails that the learner has to use the computer graphing software to construct a visual representation of some



mathematical concept. The learner is then actively involved in analysis and interpretation of the image on the computer screen as the computer graphing software, used in this instructional model, was purposefully designed as graphing tool and not as an instrument that generates answers. This involvement promotes, amongst others, learner control, self-paced instruction and the development of meta-cognitive skills (Steyn, 1998).

Astin (1985:38) points out that *the theory of student involvement encourages educators to focus less on what they do and more on what the student does.*

2. Feedback

Feedback should be used to guide and support accurate and mental connections (Thompson, Simonson & Hargrave in Ertmer & Newby, 1993). Astin (1985:36) points out that in addition to learner involvement *an excellent learning environment* is further characterised by *high expectations and assessment and feedback*. The author of this thesis regards **personal** feedback as of exceptional importance in the learning facilitation strategy proposed in this study. The author is of the opinion that the only way by which it can be known (either by the facilitator or the learner) whether mathematical concepts have been mastered is through (mathematics) communication (orally and in writing) and feedback thereon.

In the strategy for learning facilitation of mathematics proposed in this study, feedback is given through continuous communication with the learners. The communication comprises conversations on mathematics and writing of mathematics. Conversations between facilitator and learner as well as peer communication between learner and learner pose opportunities to the learner to formulate mathematics. The facilitator (lecturer or tutor) is compelled to listen carefully and guide the student to correct formulation and expression of mathematics concepts. A learner has the opportunity to write down what was orally formulated by completing answer sheets or doing assignments that are then assessed and graded. It should be stressed that the focus is not on quantitative feedback in terms of numerical right or wrong answers but to **guide and support a learner to achieve accurate concept mastering of fundamental mathematical concepts.**

Concerning the importance of feedback to improve learning, Angelo and Cross (1993:9) point out that *students need to receive appropriate and focused feedback early and often; they also need to learn how to assess their own learning.* Davis (1998:38) adds that careful



and prompt grading and feedback thereon *show students that they are a high priority*. Similarly, Astin (1993) strongly argues that feedback received directly from the lecturer (facilitator) correlates positively with most academic outcomes such as general knowledge, knowledge in the field, analytical and problem solving skills, writing skills and overall academic development. Chickering and Gamson (in Cross, 1998) regard giving prompt feedback as one of seven principles²⁷ of good practice in undergraduate education.

3.3.2 Concepts of teaching resulting from constructivism

Constructivism is aimed at making learning a more realistic and meaningful process in order to enable the learner to *create meaning from his or her own experiences* (Jonassen, 1991:10).

The assumptions and principles of constructivism relevant to the learning facilitation strategy proposed in this study can be summarised as follows:

1. According to the principle of **active involvement**, learners should be active participants in the learning activities. In the learning facilitation strategy proposed in this thesis, the learner actively has to explore mathematical concepts. This exploration is specifically structured to guide the learner in mastering concepts.
2. Constructivism postulates that **information** should be **presented in different ways**. With regard to tertiary mathematics education, this principle is addressed in calculus textbooks of the 1990s which propose an instructional approach incorporating graphical, numerical and algebraic methods (Finney *et al.*, 1994; Larson *et al.*, 1998). Elaborating on the multi-representation of calculus, Smith (1994) points out that in the early 1990s calculus reform was popularised by the 'Rule of Three' namely, the experience of calculus concepts in symbolic, numerical and graphical form. According to Smith (1994) Hughes Hallett later on proposed the 'Rule of Four', with the fourth form of representation being writing. Smith (1994) further points out that educators of calculus also suggest a 'Rule of Five' with oral communication being considered evenly balanced with graphical, numerical, symbolic and written presentations in mathematics education.

²⁷ The *Seven Principles of good practice in higher education* are listed in section 3.4.2 on page 98.



A variety in the way concepts are presented, in the use of instructional material and in the facilitation of learning is in accordance with the concept of whole brain utilisation in learning²⁸ and whole brain utilisation in teaching.²⁹

Furthermore, it can be noted that the principle of "communicating mathematics orally and in writing" was also formulated by the author resulting from experiences with first year mathematics students during 1991-1997 (Steyn, 1998).

These views of educators in mathematics and the findings of the author as expressed above underscore the constructivist principles of **verbalisation** of concepts and **communication** as an essential aspect of learning facilitation.

3. Another principle that is stressed by constructivism is that **learning content** should be experienced **in context**. This principle has culminated in the concept of "problem-based learning". Furthermore, it is generally accepted that problem-based learning should be accomplished within the context of real-life experiences (Ertmer & Newby, 1993; Barr & Tagg, 1995; Berson, Engelkemeyer, Olario, Potter, Terenzini & Walker-Johnson, 1998).

In the opinion of the author, the concept of problem-based learning, in real-life context, cannot merely be applied to learning content involving fundamental conceptual knowledge such as in the basic sciences, for example mathematics. Figure 3-4 on page 95 was constructed by the author to illustrate this view on "context based learning" and the facilitation thereof.

The author distinguishes between "subject content-based context" and "application-based context". **Subject content-based context** refers to aspects and relationships of fundamental concepts within a specific subject. It is inevitable that the composition of and structure of fundamental concepts in one subject differ from that in another. **Application-based context** refers to the appropriate application of subject content in real-life context. In the opinion of the author both application-based context and content-based context constitute **problem-based** learning and instruction. However, the author feels that the competence of a learner in fundamental subject knowledge (content-based context) is a prerequisite for successful problem solving and transfer of knowledge to application-based

²⁸ See Chapter 2, section 2.4.

²⁹ See section 3.5.



context (real-life problems). The author feels that this is most probably the case with mathematical knowledge.

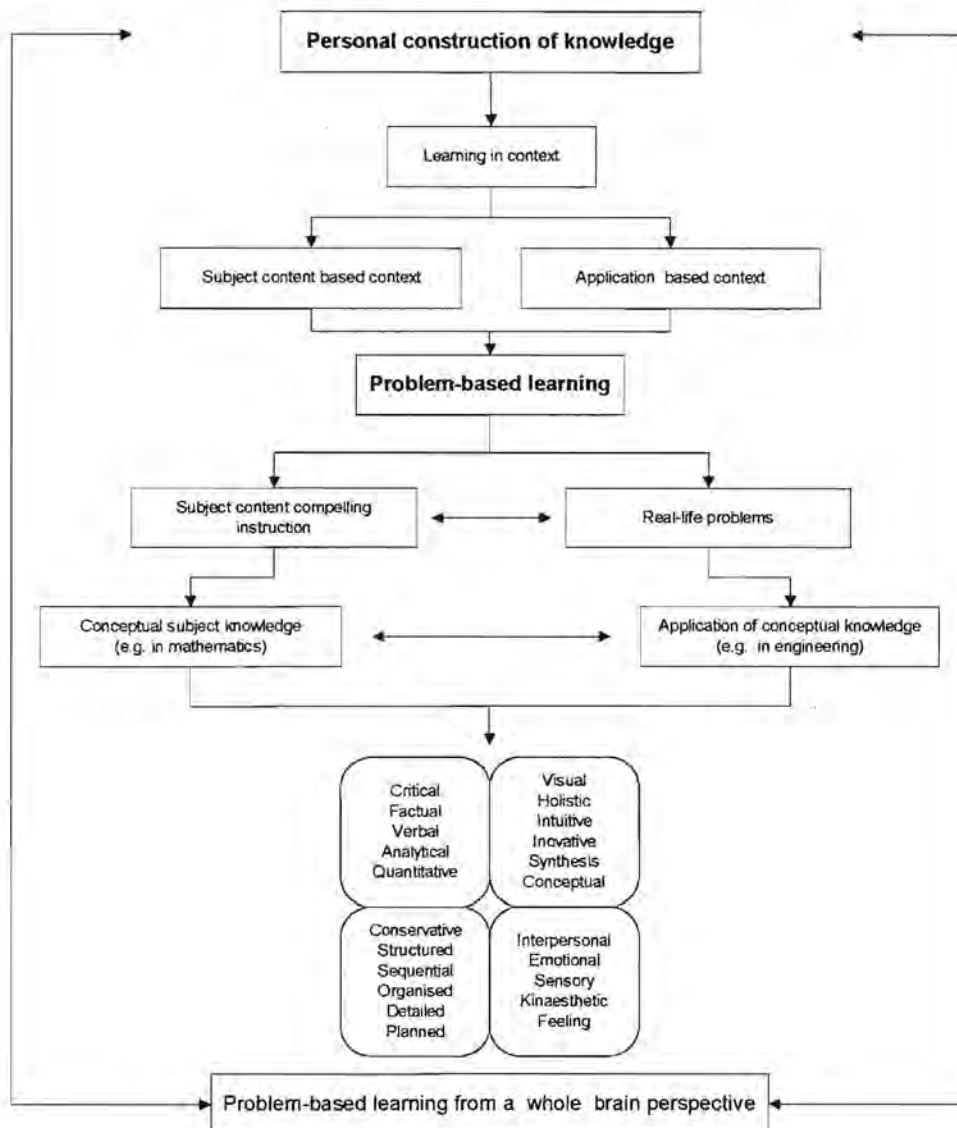
4. The **value of fundamentals** as a learning principle was articulated by the Lewis Commission at the Massachusetts Institute of Technology (MIT) in 1949 (Hansman & Silbey, 1998). In September 1998, this principle was reaffirmed in a report on student learning at MIT and elaborated on as follows:

The information revolution exacerbates the need to focus on fundamentals. Because *information will be cheap in the future, our students will need a fundamental basis to evaluate information and apply knowledge.* (Hansman & Silbey, 1998:12).

The need for conceptual knowledge is also expressed by Larson *et al.* (1998:xi) who state that *we have found no evidence that it is somehow possible to apply calculus in real-life situations without first being able to understand and "do" calculus.* Similarly, according to Bransford *et al.* (1999) point out that to develop competence in an area of inquiry, students must have a deep foundation of factual knowledge; understand facts and ideas in the context of a conceptual framework and organise knowledge in ways that facilitate retrieval and application. Bransford *et al.* (1999:50) also point out that overly contextualised knowledge (like case-based and problem-based learning) can result in learners not being able to transfer this knowledge flexibly to new situations. They continue to point out that *wide transfer of learning* and *general principles* will lead to more flexible transfer (Bransford *et al.*, 1999:51).

The author is of the opinion that learners need a thorough base of fundamental concepts in a subject in order to apply their knowledge (in real-life contexts). The mathematical content of the support course described in this study strongly focuses on fundamental concepts underpinning a study in calculus and the focus is thus on (mathematical) content based context. This, however, does not imply that application based context is ignored. For example, in the worksheet example given in Appendix B, an application-based context (a scuba dive) was used for the exploration of mathematical content-based context (two-dimensional piece-wise defined functions) after the mathematics concepts *per se* have been explored.

Figure 3-4 Conceptual knowledge in a whole brain problem-based approach



Compiled by the author of this thesis

3.3.3 Concepts of teaching derived from learning theories about adults

According to Knowles (1990) the concepts of teaching that are derived from theories of learning in studies of adults differ remarkably from those that see the role of the teacher as a shaper of behaviour. Teachers of ancient times taught adults and they perceived learning to be a process of active inquiry and not passive reception of the transmitted content. An example of this is the Socratic dialogue, whereby teaching is done by means of a series of questions and answers. The author is of the opinion that the Socratic principle is of great

importance in the facilitation of (tertiary) mathematics in a support course. Ideally, a facilitator should interpret a learner's question promptly in order to establish the learner's need for asking. The question should then be answered, not by giving an answer, but by guiding the learner to the correct solution through asking a (series of) follow-up question(s).

In the realm of adult education (Knowles:1990) the role of the teacher is in essence seen as that of a **facilitator of learning**. The author is of the opinion that the role of the tertiary educator in a support course should indeed also be that of a **facilitator of learning**.

The relevance of the concept "facilitation" for **all** education is endorsed by comparing the meaning of "facilitate" to that of "instruct". These definitions³⁰ highlight the fact that "facilitate" indicates a more **supporting approach** to aid understanding whereas "instruct" indicates a more **commanding approach** in transferring content.

The author of this thesis is of the opinion that the role of an educator as a facilitator is of great significance in the early 2000s. Tertiary education is immersed in a changing environment that is, amongst others, characterised by an overload of information and technology. The tertiary lecturer of a support course for mathematics can no longer only "transfer" mathematical content that students are supposed to have, but should aid students in verifying their understanding of fundamental concepts needed for their studies in mathematics. Therefore, the view of Rogers (in Knowles, 1990:42) dating back to the 1960s that the aim of education must be the *facilitation of learning* is still very appropriate. Rogers provides guidelines (in Knowles, 1990:78) for a facilitator of learning of which the following have significance for the purpose of this study:

- The facilitator has much to do with setting the initial mood of the class experience and in so doing create a climate for learning.
- The facilitator can help to utilise a particular individual's (student's) own motivational force for significant learning.
- As the acceptant learning environment becomes established, the facilitator increasingly also becomes a participant learner.

³⁰ The terms "facilitate" and "instruct" are defined in Chapter 2, section 1.2.2.



Barr and Tagg (1995) argue that the shift in higher education should be from teaching to (the facilitation of) learning. They point out that

- The mission of higher education is not instruction but that of producing learning.
- Student learning and success set the boundary of what institutions for higher education can do.
- Students, faculty and the institution take responsibility for learning outcomes.
- The goal for all students is not simply access to higher education, but to achieve success. Barr and Tagg (1995:15) define success as

The achievement of overall student educational objectives such as earning a degree, persisting in school, and learning the 'right' things – the skills and knowledge that will help students to achieve their goals in work and life.

3.4 Aspects of teaching resulting from experiences in higher education

3.4.1 Views of learning that reflect on teaching

In accordance with the opinions expressed in the introduction to this chapter, namely, that relevant theory and the practice of instruction in higher education should be seen in congruence, teaching aspects resulting from experiences in higher education are considered in this section.

It seems as if a trend in higher education in the late 1900s (into the early 2000s) is evolving that postulates aspects of student learning derived from practice and to use these as principles for the facilitation of learning. (Barr & Tagg, 1995; Cross, 1998; Berson *et al.*, 1998; Felder & Brent, 1999; McKeachie, 1999; Smith, 1998 & 2001). It is then presupposed that facilitators of learning in higher education should construct their instruction to promote the learning principles that they have identified.

*By applying these principles to the practice of teaching, the development of curricula, the design of learning environments, and the assessment of learning, we will achieve more powerful learning. (Berson *et al.*, 1998:15).*

It is interesting to note that most of the learning principles listed by Berson *et al.* (1998) in essence correlate with the known learning concepts postulated by the cognitive learning



theory and constructivism. However, it is noteworthy that Berson *et al.* (1998) also explicitly refer to learning principles that reflect on the contribution of neuroscience and the utilisation of brain functions to student learning. In the realm of tertiary mathematics education Smith (2001) suggests that teachers should become aware of the implication of (recent) neurobiological research for learning.

The focus on learning as a basis for instructional design is further stressed by Barr and Tagg (1995) who postulate a "learning paradigm" instead of a "teaching paradigm" as a model for undergraduate education. They argue that a learning paradigm requires a constant search for new structures and methods that work better for students' learning. They point out that

Instead of fixing the means – such as lectures and courses – the Learning Paradigm fixes the ends, the learning results, allowing the means to vary in its constant search for the most effective and efficient paths to student learning. (Barr & Tagg, 1995:21.)

The implication of the view to take learning as the premise for constructing facilitation compels the (tertiary) facilitator to take cognisance of the concepts that underpin student learning and to design his/her instruction accordingly. Obviously, in designing instruction, the special structure of the specific subject content would influence the design to some extent.

3.4.2 Examples of principles relating to 'good practice' in higher education

In 1987, Chickering and Gamson (Cross, 1998) formulated the *Seven Principles of Good Practice in Undergraduate Education* for higher education in the USA. These principles seem to have influenced higher education in the USA since their creation. Recently the implementation of these principles has also been interpreted with relevance to technology as a lever for instruction (Chickering & Ehrman, 1997). Although a detailed discussion of the seven principles is beyond the scope of this study, the author feels that they, in essence, also underpin the principles of the learning facilitation strategy proposed in this study. Chickering and Gamson's seven principles (Cross, 1998:6) are:

1. Good practice encourages contacts between students and faculty.
2. Good practice encourages cooperation among students.
3. Good practice encourages active learning.

4. Good practice gives prompt feedback.
5. Good practice emphasises time on task.
6. Good practice communicates high expectations.
7. Good practice respects diverse talents and ways of learning.

Smith (2001) points out that these principles employ six powerful forces in education, namely, activity, cooperation, diversity, expectations, interaction and responsibility. Although each of the principles can stand on their own, their effects multiply if they are all present.

3.4.3 Contributive learning

During the research activities of the author since 1991, an aspect regarding learning and teaching that differs from the known concepts of cooperative learning and collaborative learning was identified. In cooperative and collaborative learning the focus is on learning via the interaction between learners (Bruffee, 1995; Matthews, Cooper, Davidson & Hawkes, 1995). When the learner as well as the facilitator learns during the facilitation of learning, the author feels that this learning activity can best be defined by the phrase **contributive learning**.

Thompson (1995:291) defines the term "contributive" as *help to bring about a result* while Gove (1961:496) defines it as to *add (as knowledge or effort) to a common interest or activity*. Both these definitions underpin the author of this thesis' view of the term "contributive learning" which is as follows:

Contributive learning indicates the involvement of both the **learner as a learner** and the **facilitator as a learner** in a mutual process of learning. For the facilitator this learning is not confined to subject content. It can be diverse and can, for example, include aspects of student learning as well as successes and pitfalls of instructional activities.

During the mentioned research (Steyn, 1998), contributive learning occurred as a result of the restructured approach to the tuition of calculus that was followed incorporating use of computer graphing technology as a tool for learning. In this regard the author and colleagues had, amongst others, experienced how the use of technology can enhance

mathematical concepts and what the requirements for a computer graphing tool need to be to promote mathematics conceptualisation (Steyn, 1998; Steyn & Maree, 2002).

Furthermore, contributive learning can give a facilitator new insights into learners' construction and comprehension of mathematical concepts.

3.5 A whole brain approach to facilitation: an instructional design perspective

It was pointed out in Chapter 2 that the principle of whole brain utilisation may be seen as new paradigm for elucidating learning in a support course for first year tertiary students. The implications for a facilitator is to take cognisance of the functioning of the brain by noting that, on the one hand, different cognitive functions are predominantly located in different parts of the brain but, on the other, that the brain functions as a composite whole. Ideally, the facilitation of the learning must be structured so that whole brain utilisation results.

A whole brain approach has been advocated extensively in the fields of learning, creative thinking and creative problem solving (Buzan, 1991; Buzan & Buzan 1997; Dennison & Dennison, 1989 & 1997; Hannaford, 1995 & 1997; Herrmann, 1996; Jensen, 1994 & 1996; Lumsdaine & Lumsdaine, 1995b; Swartz, 1991).

Documentation on a whole brain approach to the facilitation of learning in tertiary mathematics education is scarce although an awareness for the approach is seemingly developing (Steyn, 1998; Smith, 2001).

The author of this thesis is of the opinion that a whole brain approach to teaching does not exclude conventional instructional design principles or any concepts of instruction that result from learning concepts or experiences in teaching as discussed in the preceding sections. Furthermore, the author feels that a whole brain approach to teaching utilises and endorses these principles. A facilitator of learning should thus ideally **incorporate all relevant concepts of learning and principles of instructional design within the framework of a whole brain approach.**

The following discussion gives an instructional design perspective on a whole brain approach to learning facilitation. In the opinion of the author two modes can be distinguished in this approach to learning facilitation, namely, an "accommodational mode" and a "functional mode".

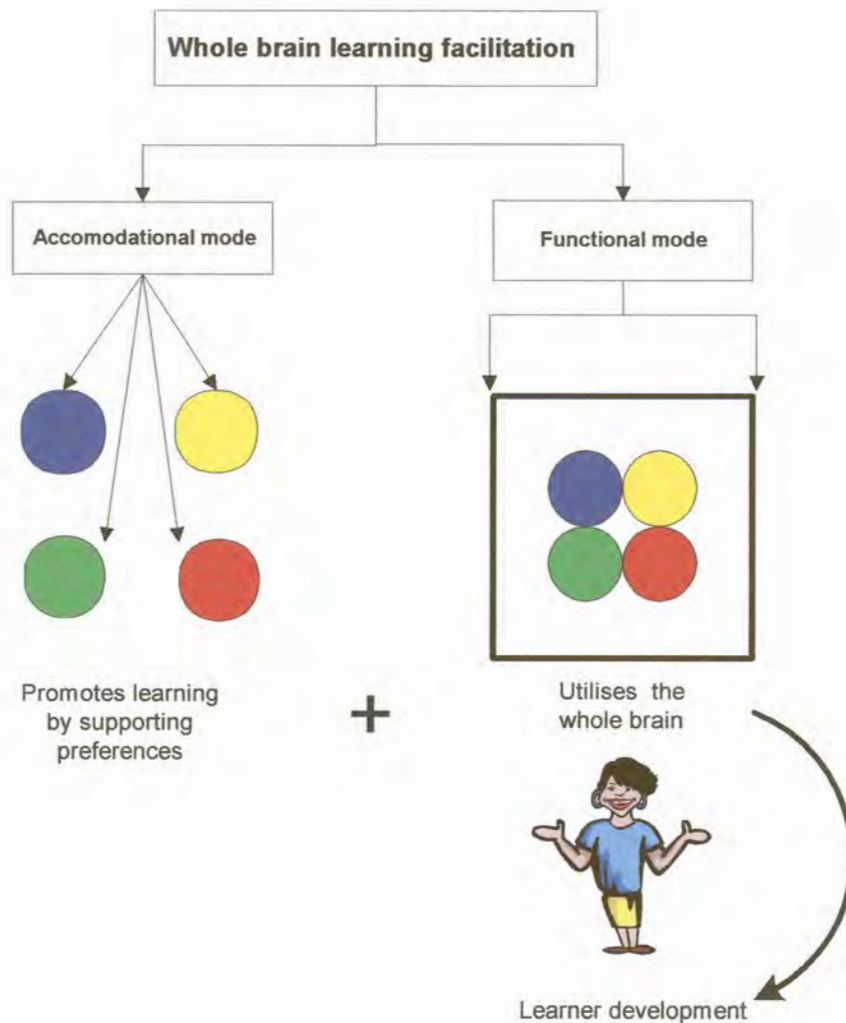
Gove (1961:12) defines the term "accommodation" as to *adapt; to bring into agreement or concord* or to *adjust*. Woolf (1975:7) defines it as *to give consideration to* or *to allow for the special interest of various groups*. Thompson's (1995:9) definition of "accommodation" is similar to that of Woolf, namely, *an adjustment or adaptation to suit a special or different purpose*. The term "functional" is defined by Gove (1961:921) as *dependently related* and as *existing or used to contribute to the development or maintenance of a larger whole*. These definitions underpin the author of this thesis' view of the terms as they are held in this study and discussed in the following paragraphs.

Figure 3-5 on page 102 illustrates the relevance of the concepts "accommodational mode" and "functional mode" for a whole brain approach to learning facilitation.

In the **accomodational mode** the focus is on designing instructional activities that promote learning by supporting the preferred thinking (learning) styles³¹ of individual learners. In the **functional mode**, the focus is on designing instructional activities in such a way that not only preferred thinking and learning styles are used but less preferred thinking and learning modes are also utilised.

³¹ Thinking (learning) style preferences are viewed in this model according to the four quadrant whole brain model of Herrmann that is discussed in Chapter 2, Section 2.3.4.

Figure 3-5 Promoting and utilising the whole brain in learning facilitation



Proposed by the author of this thesis

If the **accomodational mode** is interpreted with relevance to the Herrmann model³² it means that for learners with an A-quadrant preference, instruction should be formally and verbally structured, should include factual data and stimulate critical and analytical thinking. For learners with a B-quadrant preference, instruction should be organised sequentially, should be well planned and strictly adhere to the plan. Instruction should also be accompanied by detailed instructional material. Instruction for A- and B-quadrant learners should further be planned so that they can work individually. On the other hand, instruction for learners with a C-quadrant preference should be planned around group

³² Aspects of the Herrmann model are discussed in Chapter 2 sections 2.3.4 and 2.5.1.

interaction. Learners with a C-quadrant thinking preference value experiential opportunities that incorporate sensory stimuli. For learners with a D-quadrant preference, instruction should focus on the visual representation of data and to present a global picture of the instructional content. Furthermore, for learners with a D-quadrant preference, instruction should not be rigidly structured but should encourage intuition, self-discovery and the forming of concepts.

In the **functional mode** of the whole brain approach to teaching, the focus is not on promoting the different thinking (learning) modes but is on ensuring that **all learners** with presumably different preferences are compelled to utilise their less preferred or un-used styles of thinking and learning. This means that, for example, a learner with an A-quadrant preference will also be encouraged to respond to facilitation strategies aimed at utilising potential competencies (such as working in a group) that are predominantly classified in the other quadrants and by so doing he/she will be encouraged to utilise his/her whole brain (according to the Herrmann model).

The author feels that both the accommodational and the functional modes are relevant for learning facilitation of mathematics in a support course. Furthermore, the author feels that the **functional mode of whole brain utilisation** could also be regarded as an **instructional principle for the facilitation of fundamental mathematical concepts**. This premise underpins the instructional strategy defined in section 3.9 of the chapter.

In Table 3-3 on the following page a summary of the learning and learning facilitation considerations discussed in the foregoing sections are applied to the present study.



Table 3-3 Learning and learning facilitation considerations in the present study

Principles of learning and learning facilitation	Principles as applied to the present study
Learner involvement	Active participation of the learner in information processing and knowledge construction.
Feedback	Continuous and appropriate feedback by the facilitator to guide the learner to achieve accurate mastering of fundamental concepts underpinning a study in calculus.
Developmental	A main focus is to develop the mathematics potential of the learners.
Multiple representation	Mathematical concepts are addressed graphically, symbolically, in writing and by way of oral communication.
Value of fundamentals	Exploration activities are aimed at a thorough understanding of fundamental concepts underpinning a study in calculus.
Interaction with facilitator	Communication, assessment and feedback are highly valued.
Interaction with peers	Spontaneous discussions amongst learners in informal groups of two or three are encouraged by the exploration activities. Establishing a learning community.
Contributive learning	Learners and facilitators contribute to each others knowledge including aspects related to mathematical content, learning, teaching, cognitive developmental aspects and academic related social communication.
Self-paced	When working through the exercises on the structured worksheets, learners set their own pace exploring the content.
Time on task	Mathematics activities are scheduled so that learners can master the concepts covered in a worksheet within 60 to 90 minutes.
Study skills	Study skills for mathematics study are addressed as an aspect of the developmental approach and students strongly encouraged to apply them
Accommodate preferences	From a whole brain perspective, exploration activities include aspects of all the cognitive modes according to Herrmann's four quadrant whole brain model. Students' preferred modes as well a non-preferred modes are called upon.
Develop competencies	
Personal construction of concepts	Exploration activities and mathematical communication (orally and in writing) contributes to learners' personal construction of concepts.

Compiled by the author of this thesis



3.6 The relevance of thinking styles, learning styles and teaching styles for a learning facilitation strategy in mathematics

According to Sternberg (1994:109) thinking styles refer to *the processes used to solve a problem or to devise an answer* and learning styles refer to *the processes of obtaining knowledge and skills by means of studying, instruction or experience*. Cilliers and Sternberg (2001:14) point out that teaching styles refer to *preferred ways or methods of teaching, and are closely related to preferred styles of thinking and learning*.

It should be pointed out that thinking styles, learning styles and teaching styles do not refer to abilities but to preferences. Furthermore, people's styles of learning and behaviour differ from individual to individual and there is not any one learning approach, style or orientation that is, in itself, the best or most desirable (Zuber-Skerritt, 1992a).

It can thus readily be assumed that both learners and facilitators of learning have different preferred modes of thinking and doing. An instructor's preferences have, most probably often unknowingly, a strong impact on the way he/she designs and delivers instruction. Felder (1993:288) points out that *teachers tend to favor their own learning styles, in part because they instinctively teach the way they were taught*. Furthermore, educators (of mathematics) are not necessarily aware of the diversity in the thinking style preferences and ensuing learning style preferences of their students nor of their own style that dictates their instructional approach.

Within the realm of tertiary mathematics education, there is seemingly a growing awareness that differences in students' learning styles should be taken into consideration when designing learning experiences and instruction. The Delta '99 Symposium on Undergraduate Mathematics Education in November 1999, had the principle of "diversity" as core theme of the conference, presumably proposing an awareness of the importance of "getting to know" the learners of tertiary mathematics. Although papers specifically reflecting on learning and teaching styles were scarce (Spunde, Cretchley & Hubbard, 1999), contributions regarding learning and teaching styles that were presented at the conference (Smith, 1999; Steyn, De Boer & Maree, 1999) were well received.



Becker and Pence (1994:7) remark that

Learning styles is a major area in need of further research. In particular, we need to identify how much difference it makes if teaching methods are incongruent with a student's preferred style.

The author of this thesis acknowledges Becker and Pence's view but also feels strongly that the focus should not merely be on accommodating different thinking and learning styles but also on developing all cognitive skills available that are needed and appropriate for successful study (of mathematics). This necessitates an awareness of students' thinking and learning preferences as well as of the preferences that facilitators have. Determining students' thinking preferences and making them aware thereof, is one of the aspects addressed in the research questions stated in Chapter 1.³³

Concerning facilitators of mathematics, awareness of diverse thinking and learning styles of both students and facilitators could become a point of departure for re-thinking the way in which tertiary mathematics are learned and taught. Congruency between learning styles and instructional styles will certainly be beneficial to learners but students also need to realise that incongruence between their own style and that of an instructor can and will occur during their studies. Students will always encounter teaching environments that do not match their preferred style. Once realised that this can happen, students need to develop the skills to cope with such situations.

Felder (1988, 1993) points out that the idea is not to determine each student's learning style and then teach to it exclusively but to simply address each side of the dimension of a learning style.³⁴ Felder's view regarding learning styles endorses the accommodational as well as functional modes regarding a brain-based instructional approach proposed in section 3.5 on page 100. If learning and teaching styles are viewed in terms of Herrmann's whole brain model,³⁵ the ideal is that both students and facilitators (of mathematics) should become aware of the principles of whole brain utilisation. The author of this thesis is of the opinion that if the principle of whole brain utilisation is acknowledged, understood, accepted and implemented, the learning and learning facilitation of mathematics could be beneficial to the learners and rewarding to the facilitator.

³³ The research questions are stated in Chapter 1 section 1.4 on page 8.

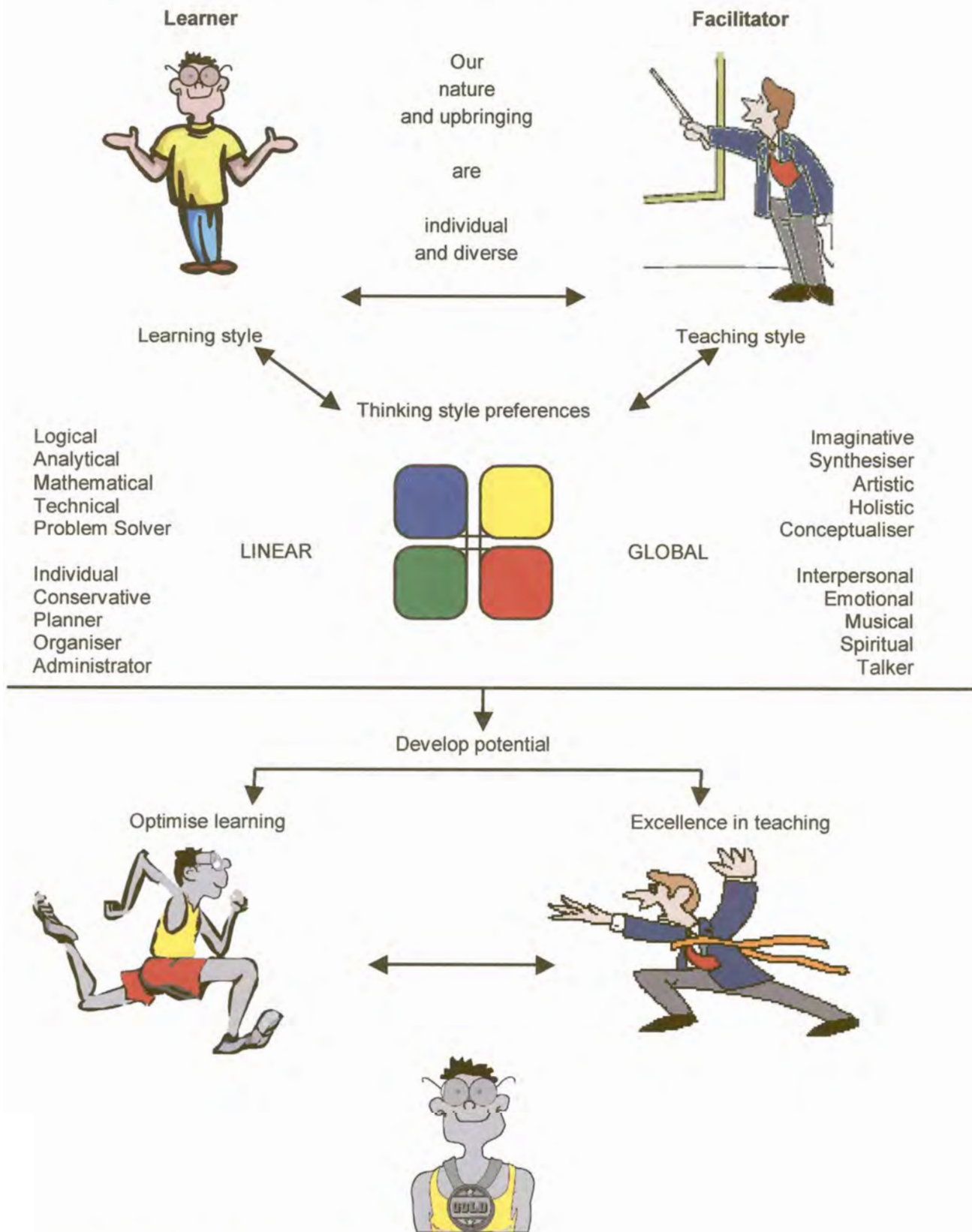
³⁴ The dimensions of Felder's learning style model are given in Chapter 2, Table 2-6.

³⁵ Herrmann's four quadrant whole brain model is presented in Chapter 2, section 2.3.4.



Figure 3-6 summarises the discourse regarding the matching of learning and teaching viewed according to the principles of the Herrmann whole brain model. What students learn seems to be less than what educators teach and what they hope students learn. Learners and facilitators are individual and diverse due to their personal nature, nurture and educational history. Learning styles and teaching styles can match or mismatch. Facilitators of mathematics need to realise that they cannot change their students' thinking and learning preferences. However, if they could align their teaching practice to accommodate and develop their students' styles of thinking when learning and doing mathematics, this could result in the development of the (mathematics) potential of the student. Ultimately, in this scenario, learning could be optimised and teaching could excel.

Figure 3-6 Matching learning and facilitation – a matter of styles



Adapted from Steyn, De Boer and Maree (1999)

3.7 The contribution of results from research during 1993-1999 to the learning facilitation of mathematics in a support course

In this section the main findings of research during 1993-1999 that formed the background to the study reported in this thesis are discussed.³⁶ The aim of the 1993-1998 research activities was to examine the use of computer graphing technology in an introductory calculus course for students on a support programme in the Faculty of Natural Sciences (Steyn, 1998). This research was done by the author of this thesis in collaboration with lecturers and tutors of the Gold Fields Computer Centre for Education and the Department of Mathematics at the University of Pretoria. During 1998 and 1999 studies were done that added further dimensions to the facilitation of mathematics introductory calculus level. The 1998 study (Steyn & De Boer, 1998) showed that mind mapping could successfully be used as a study tool for introductory calculus. The 1999 study indicated that learners of mathematics in a first course in calculus represent diverse thinking styles (De Boer & Steyn, 1999).

In the following sections the main results of the 1993-1999 studies, that are relevant for the present study, are briefly discussed.

3.7.1 Results regarding a computer graphing tool

In the 1993-1998 studies computer graphing software and pen and paper were regarded as "graphing technology" for teaching and learning mathematics. Technical, didactical and pedagogical features that a computer graphing utility should have for use in mathematics instruction were identified and were based on observing the influence that the use of technology can have on a learner as well as the effect that computer aided instruction can have on a learner's mathematical conceptualisation.

For the purpose of the 1993-1998 studies the use of graphing utilities in mathematics education were categorised as "exploratory" and "illustrative" (Steyn, 1998). In the **exploratory use** of graphing technology, technology is regarded as an aid (tool) to support a learner in the thorough investigation and examination of mathematical concepts that are represented by a graphical (visual) image. In this sense exploring a graph entails much more than only looking at the visual image. The **illustrative use** of graphing technology is

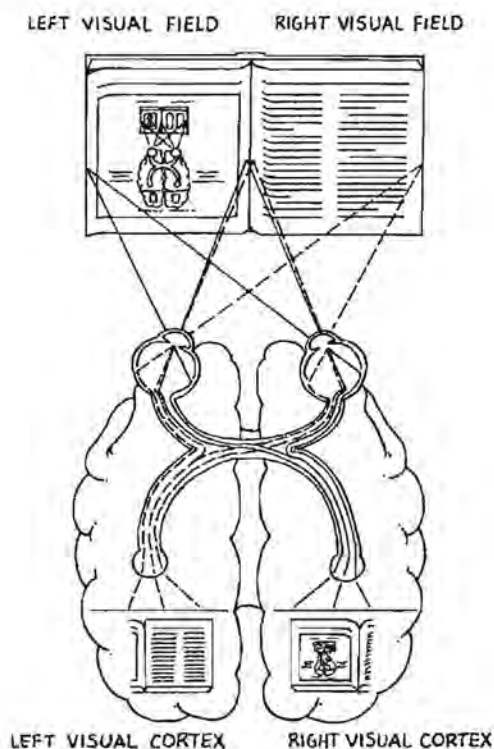
³⁶ Figure 1-3 on page 13 of Chapter 1 gives an overview of the action research cycles of the 1993-2001 studies.

defined as merely 'have-a-look-at-it'. On the one hand this implies showing a learner the visual image in order to get an overall idea of the graph. On the other, a 'have-a-look-at-it' use presupposes proficiency in the use of graphing technology as well as the necessary knowledge to interpret a graphical image. The latter scenario implies competence regarding the interpretation of graphical images that students, taking a first course in calculus, do not necessarily have.

1. Technical features of graphing utilities

The first technical aspect concerns the **user interface** of a graphing utility. The presentation of text and images on the screen should be conducive to learning in the sense that the utilisation of left and right brain functions are promoted and optimised if text is in the right visual field and visual images (graphics) are in the left visual field (Herrmann, 1995). The diagram in Figure 3-7 illustrates correct placement of text and graphics for optimising left and right brain functions.

Figure 3-7 The left and right visual fields



According to Herrmann, text should be in the right visual field and visual images (graphics, illustrations, etc.) in the left visual field to optimise brain functions.

Adapted from Herrmann (1995:14)

The layout of the user interface of the graphing utility in Figure 3-8 A is structured according to these principles but this is not generally a feature of educational graphing software.

As the human brain has an inherent fast response to colour, shape and contrast (Jensen, 1996), the use of colour in the user interface of a graphing utility should also be presented in such a way so as to aid and not to impair learning. Furthermore, care should be taken when using colour as a code for classification. This may affect interpretation of images by users who are colour blind. A further aspect pertaining to the user interface and the exploratory use of graphing technology concerns the size of the visual image. Activities during the 1993-1998 research studies contributed to the facilitators' opinions that the size of a computer screen is more conducive to true exploration than that of a graphing calculator (Steyn, 1998; Steyn & Maree, 2002).

Figure 3-8 The function $f(x) = \frac{5.1x^2 + 7.79x}{4.2x^2 - 17.64}$ displayed by two graphing utilities

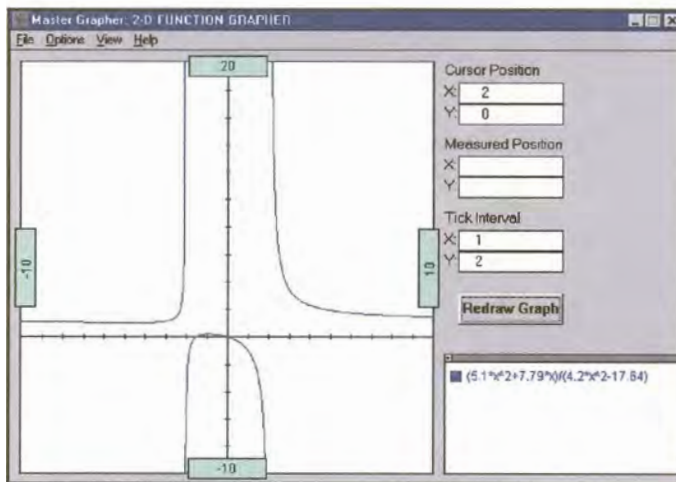


Figure 3-8 A

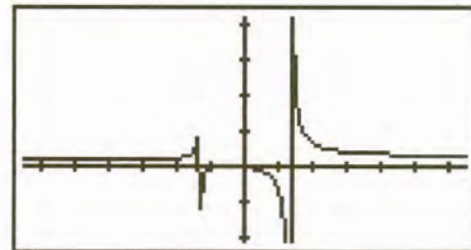


Figure 3-8 B

Compiled from Steyn (1998:114 & 123)

A second aspect required for the exploratory use of graphing technology concerns **technical capabilities**. An educational graphing tool must promote opportunities for authentic explorations and convincing observations. For example, entering functions must

be easy. It ought to be possible to display more than one function simultaneously and functions must be distinguishable from each other. Real exploratory activities, such as physically following a curve with a pointing device (mouse), can enhance one's intuitive feeling for a graphical image. Changing the dimensions (minimum and maximum values on the X and Y -axes) of the graph window (viewing rectangle), in which the graph is displayed, must be easy and toggling between consecutive graph windows should be possible. As it is not always possible to represent in only one window all the mathematical characteristics of a function one would like to visualise, an easily accessible zoom feature ought to be available. Very useful features, for the meaningful exploration of related changes in x and y -values, are moving vertical and horizontal lines as well as the ability to add fixed vertical and horizontal lines to the image displayed by the graphing utility.

2. Didactical and pedagogical features of graphing utilities

Experiences gained during the 1993-1998 research have indicated that when technology is incorporated into a mathematics tuition programme, the focus should not be on the technology. This means that the skill of using a graphing utility should be easily acquired and retained by the learners for whom the instruction is intended. Technology should add value to students' learning experiences and should facilitate rather than dictate learning.

In using graphing technology as a tool to explore mathematics, learners need to be able to get an intuitive feeling of the graph through the activity (for example, following the curve with the pointing device to get a feeling of whether a function is increasing or decreasing). This highlights the fact that the screen size of a computer and the relative ease in manipulating a pointing device (mouse) in comparison with those of graphing calculators are beneficial to authentic experiential activities. In this regard, Fuchsteiner (1997:14) points out that *intuition and concepts constitute the elements of all our knowledge ... no reform in the education of mathematics can be successful which does not focus on how we can strengthen intuition.*

A further aspect that was distinctly noticed when a graphing approach is used for the teaching and learning of fundamental concepts related to two-dimensional functions, is that the images, displayed by the graphing utility, should be accurate representations of the functions. Figure 3-8 A illustrates the graph of a rational function drawn by the computer graphing tool developed during the 1997 research (Carr & Steyn, 1998) and Figure 3-8 B illustrates the same function drawn by a graphing calculator. Images like the one in Figure

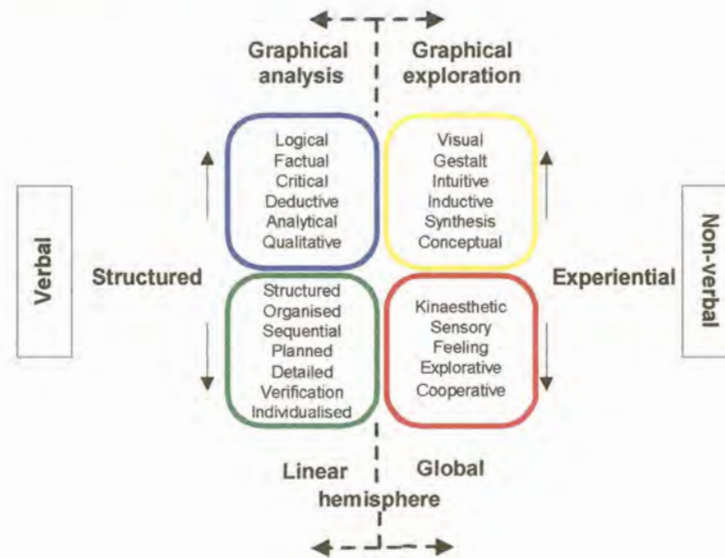


3-8 B proved to be very confusing. For example, when students had to interpret the image on the screen (as in Figure 3-8 B) and draw a freehand graph, it invariably occurred that they drew an exact copy of the image on the screen. When then asked to explain the graph, students reported that the graph had 'sharp' turning points for some x -value(s) on the interval $(-2, -1)$ and that the graph 'stopped' below and above the X -axis for x -value(s) on the interval $(1, 2)$. In an example such as this one concerning a rational function (Figure 3-8), the image as in Figure 3-8 B cannot be used to enhance concepts, e.g. in this case, the concept of vertical asymptotes or the range of the function. This ambiguity forfeits the purpose of a visual image. Experiences with students in this research have undoubtedly shown that ambiguous graphical images give rise to mathematical misconceptions.

3.7.2 Results regarding the enhancement of mathematical concepts

During the 1993-1998 research studies, the significance that the exploration of two-dimensional functions through their graphs has for enhancing conceptualisation of fundamental mathematical concepts that underpin a study in calculus was examined according to the convention illustrated in Figure 3-9, namely that graphical exploration is experiential and non-verbal and is mainly focused on the utilisation of functions of the "global" hemisphere of the human brain and graphical analysis is verbal and structured and is mainly focused on the utilisation of functions of the "linear" hemisphere of the brain. This view of learning facilitation and the distinction between the global hemisphere and linear hemisphere of the human brain are treated in Chapter 2 section 2.3.

Figure 3-9 Manifestation of mathematical concepts through graphical analysis and exploration: a whole brain perspective



Adapted from Steyn (1998:136)

The predominant categories that emerged in the research activities during 1993-1998 in determining how the exploration of two-dimensional functions through their graphs can enhance mathematical conceptualisation are the **visualisation** of functions; viewing functions as **gestalt (wholeness)** entities; a combination of **inductive** and **deductive** approaches to graphical exploration of functions and **verbalisation** in mathematics.

A major contribution that a graphing utility can make in revealing mathematical concepts lies in the visualisation of functions through their graphs. The importance of visualisation in mathematics education is extensively discussed by various authors in Zimmerman and Cunningham (1991). Visualisation interpreted from a whole brain perspective (as in Figure 3-9) also implies utilisation of a predominantly global hemisphere functions. Graphical representations further enhance conceptualisation through wholeness. This means that a mere glance at the graphical representation of a function (as in Figure 3-8 A) conveys more information regarding the features of a function than is the case when an equation alone, for example $f(x) = \frac{5.1x^2 + 7.79x}{4.2x^2 - 17.64}$ representing the function in Figure 3-8 A, is considered.



When students learn mathematical concepts by exploring examples graphically and intuitively (mainly global hemisphere utilisation³⁷), and discovering key aspects themselves, this approach can be described as inductive. The study revealed that the structured exploration of a variety of examples on the same topic leads to logical analytical deductions (mainly linear hemisphere utilisation), that should ideally lead to synthesis and conceptualisation (mainly global hemisphere utilisation). A deductive approach to graphical exploration occurs when students already know an analytical theorem or rule (mainly linear hemisphere utilisation) and then study a graph (mainly global hemisphere utilisation) illustrating the theorem (rule). The ensuing analysis of graphical images compels a learner to formulate his/her conceptualisation of the mathematics involved.

Experiences with students during the 1993-1998 studies have also shown that verbalisation (orally and in writing) with regard to the mathematics content, eventually helps learners to become sensitive to the degree of correctness or incorrectness of their own mathematical formulation and conceptualisation.

During 1996-1998 a questionnaire was administered at course end. In their responses, the students, amongst other things, indicated that the visual image contributed to their understanding of two-dimensional functions and that the concept of a graph was more meaningful after the exploration activities in the course than before; that the practical course sessions helped them to improve their ability to formulate mathematical concepts and express themselves in the language of mathematics and that their skill in writing mathematics improved. The data in Table 3-4 on the following page give the responses of students to end of course questionnaires 1996-1998. Using the binomial test and comparing the proportion of 'yes' and 'no' responses to the assumption that 50% would respond 'yes', the results were that for all the years the 'yes' response to all the questions was significantly high ($p < 0.5$ throughout).

³⁷ See Chapter 2, Table 2-5 on page 56.

Table 3-4 Responses of students to end of course questionnaire 1996-1998

		Number of student responses			
		1996	1997	1998	Total
Do you think that using visual images made the mathematical concepts more clear?	Yes	58	114	113	285
	No	7	10	5	22
Is the concept of a graph now more meaningful to you than before you used the computer graphing tool?	Yes	65	121	113	299
	No	2	3	4	9
Do you think that the practical graphing sessions helped you to improve your ability to formulate mathematical concepts and express yourself in the language of mathematics?	Yes	66	115	108	289
	No	1	9	10	19
Do you think that completing the worksheets improved your skill in writing down the mathematics correctly?	Yes	65	114	106	285
	No	2	10	12	24

Compiled from Steyn and Maree (2002)

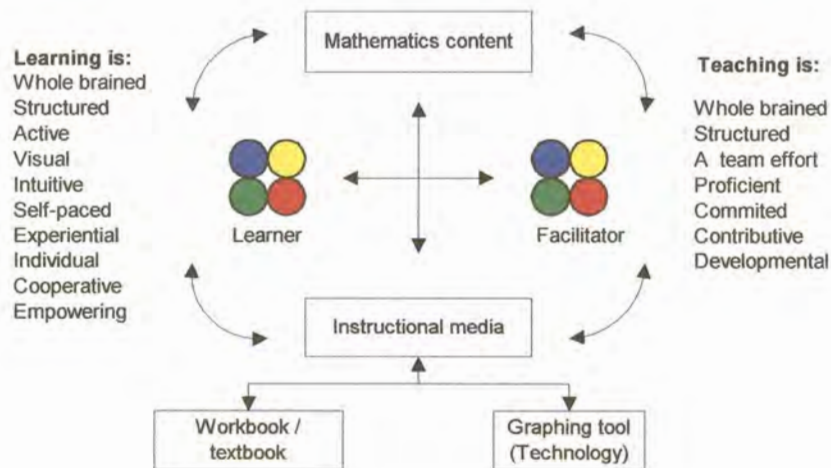
3.7.3 Results regarding an instructional strategy incorporating computer graphing technology

Experiences during the 1993-1998 studies indicated that the meaningful combination of graphical exploration and graphical analysis could be regarded as a prerequisite for the manifestation of mathematical concepts in a teaching and learning approach aimed at whole brain utilisation.

The instructional model (Steyn, 1998) in Figure 3-10 presupposes an interdependence between teaching, learning, instructional media and instructional content when technology is used in (tertiary) mathematics education. This model comprises four main components, namely, the mathematical content; the learner; the facilitation of learning and the instructional media. The fundamental premise in this model is that the components are interdependently linked. The arrows in Figure 3-10 illustrate this interconnectedness.

The mathematical content of the model in Figure 3-10 deals with aspects associated with a first course in calculus. These include principles from precalculus that underpin a study in calculus, concepts related to the Cartesian coordinate system, two-dimensional functions, the concepts of limit, continuity and differentiation and the application of the first and second derivative. In this model, the concept of whole brain utilisation is used as a paradigm for elucidating the teaching and learning of introductory calculus in a support course. The facilitation of learning in this model is also based on research and experiences related to learning discussed in Chapter 2 and the preceding sections of Chapter 3.

Figure 3-10 An instructional model for graphing technology aided mathematics tuition



Steyn (1998:128)

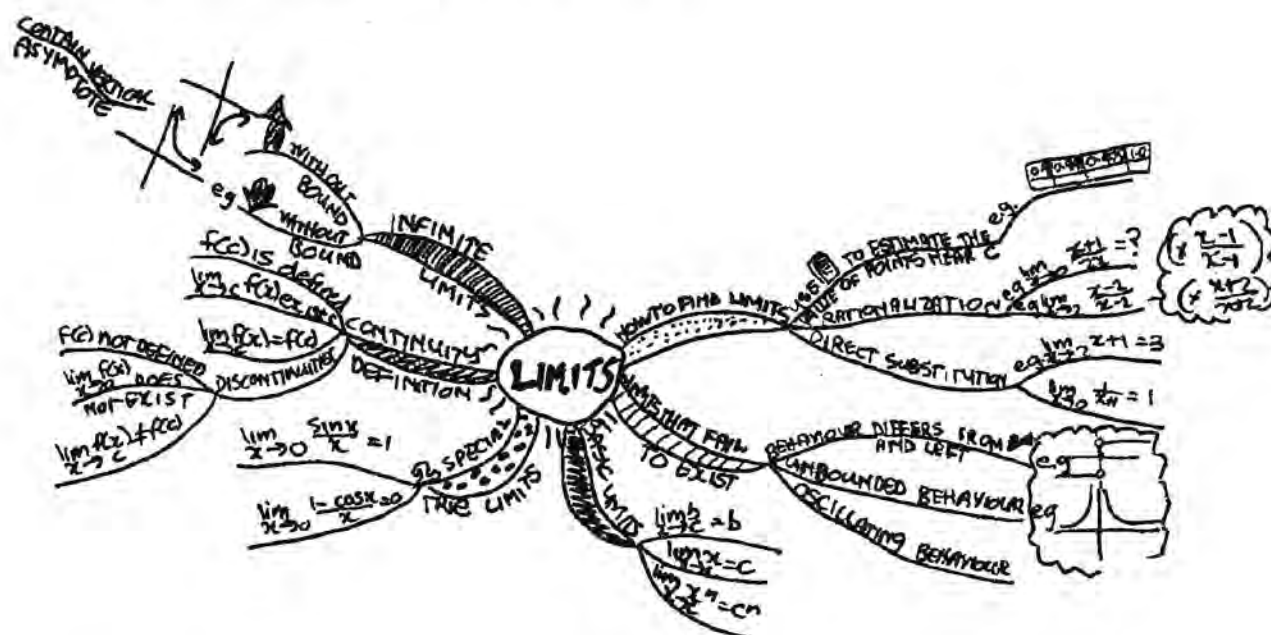
3.7.4 Results regarding mind mapping as a study strategy

During 1998 a group of students were exposed to mind mapping³⁸ as a study tool in order to foster a deeper approach to learning and to monitor the possible effect thereof on the academic disposition of the students (Steyn & De Boer, 1999). As all the students were enrolled for the calculus course in a support programme in the School of Natural Sciences, one of the mind maps they had to construct and submit for assessment had to be on their

³⁸ The term "mind mapping" is used to describe the principle as postulated by Buzan (Buzan, 1991; Buzan & Buzan, 1997).

mathematics preparation for an upcoming mathematics test. The mind map on limits in Figure 3-11 was done by one of the students.

Figure 3-11 Example of a mind map on limits



Steyn and De Boer (1998:128)

The principle findings of the study were that the students on the BSc Extended Programme lacked a workable study strategy for learning mathematics and science; that all of the students commented that previous presentations of mind mapping as a study tool lacked guidance regarding the format and structure and they had not been given the opportunity to implement the technique or to have it assessed; that the compulsory use of the mind mapping strategy resulted, in most cases, in an improvement in grades and that students experienced an empowerment of their academic disposition (Steyn & De Boer, 1998).

Melton, Reed and Kasturiarachi (2001:27) also report that results from an experiment with mind maps in a college algebra class *support the hypothesis that the use of mind maps by these students can improve their mathematical skills.*

3.7.5 Results regarding students' thinking style preferences

During 1999 the Herrmann Brain Dominance Instrument (HBDI)³⁹ was used to determine the thinking style preferences of two groups of students, both taking a first course in calculus (Steyn & De Boer 1999; Horak, Steyn & De Boer, 2000). The aims were to give the students insight into their own thinking style preferences; to determine the distribution of the students' preferred modes of thinking and to determine the homogeneity and/or diversity of the groups' preferences.

Both studies indicated that the two groups of students (one being science students on an extended programme and the other first year engineering students on a regular programme) had distinct preferences for thinking modes associated with the linear hemisphere and lesser preference (even lack of preferences) for thinking modes associated with the global hemisphere. The studies confirmed the existence of preferred and non-preferred thinking modes across the four quadrants of the Herrmann model. Aspects of the results of these studies are discussed in Chapter 6 of this thesis.

The results of the mentioned studies during 1993-1999 paved the way for the construction of the strategy for learning mathematics and the learning facilitation strategy that are proposed in this thesis.

In the next section a strategy is proposed for mathematics learning. This strategy is then applied to propose a learning facilitation strategy for mathematics in a support course for first year engineering students as it was implemented by way of action research activities during 2000 and 2001.

³⁹ The HBDI is discussed in Chapter 4, section 4.4.2.3 and in Chapter 6, section 6.6.3.

3.8 A strategy for learning mathematics in a support course

3.8.1 Components of the strategy

The strategy proposed in Figure 3-12 on page 123 comprises the following components, namely:

- The learner.
- The sense modes for learning mathematics.
- The information environment.
- The structure for processing mathematics information.

In the following paragraphs each of the components is discussed.

3.8.1.1 The learner

In the strategy for learning mathematics in a support course the learner is regarded as the active participant in learning and as someone who personally has to take action regarding his/her mathematics learning. The central focus is on the learner, his/her learning experience and his/her development as a learner. The learner is viewed as having the potential to study mathematics (in a first course in calculus). It is acknowledged that the learner comes into the support course with an existing study orientation in mathematics that results from the learner's personal background and experiences in mathematics. It is also assumed that the learner has established thinking and learning style preferences based on genetics as well as on educational experiences gained through formal schooling and also informal learning.

3.8.1.2 The sense modes for learning mathematics

From the five human senses (visual, auditory, kinaesthetic, olfactory and gustatory) the sense modes relevant to mathematics learning at introductory calculus level are visual, auditive and kinaesthetic. It should be pointed out that for the purposes of this study, the sense modes are regarded as modes that are available to all learners to be put to best use in learning and learning facilitation. However, an analysis pertaining to learners' individual

preferences or competencies in using the different modes was not done in the study reported in this thesis.

3.8.1.3 The information environment

In the strategy for learning mathematics in a support course the information environment for learning is regarded as the modes by which information is obtained from the environment. In this strategy the four modes for mathematics learning are described according to Lumsdaine and Lumsdaine's⁴⁰ view of how a tertiary student learns namely, external learning, internal learning, interactive learning and procedural learning. The attributes of these modes that constitute the information environment for mathematics learning are summarised in Table 3-5.

Table 3-5 Attributes of the information environment for mathematics learning

External learning	Procedural learning	Interactive learning	Internal learning
Technology Workbook; textbook Information by the facilitator	Procedures used for graphical exploration Exercises Assessment	Experiential activities Feedback Discussion	Visualisation Insight Intuition Concepts

Compiled by the author of this thesis

3.8.1.4 The structure for processing mathematics information

In the strategy for learning mathematics in a support course, information processing by the learner is regarded as being done by utilising cognitive modes according to the Herrmann whole brain model. This structure for processing mathematics information is inherently part of the learner and forms a focus in the strategy. Herrmann's four-quadrant whole brain model⁴¹ was adapted for the strategy defined in Figure 3-12 on page 123. These adaptations are listed in the different quadrants of the component in the centre of the strategy illustrated in Figure 3-12.

In the strategy defined in Figure 3-12 the left/right hemispheric division of the brain concerning the processing modes that are associated with each hemisphere is endorsed.

⁴⁰ See Chapter 2, section 2.5.2.

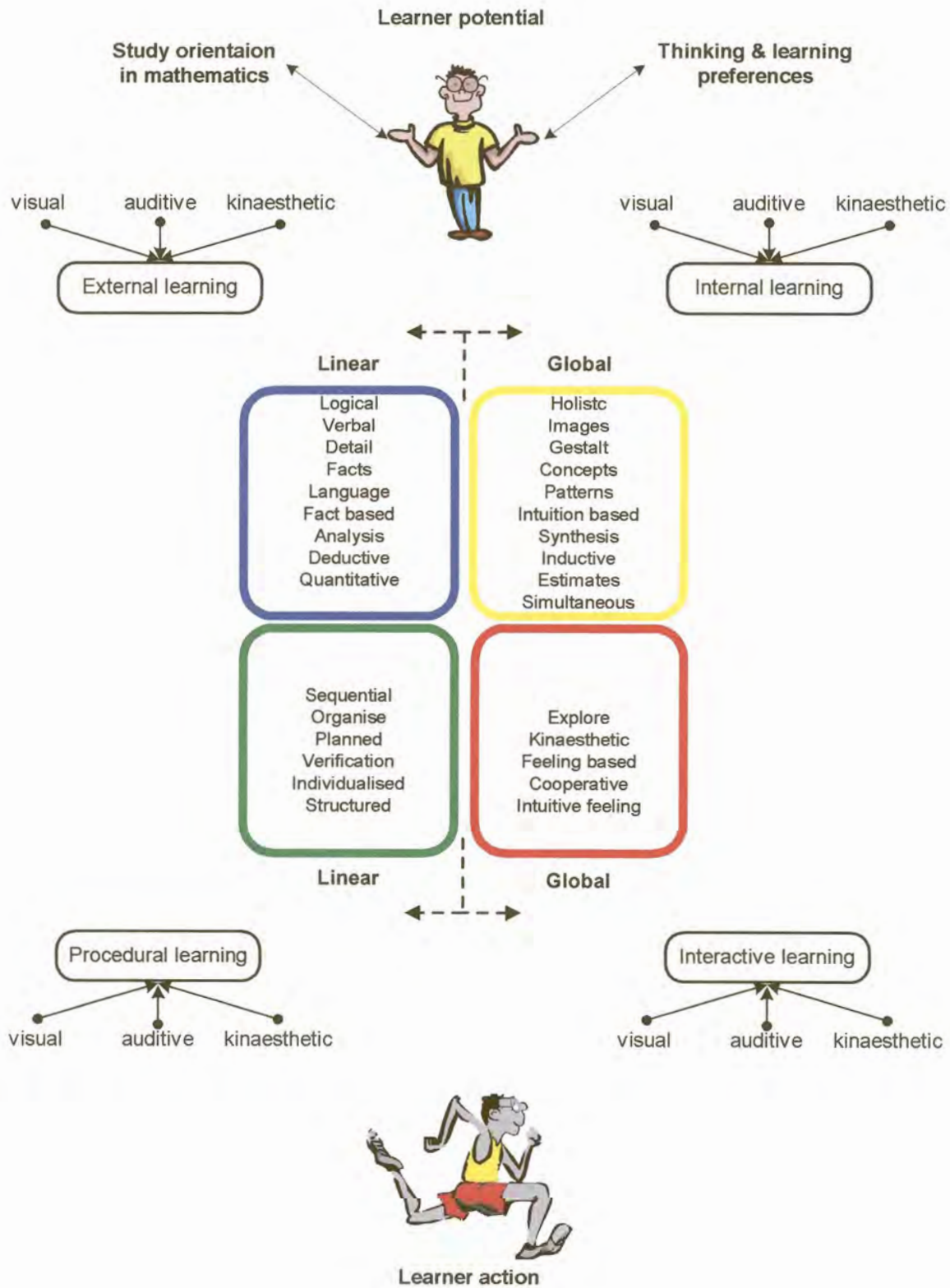
⁴¹ See Chapter 2, section 2.4.4.



However, it should be pointed out that the following terminology, describing the functions associated with a particular brain hemisphere, is used.

The term "linear" is used to describe the brain functions generally associated with the left brain hemisphere and the term "global" is used to describe those functions generally associated with the right brain hemisphere. For about 90% of all learners the "linear" hemisphere will coincide with the left brain hemisphere and the "global" hemisphere will coincide with the right hemisphere (Vander, 2001:368). For about 10% of learners the functions of the hemispheres are transposed meaning that for them the "linear" hemisphere will coincide with the physical right brain hemisphere and the "global" hemisphere will coincide with the physical left hemisphere. Using the terms "linear" and "global" thus reflect on the functions associated with the particular hemisphere and do not indicate the left or right side of the body.

Figure 3-12 A strategy for learning mathematics in a support course



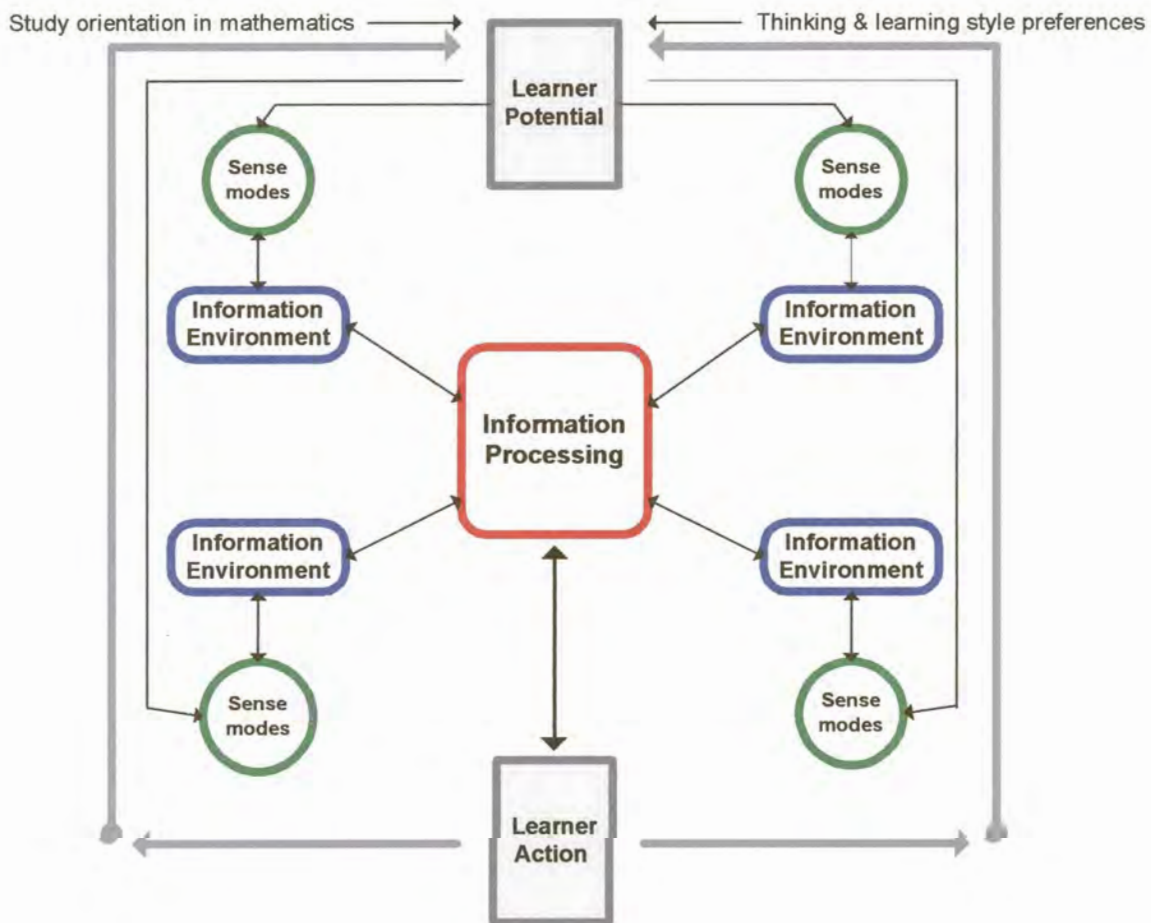
Proposed by the author of this thesis

3.8.2 Functional aspects of the strategy

Functionally, the strategy can be described in terms of information input, information processing and information output. On a physiological level Diamond (1999) points out that the activation of specific brain regions and their chemical and electrical frequency characteristics are relevant for information input, information processing and information output.

The functioning of the strategy for learning mathematics defined in the previous section is illustrated in Figure 3-13. The arrows indicate the relationship between the learner and the attributes of the strategy.

Figure 3-13 Functioning of the strategy for learning mathematics



Proposed by the author of this thesis

In the following sections the three functional processes (information input, information processing and information output) are briefly discussed. Although the author of this thesis fully acknowledges the relevance of physiological functioning, this discussion only reflects an educational perspective based on the Herrmann whole brain model, the mentioned categories of Lumsdaine and Lumsdaine and the sense modes relevant for mathematics learning.

3.8.2.1 Information input

According to the strategy in Figure 3-13 information input is through the sense modes and the information environment. The sense modes include visual, auditory and kinaesthetic input. The information environment includes input through external sources (such as information by the facilitator, appropriate technology, a workbook and textbook); through procedures (such as exercises and assessment); through interaction (such as exploration activities, discussion, feedback) and through internal cognition (such as intuition and insight).

Concerning the sense modes, Jensen (1996:131) distinguishes between internal and external input. External input is from an outside source and internal input is created by oneself, in the own mind. For example, visual external would be a person looking outward, visual internal would be a person visualising it ('seeing' it in the minds eye or making mental pictures). Auditory external is through verbal communication with others whereas auditory internal is by way of 'communication' with the self. Kinaesthetic external is 'hands-on' activity whereas kinaesthetic internal is intuitively and feeling based.

According to Hannaford (1997) when new information is taken in via the senses a learner has greater access to those senses that are directly linked to the dominant brain hemisphere. More specifically, sensory intake is ideally facilitated when the dominant eye, ear, hand and foot are on the opposite side of the body from the dominant brain hemisphere. This means that if the linear hemisphere is dominant and located in the left brain hemisphere the dominant eye, ear, hand and foot should be on the right side of the body. Although the author acknowledges these facts, it must be pointed out that the relation between the dominant hemisphere and the dominance of the senses as well as the effect hereof on mathematics learning were not considered or investigated in this study.

Stice (in Felder *et al.*, 1988:677) reports on research that determined the retention of information presented in different ways as follows. Students retain 10% of what they read, 26% of what they hear, 30% of what they see, 50% of what they see and hear, 70% of what they say and 90% of what they say as they do something. These figures illustrate that a combination of the sense modes of visual, auditive and kinaesthetic could be favourable for learning mathematics.

Ideally, information should be presented in such a way that information input fosters the functional mode (see Figure 3-5 on page 102) of whole brain utilisation that should then in turn contribute to whole brain utilisation with relevance to information processing.

3.8.2.2 Information processing

According to the strategy in Figure 3-13, information processing is dependant on input via the sense modes and the information environment. Information processing is also influenced by a learner's study orientation in mathematics and preferences related to individual thinking and learning styles. In the strategy information processing of mathematics in a support course is viewed in terms of utilising cognitive functions described by the Herrmann whole brain model that have been adapted for describing mathematics learning. The diagram in Figure 3-14 below illustrates a whole brain perspective on information processing regarding mathematics as presented in the strategy in Figure 3-12.

Figure 3-14 A whole brain perspective on information processing regarding mathematics



Adapted from Steyn (1998:136)

In the following paragraph a generic description is given that demonstrates, in a simplistic manner, how information processing of an aspect of mathematics could be interpreted from a whole brain perspective.

When students explore two-dimensional graphs generated by an appropriate computer graphing tool⁴² as used in the mathematical activities referred to in this thesis, the initial information processing concerns a visual image, gestalt input and possibly also kinaesthetic input from hand-on experiences or intuition. This processing mainly represents activities related to global hemispheric functions. This can then be followed by analysis, deductions and formulation of observations that mainly represent linear hemispheric functions. This again can be followed by inductions, estimates, synthesis and concepts (mainly global hemispheric functions) that result in verbalising the concepts orally and/or in writing (mainly linear hemispheric functions). Similarly, the sequence of information processing can start with analysis of mathematical text (mainly representing linear hemispheric functions) which can be followed by exploration activities using the graphing tool (mainly representing global hemispheric functions). It should, however, be

⁴² *Master Grapher for Windows.*

kept in mind that the human brain functions as a composite whole and these functions occur both simultaneously and iteratively.⁴³

3.8.2.3 Information output

According to the strategy in Figure 3-13, information output is viewed in terms of learner action. Learner action comprises the following attributes namely, mastering of fundamental concepts that underpin a study in calculus; a restructured study orientation in mathematics and an awareness of the effect of thinking and learning style preferences on mathematics learning.

The author of this thesis is of the opinion that a learner's study orientation in mathematics and his/her preferred style of thinking and learning have an influence on information input, information processing and information output. These aspects are addressed through the research questions stated in Chapter 1 of this thesis.⁴⁴

3.9 A learning facilitation strategy for mathematics in a support course

3.9.1 Aspects of the strategy

The main aspects regarding a learning facilitation strategy for mathematics in a support course for first year engineering students defined in this thesis comprise all the **attributes of learning** as well as their **functional aspects** that are defined in the strategy for learning mathematics and which have been discussed in the preceding sections of the chapter. In addition, a **learning environment** contributes to the mentioned strategy. Figure 3-15 on page 132 illustrates this proposed strategy.

The main aim of the proposed learning facilitation strategy, is to develop the mathematics potential of the learners. This development is attained by optimising the learning of mathematics according to the strategy illustrated in Figure 3-12 (page 123). In the strategy illustrated in Figure 3-15, the prospective optimisation is achieved within a learning environment. In the following paragraphs the concept "learning environment" and its relevance for the proposed learning facilitation strategy is discussed.

⁴³ See Chapter 2, Table 2-7 on Key brain characteristics.

⁴⁴ See Chapter 1 section 1.4.

The term "learning" was defined in Chapter 1 and further discussed in Chapter 2. The term "environment" is defined as *the conditions, circumstances, etc. affecting a person's life* (Crowther, 1995:387) and as the *aggregate of social and cultural conditions that influence the life of an individual or community* (Woolf, 1977:382). Thompson (1995:452) and Woolf (1977:382) define the term "environ" as *encircle, surround*.

These definitions of "environ" and "environment" clearly convey their meaning with regard to the learning facilitation strategy for mathematics defined in this thesis. In this strategy the "learning environment" is viewed as the *social and academic conditions that surround and influence the learning of mathematics in a support course*.

3.9.2 Learning environment

According to Smith and Ragan (1993:37) a learning environment comprises of teachers; existing curricula; instructional equipment; the institution and the larger community. Du Plessis and Quagraine (2000) identify the following aspects regarding a learning environment that influence the way in which students cope at university, namely, teaching strategies; the curriculum; the method(s) of assessment; resources; teacher characteristics; institutional administration and peer group.

The author of this thesis acknowledges the mentioned attributes but wishes not to restrict the concept "learning environment" to formalised attributes with regard to the proposed strategy. The author regards **any physical, social and/or academic condition that surrounds and influences the learning of mathematics** as part of a "learning environment" – a view that is confirmed by Du Plessis (2000). It inevitably follows that these conditions may vary according to the characteristics of the learners and the community wherein the learning takes place. The following are examples of social and academic aspects that formed part of the research activities during 2000, 2001 and 2002 when the strategy was implemented and which contributed to the learning environment.

3.9.2.1 Facilitator characteristics and activities

According to the proposed strategy, the activities of the facilitator should ideally incorporate all relevant concepts of learning (discussed in Chapter 2) and principles of instructional design and aspects of teaching (discussed in the preceding sections of this



chapter) within the framework of a whole brain approach. In a whole brain approach the complex physiology of the brain involved in learning is not the primary focus for facilitating learning but the author is of opinion that a facilitator of mathematics cannot ignore the results pertaining to the functioning of the human brain and the implications thereof for educational activities.

The author of this thesis is also of the opinion that to facilitate learning one needs to be aware of the students' existing learning experiences, their learning preferences and how they view learning. Rodgers (1983:18) points out that *teaching is more difficult than learning because what teaching calls for is this: to let learn*. Similarly Steen (1988b:91) remarks that *in a sense, no one can teach mathematics. What we hope is that a good teacher can stimulate a student to learn mathematics*.

Felder's reflections on the teaching of first year engineering students may very well also apply to the learning facilitation of mathematics in a support course and he remarks that

Teaching freshmen can be exasperating, and it's easy to conclude that it isn't worth the effort to overcome the obstacles they put in the way of their own learning and growth. If you're [the facilitator] sufficiently patient, thick-skinned, and positive, and if you maintain unshakable faith in their [the students'] ability to succeed despite themselves, they will reward you ... with understanding and skills you would not have believed possible. (Felder, 1997:16).

3.9.2.2 A learning community

The creation of a community for learning mathematics in a support course entails both social and academic matters. Landis (1997:9) identifies three stages in the building of a learning community in any class, namely, socialisation; group building and human relations training. Socialisation entails that each student should know all the other students in the class by name. Once this has been achieved, students should work and study in peer groups. Human relations training should focus on developing personal, academic and social skills that will contribute to their development as learners. These aspects regarding a learning community is endorsed by the learning strategy for mathematics discussed in this section.

Furthermore, the author of this thesis is of the opinion that learners on a support course in mathematics need to experience "support" beyond the mere understanding of mathematics content. The author feels that students need to feel valued as learners and that their academic success is a shared commitment including themselves as well the facilitator. However, learners also need to increasingly take responsibility for their own learning (in mathematics) and they need to be supported to develop the skills for self-directed and life-long learning that will hopefully not be restricted to the domain of mathematics content. Philip (1991) points out that the domains of the teacher's and the learner's responsibilities are not mutually exclusive. Philip (1991:9) regards *teachers and learners as occupying positions on a continuum extending from teacher-control at one extreme to learner-control at the other, where the deliberate surrendering of certain prerogatives by the teacher is accompanied by the concomitant acceptance of responsibilities by the learner or learners.*

3.9.2.3 Physical learning facilities

Concerning the personal physical well being of the students, they need to be made aware of the benefits of mental and physical health and a lifestyle supporting it. Landis (2000:187) specifically points out that it must be conveyed to students that they must *eat nutritionally, engage in regular aerobic exercise, get adequate sleep and avoid drugs.*

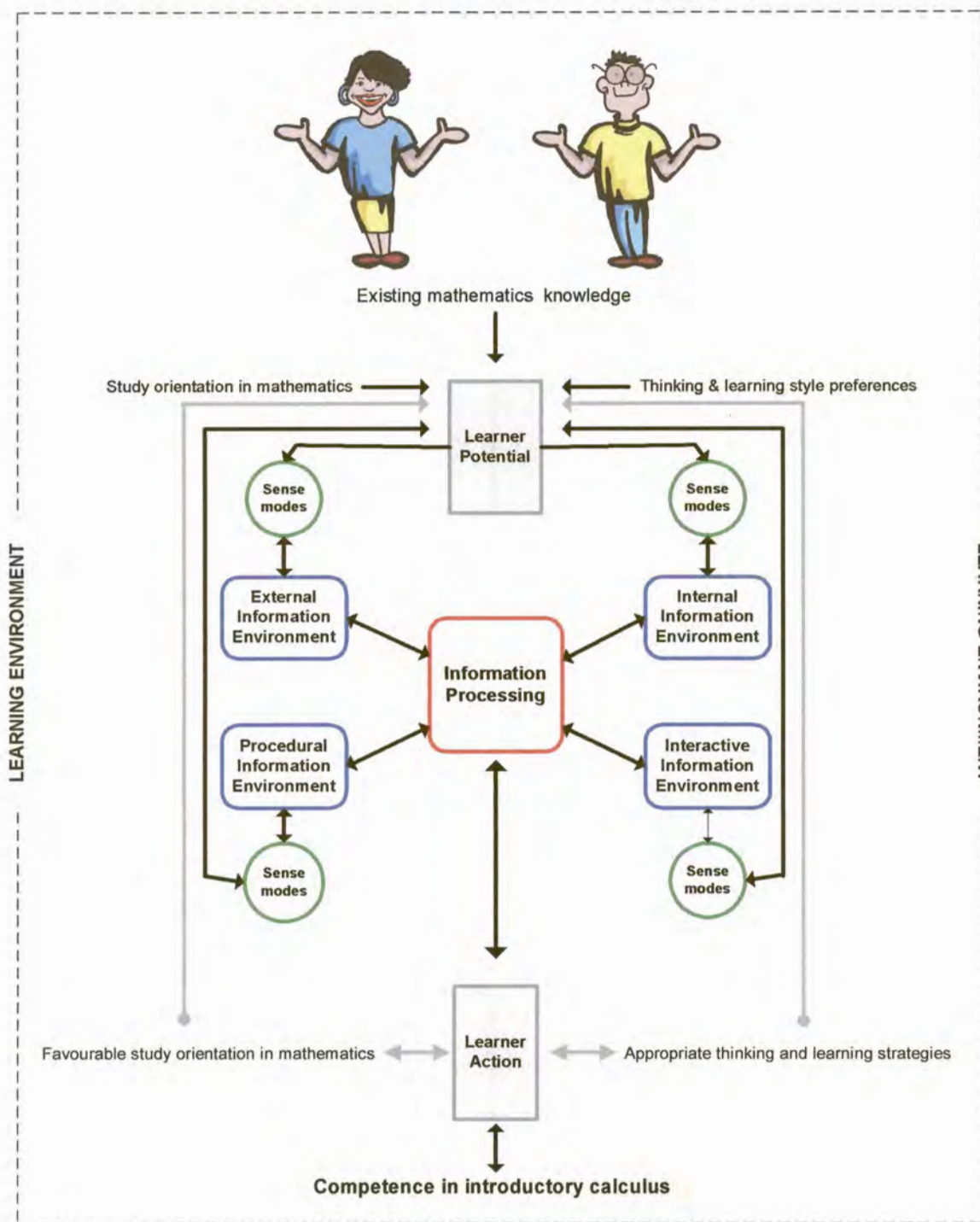
3.9.3 Implementation of the strategy

The strategy for the learning facilitation of mathematics defined in this chapter were implemented during 2000, 2001 and 2002 and assessed through action research studies. These studies are discussed in Chapter 5 and the results of data pertaining to the studies are analysed in Chapter 6 and discussed in Chapter 7. It should, however, be pointed out that only those aspects of the strategy pertaining to the research questions⁴⁵ were analysed.

Figure 3-15 on the following page illustrates the learning facilitation strategy proposed in this thesis and which was followed in the studies discussed in Chapter 5.

⁴⁵ For details of the research questions, see Chapter 1 section 1.4.

Figure 3-15 A learning facilitation strategy for mathematics in a support course for first year engineering students



Proposed by the author of this thesis

3.10 Summary

In this chapter it was indicated how principles of instruction and views of learning from various perspectives contribute to a strategy for learning facilitation that has as core principles the development of a learner's mathematics potential, learner involvement and whole brain utilisation.

It should be noted that although the strategy for learning mathematics as well as the strategy for the learning facilitation of mathematics presented in this chapter are partially embedded in the writer's own research experiences during 1993-2001 with students on support programmes, the components of the learning strategy and the principles that constitute the strategy address fundamental aspects related to human learning and learning facilitation. The writer is of opinion that the strategy may be relevant for any learning facilitation regarding mathematics and in particular in a first course in calculus.

With regard to the instruction of mathematics in tertiary education, Ruthven (1998:92) points out that:

Teachers of university mathematics courses, on the whole, have not been trained to, and do not often consider educational, didactic or pedagogical issues beyond the determination of the syllabus; few have been provided with the incentives or encouragement to seek out the results of mathematics education. In days gone by responsibility was placed on students' shoulders: it was assumed that faculty's responsibilities were primarily to present material clearly, and that good students would pass and poor ones fail. The climate today is that academic staff is considered to have greater overall responsibility for students' learning. The role of instruction (specifically, of lectures) and staff accountability are being reconsidered.