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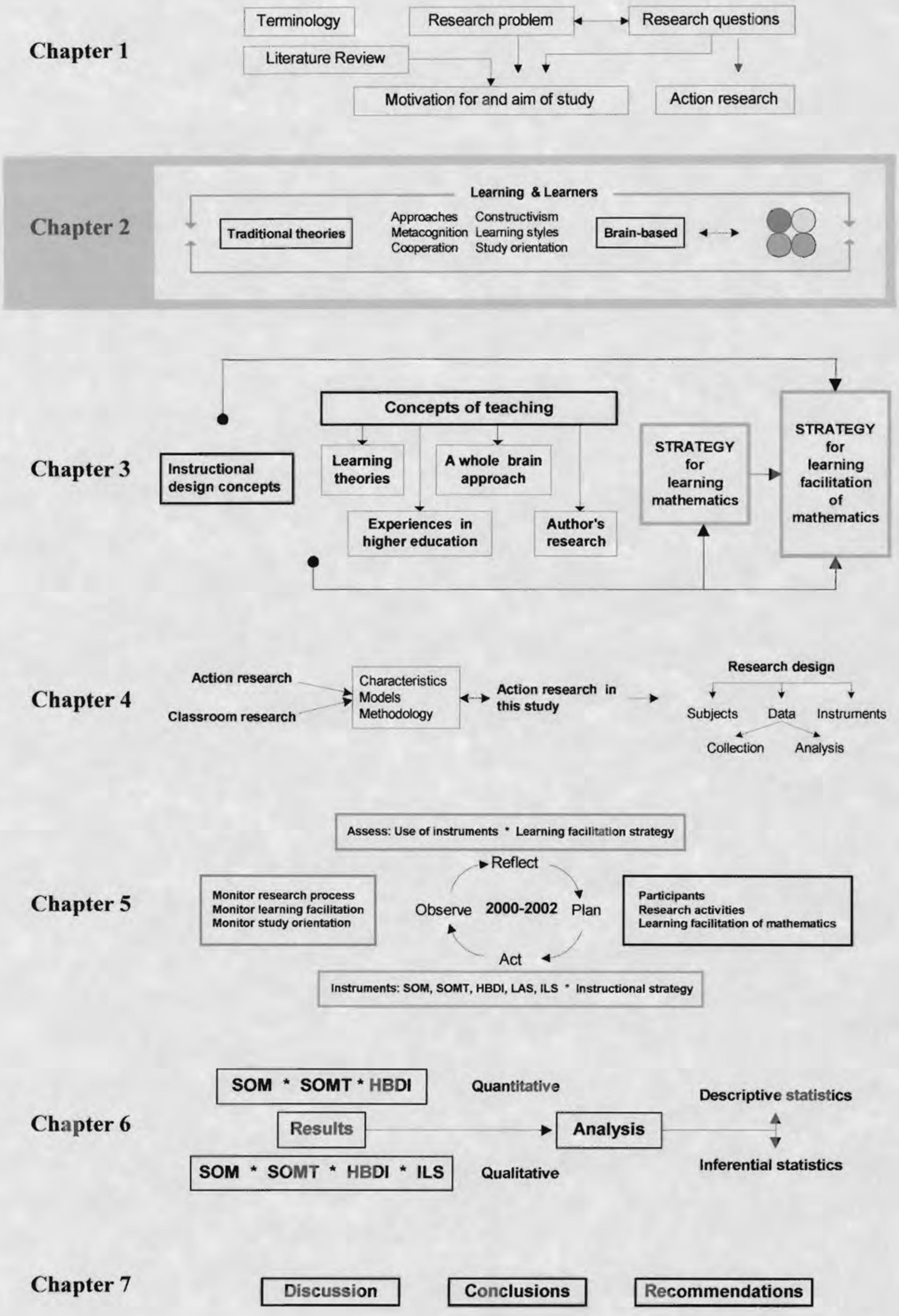

Perspectives on learning and learners: an epistemological overview

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Chapter 2

Perspectives on learning and learners: an epistemological overview

2. Introduction

According to Sylwester (1995) the dominant theory of human behaviour and resulting educational paradigms during the 1950s and 1960s came from the doctrines of Skinner. Sylwester further points out that, based on these paradigms, educators focused on visible and measurable manifestations of cognition rather than on cognitive mechanisms and processes. Educators learned to manipulate learners' environments to achieve the behaviour they desired and educators became a profession of behaviourists (Sylwester, 1995). Knowles (1990) points out that most of the 'scientific' theories of learning have been derived from studies focusing on learning by animals and children. The author feels that these learning theories cannot, as such, be applied to the learning of first year tertiary students. However, research since the 1960s on the functioning of the human brain has paved the way for educators to rethink the way their students learn and how learning should be facilitated. Sylwester (1995) argues that, in a certain sense, the increasing knowledge on the functioning of the human brain changed the paradigm for describing learning from psychology to biology.

In pursuing a theory of learning, the following views are noteworthy. If a theory is considered as a *system of ideas explaining something* (Crowther, 1995:1446) or *a set of principles* (Smit, 1995:3), a theory should not be regarded as an end in itself, but that a specific theory can serve as a frame of reference for a research question. Zuber-Skerritt, (1992a:57) remarks that

> *All theories are hypotheses created by people; they may be valid at any particular time, but may suddenly be invalid in some unforeseeable respect and replaced by a better theory.*

Reporting on new trends in undergraduate mathematics education, Kaput, Schoenfeld and Dubinsky (1996:215) rightly ask

> *Are learning theories transferable across cultural and subject matter boundaries? Can they be applied to different topics and different groups of students in different countries?*

Maree (1997) remarks that a final and conclusive theory on the learning of mathematics has not been formulated and that it probably cannot be done.

In the current study, only examples of those theories that are seemingly significant for the discussions in the study have been considered. Furthermore, the approach in this study is in accordance with the opinion of Maree (1997) who emphasises a holistic[6] approach to any research that reflects on learners in mathematics.

Following on Sylwester's line of thought, this chapter starts with an overview of traditional theories of learning relevant for tertiary mathematics education in a support course, followed by current views on learners and aspects of learning. The aspects of learning treated here include learning strategies, metacognition, cooperative learning, constuctivism, study orientation, learning styles and learning and intelligence. The discussion then focuses on brain-based learning and overviews the left-right brain hemisphere model of Sperry, the triune model of MacLean and the four-quadrant whole brain model of Herrmann. The chapter concludes with examples of thinking and learning styles that are relevant for the present study.

2.1 Overview of traditional views on learning and learners relevant to tertiary mathematics education

In this section, the possible significance that the traditional theories of learning may have for tertiary mathematics learning in a support course will be discussed.

[6] In this study the term "holistic" refers to the totality of possible factors that may be of significance with regard to a learner. The learner is viewed as a person within a (complex) situation. Aspects of this complexity are discussed in Chapter 3.

2.1.1 Behaviourism: The stimulus response (S-R) formula

Knowles (1990:147) points out that *to learn is to change* and that the scheme most commonly proposed for explaining how learning-change takes place is the **S** (**stimulus**) and **R** (**response**) formula or **some variation** thereof.

Behaviourists oversimplified the highly complex nature of human learning by defining it as a response to external stimuli. Although they extrapolated human behaviour from observation and experiments with animals, they ought to be credited for noting that change (learning) takes place following (external) stimuli. In spite of the criticism against traditional behaviourism, its underlying principles have influenced, either consciously or unconsciously, teaching to a great extent.

It seems as if the **S-R** model is a fairly good description of learning as long as it is confined to simple kinds of learning (Knowles, 1990). When learning is more complex (as in higher education) and the learner is more mature (as with tertiary students), the physical properties of the stimulus are insufficient to account for individual differences in responses. In order to describe more complex learning, Knowles (1990:147) defined an *intervening variable* as the **person** (**O**) and extended the **S-R** model to a **S-O-R** model. All other models of learning may simplistically be related to this distinct **S-O-R** model.

It should be pointed out that Knowles's **S-O-R** model is not merely an extended behaviourist model, but a descriptive model for learning and that the **intervening variable O** implies the **totality of a learner**. This totality should further be interpreted by taking **all** aspects relating to human learning into account.

2.1.2 The neobehaviourist learning theory of Gagné

Maree (1997) points out that in spite of Gagné's strong behaviourist preferences, he acknowledged and described a learner's accountability, attitude, intentions and creativity. Gagné (1977:6) remarks that change brought about by learning is *an increased capability for some type of performance ... an altered disposition of the sort called "attitude" or "interest" or "value".*

Gagné (1977, 1985) distinguishes five categories which form the core of his views on learning. He regards these categories as learning outcomes, as learner characteristics and as descriptive of possible different kinds of human performance. These five characteristics are:

- **Intellectual skills** (1977:28; 1985:47). Gagné points out that these skills enable individuals to interact with their environment in terms of symbols or conceptualisations. It thus implies a learner's knowledge and use thereof.
- **Cognitive strategies** (1977:35; 1985:48). which Gagné regards as the capabilities that governs an individual's own learning, remembering and thinking.
- **Verbal information** (1977:38; 1985:48) is information that an individual can recall in verbatim, paraphrased or summarised form.
- **Motor skills** (1977:42; 1985:48) refer to basic (physical) abilities.
- **Attitudes** (1977:44; 1985:48) constitute a persistent state that modifies the individual's choices of action.

Gagné (1985) classified all learning (including mathematics) as hierarchical, from 'signal learning' (which corresponds with the behaviourist stimulus-reaction concept) to the more difficult 'problem solving'.

Although the ideas of Gagné may be viewed in relation to tertiary mathematics education, the fundamental premise of an integrated brain-based approach proposed in this study, differs from Gagné's hierarchical structure of learning.

2.1.3 The gestalt learning theory

Although the learning theory of Köhler was based on his observations of chimpanzees, the resulting idea of *gestalt* correlates with later research findings on the functioning of the brain and the identification of *gestalt* as a (right) brain function. Köhler should be credited for his insight into the concept of wholeness that he noted long before the research evidence on the functioning of the brain. Köhler described 'learning with insight' as the formation of a whole (*gestalt*) which is constituted by the separate parts (Maree, 1997).

Kruger (in Hodgkinson, 1998:66) states that the *crux of gestalt theory is that the whole is prior to the parts*. Crowther (1995:568) defines *gestalt* as *an organised whole that is* *perceived as more than the sum of its parts*. These definitions of *gestalt* have particular significance for the learning (and teaching) of mathematics at all levels.

Maree (1992) points out that educators cannot assume that learners, who have mastered separate elements of mathematical content, have also acquired *gestalt* cognition. Maree notes further that learning achievement includes a totality of learning operations such as *the integration of visualisation and concrete representation* (Maree, 1992:55).

In the subsequent sections of this chapter on brain-based learning as well as in Chapter 3, it is pointed out that the right brain function of *gestalt* can and should be utilised in the learning (and teaching) of mathematics.

2.1.4 Cognitive learning theories: the learner as a constructor of knowledge

Where behaviourists focus mainly on the **outcome** of learning, the cognitive theorists focus on the **process** of learning. Cognitive theories of learning emphasise that learners are ultimately responsible for their own understanding through active construction of meaning (Cross *et al.*, 1996). For the purposes of this study, aspects of cognitive theories relevant to the tertiary learner will be overviewed within the framework of a brain-based approach to learning. It should, however, be borne in mind that cognitive theorists formulated their theories on learning prior to the knowledge of brain functioning that emerged following the work of Sperry and others. Zuber-Skerritt (1992a:55) stresses that

> *The learner is not merely a passive consumer of accumulated knowledge, but an active producer and reproducer of knowledge and theory.*

2.1.4.1 Constructivism and Piaget's model of learning (1970)

According to Piaget (Copeland, 1979), learning results from a tension between two processes, the process of **assimilation** and the process of **accommodation**. Through **assimilation**, events and experiences from the world are integrated into a learner's existing concepts and experiences. Changes in a learner's existing cognitive structures are brought about by **accommodation**.

The four major stages of cognitive growth (Copeland, 1979) identified by Piaget are:

- The **sensorimotor stage** lasting from birth to approximately two years of age.
- The **pre-operational stage** lasting from two until approximately seven years of age.
- The **concrete operational stage** lasting from about seven to 11 or 12 years of age. In this stage logical thought is based in part on the physical manipulation of objects.
- The **formal operational stage** starts at about 12 years of age and is regarded as an adult level of thinking (Copeland, 1982). In this stage the child *reasons or hypothesis with symbols or ideas rather than needing objects in the physical world as a basis for his thinking* (Copeland, 1979:25). The mental structures in this stage include *the propositional combination of symbolic logic-implication (if–then), disjunction (either–or, or both), exclusion (either–or) [and] reciprocal implication* (Copeland, 1979:25).

Piaget's four stages of intellectual growth refer to children from birth to about the age of 11-12. They are therefore not directly applicable to tertiary learners. However, it could be expected that in a study such as this, involving first year tertiary students in mathematics, attention should be given to Piaget's fourth developmental stage, the **formal operational** stage. According to Piaget (Copeland, 1982), learners in this stage function on an abstract level that is an adult level of thinking and these learners are not bound by concrete experiences.

Copeland (1982) questions Piaget's view regarding abstract thinking without concrete experience and points out that children of age 11-12 still need experiences with concrete material. Copeland (1982) also points out that this is too young an age to accomplish this formal operational level. He refers to a study, done by Heron in 1975, which indicated that 50% of college freshmen in the USA were functioning completely at the concrete operational level. Fewer than 25% were fully formal operational in their thoughts (Copeland, 1982).

Piaget's postulate that cognitive function on an abstract level should be accomplished without concrete experience is also questionable within the framework of the principle of whole brain utilisation.[7] If the term "concrete" is defined as *existing in a form that can be touched, felt or seen* (Crowther, 1995:238), then the concept "concrete experience" can be viewed as a cognitive function associated with the global (right) hemisphere. This does not necessarily imply that a concrete cognitive activity remains focused on the concrete image.

[7] See Section 2.3.4.

The author feels that it is in the combination of image (global brain hemisphere function) and abstraction (linear brain hemisphere function) that knowledge is formed. This aspect is of special significance in support in tertiary mathematics education where the facilitation of fundamental mathematical concepts is a primary aim of instruction. Therefore, in the opinion of the author, the learners (freshmen students) in this study cannot be categorised in terms of Piaget's fourth development phase.

The following aspects of the cognitive learning theories of Kelly and Leontiev are of special interest to the practice of higher education.

2.1.4.2 Kelly's personal construct theory (1955)

Zuber-Skerritt (1992a) points out that Kelly's theory bridges the gap between traditional theorists on the one hand and practitioners (for example students and teachers) on the other. Kelly (in Zuber-Skerritt, 1992a) believes that everyone (students, teachers, counsellors, administrators) is a 'personal scientist' engaged in a process of observation, interpretation, prediction and control. Kelly regards learning as the active, creative, rational, emotional, intentional and pragmatic construction of reality. Therefore students should not be seen as the passive receivers of information but the active constructors (or self-instructors) and interpreters of their experiences.

Kelly's theory seems to be descriptive of tertiary students' learning. With regard to this study, it can readily be assumed that specialised brain functions should be utilised during 'active construction' and 'interpretation' in the learning facilitation strategy defined in this thesis. Kelly's theory is a holistic view of the learner and an elaboration of Knowles' **S-O-R** model. It could readily be linked to the functional whole brain model of Herrmann.[8]

2.1.4.3 Leontiev's cognitive theory of action (1977)

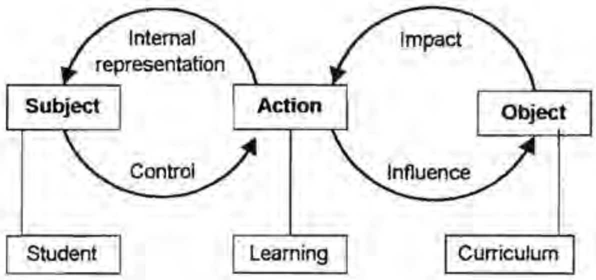
Action theories have evolved as a critical reaction to behaviourism. Action theorists regard cognition and conscious action as essential factors in human behaviour. They assume that mental processes, such as reasoning, problem-solving or decision making, as well as motor procedures, are dependent components of the macro unit 'action' (Zuber-Skerritt, 1992a).

[8] See section 2.3.4.

Where Kelly emphasises individualism, Leontiev emphasises the adaptation of the human mind to social conditions. However, in both cases, the primary importance of action is useful for human cognition and for the creation of knowledge.

Heger was among the first to apply Leontiev's theory to higher education and to develop a model for the analysis of student learning. Figure 2-1 is a model for student learning adapted from Heger by Brandt and Sell (Zuber-Skerritt, 1992a).

Figure 2-1 A model of student learning after Leontiev



Zuber-Skerritt (1992a:75)

In the model in Figure 2-1:

- the **subject** is the holistic person (e.g. student/learner) and his/her consciousness. This includes intentions, motivations, cognitive aspects (thoughts, ideas, concepts) as well as effective aspects (emotions, feelings, anxiety);
- the **action** refers to actions of studying and learning and is influenced by the personality traits and external demands;
- the **object** refers to the context of studying (a task, a problem or that which is produced by the subject) as well as all given factors within the context and system of an institution.

Zuber-Skerritt (1992a) deduced the following four principles for learning and teaching (understood as facilitation[9]) from Leontiev's theory of action:

[9] The term "facilitation" is defined in Chapter 1, section 1.2.2.

1. **The action of learning should be as concrete and practical as possible in the early stages of studying a new area of knowledge in order to facilitate internal representation.** This principle correlates, for example, with the use of the specialised instructional media[10] that is used in the learning facilitation strategy proposed in Chapter 3 and discussed in Chapter 5 of this thesis.

If this principle is interpreted form the perspective of brain-based learning (discussed in section 2.3 and in Chapter 3), the following can be noted. The pictorial visualisation of symbolic mathematical data by way of a graphical representation using a computer graphing tool to generate the graphs of two-dimensional functions is a **concrete** learning activity. **Practicality** in this sense can then be interpreted as a physical action by the learner, using a tool (computer graphing software) to represent mathematical content. This implies **experiential** and **non-verbal** cognitive activity that mainly involves right brain hemisphere functions. The **facilitation** of **internal representation** can be associated with **graphical analysis** which is verbal and structured where internal representation mainly involves right brain hemisphere and graphical analysis left brain hemisphere functions. It can therefore be assumed that Leontiev's first principle can be correlated with whole brain cognitive activity in learning mathematics.

It should also be noted that this principle for learning, deduced from Leontiev's theory of action, to an extent contradicts Piaget's fourth developmental stage as discussed in section 2.1.4.1 above.

2. **The degree of consciousness and conceptualisation depends on the extent to which actions - whether practical/concrete or mental/abstract actions - are interiorised[11] in the learner. Important constituents of interiorisation are language, communication with others and active reflection.**

Zuber-Skerritt (1992a) points out that 'interiorisation' is not only meant as a mirror-effect of the action of learning on the learner but also as the student's own effective reaction to and reflective thinking about the content and processes of learning. Mere internalisation of action (i.e. storage of information or events and accumulating them in one's memory) is only the lowest level of thinking. In order to reach higher levels of abstraction,

[10] For example, graphical exploration and analysis of 2-D functions using the textbook by Greybe, Steyn & Carr (1998).
[11] The author feels that the term "interiorised" in this case is use synonymously with the term "internalise" which *make[-s] attitudes, behaviour, etc. part of one's nature by learning or unconscious assimilation* (Crowther, 1995:712).

generalisation and theory, thinking must become progressively active, reflective, critical, self-critical and creative. Zuber-Skerritt (1992a) also points out that an instrument to reach more and more advanced levels of theory is **language** in both **oral** and **written** forms. She elaborates on this aspect and adds that one can theorise better through discussions with others and through writing a report or paper than just through thinking on one's own.

This second principle is also reflected in the aspects of metacognition and cooperative learning. These aspects are elaborated on in section 2.2.

3. The third principle of learning (and teaching) derived from Leontiev's theory of action is that **students' existing knowledge should be utilised in action learning**.

Zuber-Skerritt (1992a) explains that action learning (e.g. by doing, experiencing, participating through discussion, problem-solving, project work, etc.) and metalearning are possible because of students' previous experience and existing knowledge in most areas of learning. The principle that learners of all ages bring with them existing pre-knowledge that influences learning has repeatedly and substantially been backed by research (Bransford *et al.*, 1999).

The author's experiences with first year mathematics students have brought to light that their existing pre-knowledge of graphical representation and interpretation of two-dimensional functions is often inadequate and prone to misconceptions. However, it has also been observed that the action of learning by way of doing, experiencing, participating through discussion and problem-solving as well as metalearning not only leads to the mastering of new knowledge but aids in rectifying existing misconceptions.

4. The fourth principle of learning and teaching deduced from Leontiev's theory of action relates to the **dialectical[12] relationship between institutional requirements and the needs of students in a fast changing world**.

Zuber-Skerritt (1992a) elucidates this principle:

- Active learning and metalearning may influence the curriculum and assessment (the 'influence' arrow in Figure 2-1 on page 28). This can be done through open discussions between lecturers and students and through student evaluation of courses and teaching.

[12] Dialectic refers to a view on something in context and in contradictions.

- Institutional conditions, requirements, expectations have a great impact on learning (the 'impact' arrow in Figure 2-1). Zuber-Skerritt (1992a) illustrates this point with an example that is applicable to the learning facilitation strategy defined in this study. An assessment system with only end-of-semester or end-of-term examinations is likely to encourage rote learning and to discourage active learning. These approaches to learning can also be categorised as a 'surface' approach and a 'deep meaning' approach (see section 2.2.1). Continual assessment forms an integral part of the learning facilitation strategy defined in this study. The author feels that this form of assessment contributes to active learning and an overall 'deep meaning approach' in a support course in tertiary mathematics. The principle of continual assessment is discussed in chapters three and five as an aspect of the learning facilitation strategy proposed in this study.

2.1.4.4 Information processing and knowledge construction

Cognitive views of learning contend that new information is more easily understood and retained when it can be related to existing information. The knowledge structures that organise and store information in a learner's memory are referred to as schemata. Cross *et al.* (1996) point out that schemata can be compared to networks of facts, ideas and associations formed around related concepts.

Maree (1997) describes schemata as distinct units of physical and cognitive actions that are often repeated. Learners have their own schemata available that continuously develop, change and become more complex. These schemata are the building blocks of cognitive structures.

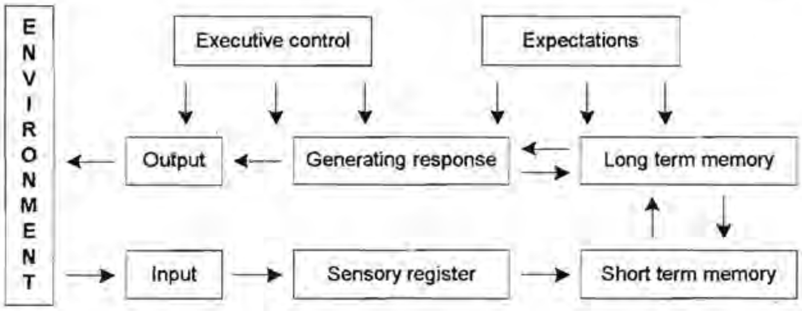
Crawford and Chaffin (1986) (in Cross *et al.*, 1996) summarise three assumptions about the way schemata work namely:

- People do not remember information exactly as they received it (mental representations are thus not 'copies' of original events).
- Schemata are generalised knowledge structures that provide the framework for and determine the nature of understanding.
- Schemata provide the learner with background knowledge from which to make deductions.

Gagné's (1983:8-10) view on information processing and knowledge construction is illustrated in Figure 2-2 and can be summarised as follows:

- The fundamental unit that is learned and stored in human memory is a 'semantic unit' which is inherently meaningful.
- The physical stimulation delivered to the senses is transformed into nerve impulses which can be viewed as 'intricate masses if information' that undergo several kinds of transformation sequentially, simultaneously or parallel.
- The kinds of transformation are called 'processes'.
- The learner has 'executive control' over the processes.
- Processing that turns external stimulation into 'learned information' is influenced by inputs from three sources, namely, the organisation of the external stimulus; the executive control processes that are available to and used by the learner and by what has previously been learned (contents of memory).

Figure 2-2 Information processing and structuring according to Gagné



Adapted from Gagné (1977:53; 1985:71)

Maree (1997) summarises the contemporary theories on information processing and points out that internal processes, which are influenced by the external organisation of stimuli, initiate the activities 'learning' and 'memory'. Learners together with their own available memory contents have control over these processes.

2.1.5 Kolb's experiential learning theory (1984)

Kolb defines learning as

> *the process whereby knowledge is created through the transformation of experience* (in Zuber-Skerritt, 1992a:105)

and Kolb's experiential learning theory suggests further a

> *holistic integrative perspective on learning that combines experience, perception, cognition and behaviour* (in Zuber-Skerritt, 1992a:100).

Although the learning of mathematics as described in this thesis is not experiential learning in the sense of learning in real-life situations, McKeachie (1999:154) points out that the criterion for experiential learning should be the *degree to which learning is transferable to other times and places*. In this sense, the learning and understanding of fundamental concepts underpinning calculus to be transferred to 'other times and places' is in essence a focus of the learning facilitation strategy described in this thesis.

Kolb has developed six major propositions that describe learning of which the following three are significant for the learning of mathematics as viewed in this thesis.

1. **Learning is best perceived as a process, not in term of outcomes** (Zuber-Skerritt, 1992a). Zuber-Skerritt points out that a process of learning will lead to an outcome, and outcomes of learning are the result of a process. She therefore rephrases Kolb's 'process proposition' as *learning is a process as well as an outcome* (Zuber-Skerritt, 1992a:103).

Kolb applies 'learning as a process' to suggest a shift away from the preoccupation with tests and examinations to a greater attention to the process of learning. He regards this to produce more personalised and lasting knowledge than the mere accumulation of knowledge in preparation for exams. This correlates with Leontiev's fourth principle mentioned above.

2. **Learning is a continuous process grounded in experience** (Zuber-Skerritt, 1992a). This proposition implies that a teacher's task is not only to expose learners to new ideas, but also to dispose of or modify old ones.

3. A further proposition of Kolb's experiential learning is that **learning is the process of creating knowledge** (Zuber-Skerritt, 1992a). According to Zuber-Skerrit, Kolb distinguishes social and personal knowledge.

The significance of these propositions of Kolb for learning activities associated with the learning facilitation of mathematics in a support course, as defined in the current study, can be summarised as follows:

- The facilitation of the mathematics component of the support course is not viewed solely in terms of outcomes. In accordance with the first proposition, it encompasses a **learning process** to enhance the conceptual understanding of mathematics and is also aimed at the development of the mathematics potential of the learner.
- In the learning facilitation of the mathematics component of the support course, computer-generated graphs of two-dimensional functions are used to explore concepts. In this way learners are exposed to **new mathematical content** and to **modify** misconceptions that may exist which underpins Kolb's second proposition of experiential learning.
- It may readily be assumed that mastered subject content knowledge, such as fundamental knowledge that underpin a study in calculus, can be related to Kolb's category of **personal knowledge**. Furthermore, an awareness of one's own thinking and learning preferences and the necessity to also utilise less preferred modes of thinking and doing contribute to social and personal knowledge and development.

2.1.6 Adult learning

Knowles (1990) distinguishes six categories to describe learners. For the purposes of this study these categories are summarised and comparatively stated for child learners and adult learners in Table 2-1 on page 36.

Knowles (1990) points out that (tertiary) educators have the responsibility to check which assumptions are realistic in a given situation, namely those reflecting on child learners or those of adult learners. Whichever strategy is realistic in a given situation for a particular (tertiary) learner with regard to a particular learning goal, should be viewed as an appropriate starting point, but **(tertiary) learners must take increasing responsibility for their own learning.**

The second assumption listed in Table 2-1, namely, a learner's self-concept is elaborated on by Knowles (1990:55) as follows

> *As individuals mature, their* ***need*** *and* ***capacity*** *to be self-directing, to utilise their experience in learning ... and to identify their own readiness to learn, increases steadily from infancy to pre-adolescence, and then increases rapidly during adolescence.*

This view has significant implications for higher education. **Tertiary learners** (as those in a support course for mathematics) should be recognised as **developing learners** and teaching initiatives should be structured accordingly.

The challenge for tertiary education in general, and with regard to learning facilitation (of mathematics) in a support course in particular, may be to think, to investigate and to write about the learners who are in transition form child learning to adult learning. It seems obvious that the learning of first year tertiary students on a support course would entail aspects of learning in both categories.

In chapter five it ought to become clear to what extent these aspects have been addressed in the learning facilitation strategy defined in this study.

Students need to be actively engaged in the learning facilitation process of mathematics. They must eventually take control of their own learning. Since understanding in mathematics is crucially important, students must learn to recognise when they understand and when they need more information or help to facilitate their understanding.

Table 2-1 A comparison between child learning and adult learning according to Knowles' assumptions

Assumption	Child learning	Adult learning[13]
1. The need to know	Learners only need to know that they must learn what the teacher teaches if they want to pass and get promoted.	Adults need to know why they need to learn something before undertaking to learn it. For the facilitator this implies, firstly, helping learners to become aware of the 'need to know' and secondly, the facilitation of experiences in which the learners discover for themselves the gaps between where they are and where they want to be.
2. (Self-) concept of learners	The teacher's concept of the learner is that of a dependent personality. The learner's self-concept remains that of a dependent person.	Adults have a self-concept of being responsible for their own decisions.
3. Role of experience	The experience that is predominant is that of the teacher, the textbook writer, and the audio-visual aids producer. Transmittal techniques, such as lectures, are the backbone of pedagogical methodology.	Adults come into educational activity with greater volume and different quality of experience, implying a wider range of individual differences. These necessitate: • individualisation of learning (and teaching) strategies; • greater emphasis on experiential learning and • acknowledgement of the learner's previous experiences as this enhances self-identity.

[13] From the viewpoint of learning, the psychological definition of adulthood is relevant: a self-concept of being responsible for one's own life and of being self-directing (Knowles, 1990:57).

4. Readiness to learn	Learners are ready to learn what the teacher tells them they must learn.	Adults are ready to learn those things they need to know and with which they will be able to cope with effectively in their real-life situations.
5. Orientation to learning	Learners have a subject-centred orientation to learning. Learning experiences are organised by the teacher according to the logic of the subject matter content.	Adults are task-centred or problem-centred.
6. Motivation	Learners are motivated to learn by external motivators – grades, the teacher's approval or disapproval as well as parental pressure.	Adults are responsive to some form of external motivators (better jobs, promotions, higher salaries, etc.). The most potent motivators for adults are internal pressures (the desire for increased job satisfaction, self-esteem, quality of life, etc.). Research by Thought (1979) (in Knowles, 1990) has indicated that all normal adults are motivated to keep growing and developing. This motivation can be blocked by barriers such as a negative self-concept (as a student) or by programmes that violate principles of adult learning.

Compiled from Knowles (1990:55-63)

2.2 Further aspects of learning

2.2.1 Learning strategies

A strategy (approach) to learning refers to the way in which cognitive skills are adapted to comply with a learning task. There are various perspectives that describe students' approaches to learning. These descriptions are not contradictory but rather complementary.

According to Du Plessis (1992) Hudson had already referred to a 'syllabus bound' and 'syllabus free' approach to learning in 1968. A student who is syllabus bound does not extend his study efforts beyond prescribed course content whereas a syllabus free student will engage in extending his focus wider than the prescribed course content.

Pask (1976) distinguishes between a 'serial' and a 'holistic' approach. A serial approach indicates a step-by-step approach whereas in a holistic approach a student first strives to get an overall view of the content. Pask points out that the most successful students are those who can adapt their strategy to the requirements of the learning task. Pask's view confirms an important principle of the learning model described in chapter three, namely the ability to utilise different cognitive modes when learning.

Marton and Säljö (1976) identify two approaches to learning, namely, a 'deep' or holistic approach and a 'surface' or atomistic approach. Marton and Säljö's categories have been widely used and adopted to describe approaches to learning. In Table 2-2 on the following page the identifying characteristics of a student who follows a 'deep' approach and of a student who follows a 'surface' approach to learning are compared.

Although deep approaches to learning are generally associated with more successful learners, Säljö (1984) (in Cross *et al.*, 1996) points out that a deep/surface distinction does not apply to all learning situations. For example, a deep approach will be more effective when reading scientific texts that present arguments, principles and constructs to be analysed whereas a surface approaches are entirely appropriate for leaning tasks that require nothing more than memorisation. Laurillard (1984, in Cross *et al.*, 1996) indicates that problem solving assignments such as word problems in algebra do not necessarily require deep approaches to learning because once students recognise that the problems follow a typical pattern (indicated by certain phrases), they may reproduce answers using formulas without thinking about underlying principles. The author of this thesis is of the opinion that this approach is not necessarily wrong because once a learner has mastered a concept, it becomes useable knowledge and is used as an automatism. Automatisms that are grounded in concepts form the base for further knowledge generation.

Table 2-2 Approaches to learning according to Marton and Säljö

A student who follows a deep approach:	A student who follows a surface approach:
• tries to understand the 'big picture';	• focuses on aspects and not on the 'big picture';
• tries to relate content in a broader context;	• sees the learning task as a short-term activity, for example, to pass a test;
• tries to synhesize aspects and integrate them within the 'big picture';	• cannot synthesize aspects and integrate them within the 'big picture';
• identifies core principles and distinguish detail;	• invariably masters content through memorization;
• seeks for the deeper meaning of course contents and	• is focused on time and the immediate completion of the task and not on a possible deeper meaning thereof and
• is committed to and enjoys the learning task.	• is not necessarily committed to the learning task or enjoyment of the process.

Compiled from Marton and Säljö (1976:20, 37, 48)

According to Entwistle and Ramsden (1983) students choose a specific learning strategy depending on their cognition of the learning task. A student may exhibit different approaches in different courses, depending on perceptions of what is expected for success. Gibbs (1981:76) refers to research by Laurillard where she states that *students will take a deep or surface approach to a task depending on the nature of the task.* It follows that students need to develop the ability to adapt their strategy to fit the learning task and content.

Entwistle and Ramsden (1983) also point out that students adapt their learning strategies through contact with other students. According to Pressley and McCormick (in McKeachie, 1999) teachers can influence the degree to which students use deep rather than surface strategies for learning through assessment that requires a deep understanding of the content.

Smith (2001) identifies course characteristics (in mathematics) that can encourage either a deep approach or a surface approach to learning. These characteristics are summarised in Table 2-3 below.

Table 2-3 Course characteristics that encourage a deep or surface approach to learning

A deep approach is encouraged by:	A surface approach is encouraged by:
• interaction – peers working in groups;	• excessive amount of material;
• well-structured knowledge base – connecting new concepts to prior experience and knowledge;	• lack of opportunity to pursue subjects in depth;
• motivational context – choice of control, sense of ownership;	• lack of choice over subjects and/or method of study;
• learner activity plus faculty connecting activity to abstract concept.	• threatening assessment system.

Compiled from Smith (2001:11)

2.2.2 Metacognition

According to Cross *et al.* (1996) metacognition implies both knowledge of cognition and control of cognition. Knowledge of cognition is an awareness of and reflection on one's own learning processes ('thinking about thinking'). The practice of observing oneself in the act of learning contributes to the ability to understand one's own learning process and to manage and control it. Biggs (1985) describes the managing of one's own learning as metalearning.

Flavell (in Maree, 1992:73) describes metacognition as follows:

> *Metacognition refers to one's knowledge concerning one's own cognitive processes and products or anything else related to them ... I am engaging in metacognition ... if I notice that I am having more trouble learning A than B; if it strikes me that I should double-check C before accepting it as a fact.*

Metacognition thus entails the planning, monitoring, controlling and evaluating of one's own learning. The ability to use metacognition as a way to improve learning is not necessarily part of a tertiary learner's frame of reference.

Flavell (1970, 1976, in Cross *et al.*, 1996) identified three main areas of knowledge about cognition that may help learners improve their learning process namely, knowledge about oneself; knowledge of the learning task and knowledge of strategies available to complete the task. The question arises if metacognition and learning strategies in mathematics (in a support course) can be developed through instruction.

Weinstein (1999:319) emphatically states that facilitators (lecturers) can have a *tremendous impact on helping students to develop a useful repertoire of learning strategies.* Students should have the opportunity to reflect on their learning (strategies) and teachers should not only ask students **what** they think but also **how** they think.

The author of this thesis strongly endorses the principle that students need to be given the opportunity to communicate in mathematics both orally and in writing. The principle of giving learners ample exercise in not only writing down mathematics but also writing down the reasoning behind it, is increasingly being implemented by mathematics educators who realise that there is more to understanding mathematics than mere performing correct calculations (Hubbard, 2001; Smith, 2001). The author of this thesis agrees that these activities promote metacognitive skills.

Metacognition refers to people's abilities to predict their performances on various tasks and to monitor their current level of mastery and understanding. Teaching practices congruent with a metacognitive approach to learning include those that focus on sense making, self assessment and reflection on what worked and what needs improving. (Schoenfeld, 1985; Bransford *et al.*, 1999). If this is true in general, it is certainly also true for the learning facilitation of mathematics.

2.2.3 Cooperative learning

Cooperative learning is generally regarded as an activity where learners work together in small groups. Davidson (in Maree 1997:77) defines cooperative learning as follows:

> *Cooperative learning involves more than just putting students together in small groups and giving them tasks. It also involves very careful thought and attention to various aspects of the group process.*

According to this definition, cooperative learning is a formal instructional methodology that implies organising groups and structuring tasks.

Davidson (1990) used cooperative learning with college level mathematics students and summarises his research findings as follows (Becker & Pence, 1994):

- students taught by small-group discovery method perform at least as well as those taught by more traditional lecture methods;
- students are actively involved;
- students have the opportunity to communicate mathematically;
- the classroom atmosphere is relaxed and informal;
- students have the freedom to ask questions;
- there is a closer student-teacher relationship;
- there is a high level of student interest;
- students' attitudes are more positive;
- students use the opportunity to pursue challenging mathematical situations.

Felder and Brent (1999) define "cooperative learning" as a subset of "collaborative learning". According to them, in collaborative learning students are interacting with one another while they learn actively and apply course material. In cooperative learning students work in groups on structured tasks under conditions that should meet the following criteria, namely:

- **Positive interdependence.** Team members must rely on one another to accomplish a goal.
- **Individual accountability.** Members are held accountable for doing their share of the work and mastering all material.
- **Face-to-face interaction.** Some or all work done by members working together.
- **Appropriate use of interpersonal skills.** Team members practise and receive instruction in leadership, decision-making, communication and conflict management.
- **Regular self-assessment of group functioning.** Teams periodically reflect on what they are doing well as a team, what they could improve, and what (if anything) they will do in future.

Felder *et al.* (1999) strongly states that cooperative learning is **not** students sitting around a table studying together or group projects with one or two students doing all the work.

2.2.4 Aspects of constructivism — a summary

Perspectives on learning and learners will fall short if the educational concept "constructivism" is not mentioned. The author views "constructivism" in contrast to "instructivism" and delineates the concepts according to the following definitions. Crowther (1995:247) and Thompson (1995:286) define "construct" as *to put or fit together*, and Woolf (1977:244) defines "forming a construct" as *mentally assembling and integrating sense-data*. Crowther (1995:619) defines "instruct" as *to teach somebody a skill* and Woolf (1977:599) defines it as *to give information*. Incorporating these definitions the author considers "instructivism" as more *teacher centred* whereas "constructivism" is more *learner centred*. Cross *et al*. (1996:43) remark that *learners are ultimately responsible for their own understanding through active construction of meaning.*

According to Maree (1997) the following aspects succinctly describe constructivism:

- Knowledge is **not passively received**.
- Learners are **active** participants in the learning process.
- **Assimilation** is the process whereby new, but recognisable, ideas are added to existing knowledge structures.
- **Accommodation** refers to the process whereby new and different ideas are incorporated to existing ideas.
- The incorporation of new ideas into existing ones presupposes the **consolidation** of previously acquired concepts.
- **Verbalisation** of concepts is a prerequisite for consolidation.
- Social constructivism focuses on **communication** between learners.
- Social constructivism is **group-based** and **process orientated**.
- Social constructivism supposes a **problem solving** and **investigatory** approach.
- Constructivist learning strategies are aimed at **understanding** subject contents in contrast to behaviourist drill and practise.
- Learners should take **charge** of their **own learning** experience.
- Learners **constitute** their world through their own experience.
- **Experiential** learning, **metalearning** and **problem-solving** are key aspects of the constructivist learning situation.
- Learning needs to be facilitated to incorporate **different** instructional approaches.

2.2.5 Study orientation in mathematics

According to Schmeck (1988:8) the term "orientation" refers to a *consistency in one's approach to learning in school and university setting* and the concept "approach" *stresses the relationship between intention, process and outcome within a specified context.* A person's orientation is thus the *result of a relation between person and environment* (Schmeck, 1988:10). Entwistle (1988) describes "orientation to studying" as the factor that summarises approaches, motives and styles and it includes elements of study methods and attitudes.

Several researchers have shown that there is a statistically significant association between aspects of study orientation in mathematics, such as anxiety, motivation, attitudes with regard to mathematics, the use of effective meta-cognitive strategies in mathematics, effective time management, concentration, the will to achieve in mathematics and the social, physical and experienced environment of learning mathematics on the one hand, and achievement in mathematics on the other (Reynolds & Wahlberg, 1992; Van Aardt & Van Wyk, 1994; Visser, 1988; Wong, 1992).

According to Maree (1997) little attention is given to pupils' (students') study orientation in mathematics when reasons for poor performance in mathematics are sought. Van Aardt and Van Wyk (1994) identify the harmful influence of inadequate study orientation among mathematics students and point out that evidence suggests that the use of effective learning and study strategies is important in determining (academic) success.

Maree (1992) points out that numerous aspects can affect study orientation in mathematics. The formation and acquisition of basic concepts is important because when conceptualisation is incomplete, problem solving in mathematics is inhibited. Students' emotions, their habits and attitudes regarding mathematics and the way in which they process mathematical information influence their study orientation in mathematics. The display of acquired, consistent and effective study methods and doing assignments form an important part of a student's study orientation in mathematics. Study attitude therefore culminates in certain study habits. A student's learning environment (social, physical and perceived), their feelings about mathematics and the way in which they experience their teachers (lecturers) all form an integrated part of a student's study orientation in mathematics.

Entwistle (1988:48) remarks that *by ensuring that students become aware of their own approaches, and the implications of adopting them, it may thus be possible to improve the quality of learning* outcomes. One of the aims of the research reported in chapters five and six of this thesis is to determine the study orientation in mathematics of a group of first year engineering students and to promote an awareness of the individual's own study orientation in mathematics.

2.2.6 Learning styles

In section 2.5 a selection of views on and definitions of styles of learning are discussed. The choice of the examples of those discussed include a selection that can be interpreted from the premise of brain-based learning as the researcher values this principle as point of departure for the learning facilitation strategy defined in this thesis.

2.2.7 Learning and intelligence

> *Intelligence boils down to your ability to know your own strengths and weaknesses and to capitalise on the strengths while compensating for the weaknesses.* (Sternberg in Jensen, 1996:175).

During the twentieth century Binet's model of intelligence dominated the educational world. This model was biased towards determining a fixed 'intelligence quotient' (IQ) that measured and quantified so-called verbal and non-verbal competencies and that was fixed for life. Gardner (1983) diverted from Binet's static model and defines human intelligences as an array comprising seven categories. He states that each individual has his/her own unique combination of these seven categories and that this combination can and does change over life. In Table 2-4 a summary of Gardner's categories for multiple intelligences is given as well as examples of indicators by which the different categories can be identified.

Table 2-4 Gardner's multiple intelligences

Intelligences	Identified by
Logical-mathematical	Strong mathematical and problem-solving skills. Sequential thinking. Handling of quantitative information.
Verbal-linguistic	Sensitivity to the meaning, sound, inflection and order of words. Enjoys reading and talking.
Musical-rhythmic	Appreciation of sounds; sensitivity to rhythm. Difference in interpretation of 'musical messages'.
Spatial	Visual and tactile ability to accurately perceive and act on objects and form in the environment. Strong imagination, read graphics, needs pictures to understand. Capacity to recognise forms, shapes and how they relate and interact. Sensitivity to the balance and composition of shapes.
Bodily-kinaesthetic[14]	Ability to handle objects skilfully, either fine or gross motor movements. Ability to control own movements for function or expression.
Intrapersonal	Enjoys working (being) alone. Has a good understanding of own strengths and weaknesses. Good at goal setting.
Interpersonal	Strong people skills.

Compiled from Sylwester (1995:108-114)

According to Sternberg (in Wiechers, 1993) intelligence and intelligent behaviour can be taught. Using this point of view and for the purposes of this study, the author of this thesis regards Gardner's dimensions of intelligence as **dimensions of potential** and defines dimensions of potential as follows:

- Potential is genetically given (based on nature).
- The development of potential is dependent on nature (physical care and maturation) and nurture (education).
- The development of given potential leads to functional competency.

[14] Sylwester (1995) remarks that bodily-kineasthetic involves the following brain structures: the basal ganglia (at the base of each hemisphere that coordinates the actions of the sensory and motor systems); the amygdala (limbic structures that provide the emotional trigger for movements); the motor cortex (narrow strips above the ears in each hemisphere) and the cerebellum (coordinates and fine-tunes automatic movement patterns). See Figure 2-7 on p 57.

- If an individual's full potential is developed, it implies 'intelligent behaviour' relevant to a specific task in a specific context.
- Intelligence in some dimension is potential that has developed to a functional state.

2.3 Brain-based learning

In this section recent views on learning from a brain-based perspective that emerged from research and insights on the functioning of the human brain are discussed. Sylwester (1995:vii) remarks:

> *Recent dramatic developments in the cognitive sciences are moving us closer to an understanding of our brain's development, organization, and operation. Increased understanding of the brain should lead to widespread discussions of the important issues that will arise out of these advances, and to the development of appropriate and effective educational applications of this knowledge.*

Key findings of research in neuroscience have indicated that *learning changes the physical structure of the brain* and *these structural changes alter the functional organisation of the brain* (Bransford *et al.*, 1999:103). Undoubtedly it can be stated that learning organises and reorganises the brain.

The strategy for learning facilitation of mathematics in a support course proposed in chapter three of the study proposes the principle of brain-based learning. Although it is beyond the scope of this study to give in-depth detail of the physiological functioning of the human brain, an overview is given of some basic structures and processes to provide a broad background for the definition of the mentioned strategy. However, the author endorses a remark by Vander, Sherman and Luciano (2001:358), namely that

> *Physiologists have only a beginning understanding of the brain mechanisms that give rise to mind or conscious experiences.*

2.3.1 Brain basics for education

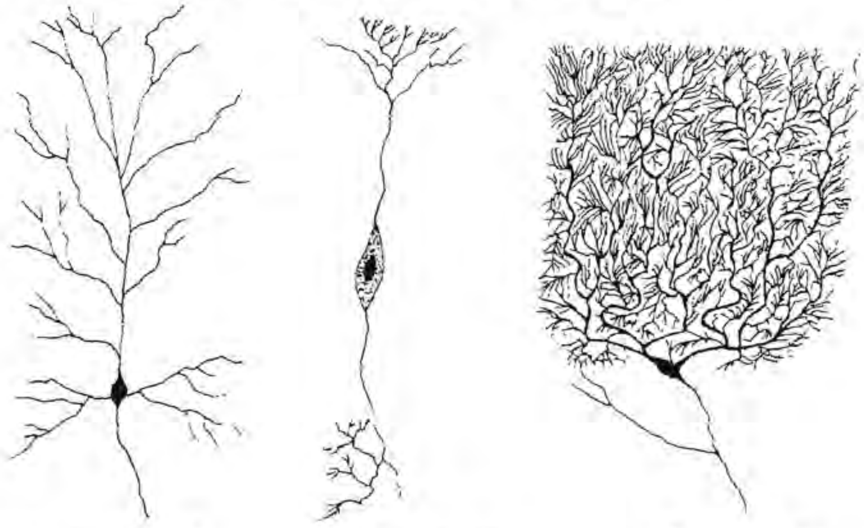
Hannaford (1995:18) points out that *learning proceeds as we interact with the world* and at a physiological level this interaction is communications among neurons. Sensory stimuli promote neurons to form dendrites to other neurons that bring the nerve cells into communication with these other neurons.

Neurons are viewed as the functional units in the brain and nervous system and are classified according to structure and function. Structurally there are three types namely, uni-polar; bi-polar and multi-polar of which the multi-polar type is the most common. In Figure 2-3 three examples of variations of multi-polar neurons in the brain are shown indicating the extensive occurrence of dendrites in some of the examples. Figure 2-4 on the following page illustrates a multi-polar neuron and the functions of the different parts. All of the structures in a neuron are involved in guiding and programming behaviour (Hannaford, 1995).

Functionally, neurons are classified as sensory, intermediate and motor. The sensory neurons bring sensory information to the brain and spinal chord from all over the body. The intermediate neurons form networks of neurons that relay information to other networks of neurons, process information and relay information via dendrites to motor neurons that carry messages from the brain and spinal chord to muscles and glands to activate their function.

Hannaford (1995) points out that by learning, neurons are activated repeatedly and the more this occurs, the more myelin (see Figure 2-4 – myelin sheet occurs at the axon) is laid down. Impulses travel faster in highly myelinated neurons. It follows that the more practise, the more myelin and the faster the processing.

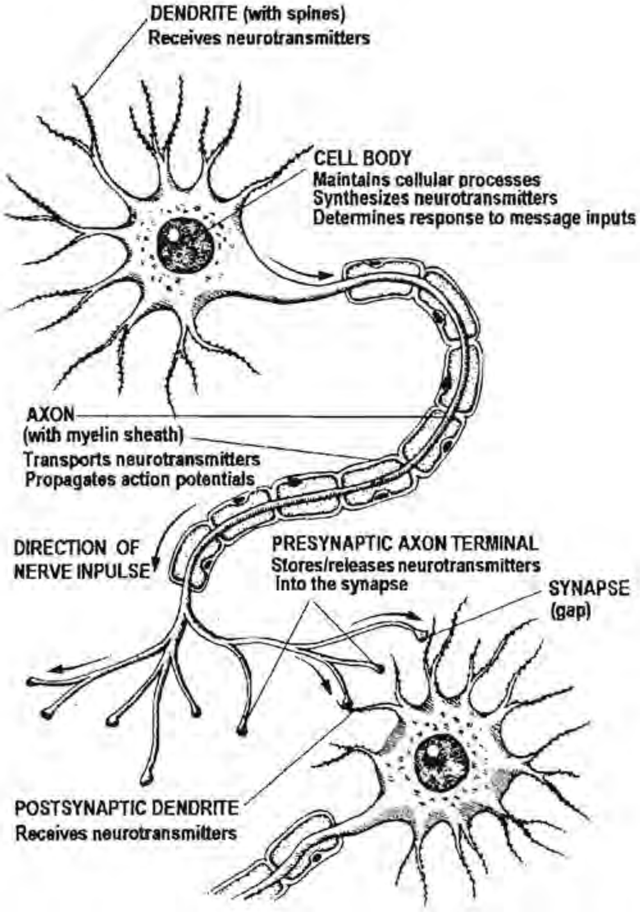
Figure 2-3 Examples of variations in neurons of the brain



cortical pyramidal cell bipolar neuron Purkinje cell

Ornstein and Thompson (1984:62)

Figure 2-4 Functional model of a multi-polar neuron

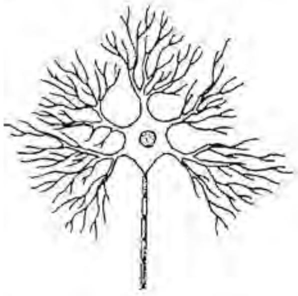


Sylwester (1995:31)

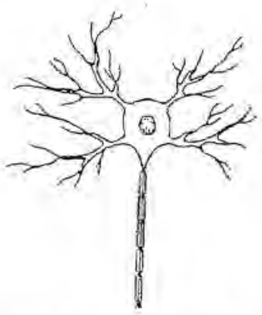
Through connections between neurons, neuronal groups are formed and continual repetitions of this process result in highly complex networks of dendrites. Simplistically put, these complex networks constitute the 'pathways' for activities in the brain. Dendritic branching occurs as a result of stimulation and synaptic connections (see Figure 2-5) are made as new learning occurs and these *linkages are paired down in a specific way that increases efficiency and thought* (Lichtman, Balice-Gordon & Katz in Hannaford, 1994). The magnitude of synaptic connections and the complexity of neuron networks are indicated by the fact that neurons have from 1000 to 10 000 synapses and may receive information from 1000 other neurons (Stevens, 1979, in Hannaford, 1994). Furthermore, it is possible to record neuron activity pulsing at a frequency of 40 to 70 Hz (Vander *et al.*, 2001).

Figure 2-5 Properties of neuron functioning

Figure 2-5 A Dendritic branching



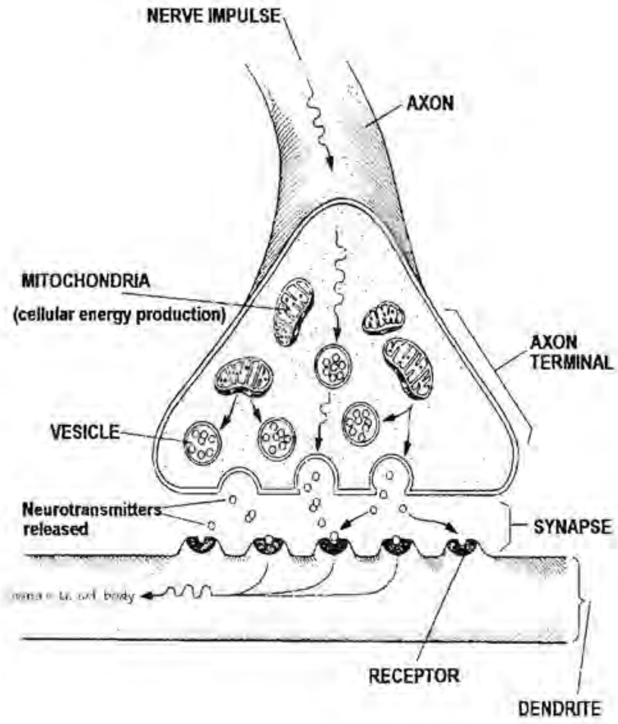
Extensive dendritic branching
(from active challenged minds)



Brain in deficient environment
shows sparse dendritic branching

Jensen (1996:144)

Figure 2-5 B Synoptic area



Sylwester (1995:34)

Crick and Koch (in Vander *et al.*, 2001) point out that conscious experiences require neural processes and that this neural activity is not restricted to a single cluster of 'consciousness neurons' but resides in a set of neurons that temporarily function together in a specific way. As we become aware of different things simultaneously, the particular set of neurons can shift among different parts of the brain dealing with, for example, visual or auditory stimuli; memories or new ideas, emotions or language. Similarly, Sylwester (1995) points out that information flow in the brain is multiplex and brain activity is distributed in the sense that several brain areas combine to process an event or object, even though certain activities are primarily executed in specific brain areas. Figure 2-9 on page 60 illustrates this phenomenon for language based activities.

Haile (1997) points out that learning occurs through the modification of brain structure although it is not yet known how this is done. The brain does not modify itself by growing new neurons but during youth the brain grows in complexity by increasing and refining connections between existing neurons (Ornstein & Thompson, 1984). Restak (in Haile, 1997) points out that the number of neurons is constant through adolescence and degenerates over the remaining lifetime with an average total loss of 10% of the original number. This loss is seemingly insignificant in relation to the brain's capacity. Selkoe (1992:101) remarks that the brain has *considerable physiological reserve to tolerate small losses of neuronal function.* From an educational perspective, Buzan (1991) points out that acquiring and using appropriate skills and practising techniques to aid learning, can contribute to ongoing mental alertness. According to Selkoe (1992:98) the brain is capable of *dynamic remodelling of its neuronal connections, even in the later years.*

Haile (1997) stresses that the brain is a self-modifying neural network and that the functions of the cerebral cortex include describing meaning to and interpretation of external and internal stimuli. He continues and points out that current knowledge of the human brain has implications for the nature of education. Haile (1997) interpreted results of research, done by Calvin, Singer and Searle in the early 1990s, regarding the role of cortical and neural activity in information processing and learning and remarks that these have implications for the nature of education.

Haile's proposals (1997:157) can be summarised as follows:

- **Learning is a natural activity** of the human mind.
- **Memory is a re-creation of information**. To be able to use what they know, students must learn cues that re-create useful neural patterns.
- Since learning creates new structures in the brain by modifying existing structures, **learning can only begin from things the student already knows**.
- The brain modifies neural connections as part of its response in those neurons that are activated to form a pattern. Learning new things amounts to a perturbation of things already known. If the perturbation is too large, then no related neural firing pattern can be created and no learning takes place. Students must be led to **new knowledge in small chunks** of information that allow the brain to modify existing neural networks. **Repetition** is then needed to strengthen new neural connections.
- **Understanding requires interconnected constellations of neurons**. Such elaborate networks allow for quick learning of new things because the vast networks offer numerous nodes that can easily be modified to assimilate new information.
- **Recognition is easier than recall** because recognition forms a meaningful pattern in response to an external stimulus while recall forms a meaningful pattern in response to an internal stimulus.
- **Learning is easier than unlearning.** During learning a neural network is modified to create a new net. Unlearning refers to correcting misunderstanding from earlier learning and involves not only the creation of a new network, but also suppressing the formation (use and strengthening) of the old erroneous pattern.
- **To learn, students must actively participate** in their own education. Only the student can modify his/her own neural networks.
- Quickness of mind versus intelligent thinking. **Intelligent thinking involves the identification of alternatives and choosing from among them**. This cannot necessarily be done quickly.

The author values Haile's views on learning as also being significant for mathematics education. Haile (1997) credibly connects important aspects of learning to neurological research findings. The relevance of Haile's statements above for the model of learning facilitation defined in this thesis is discussed in Chapter 3.

Due to the complex anatomy and physiology of the human brain, brain researchers and scholars focus their study on a specific aspect at one level. Approaches to studying the brain are either on small systems like single cellular brain mechanisms as done by neuroscientists (as in the examples mentioned in the paragraphs above) or a broader approach that focuses on complex cognitive mechanisms as done, for example, by cognitive psychologist (as in the examples noted in sections 2.1, 2.2 and 2.4).

In the following sections different models describing the architecture of the brain are discussed. These models are useful in understanding brain-based learning from an educational perspective but it should be stressed that they remain merely models of a highly complex object like the human brain.

2.3.2 The divided brain

The pioneering work of Sperry, Bogen and Vogel since the 1960s on split-brain patients (Herrmann, 1995 & 1996; Jensen, 1996; Ornstein & Thompson, 1984; Sylwester, 1996) has led to new insights about the human mind in the fields of neuroscience, psychology and anthropology.

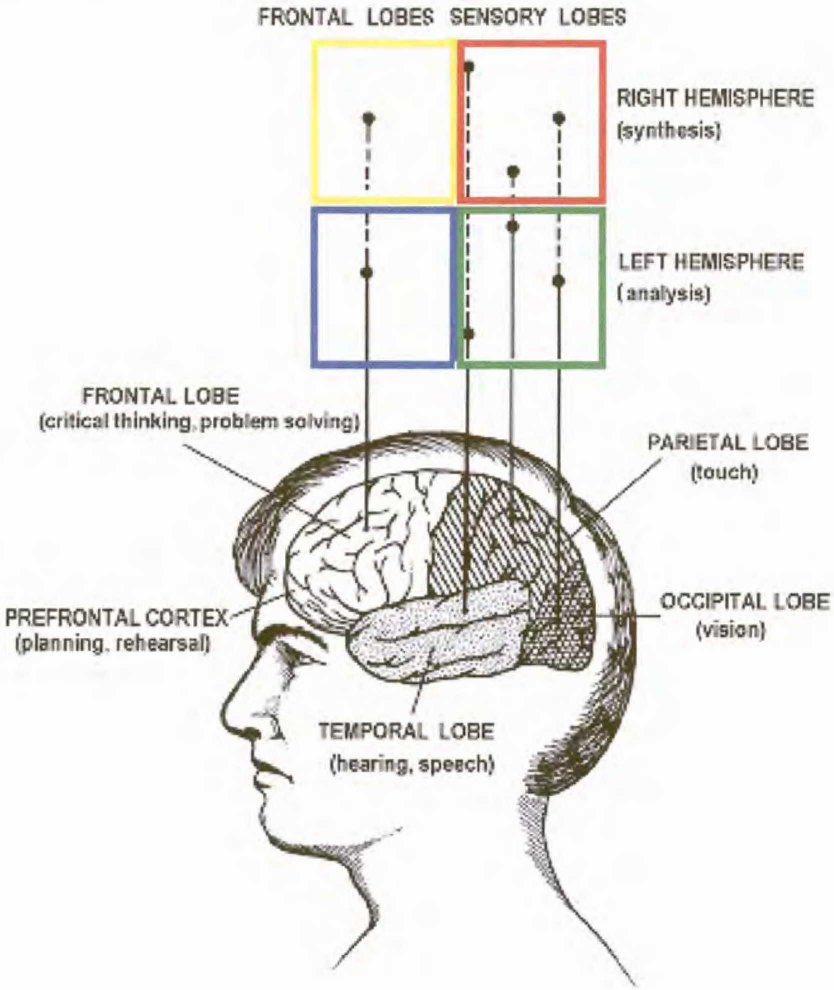
Although the work of Sperry and his colleagues was in the field of neuroscience, the implications thereof have been far reaching. Their research with patients suffering from severe epileptic seizures eventually brought to light the specialised functions of the human brain. In an effort to relieve patients of these seizures, they severed the corpus callosum (see Figure 2-7 on page 57), a thick band of nerve fibres connecting the left and right brain hemispheres. They then designed a series of tests in order to determine the functions of each brain hemispheres. The findings of these tests clearly indicated that unique cognitive functions are associated with each brain hemisphere. The diagram in Figure 2-6 on the following page shows the four brain lobes and a schematic top view of the cortex that is divided into a left and a right hemisphere and each of these is further divided into frontal and sensory lobes.

In the left brain right brain theory both the limbic brain and neocortex consist of a left and a right half. These halves are physically connected by three bundles of axionic fibres enabling the brain to coordinate the activities located in parallel regions of each cerebral hemisphere as well as connecting the activities of each limbic half with the cortex. This ensures left right as well as upper lower connections and coordination of brain function (Herrmann, 1995). Furthermore, these physical connections secure integrated brain activity.

Further research in neuroscience confirmed that functionally, the human brain can also be divided into two hemispheres, each with its own specialised functions, and much of the original findings of Sperry and co-workers remain valid (Gazzaniga, 1998; Jensen, 1994 & 1996; Vander *et al.*, 2001). These specialised functions associated with each hemisphere are listed in Table 2-5 on page 56. It has been established that for most people logical, analytical, quantitative and fact-based knowledge is located in the left hemisphere. The right hemisphere predominantly supports and coordinates intuition, emotion, spatial perception and kinaesthetic feelings.[15] About 90% of the population displays this functional distribution between the left and the right brain hemispheres and the functions are transposed for only 10% of the population (Vander *et al.*, 2001).

[15] The term "kinaesthesia" refers to *the brain's awareness of the position and movement of the body, limbs, etc., by means of sensory nerves in the muscles and joints* (Crowther, 1995:746).

Figure 2-6 The four brain lobes and schematic top view of the cortex



Adapted from Sylwester (1995:48)

Table 2-5 Specialised functions associated with each brain hemisphere

Left hemisphere (right side of body)	**Right hemisphere** (left side of body)
Speech / verbal	Spatial / music
Logical, mathematical	Holistic
Linear, detailed	Artistic, symbolic
Sequential	Simultaneous
Controlled	Emotional
Intellectual	Intuitive, creative
Dominant	Minor (quiet)
Wordly	Spiritual
Active	Receptive
Analytic	Synthetic, gestalt
Reading, writing, naming	Facial recognition
Sequential ordering	Simultaneous comprehension
Perception of significant order	Perception of abstract patterns
Complex motor sequences	Recognition of complex figures

Trotter (1976:219)

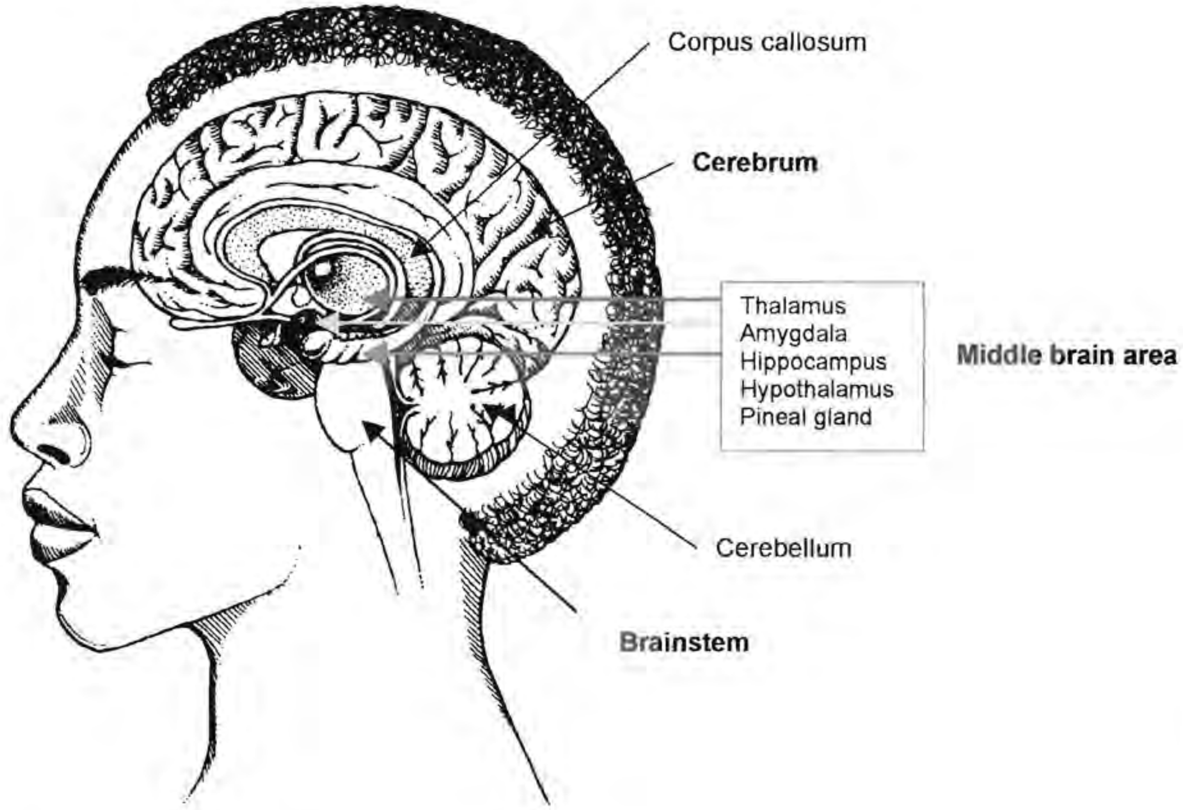
Unfortunately, enthusiasts of the left/right brain hemisphere principle simplistically categorised people as 'left brained' or 'right brained' which, from an educational perspective, resulted in a dichotomous left brain versus right brain view of 'discard the old left-brain way' and 'update to the right brain way' (Jensen, 1994 & 1996), whereas the brain should be understood as an integrated whole.

In addition to Sperry's left brain right brain model, MacLean defined the model of the triune brain (see Figure 2-10 on page 62). The triune model describes the brain as a hierarchy that evolved to process survival, emotional and rational functions. According to the triune model the brain is divided into the primitive reptilian brain, the limbic brain (paleomammilian brain) and the neocortex (neomammilian brain). The neocortex is the centre of higher cognitive activity that is unique to mankind. This model is simple, easily understandable and has been widely used in educational circles (Jensen, 1996; Sylwester, 1995). Jensen (1996:21) feels strongly that the triune model is *outdated, stereotypical and in some cases, dead wrong*. Cytowic (1993, in Sylwester, 1995) feels that the triune model is still useful as a functional metaphor of the brain's organisation but not as an exact model.

Jensen (1996) argues that certain functions of the brain are best understood if a model of the brain is described by a structural division in terms of the lower, middle and upper areas. Jensen describes this division as follows.

- The **lower area** is the brain stem area.
- The **middle brain area** contains the amygdala, hippocampus, thalamus, hypothalamus, pineal gland and other critical areas (Jensen does not specify 'other critical areas').
- The **upper area** is the cerebrum and neocortex covering it (see Figure 2-7).

Figure 2-7 Schematic model of the brain



Adapted from Sylwester (1994:42)

A summary of the functions of the brain according to the three areas proposed by Jensen (1996) is given in Table 2-6 below.

Table 2-6 Functions of the brainstem, middle brain area and cerebrum

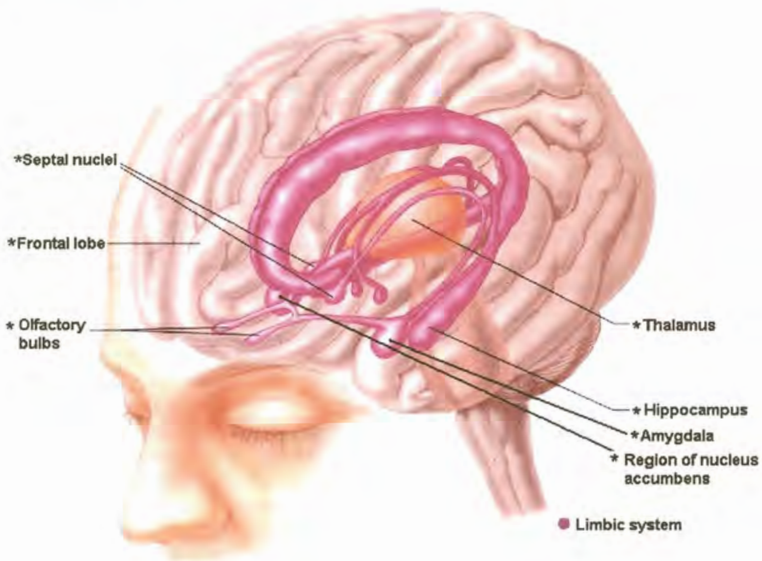
Brain area	Functions / Learner behaviour
Brainstem	Territoriality (defending the own). Social conformity and rituals. Deception (can form subverted aggression). Hierarchy.
Middle brain	Formation of memories. Long-term memory. Immediate expressiveness. Emotions (positive and negative). Sense of space and location. Attention and sleep. Hormones.
Cerebrum	Problem solving and computations. Language, writing and drawing. Thinking, reflection and consciousness. Visualising, envisioning. Reading, translating and composing. Creativity in art, music and theatre. Long range planning, forecasting. Some processing of emotions.

Compiled from Jensen (1996:21-35)

On the one hand Jensen's description of the functional areas of the brain incorporates the aspects of MacLean's triune model but on the other it points toward a 'functional whole brain'. Jensen views brain-based learning with relevance to cognitive, affective and physical aspects and uses the knowledge of the functioning of the brain areas to describe learning in terms thereof.

Selkoe (1992) points out that the limbic system is central to learning, memory and emotion. The diagram in Figure 2-8 on the following page illustrates the limbic system as well as all the brain structures that are involved in attention, emotion and motivation. It should be pointed out that emotion is not restricted to the limbic system but also involves the thalamus and the frontal lobe (Vander *et al.*, 2001). Smith (1998) points out that the formation of long-term memory strongly involves emotion whereas working memory is not related to emotions but to facts. He concludes that educators of mathematics need to stimulate emotional connections to subject matter to aid the transfer to long-term memory.

Figure 2-8 The limbic system and brain structures involved in attention, emotion and motivation



Adapted from Vander *et al.* (2001:215 & 362)

Recent research continues to point to the task specialised functioning of the brain also with regard to mathematical cognition. Using brain imaging techniques, Dehaene and Spalke (1999) report that approximate mathematical calculations take place in the brain's large-scale network involved in visual, spatial and analogical transformations. Rote arithmetic takes place in an area usually reserved for verbal tasks. Dehaene and Spalke also found that that these different kinds of mathematics problems were instantaneously assigned by the brain to their respective areas. These findings suggest that specific neuron circuits complete both the calculation itself and the decision to perform it.

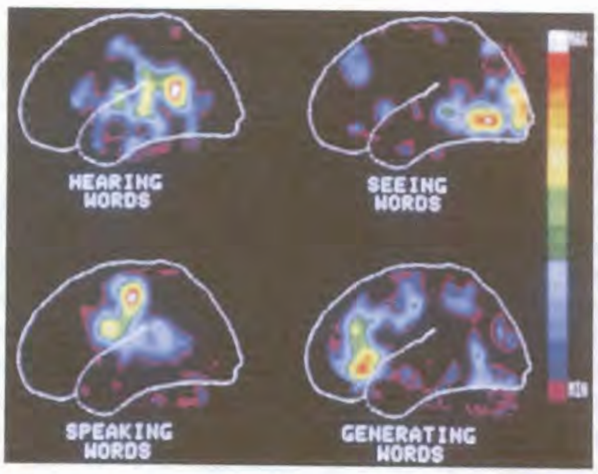
In the following section the focus is on research that indicates that the brain functions as a complex whole.

2.3.3 The integrated brain

Since the early 1990s the development of modern technology has undoubtedly contributed to further understand the complex functioning of the human brain. Computer assisted tomography (CAT) and magnetic resonance imaging (MRI) can distinguish minute brain structures; positron emission tomography (PET) and single photon emission computerised tomography (SPECT) track bloodflow (a sign of brain activity), whereas super conducting quantum interferences devices (SQUID) pick up magnetic fields (also a sign of brain function).

In Figure 2-9 PET scans reveal changes in blood flow during various language-based activities. Although similar images for distributed brain activity when doing mathematics are not readily available, the mentioned research by Dehaene and Spelke (1999) probably showed comparable images.

Figure 2-9 PET scans of different language based activities



Vander *et al.* (2001:368)

There is undoubtedly agreement that the brain is highly specialised and that different aspects of thought and action are done in different parts of the brain. In addition, there is also consensus that the brain operates simultaneously on many levels and that events occurring in one hemisphere can and do influence events occurring at the same time in a very different remote part of the other hemisphere.

Crick (in Jensen, 1996:9) affirms that the functions of the brain are *usually massively parallel* and Levy (in Jensen, 1996:14) points out that *both sides of the brain are involved in nearly every human activity.*

Much speculation has been done on how much of the brain's capacity we use. Figures quoted range from less than 2% to 20 % (Buzan, 1991; Lumsdaine and Lumsdaine, 1995a; Bransford *et al.*, 1999). These speculations are based on findings from early experiments recording electroencephalogram (EEG) that measured brain activity when subjects performed different tasks (Herrmann, 1995; Ornstein *et al.*, 1984). The author is of the opinion that the use of new technologies to reveal brain activity may change these figures significantly if PET scans as in Figure 2-9 could be used to show (distributed) brain activities when cognitive tasks are performed.

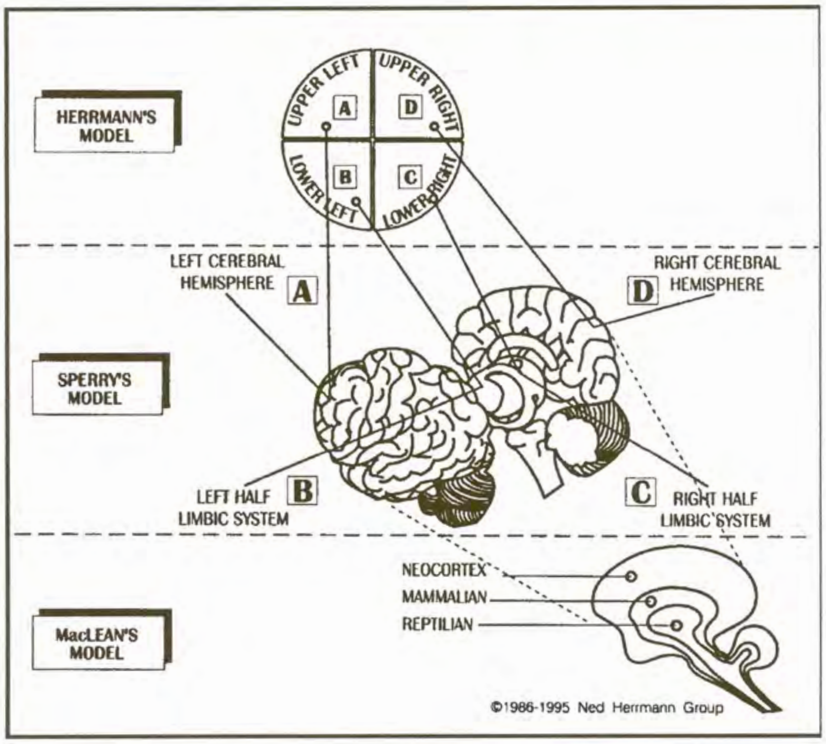
2.3.4 The Herrmann four quadrant whole brain model

In the early 1980s Herrmann (1995; 1996) combined Sperry's left-brain right-brain model of the functioning of the brain with theories of how the human brain is physiologically organised to develop a metaphoric whole brain model. Figure 2-10 illustrates how the four-quadrant concept is embedded in the left/right hemisphere brain theory as well as in the triune brain theory.

Herrmann (1995) regards the four quadrants (A, B, C and D) identified in his model as interconnected clusters of specialised mental processing modes. He points out that they function together 'situationally' and 'iteratively'.[16] Figure 2-11 gives more detail regarding the cognitive modes related to each quadrant.

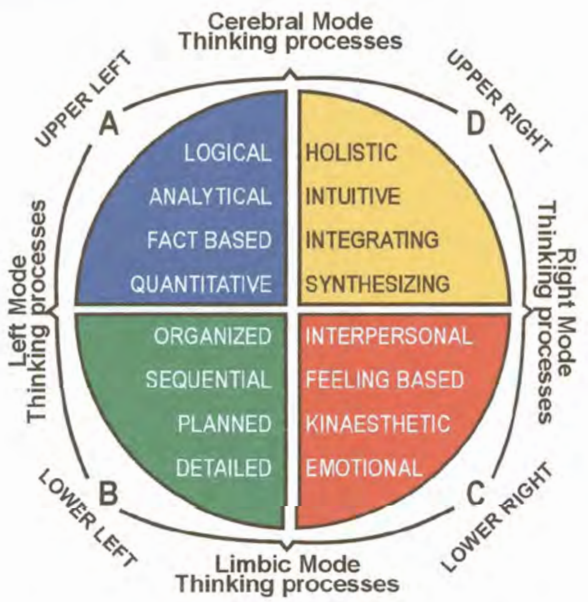
[16] The author assumes that the term "situationally" refers to *in context* (Crowther, 1995:1298) and the term "iteratively" refers to *repeatedly* (Crowther, 1995:723) indicating continuous activity in both hemispheres.

Figure 2-10 The whole brain model of Herrmann and its physiological roots



Adapted from Herrmann (1995:40 & 64; 1996:14)

Figure 2-11 The Herrmann whole brain model



Adapted form Herrmann (1995:411)

According to Hermann the upper cerebral modes function on a higher cognitive level than the lower limbic modes. The dominant processes in the left hemisphere include logic, analysis, words, numbers, sequence, organisation, detail. The dominant processes in the right hemisphere include holistic conceptualisation, synthesis, integration, intuition, imagination, colour, emotions, kinaesthetic feelings, spatial awareness, rhythm and dimension (Herrmann, 1995; Buzan, 1991). Herrmann (1995, 1996, 1998) identifies eight key characteristics of the normal human brain. These are summarised in Table 2-7.

Table 2-7 Key brain characteristics according to Herrmann

The brain is	This means that
Unique	Every human being's brain is as unique as a fingerprint.
Specialised	The human brain is specialised in obvious ways, as in the ability to talk, listen, see, feel, touch, etc. It is also specialised in how we think – the logical processes, procedural processes, emotional processes, the ability to see the whole picture, to think intuitively as well as analytically. All human beings have these specialised modes.
Situational	As one engages in a particular activity that requires, for example, talking or thinking mathematically, the part of the brain that is specialised to perform that function becomes engaged and active in it as required.
Interconnected	The specialised structures of the normal brain are interconnected by millions of axionic fibres. This fact disavows the oversimplification of 'left brain/right brain'.
Iterative	The interconnections in the normal brain provide the structure for the brain to alternate between specialised modes. One can think logically and rationally and also intuitively and conceptually and move back and forth between these separate modes.
Dominant	Dominance of paired structures occurs naturally. Similarly, for the pairs of structures in the brain one becomes dominant with respect to the other.
Malleable	The normal brain is so adaptable that there are virtually no inherent constraints.
Whole	The structures and interconnections all provide the basis for the normal brain to function in its entirety.

Compiled from Herrmann (1995, 1996, 1998) and Herrmann International (1999a).

It should be borne in mind that although Herrmann's model is based on the divisions of the physical brain, it is a metaphoric model. However, the Herrmann model remains a useful way to describe learning and learners. The principles underpinning Herrmann's model is used a point of departure for the learning facilitation strategy proposed in chapter three. Furthermore, the key characteristics listed in Table 2-7 underscores the premise that the mathematics potential of a tertiary learner on a support course can be developed.

2.4 Recent views on learning and learners

Based on the report by the Committee on the Developments in the Science of Learning in the USA (Bransford *et al.*, 1999) the following three core learning principles emerged from new developments in the science of learning.

1. Students come to the classroom with **pre-existing knowledge.**

The initial learning phase of any content (subject) is based on knowledge that people bring to the learning situation. The importance of building on previous experiences and working with existing understandings is relevant and important for learners of all ages. (Bransford *et al.* 1999).

2. To develop competence in an area of inquiry, students must have a **deep foundation of factual knowledge**, understand **facts and ideas** in the context of a **conceptual framework** and **organise knowledge** in ways that facilitate **retrieval and application** (Donovan, Bransford & Pellegrino, 1999).

The mentioned two principles for developing competence emerged from research that compared the performance of novices and experts and from research on learning and transfer. Bransford *et al.* (1999) point out that although it cannot be expected that all learners should become experts, the study of expertise shows what the results of successful learning look like. According to them, research has identified key principles of experts' knowledge that can have implications for learning and instruction. The author of this thesis is of the opinion that of these principles, the following have significance for the learning facilitation of mathematics in a support course:

- Experts notice features and meaningful patterns of information that are not noticed by novices. Research on expertise suggests the importance of providing students with learning experiences that specifically enhance their abilities to recognise meaningful patterns in information. Processing of 'patterns' forms a significant aspect of the strategy for learning mathematics that is proposed in chapter three of this thesis.

- Experts have acquired a great deal of content knowledge that is organised in ways that reflect a deep understanding of their subject matter. Donovan *et al.* (1999:12) add that an *expert's command of concepts shapes their understanding of new information.* In mathematics, experts are more likely than novices to first try to understand problems rather than simply attempt to use formulas to derive an answer (Bransford *et al.* 1999). In organising curricula, care should be taken not to overemphasise or superficially cover facts, but to give learners enough time to develop important organising concepts and promote conceptual understanding. Donovan *et al.* (1999:12) also remark that

  > *The ability to plan a task, to notice patterns, to generate reasonable arguments and explanations, and to draw analogies to other problems are all more closely intertwined with factual knowledge than was once believed.*

- Experts' knowledge cannot be reduced to sets of isolated facts or propositions but, instead, it includes a specification of the context in which it is useful. For mathematics learning in introductory calculus the implication hereof is that learners must thoroughly understand the fundamental concepts of calculus for use as reference.

- Experts are able to flexibly retrieve important aspects of their knowledge with little additional effort. Bransford *et al.* (1999) point out that people's ability to retrieve information can vary from being 'effortful' to 'relatively effortless' (fluent) to 'automatic'. They continue to say that fluency is important because effortless processing places fewer demands on conscious attention. Applying this principle to mathematics learning in introductory calculus means that learners need to acquire the knowledge of the fundamental concepts of calculus in order to access and use these (at least) fluently.

The third core principle that emerged from research on learning and learners and noting the differences between experts and novices is that

3. A **metacognitive approach** to instruction can **help students learn to take control of their own learning** by defining learning goals and monitoring their progress in achieving them. (Donovan *et al.*, 1999).

According to Bransford *et al.* (1999) experts have varying levels of flexibility in their approach to new situations. This provides an important model for learning in the sense that experts approach new situations flexibly and learn throughout their lifetimes. This presupposes a metacognitive awareness of own performance. This principle thus endorses the view expressed on metacognition in section 2.2.2 on page 40.

It seems appropriate that facilitation of learning should also encourage the development of flexibility in approach and thus foster adaptive expertise. Applied to the current study, this view is congruent with the principle to 'develop a learners' mathematics potential' encompassing the ability to learn and do mathematics using all thinking modes as defined in the proposed strategy for learning facilitation of mathematics.

2.5 Learning styles

This section by no means attempts to give a comprehensive account of learning style models that are currently available. The following discussion only gives an overview of thinking and learning styles that contribute to the strategy for learning mathematics proposed in chapter three of this thesis.

Schmeck (1988) describes the term "style" as an inclination to use the same strategy in varied situations and Entwistle and Ramsden (1983) uses "style" to refer to a stable, trait like consistency in one's approach to attending, perceiving and thinking.

In the following sections the Herrmann Brain Dominance model, the Lumsdaine and Lumsdaine model and the Felder Silverman model concerning thinking and learning styles as well as lateral dominance profiles are discussed. The Herrmann Brain Dominance (HBD) model and the Lumsdaine and Lumsdaine model for thinking and learning styles were selected because of their strong relevance to the principle of brain-based learning. The Felder Silverman model was chosen because it emerged from experiences specifically with engineering students. Although the effect of lateral dominance on thinking and learning styles was not researched in this study, the author is of the opinion that this aspect is worth referring to as it can contribute to improve the learning facilitation of mathematics.

2.5.1 The Herrmann model

Herrmann used his metaphoric whole brain model (section 2.3.4) as a basis for describing thinking style preferences. He sees thinking styles as a 'coalition of four different thinking selves' (Herrmann, 1996). These four different selves are shown in Figure 2-12 A on the following page.

According to Herrmann people develop preferred modes of thinking that can be related to use of the dominant structures in the brain. Lateral dominance (between brain hemispheres, eyes, ears, hands and feet) is innate and influences the way in which the brain and body handle and process information. In the case of hands and feet these differences are quite obvious and the more frequent and repeated use of, for example, the dominant hand, makes it stronger and more capable. Herrmann argues (1995, 1996, 1998) that in the case of the brain, one's preference to think in particular ways results in more frequent use of a particular brain area with the resulting development of greater competency for the set of mental activities that are located in that specialised area. Herrmann continues to say that at birth, the brain is without developed preferences and is therefore essentially 'whole'. As a result of an individual's life experiences, the brain acquires preferences as it matures. The developing brain can be seen as an evolving coalition of many different preferences. The degree of a one's preference for each of the four styles is unique to every person. Herrmann interprets these preferences from different perspectives. Using the perspectives as in Figure 2-12 A and in Figure 2-12 B on the following page, Herrmann describes learners as **analysers**, **organisers**, **personalisers** and **visualisers**.

Figure 2-12 The Herrmann model for learning styles

Figure 2-12 A Different selves

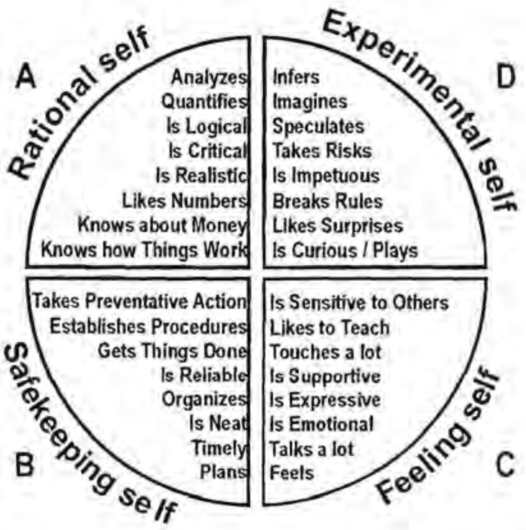
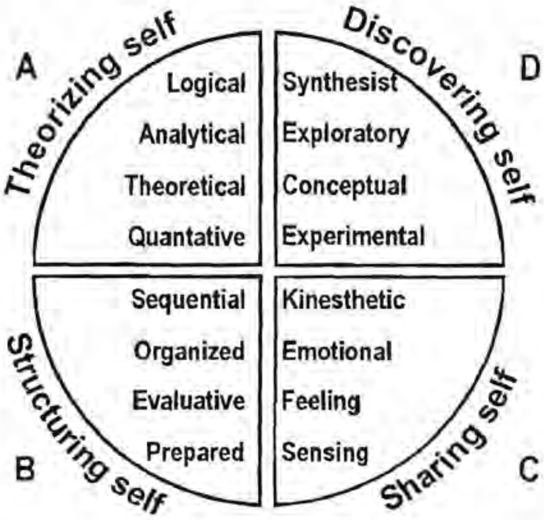


Figure 2-12 B Learning selves



Herrmann (1996:30) and Herrmann International (1999a)

According to Herrmann (1995; 1996):

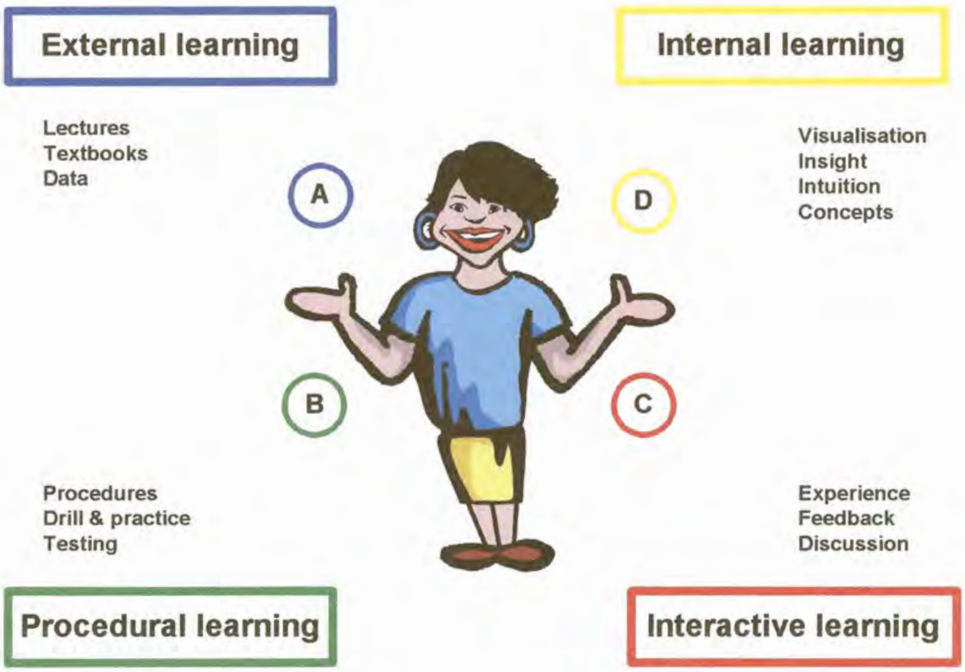
- The A-quadrant **analyser** is characterised by logical thinking, analysis of facts and processing numbers.
- The B-quadrant **organiser** is characterised by planning approaches, organising facts and detailed review.
- The C-quadrant **personaliser** is interpersonal, intuitive and expressive.
- The D-quadrant **visualiser** is imaginative, thinks in big pictures and can conceptualise.

2.5.2 The Lumbsdaine and Lumbsdaine model

Applying the principles of Herrmann's model Lumsdaine and Lumsdaine (1995a) and Lumsdaine, Lumsdaine and Schellnut (1999) distinguish four modes of how tertiary students learn, namely **external** learning, **internal** learning, **interactive** learning and **procedural** learning. An adapted version of the Lumsdaine and Lumsdaine model is illustrated in Figure 2-13 on the following page. The connection of the Lumsdaine and Lumsdaine model with the Herrmann model is indicated in Figure 2-13 and outlined in the following definitions.

- **External learning** is related to teaching from authority through lectures and textbooks. This is predominantly A-quadrant learning in terms of the Herrmann model.
- **Internal learning** is related to learning through insight, understanding concepts globally and intuitively, synthesis of data and personalising content into context. This is predominantly D-quadrant learning in terms of the Herrmann model.
- **Interactive learning** comes from experience, hands-on activities, discussion and feedback (with an opportunity for encouragement). Interactive learning is predominantly C-quadrant learning.
- **Procedural learning** is characterised by a methodical approach, practice and repetition to improve skills and testing. Procedural learning is predominantly B-quadrant learning in terms of the Herrmann model.

Figure 2-13 Lumsdaine and Lumsdaine's four modes of student learning



Adapted from Lumsdaine and Lumsdaine (1995a:97); Lumsdaine *et al.* (1999:63)

2.5.3 The Felder Silverman model

Felder and Silverman (1988:674) point out that *learning in a structured educational setting may be thought of as a two-step process involving the reception and processing of information.* They continue and indicate that reception involves external as well as internal information and processing encompasses memorisation, reasoning and some kind of action. The author of this thesis is of the opinion that this process can also be described as a three-step process of reception, processing and action.

Felder *et al.* (1988) synthesised findings from a number of studies and formulated a learning style model with five dimensions that are particularly relevant to engineering (and science) education. The five dimensions of this model are listed in Table 2-8. Felder *et al.* point out that the dichotomous learning style dimensions of the model are continua and not either/or categories (Felder, 1993).

Table 2-8 Categories of the Felder Silverman model

1	A learner's preference for the type of information that is perceived:	
	Sensory (external) through sights, sounds, physical sensations	**Intuitive** (internal) through possibilities, insights, hunches
2	The modality through which sensory information is effectively perceived:	
	Verbal through words and sounds	**Visual** through pictures, diagrams, graphs, demonstrations
3	How the learner processes information:	
	Reflectively through introspection	**Actively** through engagement in physical activity or discussion
4	How the learner progress towards understanding:	
	Sequentially in continual steps	**Globally** in large jumps (holistically)
5	The organisation of information that is most comfortable to the learner	
	Deductive - principles are given, applications deduced	**Inductive** -observations are given, principles are inferred

Compiled from Felder (1993), Felder *et al.* (1999), Felder *et al.* (1988)

According to Felder (1993) and Felder *et al.* (1988), **sensors** like facts, data, detail, repetition and well-established procedure. They are careful in what they do but may be slow. **Intuitors** like principles, complexities, words and symbols and are quick but may be careless. **Verbal learners** get more information from written and spoken words and mathematical formulas whereas **visual learners** prefer information given in graphs, diagrams, schematics, etc. **Reflective learners** do much of their processing introspectively by thinking things out on their own. **Active learners** like to work in groups and to experiment. **Sequential learners** progress linearly working in small steps and are good at analysis. **Global learners** progress through intuitive leaps and are good at synthesis.

2.5.4 Dominance Profiles

The concept of a profile that depicts lateral dominance and that can be used as a learning style assessment system originated in the early 1970's from studies by Dennison with so-called 'learning disabled' children in which he found that deficits mostly occurred in physical/perceptual abilities (Dennison & Dennison, 1997).

The dominance profiles described by Dennison and Dennison (1997) and Hannaford (1995 & 1997) identify the lateral dominance of eyes, ears and hands in relation to the genetic dominant brain hemisphere. Hannaford (1997:10) points out that a person's lateral dominance is basically innate and that these patterns of lateral dominance *greatly influences the way that information is internally processed by an individual and consequently the kind of learning activities he or she prefers.*

Hannaford (1995, 1997) also points out that a lateral dominance profile gives information about how a person takes in, assimilates and processes sensory information and then how a person responds to and express new learning. A person's sensory intake is ideally facilitated when the dominant eye, ear, hand and foot are on the opposite side of the body from the dominant brain hemisphere. However, Hannaford emphasises (1997) that it is necessary to use both hemispheres of the brain to optimise proficiency at anything.

In section 2.3 of this chapter it was mentioned that increased neuron activity stimulates the branching of dendrites and the more branching occurs, the more connections are formed and the faster processing can occur. Hannaford (1997:18) points out that this principle also applies to the connections between the two brain hemispheres and that the faster the processing between the hemispheres is, the more *intelligently we are able to function.*

Hannaford (1995:112) notes that *every learning situation deals basically with the same steps: sensory input, integration and assimilation, and action.* In order to facilitate the perceptual and physical functions necessary for learning, Dennison developed a set of so-called Brain Gym[17] exercises. According to Dennison and Dennison (1997) Brain Gym exercises are aimed at neural and cortical activation and increasing the supply of oxygenated blood to the brain. Brink (in Hannaford, 1997:112) points out that *a coordinated series of movements produces increased neurotrophins (natural neural growth factors) and a greater number of connections among neurons.*

It should be noted that Dennison and Dennison's Dominance Profiles and Brain Gym exercises were not included in the research activities reported in this study.

2.6 Profile of a learner

The diagrams in Figure 2-14 summarises the view proposed by the author of this thesis concerning the facets (characteristics) regarding a learner of mathematics enrolled for a support course in the first year of tertiary study. For the purposes of this study Woolf's definition (1977:410) the term "facet" is accepted, namely that "facet" indicates *any of the definable aspects that make up a subject (as of contemplation) or an object (as of consideration).*

The first facet concerns the (mathematics) potential of a learner. The second facet represents the way in which humans think and learn. The third facet concerns the way in which information is processed. The third facet is recognised but was not researched in the study reported in this thesis. Study orientation in mathematics is regarded as the fourth facet. These four facets are embedded in the learner's total environment comprising further multiple facets including aspects related to the learner's physical, social, academic and political circumstances. It inevitably follows that although these facets are identified, their composition is uniquely defined for every individual learner.

[17] Brain Gym is a registered trademark of the Educational Kinesiology Foundation, California, USA.

Furthermore, the author of this thesis is of the opinion that the development of the (mathematics) potential of a learner could be viewed in the light of Maslow's hierarchy of needs. This hierarchy (Schunk, 1996:289-290), consists of *physiological needs* (the lowest level), *safety needs*, *belongingness (love) needs*, *esteem needs* (including self-esteem and esteem from others) and *self-actualization* (the highest level). Maslow (in Schunk, 1996:290) points out that self-actualisation is *the ongoing actualization of potentials, capacities and talents* (in mathematics) and can only be achieved once the lower-level needs have been met. Although the author of this thesis acknowledges that unmet lower-level needs can have an effect on the development of a learner's mathematics potential, this aspect was not explicitly addressed in this study.

The comprehensive profile comprising the characteristics of a learner of mathematics in a support course is illustrated in Figure 2-15. This profile should be interpreted as a dynamic profile in the following way. In the second facet (whole brain learning and thinking learning and thinking) the left side (the sections logical/factual and sequential/structured) and the right side (the sections holistic/conceptual and explorative/intuitive) can be transposed. This is to accommodate the 10% cases of the population where linear cognitive processing is located in the right brain hemisphere and global cognitive processing is located in the left brain hemisphere.[18] To ensure that the positioning (alignment) of the second facet is correct, namely that both sections combined and identified as "linear" and both right sections combined as and identified "global", a triangle and circle are used as markers. The top triangles and bottom circles should always be aligned. The third and fourth facets, representing the information processing modes and aspects of study orientation respectively, can each rotate independently. Each of the aspects in all the facets can be thus aligned (or not aligned) to another.

It follows that the possible combinations of alignment between the aspects in the different facets of this learner profile are multiple. In a way this illustrates the complexity of how a learner's preferences and competencies may be matched or mismatched. One can only speculate about the influence that the degree of such matching or mismatching can have on learning and learning facilitation of mathematics in a support course on first year tertiary level.

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[18] See section 2.3.2

Figure 2-14 Facets describing learners of mathematics in a support course

First facet: Learner potential

The learner enrolled for a support course has the potential to succeed in a first semester standard calculus course. The dimensions of potential defined in this chapter[19] are thus reaffirmed. These include that

- potential is genetically given
- the development of potential is dependant on physical care as well as on education
- developed potential leads to functional competency
- developed potential (in mathematics) implies intelligent behaviour (in mathematics).

Second facet: Four quadrant whole brain

The learner is viewed as somebody who has a preferred way of thinking and learning. For the purposes of the research reported in this study thinking preferences are mainly categorised according to the Herrmann four quadrant whole brain model.

A four quadrant whole brain approach entails not only accommodating one's preferences for specific cognitive modes but also developing less preferred modes for thinking and learning.

Third facet: Information processing

The third facet represents the modes by which information is processed. These modes are acknowledged and briefly discussed in this chapter but were not researched in this study.

Fourth facet: Study orientation

This facet focuses on six aspects pertaining to study orientation in mathematics. These aspects include study attitude, mathematics confidence, study habits, problem solving behaviour, study environment and information processing.

Proposed and compiled by the author of this thesis.

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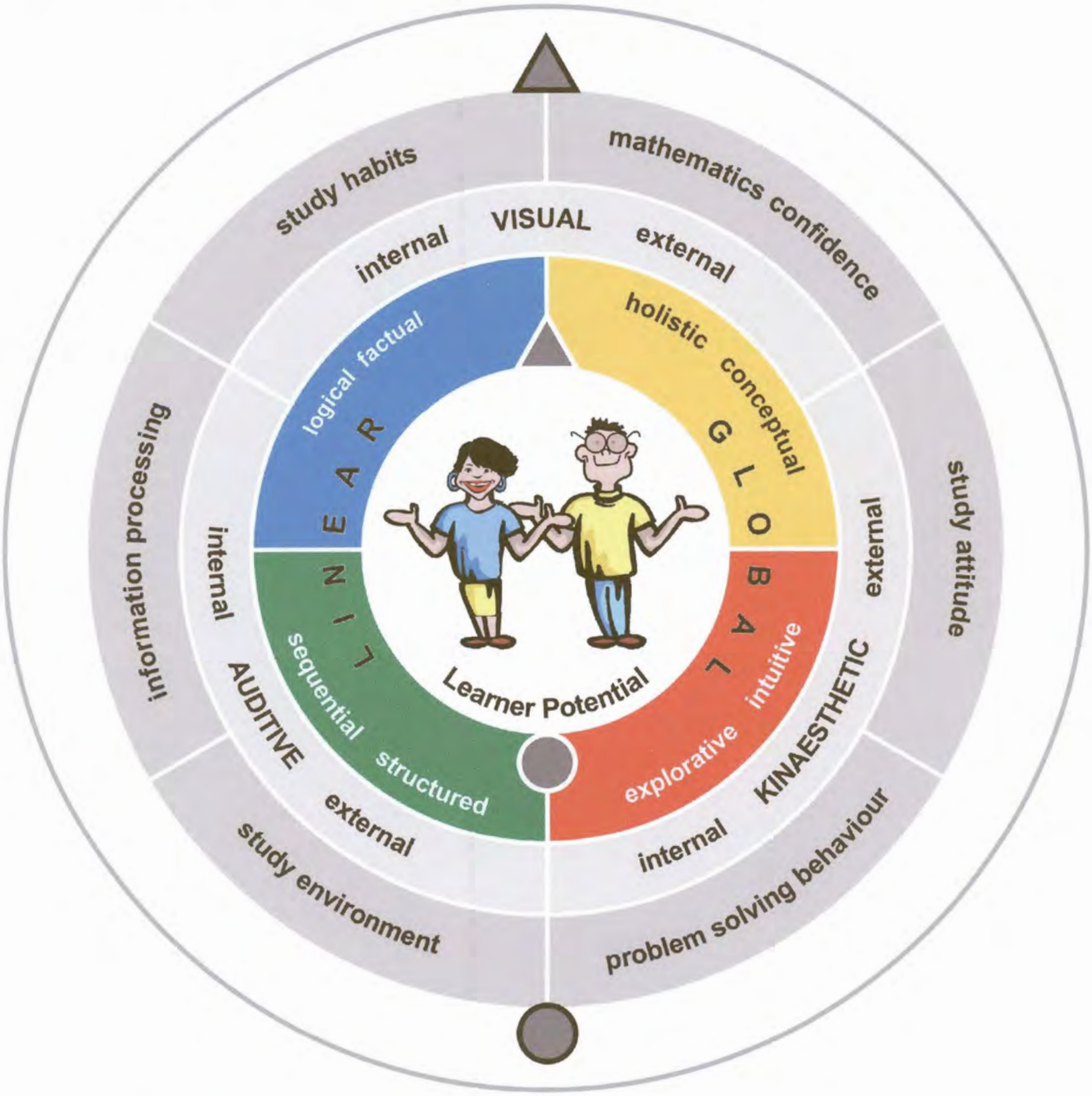
[19] See page 46.

Figure 2-15 Profile of learners of mathematics in a support course



Proposed and compiled by the author of this thesis.

During the research conducted in this study, the learners (participants in the study) were viewed according to the profile proposed in Figure 2-15. Against this background and within the context of the aspects of learning addressed in this chapter, a strategy for the learning facilitation of mathematics in a support course is proposed in the following chapter.

2.7 Summary

The extent of the complexity of human learning is such that it cannot be described by a single definition. It encompasses more than the physical, functional, psychological, developmental and social aspects of an individual. These aspects can be (and have been) described and interrelated by numerous researchers. Justly, Cross (1998:1, 8) remarks that

> *We have more information on learning available to us than ever before in the history of the world, and the amount of research on learning is escalating at an alarming rate. ... Looking carefully at how even one student learns is often quite revealing. ... Every student ... has a lesson to teach us about how students learn. ... If we are to take learning seriously, we will need to know what to look for (through research), to observe ourselves in the act of life-long learning (self-reflection), and to be much more sensitively aware of the learning of the students that we see before us everyday.*

A first year engineering student enrolled for a support course (as is the case with the POSC students who participated in this study) can be viewed as a developing learner that has the potential of multiple intelligences. Traditional theories of learning alone can no longer be used to describe the cognitive activity of such a learner in the early 2000s. The learner should holistically be viewed as an active, creative, rational and emotional human being who is part of a personalised social environment but engulfed in a new tertiary academic environment.