Chapter 5

Overview of Reduced-Form Models

5.1 Introduction

Reduced-form models typically take as the basic ingredients the behaviour of default-free interest rates, the fractional recovery of defaultable bonds at default, as well as a stochastic intensity process λ for default. The intensity λ_t may be viewed as the conditional rate of arrival of default. For example, with constant λ, default is a Poisson arrival. In these models, the intensity process and recovery rates are modelled exogenously and hence the need to directly model the assets of the firm and understand the priority structure of the firm's funding is eliminated. The reduced-form models have been implemented in a commercial software package. The model is called *Credit Risk+* and it was developed by Credit Suisse Financial Products as a tool for the portfolio management of credit risk. In this model a default is triggered by the jump of a Poisson process whose intensity is randomly drawn for each debtor class.

A reduced-form model requires characterization of the following:

1. Issuer's default process (and/or corresponding intensity process).

2. Recovery process.

3. Default-risk-free interest rate process.

4. Correlation between the default-risk-free interest rate process and the default process.

Our investigation focuses on two reduced-form models that are designed to price default-risky bonds: Jarrow, Lando and Turnbull (1997), and Duffie and Singleton (1999). Before reviewing these two models, we will develop a pricing formula for a general contingent claim (that also includes the possibility of default), U. Following Duffie and Singleton's development, we define a defaultable claim to be a pair $((X,T),(X',T'))$ where the issuer is obligated to pay X (possibly a random variable) at time T. The second part of this pair says that T' is a (exogenously specified) stopping time at which the issuer defaults and claimholders receive X' (exogenously specified recovery). This means that a contingent claim (Z,τ) generated by a defaultable claim $((X,T),(X',T'))$ is defined by

$$\tau = \min(T,T'); \quad Z = XI(T' > T) + X'I(T' \leq T) \tag{5.1}$$

where τ is a stopping time at which Z is paid.

Under the assumption of arbitrage-free markets, there exists an equivalent martingale pricing measure $\widetilde{P}$ relative to the short-rate process r. We also assume that Z is F_τ measurable (which allows us to assume that Z can be determined given the information up to and including τ). This means, under the pricing measure $\widetilde{P}$, the price process for any contingent claim U described by (Z_τ,τ) is defined by $U_t = 0$ for $t \geq \tau$ and

$$\begin{aligned} \frac{U_t}{e^{\int_0^t r_u du}} &= \widetilde{E}\left[\frac{Z}{e^{\int_0^\tau r_u du}} \mid F_t\right] \\ U_t &= \widetilde{E}\left[e^{-\int_t^\tau r_u du}(XI(T' > T) + X'I(T' \leq T)) \mid F_t\right] \end{aligned}$$

$$
\begin{aligned}
&= \widetilde{E}\left[e^{-\int_t^T r_u du} X I(T' > T) + e^{-\int_t^{T'} r_u du} X' I(T' \leq T) \mid F_t\right] \qquad (5.2)\\
&= \widetilde{E}\left[e^{-\int_t^T r_u du} X I(T' > T) \mid F_t\right] + \widetilde{E}\left[e^{-\int_t^{T'} r_u du} X' I(T' \leq T) \mid F_t\right]
\end{aligned}
$$

When the interest rate process, r_u, the default process T' and the recovery process X' are specified, equation (5.2) fully characterizes the price of the contingent claim U. Also, the differences between reduced-form models are due to their assumptions for the processes followed by these parameters.

5.2 Jarrow, Lando and Turnbull (1997)

Jarrow, Lando and Turnbull (henceforth, JLT) present an arbitrage-free model of credit risk which characterizes the default process as a finite state Markov process in the firm's credit ratings. The authors begin the construction of their model by assuming that the markets for risk-free and risky debt are complete and arbitrage-free. The JLT model has three important characteristics:

- Different seniority debt for a particular firm can be modelled by assuming different recovery rates in the event of default.
- It can be combined with any default-free term structure model.
- Pseudo-probabilities (martingale, risk adjusted) for valuation are determined from historic transition probabilities for different credit rating classes.

For implementation of this model, the authors impose one major simplifying assumption. It is assumed that the process of the default-free term structure and the firm's bankruptcy (or more generally, financial distress) are statistically independent under the pseudo-probabilities. This means the Markov process for credit ratings is independent of the level of interest rates. The authors reference studies that show that while this assumption may hold for investment grade debt, it is not feasible for speculative grade debt.

The authors assume that default-risky discount bonds pay one dollar at maturity if there is no default, and pay $\delta < 1$ dollars at maturity in the event of default. δ represents the recovery rate on the bond and is taken to be an exogenously given constant. Under the JLT model, default-risky discount bonds are valued as follows:

$$v(t,T) = p(t,T)(\delta + (1-\delta)\widetilde{Q}_t(\tau^* > T)) \tag{5.3}$$

where $p(t,T)$ is the time t price of a default-free discount bond, $v(t,T)$ and is the time t price of a default-risky discount bond. τ^* represents the random time at which bankruptcy occurs and $\widetilde{Q}_t(\tau^* > T)$is the probability (under the martingale measure) that default occurs after date T . From equation (5.3) we see that the term structure of default-risky debt will be uniquely determined by specifying a distribution for the time of bankruptcy under the pseudo probabilities. JLT model the distribution of the time of bankruptcy as the first hitting time of a continuous time Markov chain with discrete states that consist of the different credit ratings and default (the absorbing state).

The authors use the following methodology to specify the bankruptcy process. They define a finite state space $S = \{1, \ldots, K\}$, which represents all of the possible classes of credit ratings, with state 1 being the highest, state $K-1$ being the lowest state and state K being the bankruptcy state. Examples of these different rating schemes can be seen in Table 2.1 on page 20 of this dissertation. They then specify a continuous time, time-homogenous Markov chain $\{\eta : 0 \leq t \leq \tau\}$ in terms of its $K \times K$ generator matrix

$$\Lambda = \begin{pmatrix} \lambda_1 & \lambda_{1,2} & \lambda_{1,3} & \cdots & \lambda_{1,K-1} & \lambda_{1,K} \\ \lambda_{2,1} & \lambda_2 & \lambda_{2,3} & \cdots & \lambda_{2,K-1} & \lambda_{2,K} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_{K-1,1} & \lambda_{K-1,2} & \lambda_{K-1,3} & \cdots & \lambda_{K-1} & \lambda_{K-1,K} \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix} \tag{5.4}$$

where $\lambda_{i,j\geq 0}$ $\forall i,j$ and

$$\lambda_i = -\sum_{i\neq j=1}^{K} \lambda_{i,j}$$

The off-diagonal terms of the generator matrix, $\lambda_{i,j}$, represent the transition rates of jumping from credit class i to credit class j. To estimate the empirical generator matrix Λ, the authors suggest using historical results from Moody's or Standard & Poor's which are typically quoted in an annual fashion. The last row of zeros implies that bankruptcy (state K) is absorbing (i.e. once you enter it you can never leave). The $K \times K$ $t-$period probability transition matrix (under the real-world measure) for η is given by

$$Q(t) = \exp(t\Lambda) = \sum_{k=0}^{\infty} \frac{(t\Lambda)^k}{k!} \tag{5.5}$$

Although these real-world transition probabilities are Markovian, the transition probabilities under the martingale pricing measure could depend on the entire history of the process up to time t (i.e. non-Markovian). To facilitate empirical estimation and implementation, JLT assume that the transition probabilities are Markovian under the martingale pricing measure. In particular, they assume that the generator matrix under the martingale pricing measure is given by

$$\widetilde{\Lambda}(t) \equiv U(t)\Lambda \tag{5.6}$$

where $U(t) = diag(\mu_1(t), \ldots, \mu_{K-1}(t))$ is a $K \times K$ diagonal matrix whose first $K-1$ entries (corresponding to the $K-1$ credit ratings) are strictly deterministic functions of t that satisfy

$$\int_0^T \mu_i(t)dt < \infty, i = 1, \ldots, K-1 \tag{5.7}$$

Under the assumption in equation (5.6), the credit rating process is still Markov, but it is no longer time-homogenous. Although homogeneity is desirable, the authors make this trade-off so that the model can match any given initial term structure of credit risk spreads. The $\mu_i(t)$ are interpreted as risk premia, that is, the adjustments for risk that transform the actual probabilities into pseudo-probabilities suitable for valuation processes. To estimate the risk premium one could use the market price of the firm's default-risky discount bonds and back-out the implied risk premiums. This method calibrates the model to market prices in much the same way as arbitrage-free models of the risk-free term structure.

The $K \times K$ transition matrix from time t to time T for the Markov chain, η, under the equivalent martingale measure is given as the solution to the Kolgoromov differential equations

$$\frac{\partial \widetilde{Q}(t,T)}{\partial t} = -\widetilde{\Lambda}(t)\widetilde{Q}(t,T); \tag{5.8}$$

$$\frac{\partial \widetilde{Q}(t,T)}{\partial T} = \widetilde{\Lambda}(T)\widetilde{Q}(t,T); \tag{5.9}$$

$$\widetilde{Q}(t,t) = I \tag{5.10}$$

where I is the $K \times K$ identity matrix. We will denote the (i,j)th entry of $\widetilde{Q}(t,T)$ by $\widetilde{q}_{ij}(t,T)$. If we let the firm be in state i at time t, that is $\eta_t = i$, and define $\tau^* = \inf\{s \geq t; \eta_s = K\}$, then we have

$$\widetilde{Q}(\tau^* > T) = \widetilde{Q}[\tau^* > T|\eta_t = i] = \sum_{j \neq K} \widetilde{q}_{i,j}(t,T) = 1 - \widetilde{q}_{i,K}(t,T) \tag{5.11}$$

To facilitate their exposition, JLT assume a *recovery of treasury* (RT) recovery process that is given by

$$\varphi_{\tau^*} = \delta P(\tau^*, T) \tag{5.12}$$

where δ, the recovery rate is an exogenously specified constant and $P(\tau^*, T)$ is the price at time τ^* of an otherwise equivalent, riskless discount bond maturing at time T. Equation (5.12) says that claimholders receive \$1 at time T if default does not occur by T, and otherwise they receive δ dollars at time T. This is equivalent in saying that the claimholders invested the $\delta P(\tau^*, T)$ in a riskless discount bond that matures at time T.

Under these simplifying assumptions, equation (5.2) becomes

$$\begin{aligned} U_t &= \widetilde{E}\left[e^{-\int_t^T r_u du}(XI(\tau^* > T) + \delta XP(\tau^*, T)I(\tau^* \leq T)) \mid F_t\right] \\ &= \widetilde{E}\left[e^{-\int_t^T r_u du}XI(\tau^* > T) + e^{-\int_t^T r_u du}\delta XP(\tau^*, T)I(\tau^* \leq T) \mid \eta_t = i\right] \\ &= \widetilde{E}\left[e^{-\int_t^T r_u du}XI(\tau^* > T) + e^{-\int_t^T r_u du}\delta XI(\tau^* \leq T) \mid \eta_t = i\right] \end{aligned}$$

$$
\begin{aligned}
&= \widetilde{E}\left[e^{-\int_t^T r_u du} X(I(\tau^* > T) + \delta I(\tau^* \leq T)) \mid \eta_t = i\right] \\
&= \widetilde{E}\left[e^{-\int_t^T r_u du} X(1 - I(\tau^* \leq T) + \delta I(\tau^* \leq T)) \mid \eta_t = i\right] \\
&= \widetilde{E}\left[e^{-\int_t^T r_u du} X(1 - (1-\delta) I(\tau^* \leq T)) \mid \eta_t = i\right] \\
&= \widetilde{E}\left[e^{-\int_t^T r_u du} \mid F_t\right] \widetilde{E}\left[X(1 - (1-\delta) I(\tau^* \leq T)) \mid \eta_t = i\right] \\
&= P(t,T)\widetilde{E}\left[X(1 - (1-\delta) I(\tau^* \leq T)) \mid \eta_t = i\right] \qquad (5.13)
\end{aligned}
$$

where the second last equality holds because JLT assume that the process for default and the default-free term structure are independent under the martingale pricing measure. The third equality uses the fact that

$$
P(\tau^*, T) = \widetilde{E}\left[e^{-\int_{\tau^*}^T r_u du} \mid F_t\right]
$$

If the contingent claim U is a default-risky discount bond (i.e. $X = 1$), and $v^i(t,T)$ is the price of a default-risky discount bond that is now in credit class i , then equation (5.13) becomes

$$
\begin{aligned}
v^i(t,T) &= P(t,T)\widetilde{E}\left[X(1 - (1-\delta) I(\tau^* \leq T)) \mid \eta_t = i\right] \\
&= P(t,T)(1 - (1-\delta)\widetilde{E}\left[I(\tau^* \leq T)) \mid \eta_t = i\right]) \\
&= P(t,T)(1 - (1-\delta)\widetilde{Q}\left[\tau^* \leq T \mid \eta_t = i\right]) \\
&= P(t,T)(1 - (1-\delta)(1 - \widetilde{Q}\left[\tau^* > T \mid \eta_t = i\right])) \\
&= P(t,T)(\delta + (1-\delta)\widetilde{Q}\left[\tau^* > T \mid \eta_t = i\right]) \\
&= P(t,T)(\delta + (1-\delta)\widetilde{Q}_t^i\left[\tau^* > T\right]) \qquad (5.14)
\end{aligned}
$$

Equation (5.14) indicates that the higher the probability of default not occurring before maturity, the higher the value of the default-risky bond and therefore the lower the credit spread is.

Given that the forward rate for the default-risky discount bond in credit class i is defined by

$$
f^i(t,T) \equiv -\frac{\partial}{\partial T} \ln v^i(t,T), \qquad (5.15)
$$

equation (5.14) yields

$$\begin{aligned} f^i(t,T) &= -\frac{\partial}{\partial T}\ln\left(P(t,T)(\delta+(1-\delta)\widetilde{Q}_t^i[\tau^*>T])\right) \\ &= f(t,T)-\frac{\partial}{\partial T}\ln\left(\delta+(1-\delta)\widetilde{Q}_t^i[\tau^*>T]\right) \\ &= f(t,T)-I(\tau^*>t)\left(\frac{(1-\delta)\frac{\partial}{\partial T}\widetilde{Q}_t^i[\tau^*>T]}{\delta+(1-\delta)\widetilde{Q}_t^i[\tau^*>T]}\right) \\ &= f(t,T)+I(\tau^*>t)\left(\frac{(1-\delta)\lambda_{i,K}\mu_i(t)}{\delta+(1-\delta)\widetilde{Q}_t^i[\tau^*>T]}\right) \end{aligned} \quad (5.16)$$

where

$$\frac{\partial}{\partial T}\widetilde{Q}_t^i(\tau^*>T)=\frac{\partial}{\partial T}\widetilde{Q}_t^i(\tau^*>T)=\lambda_{i,K}\mu_i(t) \quad (5.17)$$

From the definition of $I(\tau^*>t)$, it follows that in bankruptcy,

$$f^i(t,T)=f(t,T) \quad (5.18)$$

The credit risk spread on the short rate is given by

$$\begin{aligned} r^i(t)-r(t) &= \lim_{T\to t}\left(f^i(t,T)-f(t,T)\right) \\ &= \lim_{T\to t}\left(I(\tau^*>t)\left(\frac{(1-\delta)\lambda_{i,K}\mu_i(t)}{\delta+(1-\delta)\widetilde{Q}_t^i[\tau^*>T]}\right)\right) \quad (5.19) \\ &= I(\tau^*>t)(1-\delta)\lambda_{i,K}\mu_i(t) \quad (5.20) \end{aligned}$$

Equation (5.20) follows from equation (5.19) since

$$\lim_{T\to t}\widetilde{Q}_t^i(\tau^*>T)=1 \quad (5.21)$$

$\lambda_{i,K}\mu_i(t)$ is the pseudo-probability of default. Contrary to market evidence, equation (5.20) implies that the credit risk premium is identical for all firms in a given credit class (rating category).

The main strengths of the JLT approach to credit risk modelling are its simplicity and computational tractability. The modelling of default based on credit rating transitions is intuitive, explicitly accounts for default risk and is not very computationally intensive. However, this simplicity is achieved through some assumptions whose validity is questionable. Most notable is the assumption of independence of the default process and the process for the default-free short rate under the martingale measure. This assumption certainly does not hold for lower rated bonds. Also, it is hard to believe that all bonds within a given credit rating class have identical transition probabilities. Clearly, some bonds will be more risky than others within a given class. Finally, there is the question of whether the transitions between credit classes are actually governed by a continuous-time Markov chain, since in practice there appears to be a tendency for a firm to continue to fall through changes in credit class. Also, modelling the transitions between credit classes as a continuous-time Markov chain means that the times in rating classes will be exponentially distributed, but more importantly, the probability of a downgrade given that the firm has just experienced one is higher than for a firm that has been in that class for some time. This is not supported by evidence. Therefore, it is clear that the JLT model will not be useful in making investment decisions among bonds of equal credit ratings, although it could be used to back out the relative credit risk imputed by the market. However, this model might be useful in discovering the term structure of credit risk for a given bond issuer and facilitate investment decisions and pricing and hedging of derivatives for that family of bonds.

5.3 Duffie and Singleton (1999)

JLT made some strong assumptions about the independence of the default process and the process for the riskless short rate under the pricing measure, and this led to a neat formula for the price of a default-risky discount bond. Under the JLT model, the default process is governed by a Markov process[1] under the pricing measure.

Unlike JLT, Duffie and Singleton (henceforth, DS) abstract from specifying the details of the default process. They treat default as an unpredictable event governed by an intensity-based or hazard-rate process and focus on the assumption made about the recovery process, which they assume obeys *recovery of*

[1] A Markov process is a stochastic process where only the present value of a variable is relevant for predicting its future.

market value (RMV). Under this assumption, for contingent claim U we have, for default at time τ

$$X' := (1 - L_\tau)U_{\tau-} \tag{5.22}$$

where X' is the payment claimholders receive given a default at time τ and $U_{\tau-} = \lim_{s\uparrow\tau} U_s$ is the price of the contingent claim "just before" default. The DS framework summarized below assumes the existence of the processes L_t, U_t, r_t, and h_t. The distribution of $X \equiv X_T$ under the pricing measure is also taken as given.

- h_t = risk-neutral hazard rate for default at time t
- $h_t\Delta t$ = conditional risk-neutral probability at time t of default over small time interval Δt, given no default before t
- L_t = loss in market value given a default
- $h_t L_t$ = risk-neutral conditional expected loss rate of market value
- r_t = risk-free short rate process
- $R_t = r_t + h_t L_t$ = default-adjusted short rate process

Let $A^t_{\Delta t}$ represent the event of a firm defaulting on its obligation for the first time in the interval $[t, t + \Delta t]$. Then a hazard rate of implies that

$$h_t = \lim_{\Delta t \to 0} \frac{\widetilde{E}[I(A^t_{\Delta t}) \mid F_t]}{\Delta t} \tag{5.23}$$

where $\widetilde{E}[\cdot]$ indicates the expectation under the equivalent martingale measure. One may also think of h_t as the jump arrival intensity at time t (under the equivalent martingale measure) of a Poisson process[2] whose first jump occurs at default.

The fundamental idea behind the hazard rate approach is that default comes by *surprise* (i.e., default involves a sudden loss in market value of an asset) and we only need to model the *intensity* or infinitesimal likelihood of a default. To incorporate this element of surprise, we define a default process that is

[2]See Appendix A

independent of the processes L_t, U_t, r_t, h_t, and of X. Thus, for the default process Λ_t which is 0 before the default and 1 afterward,

That is,

$$\Lambda_t = I(\tau \leq t) \tag{5.24}$$

the intensity process is given by

$$\{h_t(1 - \Lambda_{t-}) : t \geq 0\} \tag{5.25}$$

The assumption by DS that h_t and L_t do not depend on the value U_t of the contingent claim is typical of reduced-form models. The authors also assume that U does not jump at default τ. This means that, although there may be *surprise* jumps in the conditional distribution of the market value of the default-free claim (X, T), h, or L , these surprises occur precisely at the default time with probability zero. At this point, DS apply Itô's formula for jumping processes to the discounted gains process (which is a martingale under the pricing measure) to verify that

$$U(t,T) = \widetilde{E}\left[\exp\left(-\int_t^T (r_u + h_u L_u)du\right) X \mid F_t\right] \tag{5.26}$$

where the discounted gains process G is defined by

$$G_t = \exp\left(-\int_0^t r_u du\right) U_t(1 - \Lambda_t) + \int_0^t \exp\left(-\int_0^u r_s ds\right)(1 - L_u)U_{u-}d\Lambda_u \tag{5.27}$$

Equation (5.27) has the following intuitive meaning. The first term is the discounted price of the claim; the second term is the discounted payout of the claim upon default.

Instead of following DS's development of equation (5.26) using Itô's formula for jumping processes, we will follow Lando's (1998), more intuitive development of equation (5.26).

Lando (1998), showed that for the case of zero recovery, (i.e.,$L_\tau \equiv 1$) the expression of the contingent claim (equation (5.2)) is

$$
\begin{aligned}
U(t,T) &= \widetilde{E}\left[e^{-\int_t^T r_u du} X I(T' > T) + e^{-\int_t^{T'} r_u du} X' I(T' \leq T) \mid F_t\right] \\
&= \widetilde{E}\left[\exp\left(-\int_t^T r_u du\right) X I(\tau > T) \mid F_t\right], \tau = T', t < \tau \\
&= \widetilde{E}\left[\exp\left(-\int_t^T (r_u + h_u) du\right) X \mid F_t\right] \qquad (5.28)
\end{aligned}
$$

This establishes the result (5.26) in the special case $L \equiv 1$. We now use heuristic reasoning to establish the result (5.26) for all L. Suppose that the default time, happens exactly as before, with intensity h_t. Receiving a fraction $1-L_t$ of pre-default value in the event of default (at t) of a contract is equivalent, from a pricing perspective, to receiving the outcome of a lottery in which the full pre-default value is received with probability (under the martingale measure) $1 - L_t$ and 0 is received with probability L_t, i.e. the event of default has been retained with probability L_t. This in turn may be viewed as a default process in which there is 0 recovery but where the default intensity has been thinned using the process L, producing a new default intensity of $h_t L_t$. Clearly, this way of thinking does not change the expectation in equation (5.2). However, we now can think of two types of default. Harmless default that occurs with intensity $h_t(1 - L_t)$, and lethal default that occurs with intensity $h_t L_t$. As far as valuing the contingent claim prior to default is concerned, we are clearly only interested in lethal defaults, and we therefore price using the intensity of lethal defaults. Equation (5.28) becomes

$$
U(t,T) = \widetilde{E}\left[\exp\left(-\int_t^T (r_u + h_u L_u) du\right) X \mid F_t\right] \qquad (5.29)
$$

By discounting at the adjusted short rate R, both the timing and probability of default, as well as the effects of losses on default are all accounted for. Using this approach, defaultable contingent claims are treated as default-free when they are discounted at the default-adjusted short rate.

The key feature of the DS model is that h_t and L_t are exogenously specified. This allows the authors to derive a term structure model for default-risky debt which can be used in conjunction with common term structure models for risk-free debt such as BDT, Vasicek, CIR, and the HJM approach. The DS model does not allow for the effects of h and L separately since they enter the adjustment for default in the discount rate $R = r + hL$ in the product form hL. While it is clear that hL represents a credit spread between default-risky and risk-free debt, it is not clear what the individual contributions are to this spread. In order to learn more about the hazard and recovery rates in market prices, the loss percentage L could be modelled using historic default recovery rates, such as those in the Longstaff-Schwartz section of this dissertation and the default probability h could be estimated historically by studying the number of defaults within different classes of bonds. Another way to estimate these two parameters would be to back them out of the market prices of derivatives such as default-risky bond options whose payoffs depend nonlinearly on h and L. However, without a wide range of debt securities deriving value from the same issuer (e.g. liquidly traded bonds, credit derivatives), the components of the mean loss rate cannot be estimated separately. Given the paucity of credit data, efficient estimation of each individual parameter in the DS modelling framework can be a daunting task.

By modelling the default-adjusted rate $R_t = r_t + h_t L_t$ instead of the usual short rate r_t, more non-default factors which influence credit spreads may be incorporated in the model. Some of these factors could be due to liquidity, demand and supply, tax costs and embedded options. DS propose that all these non-default factors, or "liquidity" effects, be modelled with a stochastic process l, which represents the fractional carrying cost of the default-risky debt. The new adjusted short rate would then be adjusted for default and liquidity as follows:

$$R = r + hL + l \tag{5.30}$$

To gain insight into the term structure of $hL + l$, we could fit both a defaultable zero curve and a default free zero curve and compare the respective yields. However it will be difficult to infer anything about h, L and l individually. Responding to this, the authors suggest "extracting" information about the mean-loss-rate process hL from defaultable bond prices (before default) to infer the contribution of hL to the credit spread. The idea of relating credit spreads to firm-specific or macroeconomic variables such as stock prices, investor sentiment and capital investment is suggested as one possibility.

Although DS favour a reduced-form of credit risk model, they do mention that a general formula can be given for the hazard rate h_t in terms of the default boundary for assets, the volatility of the underlying asset process V at the default boundary and the risk-neutral conditional distribution of the level of the assets given the history of information available to investors. This brings us back in some fashion to the framework of the structural model where default is triggered by the firm value process.

DS's modelling approach is described in somewhat general terms, but they give various examples of how their framework can be applied to the valuation of default-risky bonds (callable and non-callable) and the pricing of credit derivatives such as credit-spread put options on default-risky bonds. The authors discuss several approaches to pricing default-risky bonds using equation (5.26). For example, one can either parametrize R directly, or parametrize the component processes r, h, and L. Pricing models that focus directly on R combine the effects of the changes in the default-free short rate r and the mean loss-rate process hL on bond prices. In contrast, pricing models that parametrize R and hL separately are able to "extract" information about mean loss rates from historical default-risky bond yields. Alternative specifications of the DS model focus on the different assumptions regarding the processes governing h_t, r_t, L_t and l_t.

The RMV assumption is central to the DS approach to modelling credit risk. We now define two other recovery assumptions before discussing the tractability of the RMV assumption. Let φ_τ denote the amount recovered (for every \$1 of face value owed) if default occurs at time τ. Under the *recovery of face value* (RFV) framework, the creditor receives a fraction $\varphi_\tau = (1 - L_\tau)$ immediately upon default. Under the *recovery of treasury* (RT) framework, the creditor receives a fraction $\varphi_\tau = (1 - L_\tau)P(\tau, T)$ immediately upon default. $P(\tau, T)$ is the time price of an otherwise equivalent, default-free bond.

The RMV assumption is accurate for products such as interest rate swaps, cross-currency swaps and discount bonds. These types of products are usually marked-to-market on a daily basis, and one could expect to receive a fraction of what the product was marked at just prior to default. Indeed, DS comment that "the RMV assumption is well matched to the legal structure of swap contracts in that standard agreements typically call for settlement upon default based on an obligation represented by an otherwise equivalent, non-defaulted, swap".While there may be cases where RT is more realistic than RMV, DS emphasize that under the RT assumption, the computational burden of computing equation (5.2) can be substantial. Largely for this reason, various simplifying assumptions regarding the relationships between h, r, and L have to be made. Finally, DS note that if one assumes liquidation at default and that absolute priority applies, then the RFV assumption may be more realistic since it implies equal recovery for bonds of equal seniority of the same issuer. The main attraction of the RMV model is that it is easier to use, because standard default-free term structure modelling techniques can be applied. The key thing to remember is what simplifications or assumptions one has made, and how this will affect the pricing of real world securities.