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Chapter 2

Fundamentals of Credit Modelling

2.1 Introduction

Most default-risky bonds will deliver some cash flow other than the promised cash flow when default occurs. This will necessarily have a present value that is less than that of the promised cash flows. If this happens, then we say that there is a *partial recovery*.[1] The present value of the default cash flows at the time of the default is often referred to as the *recovery value of the bond*.

If there are no-arbitrage opportunities in the market, default-risky bonds must trade at values that are less than their risk-free counterparties.[2] This implies that their yields will be higher than the corresponding risk-free yield. The difference in yields is referred to as a *credit spread* (or sometimes just spread).[3]

Default risk and recovery risk together determine credit spreads on a bond. It

[1] For corporate debt, partial recovery is usually awarded in a bankruptcy settlement well after the promised cash flows are due. For sovereign debt, partial recovery is usually in the form of a "restructuring," which means that the country pays part of its obligations with new debt of a lesser value.

[2] That is risk-free bonds with the same promised cash flows.

[3] In most cases spreads can be thought of as the market's "view" of the likelihood of default.

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is important to segregate them since credit derivatives may be written on either or both risks - hence from a modelling viewpoint, it is essential to not treat them as one composite entity. Moreover, the sources of empirical information in the modelling process will be quite disparate. Ratings and other industry-level information are quite effective in providing market participants with a good idea of default likelihood, whereas a more accurate assessment of the individual firm's recovery risk is necessary in understanding why spreads tend to be different for firms with the same rating category and industry.

An important variable in credit modelling is the time of default. This can be difficult to define in practice. There are many scenarios in which one could interpret the time of default in multiple ways. For example, let us say that an issuer declares its intention to default on a certain obligation before any payment is due. Does default occur at the time of the announcement or at the next payment date (assuming that the full cash flow is not delivered then)? This sort of issue needs to be dealt by on a case by case basis. Credit models must answer these questions when they define default. Actual credit derivatives must specify what they mean by default in their defining contracts.[4]

It is desirable that credit risk models possess the following attributes. First, they should be arbitrage-free and they should reflect current market information. That is, we should be able to fit a credit risk model to the current term structure of credit spreads. This is akin to fitting an interest rate model to the current term structure of interest rates.

Second, the models should produce default rates (sometimes called hazard rates) that are plausible. Third, models should be computationally tractable where the inputs to the model are readily estimable.

Now that we have the fundamentals, we are ready to present methods of credit risk modelling. The current methods of modelling can be divided into two distinct approaches, namely "traditional" and "market based" models.

[4]This question is relevant for many credit derivatives. An example is a credit default swap.

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2.2 Traditional Credit Models

Traditional models use historical data to determine both default probabilities and recovery rates specific to certain debt classes. Traditional credit analysis for a debt class is a combination of both industrial and financial analyses. Industrial analysis focuses on economic cyclicality, growth prospects, R&D expenses, competition, source of supply, degree of regulation, labour, and company accounting factors, whereas financial analysis looks closely at various financial ratios, equity returns, foreign exposure, management quality and other factors. Credit analysts compare these factors to their historical values as well as for competing companies in the same industry when drawing conclusions about the creditworthiness of a company

Rating agencies like Standard and Poor's (S&P), Fitch Ibca, Duff and Phelps and CA Ratings[5] use traditional credit analysis in assigning ratings to borrowers. The ratings are ordinal in nature and do not quantify the default probability. The rating agencies publish observed historical defaults that can be used to infer the default probability for a specific rating. The higher credit ratings exhibit extremely low observed default frequencies, and therefore the historical experience is only really statistically significant for lower quality credits. For example, for the period 1981 to 1995, Standard & Poor's only had one default within one year of an A-rated or better company. The rating designations are shown in Table 2.1 below.

2.3 Market Based Models

The market based models use information from the market (equity values and credit spreads) to derive values for the default probabilities and recovery rates. Market based models attempt to describe the dynamics of default within the rigorous framework of financial mathematics. The key ingredients of this approach are credit events (e.g. defaults or downgrades) and payments on contracts made at such events. The mathematical modelling of credit risk involves making assumptions about the stochastic process driving default, the process generating the payoff upon default, and the evolution of risk-free interest rates.

[5]South Africa only.

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Moody's	S&P	Meaning
Aaa	AAA	Highest Quality. Smaller degree of risk. Interest payments protected by a large or stable margin.
Aa	AA	High Quality. Margin of protection slightly lower than Aaa (AAA).
A	A	Upper Medium Grade. Adequate security of principal and interest. May be susceptible to impairment in future.
Baa	BBB	Medium Grade. Neither highly protected nor poorly secured. Adequate security for the present. Speculative features.
Ba	BB	Lower Medium grade. Speculative elements. Future not well secured.
B	B	Speculative. Lack characteristics of desirable investment.
Caa	CCC	Poor standing. May be in default or danger with respect to principal or interest.
Ca	CC	High degree of speculation. Often in default.
C	C	Lowest-rated class. Extremely poor chance of ever attaining any real investment standing.
-	D	In default.

Table 2.1: Ratings assignments and their meanings

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In an arbitrage-free complete financial market, the price at time t, X_t, of a promised payoff X paid at a terminal time T is

$$X_t = \widetilde{E}\left[X \exp\left(-\int_t^T r_u du\right) \mid F_t\right] \tag{2.1}$$

where $(r_s, s \geq 0)$ is the spot interest rate and $\widetilde{E}[\cdot]$ is the expectation taken under the *equivalent martingale measure* (Harrison and Kreps (1979), and Harrison and Pliska(1981)), and F_t is the information available to agents at time t.

In the default risk framework, a default appears at some random time τ. We denote by $I(T < \tau)$ the indicator function of the set $\{T < \tau\}$ equal to 1 if the default occurs after T and equal 0 to otherwise. A default free contingent claim consists of a nonnegative random variable which represent the amount of cash paid at a pre-specified time to the owner of the claim. For a defaultable contingent claim, the promised payment is actually done only if the default did not occur before maturity. If the default occurs before maturity, some payment other than the promised payment is done. In general, the payment of a defaultable claim consists of two parts:

1. Given a maturity date $T > 0$, a random variable X, which does not depend on τ represents the promised payoff - that is, the amount of cash the owner of the claim will receive at time T, provided that the default has not occurred before the maturity date T.

2. A predictable process φ, prespecified in the default-free world, models the payoff which is received if default occurs before maturity. This process is called the *recovery process*.

The value of the defaultable claim is, provided that the default has not occurred before time t,

$$X_t = \widetilde{E}\left[XI(T < \tau)\exp\left(-\int_t^T r_u du\right) + \varphi_\tau I(\tau \leq T)\exp\left(-\int_t^T r_u du\right) \mid F_t\right] \tag{2.2}$$

where F_t is the information at time t. It is assumed that the owner of the contingent claim knows when the default appears. At time t, the owner of

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the claim knows if the default has occurred before; if the default has not yet occurred, he has no information on the time when it will happen.

As mentioned before, the time of default τ is an important variable in default risk modelling and market based models differ in their modelling of the default time. Once an assumption has been made on the evolution of the default time, the valuing of the defaultable claim (equation (2.2)) reduces to the problem of computing the expectation of $XI(T < t)$ under the risk-neutral probability.

2.3.1 Types of Market Based Models

The problem of modelling default risk is well represented in the literature. There are two distinct approaches. The first, pioneered by Merton (1974), attempts to model the default process by specifying two processes: one for the market value of the firm's assets and one for a benchmark of default. This benchmark of default is related to the firm's liabilities, and default is said to occur when the value of the firm's assets falls below this benchmark. Models of this type are often called *structural models*. The difficulty of modelling both the conditions under which default occurs, and in the event of default, the division of the value of the firm among claimants has led to the development of an alternative modelling approach.

Under the alternative approach, no direct reference is made to a firm's asset value; instead, default is modelled as an unpredictable event governed by a hazard rate process. The hazard rate process and the recovery rate are exogenously specified. Models of this type are often called *reduced-form models*. Reduced form models are especially practical when it is difficult to gather the asset and liability information needed by a structural model.

The distinction between structural and reduced-form models is only one of the many distinctions one must take into account when developing a model of credit risk. One must also model the type of payoff upon default. Different approaches have provided for fractional recoveries of par, a default-free version of the bond, or the market value at time of default. Recovery can also be modelled as a function of the debt's priority of claim in the capital structure (e.g. senior or subordinated) or the credit rating that is given to the debt by one of the major credit rating agencies. Consideration may also be given to the

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type of default. Both business cycles and firm-specific events influence defaults. However, firm-specific defaults can be unrelated to the business cycles. These may arise from events related to a firm's business activities or product liability lawsuits. Therefore, default may be triggered by some unexpected information that cannot be observed from economic variables only. Clearly, the modelling of default is very complex and should take into account as many of these issues as possible.

Another consideration in modelling credit risk is whether to use an *equilibrium* or *arbitrage-free* model. Equilibrium models focus on investor preferences, and assume that the economy tends to gravitate towards a state where all investors have allocated their resources optimally. In any other circumstance, investors with suboptimal allocations will attempt to improve their positions, thus creating instability. This instability will only disappear once the economy enters into a steady-state where no market participants are motivated to cause disturbances (i.e. when all investors have achieved optimal wealth creation). Equilibrium models require that the parameters to a given model be estimated empirically. The equilibrium model is then used to price securities. Equilibrium models require significant data and econometric techniques to estimate, and their output will not equate to market prices in all (or any) cases.

In contrast to the equilibrium models, arbitrage-free models begin by assuming that the prices of a small number of securities are given, and then deduce prices of other instruments by attempting to match their behaviour with these "basic" securities. The main assumption employed is that markets are free from opportunities to earn riskless profits (i.e. arbitrage). This leads to the result that any two portfolios producing identical payoffs under all scenarios have the same price (otherwise, riskless profits are possible by purchasing the cheaper portfolio and selling the more expensive one). Arbitrage-free models use market prices of securities to infer a model's parameters. Therefore, the arbitrage-free model will be calibrated in such a fashion as to produce the given market prices. Arbitrage-free models are easy to get data for and are useful in hedging derivatives. However, they can be misled by market imperfections such as illiquidity.

There are two main advantages possessed by the arbitrage-free pricing methodology over its equilibrium counterpart. The first is that arbitrage pricing does not require any assumptions regarding investor preferences, aside from the basic

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axiom that market participants will always prefer more wealth to less (referred to in economics as *insatiability*). While equilibrium models are suitable for economic theory, in applications to derivatives pricing it is not often justifiable to assume a specific form of preference function for a given investor. The second advantage of arbitrage pricing is that it provides an explicit algorithm whereby the violation of the arbitrage-free results will lead (at least in theory) to a risk-free profit. The key to choosing between these two types of models is whether one is more concerned about estimating intrinsic value (equilibrium models) or value relative to current market prices (arbitrage-free models).

When valuing defaultable debt, it is important not only to model the credit risk, but also the interest rate risk. For that reason, most models of credit risk are integrated with an interest rate model. There are two distinct approaches in the literature on the models of the interest rate curve. The first is the Vasicek (1977), and its variants, which focus on the dynamics of the short-term interest rate from which the whole yield curve is reconstructed. Models of this form are commonly referred to as *short rate models*. The second approach initiated by Heath, Jarrow and Morton (1992), takes the full forward rate curve as a dynamic variable, driven by one or several continuous-time Brownian motions. Models of this form are commonly referred to as *Heath, Jarrow and Morton models* (HJM models). Most interest rate models are based more on their mathematical tractability rather than on their ability to describe the data. Since it is difficult to build a model of credit risk, most people choose relatively simple models of interest rate risk to accompany the model of credit risk. The simplest approach assumes constant interest rates.

Another important question when implementing a model of credit risk is what technique will be used to calculate prices? The most popular approaches are analytic (or closed form) solutions, a lattice (or tree) framework, finite difference methods and Monte Carlo methods. Analytic solutions are convenient to use and provide quick intuition on important variables, but usually are too simple or too inflexible in practical situations where complex payout or exercise contingencies are present. The lattice framework provides more flexibility and is computationally feasible if the problem can be solved with a recombining binomial or trinomial tree. Finite difference methods are suitable for problems with two or three random factors. They are similar to lattice framework in that

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the computations work back from end of the life of the security to the beginning. However, they are more flexible than the lattice framework because there are many ways to improve finite difference methods making them faster and more accurate. Finally, Monte Carlo simulation provides the most flexibility and is useful for solving complex path-dependant problems or high-dimensional problems. However, Monte Carlo analysis can be slow and computationally intensive. The method that is chosen in practice is likely to depend on the characteristics of the problem being evaluated and the accuracy required.