Chapter 3

Optimal scheduling

3.1 Introduction

In train handling, different kinds of scheduling approaches are proposed in the literature. However, the differences among them are not discussed. It is assumed that there are redundancies in optimization of scheduling. Optimal scheduling may improve the performance of a train. It is said in chapter 1 that the ECP braking system can improve train performance. In this chapter, the difference between trains equipped with a pneumatic braking system and an ECP braking system is first compared and then the difference between optimal scheduling and heuristic scheduling is discussed.

In the first part of this chapter, three control strategies are proposed for train handling. In the second part, the performances reached with optimal scheduling with in-train forces taken into initial consideration on trains equipped with a tradition pneumatic braking system and an ECP braking system are compared. ECP braking systems show superb performance compared with pneumatic braking systems. Thus, in the rest of this thesis, the handling of trains equipped with ECP braking systems is studied.

It is hoped that optimal scheduling can improve the performance of train handling. Optimal scheduling, taking in-train forces and energy consumption into consideration, is compared with the heuristic scheduling proposed in [15] in the third part of this chapter. It is shown that optimal scheduling presents a better start for closed-loop controllers. The work of this part is seen in [20].

In the last part, a closed-loop controller combining the optimal scheduling and LQR controller in [15], employing a linear system theory, is adopted to compare with the closed-loop controller in [15]. Optimal scheduling is used to calculate the equilibria, while a closed-loop controller based on LQR is employed to bring the trajectory of the train to the equilibria. It is confirmed from the simulation that Optimal scheduling actually improves the performance of the closed-loop controllers and that the 2-2 strategy, ECP/iDP-only strategy yields the best performance of all strategies. The work of this part can be seen in [21].

3.2 Control strategies

A traditional heavy haul train with a pneumatic controlled braking system is controlled by drivers in the leading locomotive. A single air pipe runs throughout the whole train and is responsible for supplying pressure to the braking system in each wagon as well as transmitting braking control signals. The driver controls the leading locomotive's effort while other locomotives' efforts follow that of the leading one. Because of the pressure wave propagation speed, the front wagons are responsible for most of the braking owing to the signal propagation delay and the pressure gradient. From a control point of view, there are only two control signals in this kind of strategy, one for locomotive effort and the other one for wagon brake.

When the locomotives' efforts are controlled independently and separately, this is referred to as multi-powered [11] or distributed powered. In this strategy, every locomotive or every locomotive group (some locomotives connected with rigid drawbar) has an independent control signal.

While the train is equipped with an ECP braking system, the braking control signal is transmitted electronically. There is nearly no time delay for the braking signal transmission. When the above two control strategies are implemented with an ECP system, the braking signals are not delayed.

An ECP braking system adds a new dimension to control strategy: it allows individual wagon braking. So in a fully ECP/iDP mode, every car, including locomotives and wagons, has its own independent control signal.

Summarizing the above, three major types of control are discussed in this study:

- 1-1 strategy
  One control signal is for all locomotives and one braking control signal for all wagons. Currently, this control strategy is still in use on the heavy haul trains of Spoornet equipped with ECP braking systems, for this control strategy was in use before the application of ECP braking systems and was well designed for short heavy haul trains. However, this strategy hinders the expansion of the train's length.

- 2-1 strategy
  Different control signals are for different locomotives while the same braking control signals server for all wagons. This is an iDP-only strategy.

- 2-2 strategy
  Every car has its own control signal. This is an ECP/iDP-only strategy.

3.3 Optimal scheduling on trains equipped with different braking systems

In chapter 1, it is said that a train equipped with an ECP braking system performs better compared with the traditional pneumatic braking system. The main difference between these two braking systems is the braking command signals. In a pneumatic braking system, the braking signals for all the wagons are the same and there are different time delays for the wagons to receive them. In an ECP braking system, the signal for all wagons may be different and it is received by all wagons simultaneously. In this chapter optimal scheduling on the trains equipped with these two braking systems will respectively be simulated and the difference between them will be compared. Here only the result of in-train forces is compared, while other advantages of the application of an ECP braking system are not shown. From the simulation, the ECP braking system shows superb performance compared to the pneumatic braking system on the one hand. On the other hand, the application of an ECP braking system enriches the control strategy of train handling.

An optimization procedure is applied to schedule cruise control by taking the in-train forces into initial design consideration. It is hoped that an optimal open loop controller design will present a better starting point for a closed-loop controller design. To demonstrate the open loop control design, the throttling and braking are constrained and three different operational strategies of heavy haul trains are distinguished.

3.3.1 Formulation of the optimal problem

Transient control

The inputs in (2.2) are insensitive to the change in the reference speed. To get a rapid response to the reference speed change, transient control is designed through an acceleration profile in the following open loop scheduling. When a closed-loop controller is considered, this step is unnecessary. An acceleration profile is calculated according to the velocity profile with a parameter, the acceleration limit, a_{rr}. For example, at the travel distance $dis = 1,000$ m, the reference velocity is changed from 12 m/s to 15 m/s and at the distance $dis = 5,000$ m, it is changed to 10 m/s, then the acceleration profile is as in Table 3.1, where s_1, s_2 are calculated as $s_1 = 1,000 + (15^2 - 12^2)/(2a_{rr})$, $s_2 = 5,000 + (15^2 - 10^2)/(2a_{rr})$. Thus from the point 1,000 m to the point s_1 and from the point 5,000 m to the point s_2, the open loop scheduling should maintain the accelerations.

Table 3.1: Acceleration profile

distance	$\cdots$	1,000	s_1	5,000	s_2	$\cdots$
a	0	a_{rr}	0	$-a_{rr}$	0	$\cdots$

Performance function of optimal control

The objectives of a train control project are that with optimal control, 1) the train can travel a given distance within a given period; 2) energy consumption is reduced; and 3) the range of in-train forces is in the admission range of the train couplers. At the equilibrium point, where the speeds of the cars and the displacements of couplers are constant, that is, $\dot{v}_i = 0, \dot{x}_j = 0, i = 1, 2, \cdots n, j = 1, 2, \cdots, n-1$, the energy consumption of all control strategies is nearly equal, for most of the energy is used to conquer the resistance of drag forces, which are determined by the speed profile, track profile and the train. So the second objective can be ignored in scheduling the open loop controller. The first objective is more closely related to the speed profile and speed holding. In scheduling the open loop controller, it is assumed that the desired speed is reached and held. In this chapter, the objective, therefore, is taken as

$$J = \sum_{i=0}^{n-1} f_{in_i}^2, \tag{3.1}$$

where n is the number of cars in the train. That is, the purpose of the scheduling is to minimize the in-train forces.

In the following analysis, the train is assumed to consist of n cars, in which there are k locomotives. The cars are numbered from the front to the back with 1 to n. The locomotives' numbers are from l_1 to l_k.

Constraints of the optimal problem

For open loop control, the dynamic process in the train is ignored and the reference velocity is reached or the acceleration is maintained, that is,

$$\begin{aligned} \frac{dv_i}{dt} &= a, \qquad i = 1, 2, \cdots, n \\ \frac{dx_j}{dt} &= 0, \qquad j = 1, 2, \cdots, n-1, \end{aligned} \tag{3.2}$$

where a is the acceleration, which is zero when the train is cruising and is $a_r(-a_r)$ when the train is running within a scheduled acceleration (deceleration) period.

Applying (3.2) to equations (2.2) and (2.3), one has

$$u_s + f_{in_{s-1}} - f_{in_s} - f_{a_s} - m_s a = 0, \quad s = 1, 2, \cdots, n. \tag{3.3}$$

From the first $(n-1)$ equations of (3.3), the in-train forces can be calculated as

$$f_{in_s} = \sum_{i=1}^{s} u_i - \sum_{i=1}^{s} (f_{a_i} + m_i a), \quad s = 1, 2, \cdots, n-1. \tag{3.4}$$

From the last equation of (3.3), one has

$$\sum_{i=1}^{n} u_i - \sum_{i=1}^{n} (f_{a_i} + m_i a) = 0. \tag{3.5}$$

In train operations, the inputs and the in-train forces have some constraints:

$$\begin{aligned} \underline{U}_i \leq u_i \leq \overline{U}_i, \qquad & i = 1, 2, \cdots, n; \\ \underline{F}_{in_j} \leq f_{in_j} \leq \overline{F}_{in_j}, \qquad & j = 1, 2, \cdots, n-1, \end{aligned} \tag{3.6}$$

where $\underline{U}_i, \overline{U}_i$ are the upper and lower constraints for the ith input, and $\underline{F}_{in_j}, \overline{F}_{in_j}$ are the up and lower constraints for the jth in-train force, respectively. For a wagon, $\overline{U}_i = 0$ and the value of $\underline{U}_i$ depends on the capacity of the wagon's brake. For a locomotive, the constraints $\underline{U}_i, \overline{U}_i$ depend on the locomotive's capacity in traction effort. The notch should be changed step by step, and every notch should be kept for longer than a fixed time interval before it is changed. The constraints $\underline{F}_{in_j}, \overline{F}_{in_j}$ are limited because of the requirement of safe operation and limiting of maintenance cost.

Thus open loop scheduling is an optimization problem of the objective function (3.1) with equality constraints (3.3) and inequality constraints (3.6).

3.3.2 Optimization algorithm

In the following, the optimization algorithm of the 1-1 strategy is given as an example. The other two are similar and are omitted.

With the 1-1 strategy, all the locomotives share the drag forces equally and the brake forces of all the wagons are equal. This imposes additional constraints on the optimization problem.

$$\begin{aligned} u_{l_1} = u_{l_2} = \cdots = u_{l_{m-1}} = \cdots = u_{l_k} \triangleq u_t, \\ u_i \triangleq u_b, \qquad i = 1, \cdots, n; i \neq l_j, j = 1, \cdots, k. \end{aligned} \tag{3.7}$$

It is distinguished between two cases.

1) The last locomotive is not at the rear of the train. In this case, $l_k < n$. Combining (3.7) with (3.4), one has

$$f_{in_i} = \begin{cases} iu_b - \sum\limits_{j=1}^{i}(f_{a_j} + m_j a), & 1 \leq i < l_1, \\ (i-1)u_b + u_t - \sum\limits_{j=1}^{i}(f_{a_j} + m_j a), & l_1 \leq i < l_2, \\ \cdots \\ (i-k)u_b + ku_t - \sum\limits_{j=1}^{i}(f_{a_j} + m_j a), & l_k \leq i < n, \end{cases}$$

and

$$ku_t + (n-k)u_b = \sum_{i=1}^{n}(f_{a_i} + m_i a).$$

The objective function is rewritten as

$$\begin{aligned} J = \sum_{i=1}^{n-1} f_{in_i}^2 = \sum_{i=1}^{l_1-1}\left(iu_b - \sum_{j=1}^{i}(f_{a_j} + m_j a)\right)^2 + \cdots + \\ + \sum_{i=l_{k-1}}^{l_k-1}\left((i-k+1)u_b + (k-1)u_t - \sum_{j=1}^{i}(f_{a_j} + m_j a)\right)^2 \\ + \sum_{i=l_k}^{n-1}\left((i-k)u_b + ku_t - \sum_{j=1}^{i}(f_{a_j} + m_j a)\right)^2. \end{aligned}$$

The optimization with equality and inequality constraints can be solved with the Lagrange multiplier approach [26]. The equality constraints can be taken care of with the following extended objective function with a Lagrange multiplier:

$$\bar{J} = J + 2\lambda \left(ku_t + (n-k)u_b - \sum_{j=1}^{n} (f_{a_j} + m_j a) \right). \tag{3.8}$$

First, one calculates

$$\begin{aligned} \frac{1}{2}\frac{\partial J}{\partial u_t} &= \sum_{j=1}^{k} \left(\sum_{i=l_j}^{l_{j+1}-1} jQ_{i,j} \right), \\ \frac{1}{2}\frac{\partial J}{\partial u_b} &= \sum_{j=0}^{k} \left(\sum_{i=l_j}^{l_{j+1}-1} (i-j)Q_{i,j} \right), \end{aligned} \tag{3.9}$$

where $Q_{i,s} = (i-s)u_b + su_t - \sum_{j=1}^{i} (f_{a_j} + m_j a)$, $s = 0, \cdots, k$, $l_0 = 1, l_{k+1} = n$ and, denotes them as

$$\begin{aligned} \frac{1}{2}\frac{\partial J}{\partial u_t} &= T_b u_b + T_t u_t + \sum_{i=1}^{n-1} T_i (f_{a_i} + m_i a), \\ \frac{1}{2}\frac{\partial J}{\partial u_b} &= B_b u_b + B_t u_t + \sum_{i=1}^{n-1} B_i (f_{a_i} + m_i a). \end{aligned} \tag{3.10}$$

The necessary condition for extremality of $\bar{J}$ is:

$$\begin{aligned} \frac{1}{2}\frac{\partial \bar{J}}{\partial u_t} &= \frac{1}{2}\frac{\partial J}{\partial u_t} + \lambda k = 0, \\ \frac{1}{2}\frac{\partial \bar{J}}{\partial u_b} &= \frac{1}{2}\frac{\partial J}{\partial u_b} + \lambda(n-k) = 0, \\ ku_t + (n-k)u_b - \sum_{i=1}^{n} (f_{a_i} + m_i a) &= 0. \end{aligned} \tag{3.11}$$

From them, one can get the following equations:

$$\begin{aligned} &P_{bb}u_b + P_{tt}u_t = \sum_{i=1}^{n-1} \left(kB_i - (n-k)T_i \right) F_i, \\ &P_{bb} = (n-k)T_b - kB_b, \\ &P_{tt} = (n-k)T_t - kB_t, \\ &(n-k)u_b + ku_t = \sum_{i=1}^{n} F_i, \end{aligned} \tag{3.12}$$

where $F_i = f_{a_i} + m_i a$ and from which one can get the solutions of u_b, u_t. In applying this solution to (3.6), if no constraint is violated, this solution is the optimal value. If some constraints are violated, then one takes these violated inequality constraints as equality constraints, and re-solves the optimization problem. For example, one may wish to minimize $J(x)$ subject to $f(y) \leq 0$, where f, J and x are vectors of different dimensions. Suppose that x has p components and that n components of the inequality constraints are violated, that is, $f_i(x) > 0, \quad i = 1, 2, \cdots, n$. The other constraints, $f_i(x) \leq 0, \quad i = n+1, \cdots$, may be disregarded. Define a new function, $\bar{J} = J + \lambda^T F$, where $\lambda^T = [\lambda_1 \quad \cdots \quad \lambda_n]$, $F = [f_1(x) \quad \cdots \quad f_n(x)]^T$ to replace J. Solving this minimization problem, one can get a new solution which is more admissible. The above process is repeated if necessary. This procedure of solving a constrained optimization problem is described in detail in [26].

2) The last locomotive is at the rear of the train. In this case, $l_k = n$.

In the above calculation ($l_k < n$), one could consider $l_{k+1} = n$. So one can replace k in the above case with $k - 1$ in this case.

$$f_{in_i} = \begin{cases} iu_b - \sum_{j=1}^{i}(f_{a_j} + m_j a), & 1 \leq i < l_1, \\ (i-1)u_b + u_t - \sum_{j=1}^{i}(f_{a_j} + m_j a), & l_1 \leq i < l_2, \\ \cdots \\ (i-k+1)u_b + (k-1)u_t - \sum_{j=1}^{i}(f_{a_j} + m_j a), & l_{k-1} \leq i < n, \end{cases}$$

and

$$ku_t + (n-k)u_b = \sum_{i=1}^{n}(f_{a_i} + m_i a).$$

One can get similar results.

For instance, $n = 52$, $l_1 = 1$, $l_2 = 52$, then

$$\begin{aligned} u_t &= \frac{1}{2262}\sum_{i=1}^{52}\frac{-3888 - 25i + 5i^2}{2}(f_{a_i} + m_i a), \\ u_b &= \frac{1}{2262}\sum_{i=1}^{52}\frac{1230 + 5i - i^2}{10}(f_{a_i} + m_i a). \end{aligned} \tag{3.13}$$

The mathematic developments for the other two control strategies are similar to the above one.

3.3.3 Simulation of different braking systems

In simulation, one assumes that the train consists of 200 wagons. Every four wagons (a rake) are linked with rigid drawbars of which the in-train forces are not considered and are regarded as one unit. There are two locomotives at the front and two at the rear. The neighbouring locomotives are linked with rigid drawbars and regarded as one unit too. So the train can be regarded as consisting of 50 wagons between two locomotives.

The parameters of the train are given in the following tables [15].

Table 3.2: Locomotive group parameters

m(ton)	c_0(m/s^2)	c_1(1/s)	c_2(1/m)	L(m)
252	7.6685e-3	1.08e-4	2.06e-5	40.94

Table 3.3: Wagon group parameters

m(ton)	c_0(m/s^2)	c_1(1/s)	c_2(1/m)	L (m)	F_b(kN)
417	6.3625e-3	1.08e-4	1.492e-5	48.28	720

In the tables, F_b represents the capacity of brake force, and L is the longitudinal length of a locomotive or wagon group. Fig. 2.5 shows the locomotive (group) effort (7E1) corresponding to a particular notch level and velocity. These data, including the track profile, are based on the COALink trains operated in South Africa by Spoornet. The relation between the displacement and the static force (without damping) of the coupler is shown in Fig. 2.3. The damping coefficient is notoriously difficult to estimate because the train speed is limited by dominant quadratic resistance term. According to [11] it can be as high as $\frac{1}{34}$ of the spring coefficient. Since the damping coefficient is not available in this study, it is taken as $\frac{1}{100}$ of the spring coefficient in the train model, and ignored in the control design.

In (3.6), $\overline{F_{in_i}} = -\underline{F_{in_i}} = 1600$ kN. There are some constraints with the locomotive notch operation. Firstly, the notch could only be changed stepwise; secondly, the locomotive engine should stay at a notch for at least 10 seconds, and when the locomotive's effort changes from traction to dynamic braking or the other way round, the first notch should last at least 20 seconds. The acceleration limit a_r is 0.07 m/s^2. The reference velocity is 36 km/h from the simulation starting point -2 km to 3 km and then it is 43.2 km/h. At the point 6 km, it is changed to 54 km/h, while it is changed to 43.2 km/h again at the point 8 km. Some distances are negative because the reference point is chosen in the middle of the track and the distance values are relative.

The initial state is that the train is in its steady state with all the cars' velocities 10.5 m/s and all the in-train forces zeros. For a traditional train, the time delay for a wagon's braking force is calculated with the wagon's distance to the first locomotives divided by the velocity of sound.

The simulation is processed with MATLAB. The train model runs continuously and the control signal is updated every second.

The track profile is shown in Fig. 3.1. All simulations in this study, without special description, are processed on this track profile and the track profile in the rest is omitted.

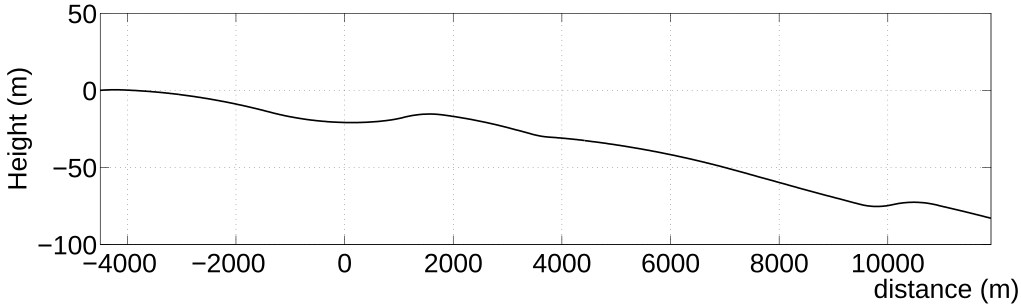

Figure 3.1: Track profile

Simulation results are shown in the following figures.

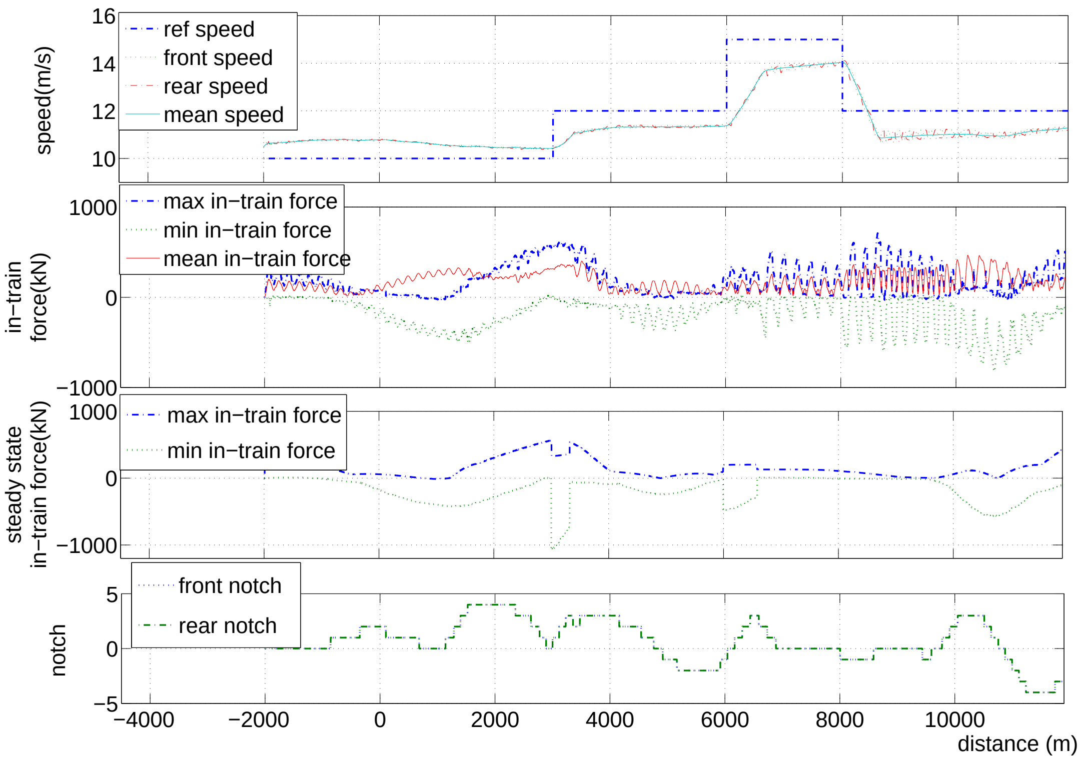

Figure 3.2: 1-1 strategy without ECP

Fig. 3.2 and Fig. 3.3 show the applications to traditional heavy haul trains (with pneumatic braking system) of 1-1 strategy controller and 2-1 strategy controller, respectively. The train is not equipped with an ECP braking system and therefore there are time delays for the wagons' control signal transmission.

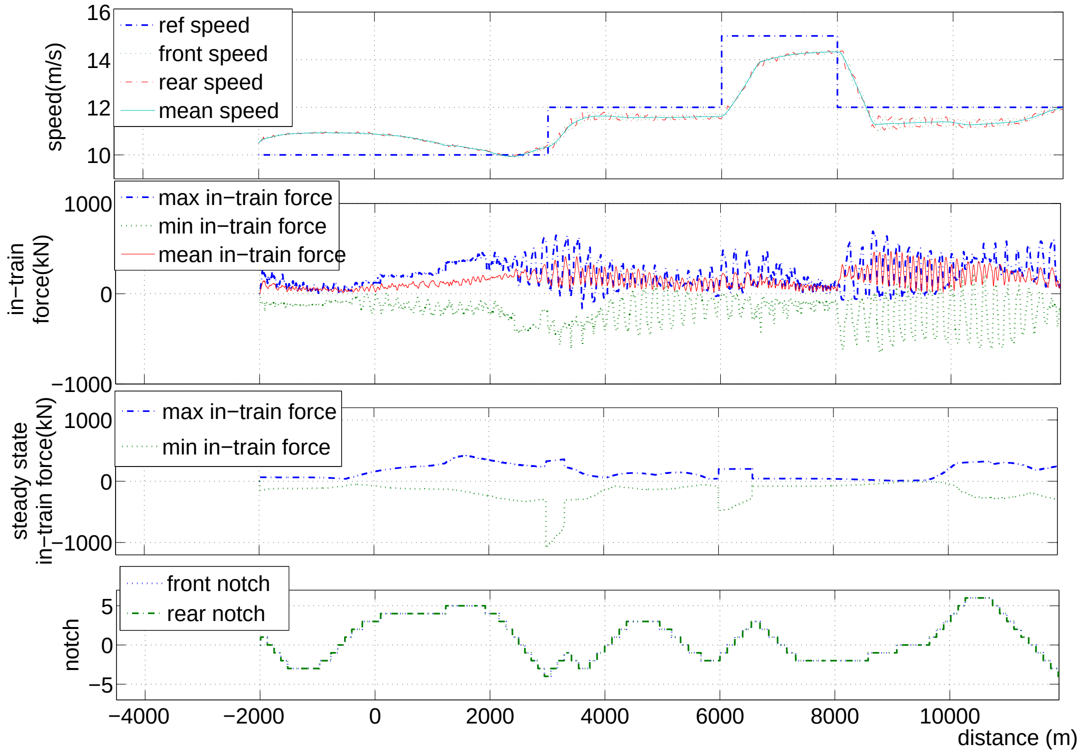

Figure 3.3: 2-1 strategy without ECP

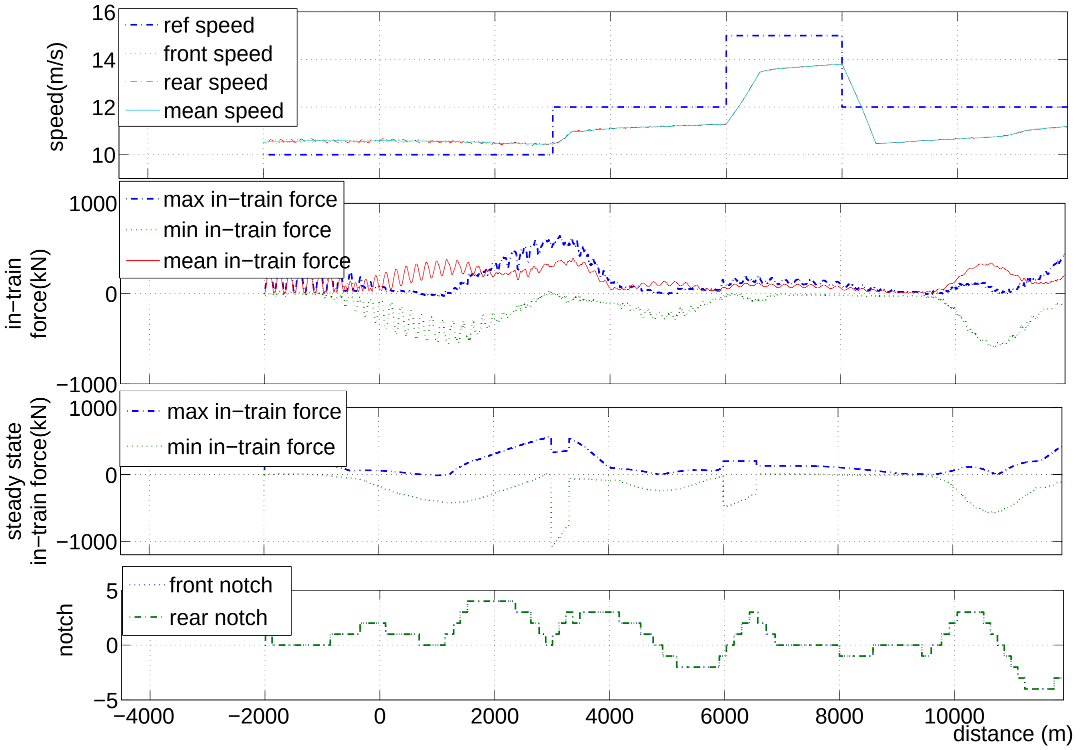

Figure 3.4: 1-1 strategy with ECP

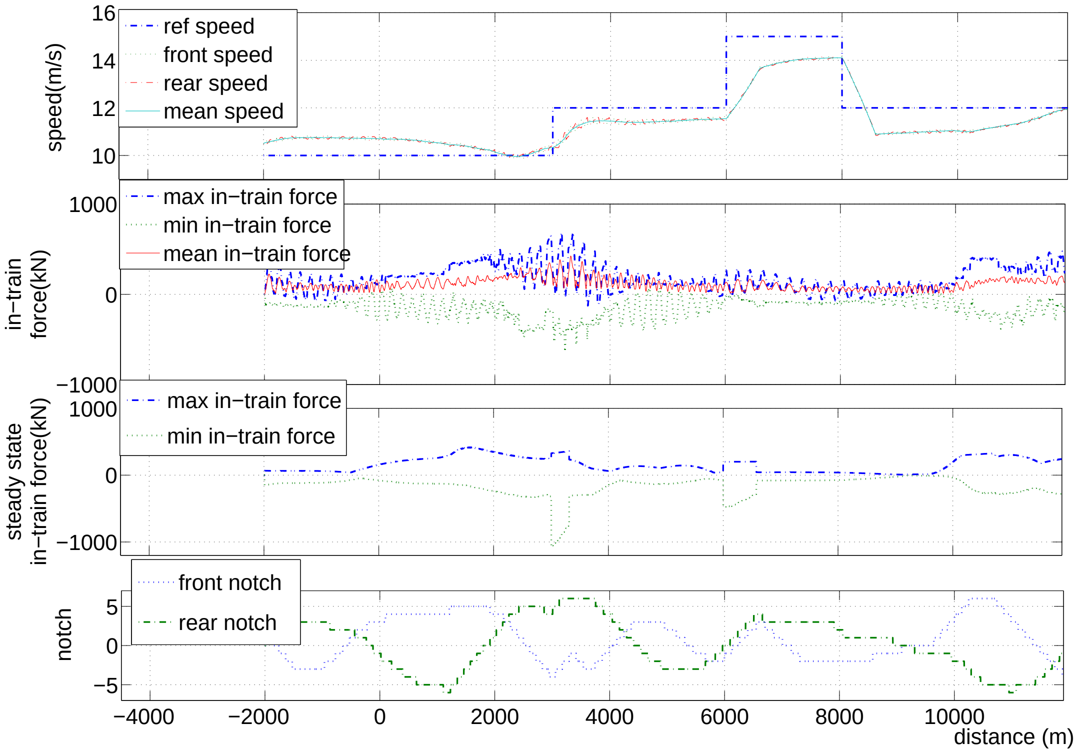

Figure 3.5: 2-1 strategy with ECP

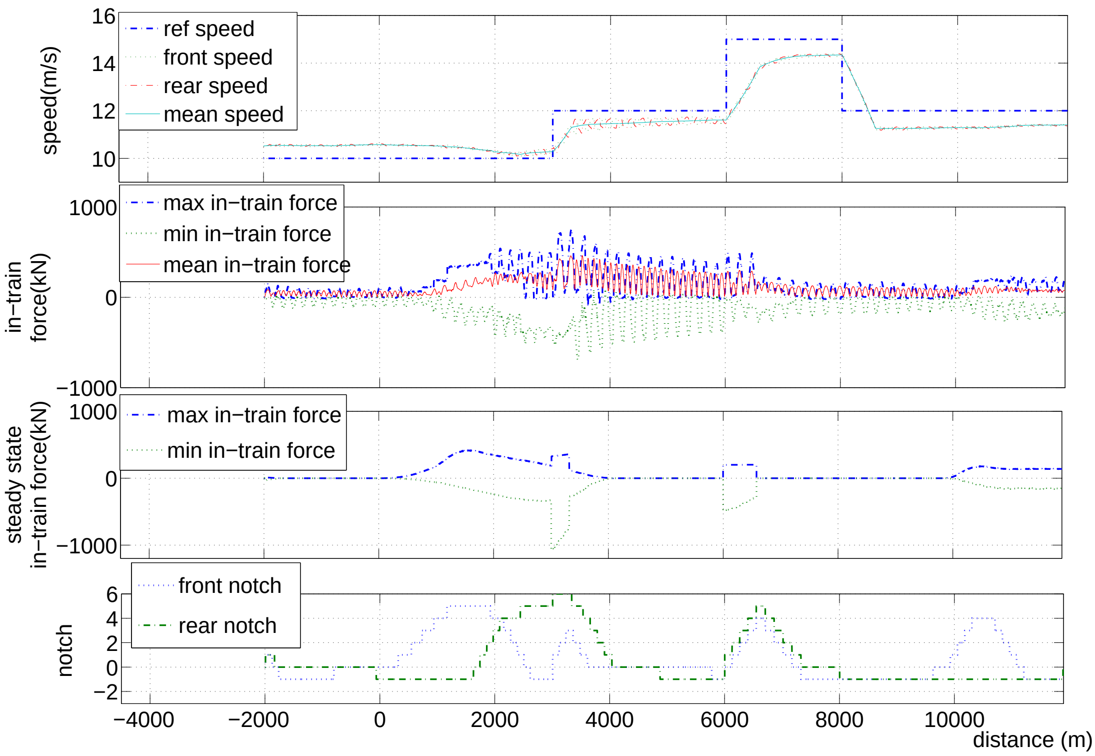

Figure 3.6: 2-2 strategy with ECP/iDP

The applications to a heavy haul train with an ECP braking system installed of 1-1 strategy controller, 2-1 strategy controller and 2-2 strategy controller are shown in Fig. 3.4, Fig. 3.5 and Fig. 3.6, respectively. The control inputs in Fig. 3.4 are the same as in Fig. 3.2 and those in Fig. 3.5 are the same as in Fig. 3.3. However, because of the installation of an ECP braking system, there is no time delay for the wagon control signal transmission.

In these figures, the first subplot is the front locomotive group speed, rear locomotive group speed and the mean speed of all the cars with respect to the distance from the starting point. The second subplot is maximum and minimum in-train forces and mean in-train force (the mean value of the absolute values of all the in-train forces at a specific time with respect to the distance). The third is the steady in-train forces, which are calculated by applying the efforts of the cars to the train model, with the reference speed (and the acceleration) maintained and the dynamic process ignored.

Table 3.4 is the performance comparison of Fig.s from 3.2 to 3.6. The variable $|\delta\bar{v}|$ is the absolute value of the difference between the reference velocity and the mean value of all the cars' velocities at a specific point. $\overline{|f_{in}|}$ is the mean value of the absolute values of all the couplers' in-train forces at a specific point. Item E is the energy consumed during travel. The items max, mean and std are the maximum value, mean value and standard deviation of the statistical variable, respectively.

Table 3.4: Comparison of braking systems: pneumatics vs. ECP

	$\lvert\delta\bar{v}\rvert$(m/s)			$\overline{\lvert f_{in}\rvert}$(kN)			E
	max	mean	std	max	mean	std	(MJ)
Fig. 3.2	3.6225	0.9179	0.51	466.46	173.17	96.61	1,170
Fig. 3.4	3.7063	0.9650	0.53	391.82	146.70	101.79	1,180
Fig. 3.3	3.3715	0.7032	0.54	467.16	151.20	88.26	2,110
Fig. 3.5	3.4530	0.7401	0.50	424.50	113.51	63.05	2,130
Fig. 3.6	3.3901	0.6510	0.42	461.86	111.18	90.95	1,370

When comparing Fig. 3.2 with Fig. 3.3, it can be seen that the locomotive speed error is smaller in the 2-1 strategy than in the 1-1 strategy. The absolute values of the maximum and the minimum in-train forces are smaller with the 2-1 strategy when it comes to steady running. However, the energy consumption with the 1-1 strategy is a little less than with the 2-1 strategy. This is because some energy is used to overcome the in-train forces' fluctuation and larger brake forces are applied. The same result can be seen when comparing Fig. 3.4 with Fig. 3.5.

When comparing Fig. 3.5 with Fig. 3.6, the locomotive speed fluctuation and error with the 2-2 strategy are smaller than those with the 2-1 strategy. The absolute values of maximum and minimum in-train forces with the 2-2 strategy are also a little lower than with the 2-1 strategy. The energy consumption with the 2-1 strategy is a little higher than with the 2-2 strategy. The steady in-train forces with the 2-2 strategy are nearly zero when the train is running in its steady state without velocity accelerations or decelerations.

When comparing Fig. 3.2 with Fig. 3.4, both with the 1-1 strategy, the former train is equipped with a pneumatic braking system and the latter is equipped with an ECP braking system. The speed fluctuation in Fig. 3.2 is greater than in Fig. 3.4. The absolute values of maximum and minimum in-train forces and the mean in-train forces are greater in Fig. 3.2 than in Fig. 3.4. The energy consumption is almost equal in both figures.

When comparing Fig. 3.3 with Fig. 3.5, one can reach similar conclusions than when comparing Fig. 3.2 with Fig. 3.4.

It can be seen that the velocity error exists in all the results when open loop scheduling is used. When comparing the steady in-train forces, which represent the reference value for closed-loop control, the performance of the 2-2 strategy is the best among the three control strategies and the performance of the train equipped with an ECP braking system is better than that of a traditional train. With the introduction of the acceleration profile, the speed variations lead to larger in-train forces, especially within the speed acceleration periods. However, the accelerations decrease the speed tracking error. The transient control is a “trade-off” between the two aspects.

From the simulation results, the following conclusions can be drawn:

1) The scheduling with the averagely distributed power among the locomotives is not optimal for train performance.
2) The higher the number of controllable inputs is, the better the train performance.
3) The ECP braking system has demonstrated superb performance compared with a pneumatic braking system.
4) The 2-2 strategy is the best among the strategies for heavy haul trains equipped with ECP braking systems.
5) Open loop scheduling cannot yield satisfactory performance, but may be a good reference for closed-loop control, which is the purpose of this chapter.

3.4 Scheduling

An open loop controller is used to calculate the inputs when a train is running in its steady state with the reference velocity and acceleration maintained.

In [11], the off-line schedule for the throttling and braking inputs is chosen in such a way that the train is in its steady state with the reference velocity maintained. The settings do not contribute to additional accelerations/decelerations of the train. The schedule determines the sequencing and the amplitudes of the inputs in case there are continuous input variations and no power limits. The applied inputs ${}^{o}u(t)$ are nonlinear functions of the schedule parameters p (grade of the track, velocity profile and train data) and the travelled distance z of the train: ${}^{o}u = f_u(z, p)$. The inputs are approximated by step functions of variable amplitudes. Such an optimal problem can be solved with MISER developed Toe in [74]. The sequence of the steps is predetermined and tuned, and the time instants of the step functions at which the steps are applied are decided on line. It is obvious that this off-line schedule is heuristic and subject to the pre-determined control sequence, so it will not be discussed further in this study.

The transient control is the same as in section 3.3.1.

3.4.1 Heuristic scheduling

According to [15], open loop control is chosen as follows, with $Beq = \sum_{i=0}^{n}(f_{a_i} + m_i a)$,

$$\begin{array}{lll} u_l = Beq/k, & u_b = 0 & Beq \geq 0, \\ u_l = Beq/n, & u_b = Beq/n & Beq < 0, \end{array} \tag{3.14}$$

where u_l is the locomotives' effort and u_b is the wagons' effort, and the variables k and n are the respective total numbers of locomotives and cars. The acceleration $a = 0$ in the cruising period while $a = \pm a_{rr}$ in the scheduled acceleration/decelertation periods. The power distribution is heuristic, so one calls it heuristic scheduling.

3.4.2 Optimal scheduling

According to the three operational strategies described in Section 3.2, there are three corresponding optimal open loop controllers for the train. In the following, the performance is a function of the in-train forces and the energy, which can be written as

$$J = \sum_{i=0}^{n-1} K_f f_{in_i}^2 + \sum_{i=0}^{n} K_e u_i^2, \tag{3.15}$$

where the weights of the in-train force and energy consumption are K_f and K_e, respectively. Optimal power distribution is characteristic of this scheduling, so one calls it optimal scheduling.

The energy cost is not proportional to u^2, but items proportional to u^2 are also evaluations of energy. From the energy point of view, it is not rational to include the minus input in the energy consumption function. However, considering the braking application for the cost of maintenance of the rail track and the train wheels, it is retained in the performance function although it is not suitable to be classified into energy. On the other hand, if necessary, it is not difficult to separate the positive inputs and negative inputs in the performance function. In this study the energy consumption and the cost of braking are processed in a simple way.

For open loop control, the dynamic process in the train is ignored and the system is assumed to be in its steady state with acceleration maintained, that is,

$$\frac{dv_i}{dt} = a, \frac{dx_j}{dt} = 0, \quad i = 1, \cdots, n, \quad j = 1, \cdots, n-1. \tag{3.16}$$

Applying (3.16) to (2.2), and assuming $f_{in_0} = 0$, one has

$$u_s + f_{in_{s-1}} - f_{in_s} - f_{a_s} - m_s a = 0, \qquad s = 1, \cdots, n. \tag{3.17}$$

In train operations, the inputs, $u_i, i = 1, \cdots, n$ and the in-train forces f_{in_i} have some constraints.

$$\begin{aligned} \underline{U}_i \leq u_i \leq \overline{U}_i, \qquad & i = 1, \cdots, n; \\ \underline{F}_{in_j} \leq f_{in_j} \leq \overline{F}_{in_j}, \qquad & j = 1, \cdots, n-1, \end{aligned} \tag{3.18}$$

where $\underline{U}_i, \overline{U}_i$ are the upper and lower constraints for the ith input, and $\underline{F}_{in_j}, \overline{F}_{in_j}$ are the upper and lower constraints for the jth in-train force, respectively. For wagons, $\overline{U_i} = 0$ and the values of $\underline{U_i}$ depend on the braking capacities of the wagons. For locomotives, the constraints $\underline{U}_i, \overline{U}_i$ depend on the locomotives' capacities in traction efforts and the running states. The constraints $\underline{F}_{in_j}, \overline{F}_{in_j}$ are limited because of the requirement of safe operation and maintenance cost.

Thus optimal scheduling is a standard quadratic programming (QP) problem with objective function (3.15), equality constraints (3.17), inequality constraints (3.18) and some additional equality constraints.

With the 1-1 strategy, the additional constraints imposed on the optimization problem are

$$u_{l_j} \triangleq u_t, \qquad u_i \triangleq u_b, \quad i = 1, \cdots, n; \quad i \neq l_j, j = 1, \cdots, k. \tag{3.19}$$

With the 2-1 strategy, the additional constraints are the following

$$u_i \triangleq u_b, \qquad i = 1, \cdots, n; \quad i \neq l_j, j = 1, \cdots, k. \tag{3.20}$$

With the 2-2 strategy, there is no additional constraint.

3.4.3 Simulation of heuristic scheduling vs. optimal scheduling

There is only one open loop operational strategy for heuristic scheduling, as shown in Fig. 3.7. Figs. 3.8, 3.9 and 3.10 are the optimal scheduling of 1-1 strategy, 2-1 strategy and 2-2 strategy respectively, with $K_e = 1,\ K_f = 1$.

In these figures, the first subplot is the front locomotive group speed, rear locomotive group speed and the mean speed of all the cars with respect to the distance from the starting point. The second subplot is maximum and minimum in-train forces and the mean value of the absolute values of all the in-train forces in a specific time with respect to the distance.The third is the steady in-train forces, which are calculated by applying the efforts of the cars to the train model with the reference speed (and the acceleration) maintained and the dynamic process ignored. As can be seen there are dips in the third subplots of these figures when the reference speed changes. This is because the steady-state in-train forces in the third subplots are the calculation results of the algebraic equations. When the reference speed has a step-type change in the algebraic equations, the other variables, such as in-train forces, unavoidably have step-type changes, which results in the dips.

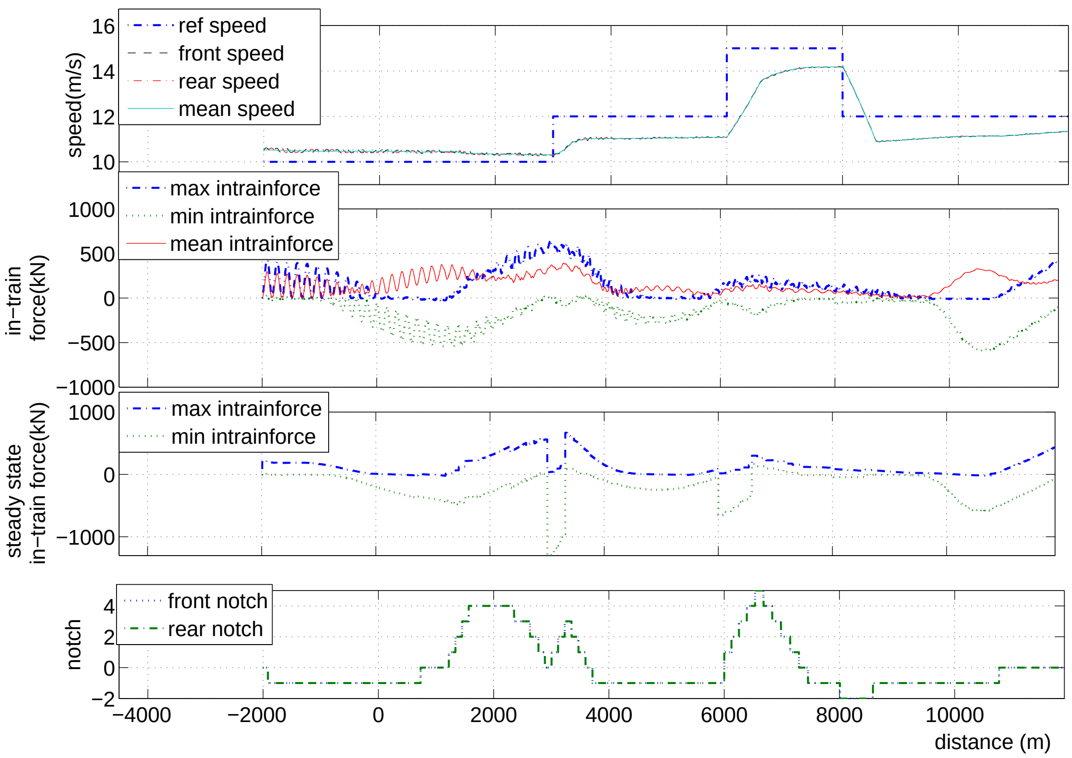

Figure 3.7: Heuristic scheduling

Table 3.5 is the performance comparison of Fig. 3.7 to Fig. 3.10. The variable $|\delta \bar{v}|$ is the absolute value of the difference between the reference velocity and the mean value of all the cars' velocities at a specific point. $\overline{|f_{in}|}$ is the mean value of the absolute values of all the couplers' in-train forces at a specific point. The item E is the energy consumed during travel. The items max, mean and std are the maximum value, mean value and standard deviation of the statistical variable, respectively.

The running results of the open loop scheduling show that the velocity tracking

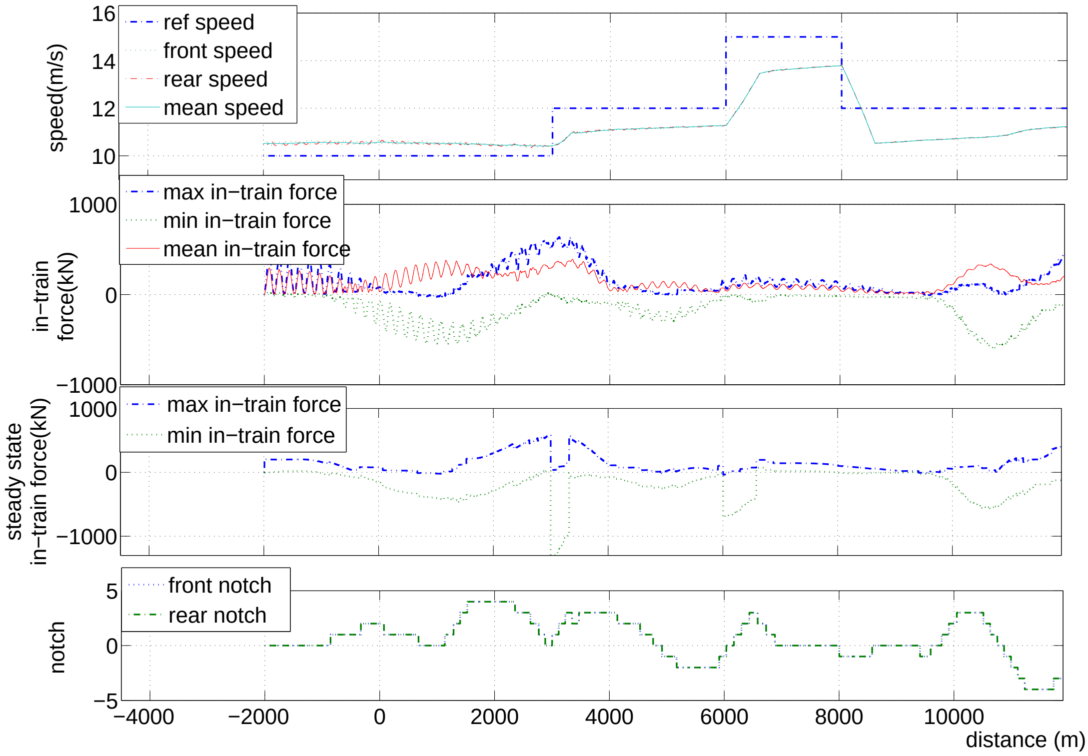

Figure 3.8: 1-1 strategy optimal scheduling

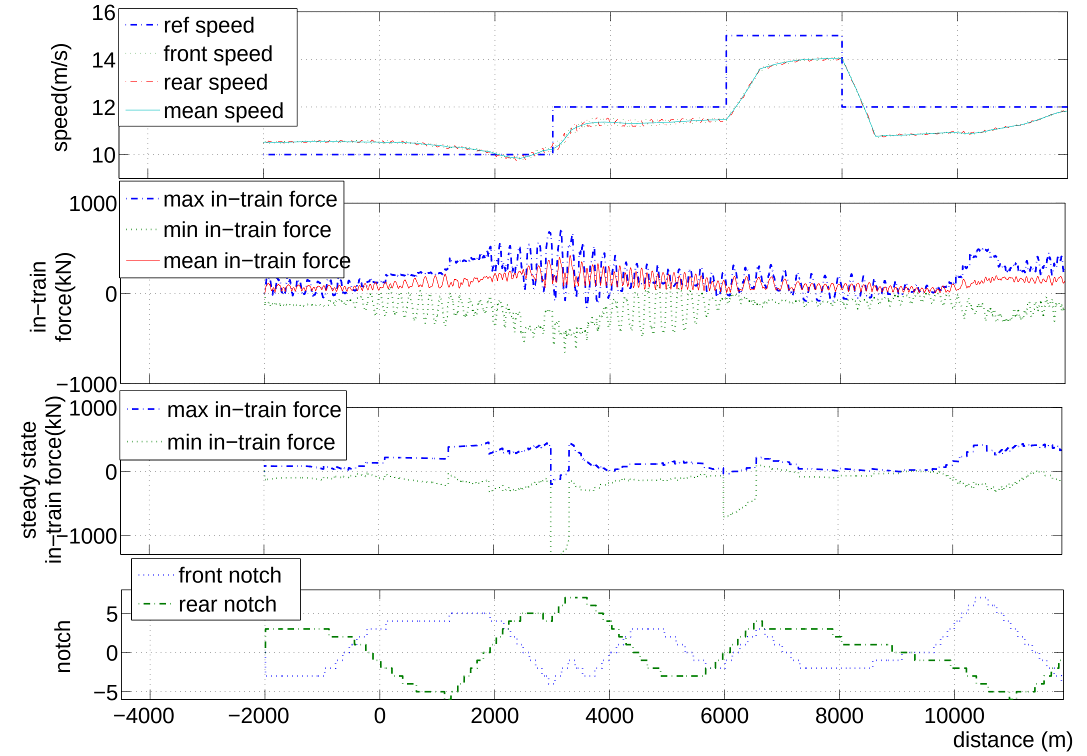

Figure 3.9: 2-1 strategy optimal scheduling

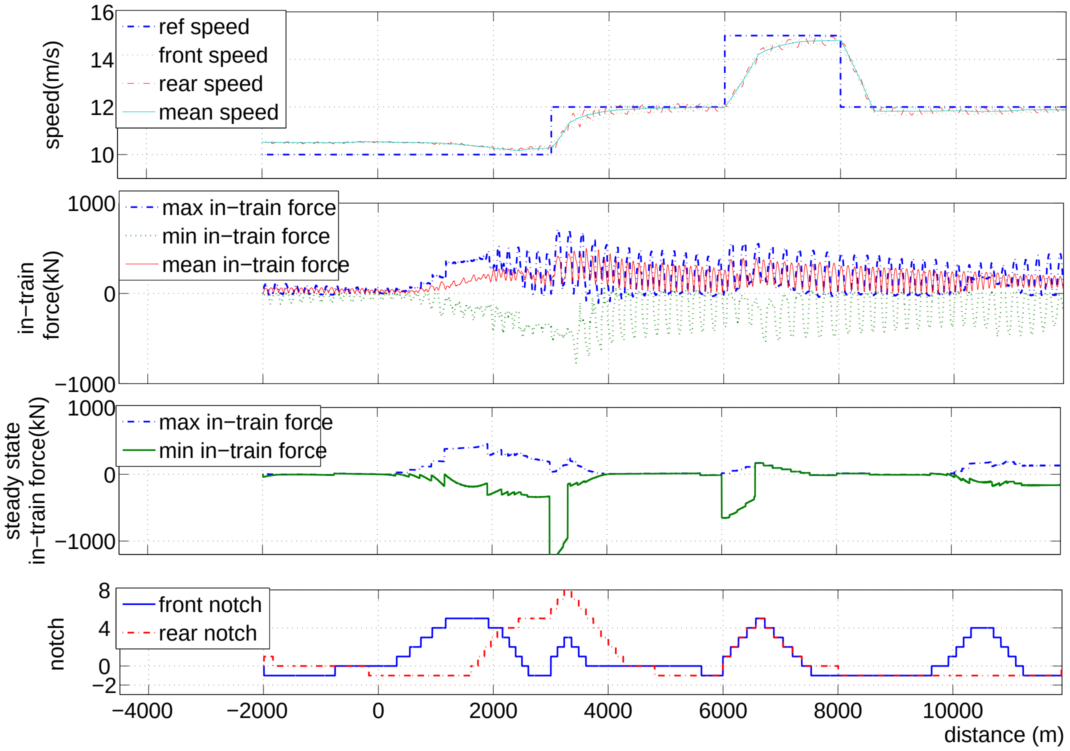

Figure 3.10: 2-2 strategy optimal scheduling

Table 3.5: Comparison of optimal scheduling vs. heuristic scheduling

	$\lvert\delta\bar{v}\rvert$(m/s)			$\overline{\lvert f_{in}\rvert}$(kN)			E
	max	mean	std	max	mean	std	(MJ)
Fig. 3.7	3.9179	0.8225	0.53	390.23	143.85	98.86	8,520
Fig. 3.8	3.7187	0.9410	0.54	392.13	144.81	101.83	11,400
Fig. 3.9	3.5277	0.7460	0.54	420.31	118.51	70.42	23,300
Fig. 3.10	3.0195	0.4152	0.46	498.59	141.43	103.73	16,400

error exists in all the scheduling. The performance of the heuristic scheduling and 1-1 strategy optimal scheduling are similar. The performance of in-train force of 2-2 strategy optimal scheduling is the worst because of oscillation, while its speed tracking error is the smallest. The velocity tracking error and the possibility of oscillation are the drawbacks of an open loop controller.

However, the performance of the steady-state in-train force of the 2-2 strategy optimal scheduling is best. The performance of the steady-state in-train force of 1-1 strategy optimal scheduling is similar to that of heuristic scheduling. The performance of the steady-state in-train force of 2-1 strategy optimal scheduling is also better than that of 1-1 strategy optimal scheduling, except within the acceleration/deceleration periods, where the states of the train change abruptly. Actually, the entire state of the train should change continuously, which leads to smoother change. The open loop scheduling does not consider the real running state, and it is difficult to say which scheduling is best. However, the open loop scheduling provides a reference to the closed-loop controller, so the steady state calculated by the scheduling is more important than the real running result. From this point of view, one can see that the performance of the 1-1 strategy optimal scheduling is similar to that based on heuristic scheduling, and the performance of 2-2 strategy optimal scheduling is best.

3.5 LQR controller

3.5.1 LQR closed-loop controller

With the calculation of open loop scheduling (optimal scheduling or heuristic scheduling), the steady state and input of the train can be denoted as $f^0_{in_j}(x^0_j), v^0_i(v_r), u^0_i, j = 1, \cdots, n-1, i = 1, \cdots, n$, which are the in-train forces (static displacement of coupler), the velocities and the traction forces or braking forces of the cars. The static displacement x^0_j is interpolated from $f^0_{in_j}$. Then one can rewrite the train model with the following equations.

$$\begin{aligned} &\delta\dot{v}_s = (\delta u_s + \delta f_{in_{s-1}} - \delta f_{in_s} - \delta f a_s)/m_s, \qquad s = 1, \cdots, n, \\ &\delta\dot{x}_j = \delta v_j - \delta v_{j+1}, j = 1, \cdots, n-1, \end{aligned} \tag{3.21}$$

where $\delta v_s = v_s - v^0_s = v_s - v_r, \delta u_s = u_s - u^0_s, \delta f_{in_s} = f_{in_s} - f^0_{in_s}, \delta x_j = x_j - x^0_j$. The variable v_r is the reference speed. When the damping of the coupler is ignored, this model can be linearized as follows:

$$\begin{aligned} &\delta\dot{v}_s = (\delta u_s + k_{s-1}\delta x_{s-1} - k_s \delta x_s)/m_s - (c_{1_s} + 2c_{2_s} v_r)\delta v_s, \quad s = 1, \cdots, n, \\ &\delta\dot{x}_j = \delta v_j - \delta v_{j+1}, \quad j = 1, \cdots, n-1, \end{aligned} \tag{3.22}$$

where k_s is the linearized coefficient of the coupler with the assumption $k_0 = 0$. The model can be written as

$$\dot{X} = AX + BU,$$

where $X = [\delta v_1, \cdots, \delta v_n, \delta x_1, \cdots, \delta x_{n-1}]^T$, $U = [\delta u_1, \cdots, \delta u_n]^T$, $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, $A_{11} = -\text{diag}(c_{1_1} + 2c_{2_1}v_r, \cdots, c_{1_n} + 2c_{2_n}v_r)$, $A_{22} = 0_{(n-1)\times(n-1)}$, $B = \text{diag}(\frac{1}{m_1}, \cdots, \frac{1}{m_n})$.

$$A_{12} = \begin{bmatrix} -\frac{k_1}{m_1} & 0 & \cdots & 0 & 0 \\ \frac{k_1}{m_2} & -\frac{k_2}{m_2} & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \frac{k_{n-2}}{m_{n-1}} & -\frac{k_{n-1}}{m_{n-1}} \\ 0 & \cdots & 0 & 0 & \frac{k_{n-1}}{m_n} \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix},$$

The variables k_i, $i = 1, \cdots, n-1$ are chosen to be constant. Although different scheduling has different equilibria, the coefficients in the linearized model (3.22) are identical.

In simulation, however, the original nonlinear model has been used for the train model.

When an LQR controller is to be designed with the approach in [16], the performance function is chosen as

$$\begin{aligned} \delta J &= \int (X'QX + U'RU)\, dt \\ &= \int \left(\sum_{i=0}^{n-1} K_f^o \delta x_i^2 + \sum_{i=0}^{n} K_e \delta u_i^2 + \sum_{i=0}^{n} K_v^o \delta v_i^2 \right) dt, \end{aligned} \tag{3.23}$$

where K_f^o, K_e, K_v^o are the weights for in-train forces, energy consumption and velocity tracking, respectively. When the coefficients K_f^o, K_e, K_v^o are chosen so that the first item of (3.23) dominates, the controller is an in-train force emphasized one. When the second item of (3.23) dominates, the controller is an energy emphasized one. It is speed emphasized control if the third item dominates.

Based on the optimization approach [27], one can get feedback control $U = -KX$, and the complete closed-loop control is

$$u = U + u^0. \tag{3.24}$$

3.5.2 Anti-windup technique

Within a closed-loop controller in this thesis, open loop scheduling is used to calculate the inputs when a train is running in its steady state with the reference velocity maintained and the input constraints are not considered in open loop scheduling. Since the throttle of the locomotives takes discrete values and the braking capacities of the wagons are constrained, when the control inputs u of a closed-loop controller are applied to the train, an anti-windup technique is employed. For the wagons, the application of the anti-windup technique is very simple. For the locomotives, the inputs are discrete with some operation constraints. Similar methods as described in [29] are used to smooth continuous control inputs. Assuming the required force of jth locomotives is F_j and the output of the kth notch with the current velocity v_j is $g(k, v_j)$, the output force F_j^r for the jth locomotives can be defined as

$$\begin{aligned} F_j^r &= g(k, v_j) \quad \text{if} \quad G(k-1, v_j) \leq F_j < G(k, v_j), \\ G(k, v) &= g(k, v_j) + \alpha \left(g(k+1, v_j) - g(k, v_j) \right). \end{aligned} \tag{3.25}$$

In (3.25), $G(k, v_j)$ and $G(k-1, v_j)$ are the upper and lower boundaries of the admitted notch k, respectively. The variable α is the ratio of the separation for the boundary. In simulation, α is chosen as 0.5.

3.5.3 Simulation settings of LQR controllers

In simulation, the model parameters of the train, the track profile and the reference speed profile are the same as those in the previous chapter, as well as the locomotive notch operation constraints.

The track profile shown in Fig. 3.1 is from the COALink line, which is typically downhill when the train is loaded. In this section of track, there are also two uphill segments, which make it difficult to drive a long train that may extend over several different gradient sections at any given time. The largest incline degree is 0.09152 and the largest decline one is -0.1, which are very similar to the slope degree (+/- 0.1) in [3].

A piecewise linear function is used to approximate the non-linear function of a coupler. In the controllers one chooses a greater value, namely 3×10^7 N/m for all the couplers' linearized coefficients in (3.22).

A safe-operation requirement for a train on the COALink is that the in-train forces should be less than $\pm 2,000$ kN. In simulation, $\overline{F_{in_i}}, \underline{F_{in_i}}$ are chosen as $1,200$ kN, considering the redundancy for a longer train with 800 wagons.

In the simulation, the weights for in-train force, energy and velocity are K_f, K_e, K_v, respectively, and $K_f^o = 3 \times 10^8 K_f, K_v^o = 5 \times 10^6 K_v$, which gives the same value for the in-train forces, speed deviation and input in (3.23) as would be obtained when $\delta x = 0.01m, \delta v = 0.1$ m/s$^2, \delta u = 200$ N with the weights $K_f = K_e = K_v$.

The acceleration limit a_{rr} is 0.07 m/s^2. This value is calculated on the assumption that the train is running on a flat track and all the traction power of the locomotives is used to accelerate. The maximum acceleration can be $760 \times 2/(252 \times 2 + 417 \times 50) = 0.07118$ m/s^2. The maximum deceleration is more than the maximum acceleration, but in the simulation they are assumed to be the same for the sake of simplicity.

The initial state of the train is that the train is in its steady state with all the cars' velocities 10.5 m/s and all the in-train forces equal to zero.

3.5.4 Simulation results of LQR controllers

Simulation results of the three different strategies' closed-loop controllers based on the heuristic scheduling and optimal scheduling are shown from Fig. 3.11 to Fig. 3.16, where the weights are $K_f = 1$, $K_e = 1$, $K_v = 1$. The energy consumption in these figures is shown in Table 3.6.

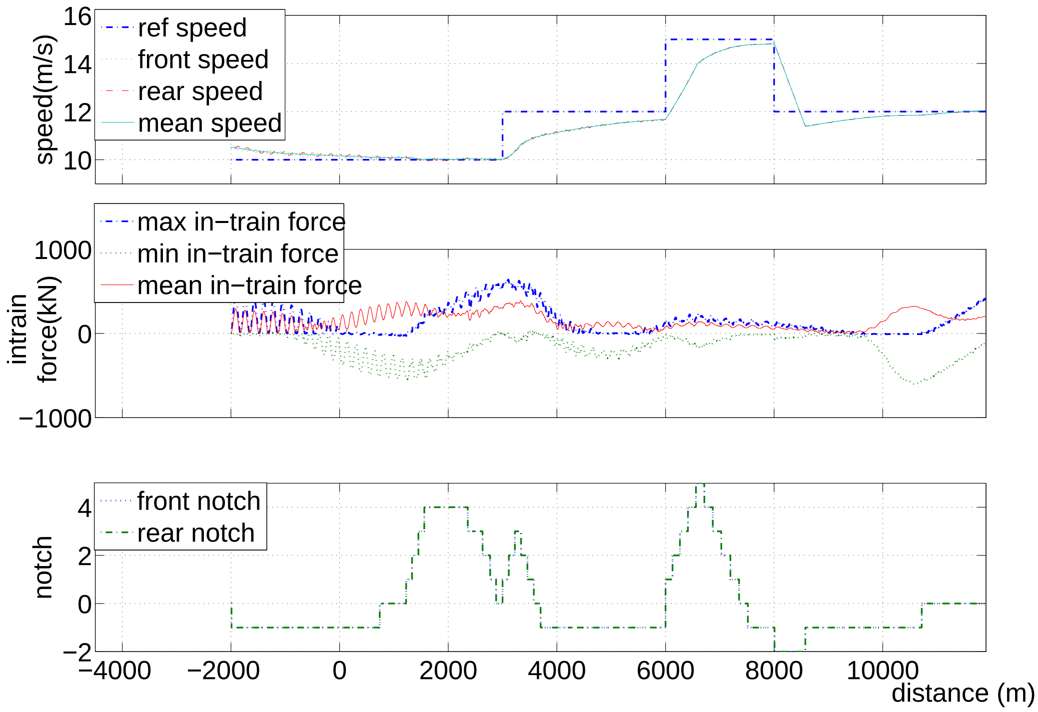

Figure 3.11: 1-1 strategy closed-loop control based on heuristic scheduling

When comparing the figures of the closed-loop controllers with those of the open loop scheduling, it is obvious that the steady velocity error is much smaller and better in closed-loop controllers than in open loop scheduling. For heuristic scheduling, the performances of the in-train force and the energy consumption of the open loop scheduling are similar to those of closed-loop controllers. For optimal scheduling, the performances of the energy consumption with the closed-loop controllers of the three

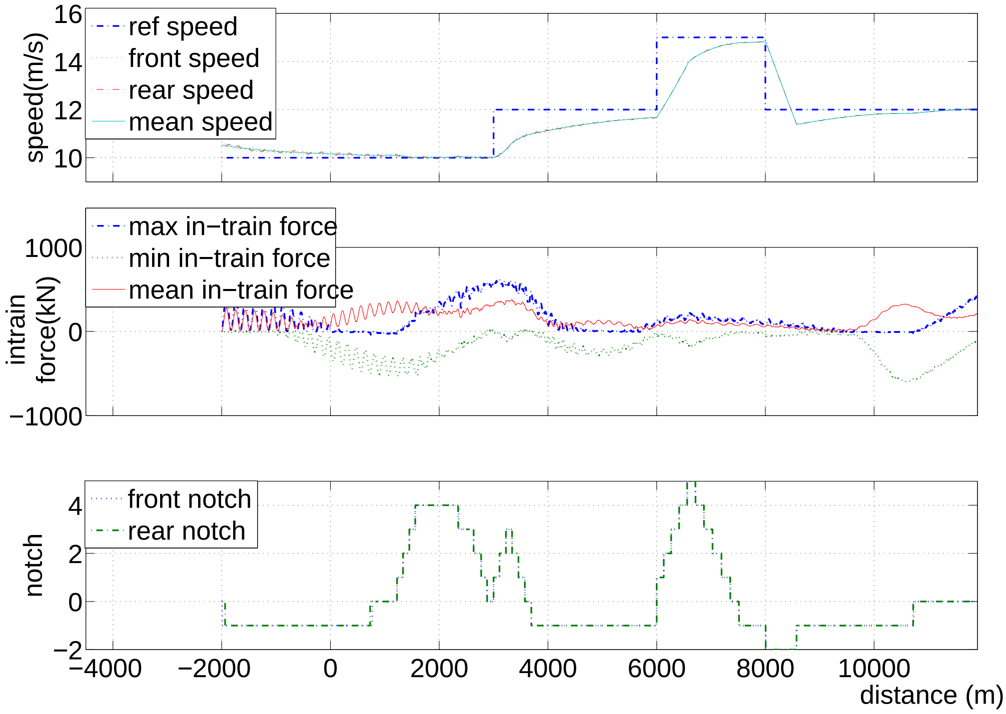

Figure 3.12: 2-1 strategy closed-loop control based on heuristic scheduling

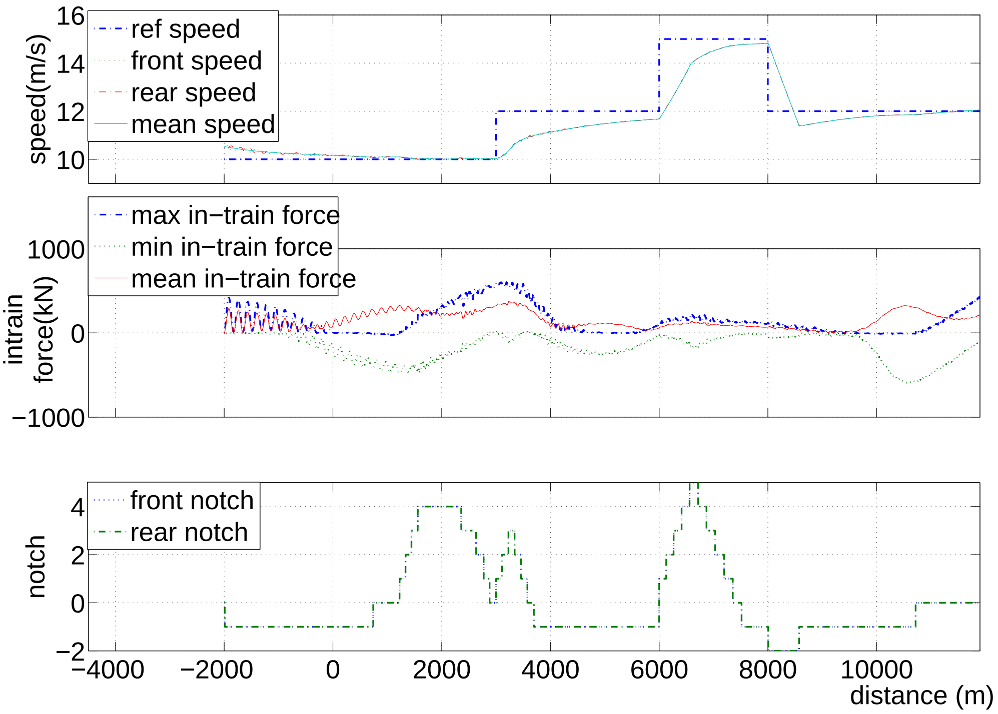

Figure 3.13: 2-2 strategy closed-loop control based on heuristic scheduling

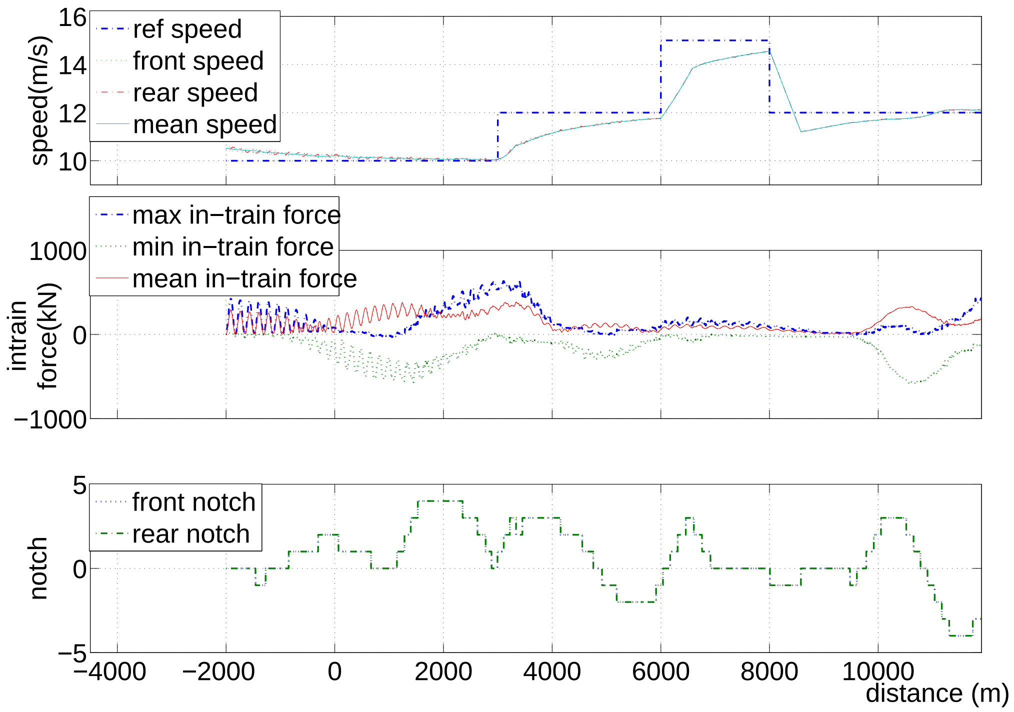

Figure 3.14: 1-1 strategy closed-loop control based on optimal scheduling

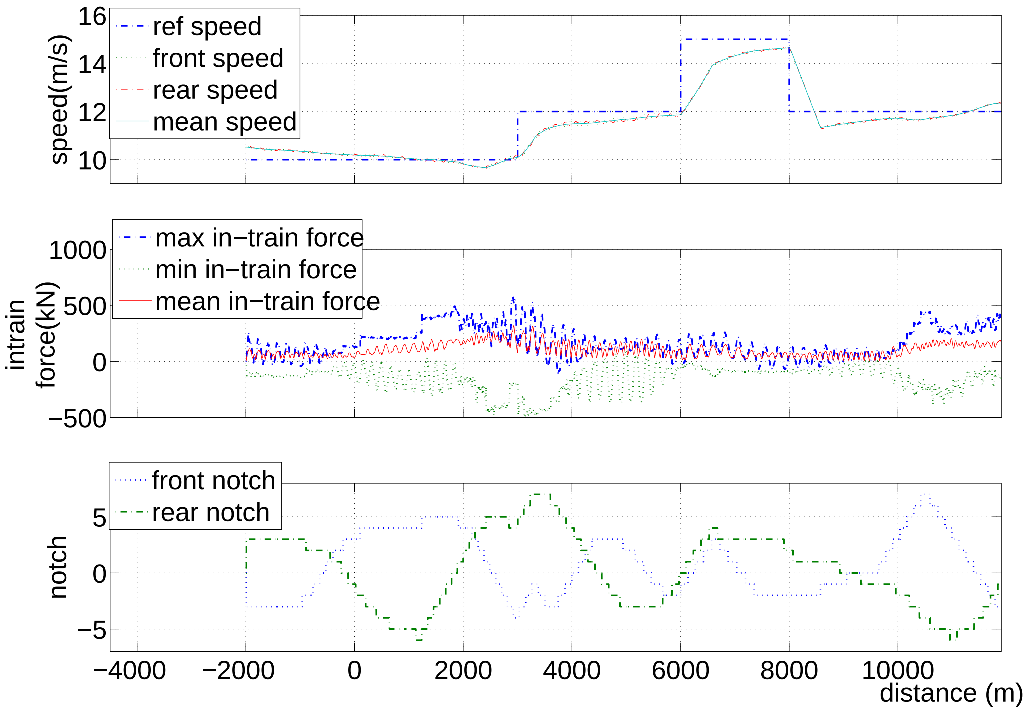

Figure 3.15: 2-1 strategy closed-loop control based on optimal scheduling

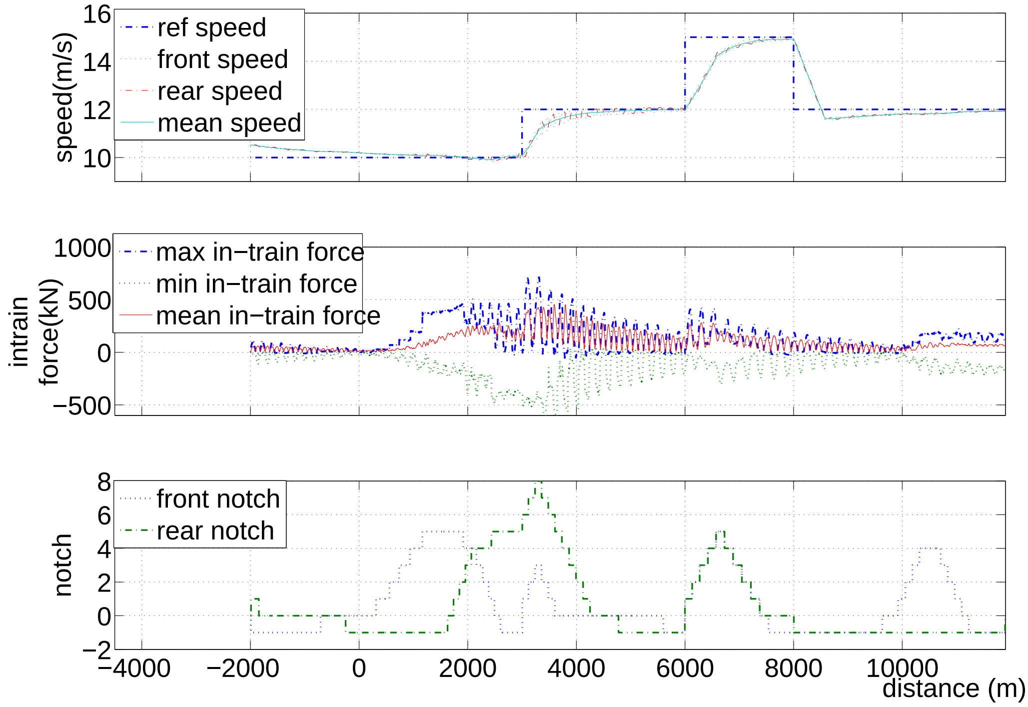

Figure 3.16: 2-2 strategy closed-loop control based on optimal scheduling

strategies are similar to those of the corresponding open loop scheduling. The 2-1 strategy and 2-2 strategy closed-loop control give better in-train force performances than the corresponding scheduling.

When comparing the three different strategies' closed-loop controllers based on heuristic scheduling, the performances are very similar.

When comparing the closed-loop controllers of the three different strategies based on corresponding optimal scheduling, the performances of the velocity and in-train force with the 2-2 strategy are best and those with the 1-1 strategy are worst. The energy consumption with the 1-1 is a little better than that with the 2-2 strategy, which is also a little better than that with the 2-1 strategy.

When comparing the corresponding strategy closed-loop controllers based on optimal scheduling and heuristic scheduling, the energy consumption with the three different strategies based on heuristic scheduling is less than that based on the corresponding optimal scheduling. The performances of the velocity and the in-train force with the 2-2 strategy based on optimal scheduling are better than those based on heuristic scheduling. The performance of the velocity with the 1-1 strategy based on optimal scheduling is worse than that based on heuristic scheduling, while the performances of the in-train force based on the two scheduling approaches are similar. The performances of the velocity with the 2-1 strategy based on the two scheduling approaches are similar and the performance of in-train force with the 2-1 strategy based on optimal scheduling is better than that based on heuristic scheduling.

From the above comparison, it can be seen that the performances of the in-train force and the velocity with the 2-2 strategy based on optimal scheduling are best. In this strategy, it is very interesting to see, as depicted in Fig. 3.16, the variation of the traction efforts of the front and rear locomotives (groups) when the train travels from 0 m to $4,000$ m and from $9,500$ m to $11,000$ m; those sections are hills in the track. When the front locomotives (groups) are climbing uphill and the rear ones are driving downhill, the front locomotives make increasing traction efforts and the rear ones are braking. When more and more cars are climbing uphill, the rear locomotives begin to make traction efforts, increasing gradually. When the front locomotives pass the top of the hill and begin to drive down, their efforts begin to decrease and the rear ones increase their efforts gradually. When the front locomotives are driving downhill and the rear ones are climbing uphill, the front ones are braking and the rear ones make traction efforts. At $3,000$ m, the train begins to accelerate from 10 m/s to 12 m/s. The front and rear locomotives begin to increase their traction efforts simultaneously, which can also be seen from distance points $6,000$ m and $8,000$ m. That is consistent with common sense.

Table 3.6: Performance with $K_e = 1, K_f = 1, K_v = 1$

	$\|\delta\bar{v}\|$(m/s)			$\overline{\|f_{in}\|}$(kN)			E
	max	mean	std	max	mean	std	(MJ)
C01	3.3241	0.4573	0.58	386.94	145.82	100.27	8,700
C02	3.3244	0.4539	0.57	376.78	145.30	99.32	8,610
C03	3.3241	0.4613	0.58	373.60	144.45	97.50	8,470
C1	3.2274	0.4992	0.56	387.04	147.52	102.65	11,760
C2	3.1405	0.4585	0.53	318.97	106.16	59.35	22,100
C3	3.0182	0.3166	0.48	454.50	97.40	86.44	16,500

Table 3.7: Performance with $K_e = 1, K_f = 1, K_v = 10$

	$\|\delta\bar{v}\|$(m/s)			$\overline{\|f_{in}\|}$(kN)			E
	max	mean	std	max	mean	std	(MJ)
C01	3.0412	0.3062	0.55	394.39	145.72	99.57	8,620
C02	3.0413	0.3080	0.55	394.50	144.54	100.07	8,550
C03	3.0412	0.3085	0.55	369.24	144.61	96.63	8,586
C1	3.0070	0.3372	0.57	382.57	147.38	102.40	11,100
C2	2.9891	0.3629	0.53	344.95	103.57	67.20	21,800
C3	3.0225	0.2443	0.50	408.70	74.07	76.34	16,500

Tables 3.6, 3.7, 3.8 and 3.9 are the simulation results of the six closed-loop controllers with different weights in the performance function for the in-train force, the energy consumption and the velocity tracking. Table 3.6 is the performance comparison of Fig.s from 3.11 to 3.16. In these tables, C01, C02 and C03 are 1-1 strategy, 2-1

Table 3.8: Performance with $K_e = 1, K_f = 10, K_v = 1$

	$\lvert\delta\bar{v}\rvert$(m/s)			$\overline{\lvert f_{in}\rvert}$(kN)			E
	max	mean	std	max	mean	std	(MJ)
C01	3.3251	0.4611	0.57	385.83	146.17	100.43	8,790
C02	3.3234	0.6609	0.57	377.25	145.96	98.68	8,610
C03	3.3243	0.4604	0.58	368.28	144.61	96.40	8,460
C1	3.2663	0.5312	0.58	384.58	147.53	101.76	11,460
C2	3.1379	0.4542	0.52	331.48	105.82	62.57	22,300
C3	3.0090	0.3670	0.47	405.70	70.77	78.04	15,000

Table 3.9: Performance with $K_e = 100, K_f = 1, K_v = 1$

	$\lvert\delta\bar{v}\rvert$(m/s)			$\overline{\lvert f_{in}\rvert}$(kN)			E
	max	mean	std	max	mean	std	(MJ)
C01	3.8039	0.7383	0.56	392.08	145.01	99.50	8,795
C02	3.8056	0.7361	0.56	389.83	146.27	99.24	8,600
C03	3.8041	0.7415	0.56	386.49	144.19	99.08	8,560
C1	3.6744	0.6889	0.57	390.46	143.61	101.44	9,550
C2	3.4603	0.6388	0.53	300.06	110.47	61.90	16,800
C3	3.247	0.4918	0.47	297.27	78.90	63.27	13,400

strategy and 2-2 strategy closed-loop controllers based on heuristic scheduling, and C1, C2 and C3 are 1-1 strategy, 2-1 strategy and 2-2 strategy closed-loop controllers based on optimal scheduling. $\lvert\delta\bar{v}\rvert$ is the absolute value of the difference between the reference velocity and the mean value of all the cars' velocities at a specific point. The variable $\overline{\lvert f_{in}\rvert}$ is the mean value of the absolute values of all the couplers' in-train forces at a specific point. The item E is the energy consumed during travel. The items max, mean and std are the maximum value, mean value and standard deviation of the statistical variable, respectively.

From these tables, it can be seen that the three strategies based on heuristic scheduling have similar performances. This is because their scheduling is the same. However, based on optimal scheduling, the 2-2 strategy yields a better performance in terms of velocity, in-train force and energy consumption than the 2-1 strategy with the same parameters. The performance of velocity in Table 3.7, which is with the velocity emphasized optimal parameters, is the best compared with the corresponding operational strategy of the other tables. The performance of the in-train force based on heuristic scheduling of the corresponding operational strategy is approximate in the four tables, while the performance of the in-train force based on optimal scheduling is best in Table 3.8, which is with the in-train force emphasized parameters, but only the improvement of the 2-2 strategy is obvious and that of the other two is only approximate. In Table 3.9, with the energy consumption emphasized parameters, the performance of energy consumption with all the corresponding controllers based on optimal scheduling, is the best among the four tables. On the whole, local optimization does work and leads to global optimization in some degree.

From simulation, it is shown that the train tracks the reference speed quickly when the reference speed changes and tracks the reference speed very well when the train is cruising. So the track length is enough for simulation of the driving profile. On a longer track, the same result can be reached, which will be shown in the next chapter. However, it should be pointed out that when the objective is to test the optimization combination of a driving profile and a reference speed profile, a longer track might be necessary.

Based on the observation of the 2-2 strategy, another approach to controller design is proposed in the following chapter by assuming only the locomotives' speed measurement. Performance comparisons are detailed further in the following chapter.

3.6 Conclusion

This chapter emphasizes open loop scheduling for the handling of heavy haul trains, which constitutes a basic problem about the trim point. In this chapter the cascade-mass-point model is adopted for a long heavy haul train. Three control strategies are proposed and then followed by the open loop optimal scheduling algorithms for them. Simulation results of these control strategies for a traditional heavy haul train and a train equipped with an ECP braking system are shown. It is noticed that ECP braking systems show superb performance compared with pneumatic braking systems.

Optimal scheduling and heuristic scheduling in [15] are compared when they are applied to trains equipped with ECP braking systems. It is shown that optimal scheduling can improve performance and the ECP/iDP is the best of the three strategies. A closed-loop controller based on an LQR approach is used to verify the result.