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APPENDICES

APPENDIX A:

SAMPLE GAMBIT JOURNAL FILE (GRID GENERATION AND MESHING): SINGLE CHANNEL PEM FUEL CELL.

```
/
$htot = 0.8
$wtot = 3
$offr1x = ($wtot/2)
$offr1y = ($htot/2)
$ys = 0.6
$xs = 0.5
$offsy = $htot - $ys
$offsx = ($wtot/2) - ($xs/2)
$offr2x = ($xs/2)
$offr2y = ($ys/2)
face create width $wtot height $htot offset $offr1x $offr1y 0 xyplane rectangle
face create translate "edge.3" vector 0 0.21 0
face create translate "edge.7" vector 0 0.012 0
face create translate "edge.10" vector 0 0.036 0
face create translate "edge.13" vector 0 0.012 0
face create translate "edge.16" vector 0 0.21 0
face create translate "edge.19" vector 0 $htot 0
face create width $xs height $ys offset $offr2x $offr2y 0 xyplane rectangle
face move "face.8" offset $offsx $offsy 0
face cmove "face.8" multiple 1 offset 0 1.08 0
face split "face.1" connected faces "face.8"
face split "face.7" connected faces "face.9"
undo begingroup
```

```
edge modify "edge.4" "edge.20" "edge.21" backward
edge picklink "edge.4" "edge.20" "edge.21" "edge.2"
edge mesh "edge.2" "edge.4" "edge.20" "edge.21" successive ratio1 1 intervals \
  20
undo endgroup
undo begingroup
edge modify "edge.22" backward
edge picklink "edge.22" "edge.1"
edge mesh "edge.1" "edge.22" successive ratio1 1 intervals 30
undo endgroup
undo begingroup
edge modify "edge.33" "edge.40" "edge.38" backward
edge picklink "edge.33" "edge.40" "edge.38" "edge.36" "edge.39" "edge.19" \
  "edge.3" "edge.31" "edge.32" "edge.34" "edge.37" "edge.35"
edge mesh "edge.35" "edge.37" "edge.34" "edge.32" "edge.31" "edge.33" \
  "edge.3" "edge.19" "edge.39" "edge.40" "edge.38" "edge.36" successive \
  ratio1 1 intervals 10
undo endgroup
undo begingroup
edge picklink "edge.7" "edge.10" "edge.13" "edge.16"
edge mesh "edge.16" "edge.13" "edge.10" "edge.7" successive ratio1 1 \
  intervals 30
undo endgroup
undo begingroup
edge modify "edge.14" "edge.15" backward
edge picklink "edge.14" "edge.15" "edge.9" "edge.12" "edge.11"
edge mesh "edge.14" "edge.11" "edge.15" "edge.12" "edge.9" successive ratio1 \
  1 intervals 4
undo endgroup
undo
/Undone to: undo begingroup
```

```
undo begingroup
edge modify "edge.14" "edge.15" backward
edge picklink "edge.14" "edge.15" "edge.9" "edge.12" "edge.8" "edge.11"
edge mesh "edge.14" "edge.11" "edge.8" "edge.15" "edge.12" "edge.9" \
  successive ratio1 1 intervals 4
undo endgroup
undo begingroup
edge modify "edge.6" backward
edge picklink "edge.6" "edge.18" "edge.17" "edge.5"
edge mesh "edge.5" "edge.17" "edge.6" "edge.18" successive ratio1 1.15 \
  intervals 10
undo endgroup
face mesh "face.1" "face.2" "face.3" "face.4" "face.5" "face.6" "face.9" \
  "face.8" "face.7" submap size 1
undo
/Undone to: face mesh "face.1" "face.2" "face.3" "face.4" "face.5" "face.6" "face
undo
/Undone to: undo begingroup
undo begingroup
edge modify "edge.5" "edge.6" backward
edge picklink "edge.5" "edge.6" "edge.18" "edge.17"
edge mesh "edge.17" "edge.18" "edge.5" "edge.6" successive ratio1 1 intervals \
  4
undo endgroup
face mesh "face.1" "face.2" "face.3" "face.4" "face.5" "face.6" "face.9" \
  "face.8" "face.7" submap size 1
edge create translate "vertex.16" vector 0 0 125
undo begingroup
edge picklink "edge.41"
edge mesh "edge.41" successive ratio1 1.1 ratio2 1.1 intervals 60
undo endgroup
```

```
volume create translate "face.1" "face.2" "face.3" "face.4" "face.5" "face.6" \
  "face.9" "face.8" "face.7" onedge "edge.41" withmesh
window modify invisible mesh
window modify visible mesh
window modify invisible mesh
physics create "inlet-a" btype "MASS_FLOW_INLET" face "face.54"
physics create "inlet-c" btype "MASS_FLOW_INLET" face "face.8"
physics create "outlet-a" btype "PRESSURE_OUTLET" face "face.9"
physics create "outlet-c" btype "PRESSURE_OUTLET" face "face.59"
physics create "wall-terminal-a" btype "WALL" face "face.67"
physics create "wall-terminal-c" btype "WALL" face "face.12"
physics create "wall-ch-a" btype "WALL" face "face.51" "face.53" "face.52"
physics create "wall-ch-c" btype "WALL" face "face.16" "face.14" "face.17"
physics create "wall-ends" btype "WALL" face "face.1" "face.2" "face.3" \
  "face.4" "face.5" "face.6" "face.7" "face.20" "face.27" "face.32" "face.37" \
  "face.42" "face.49" "face.68"
physics create "wall-gdl-a" btype "WALL" face "face.48" "face.46"
physics create "wall-gdl-c" btype "WALL" face "face.18" "face.19"
physics create "wall-sides" btype "WALL" face "face.13" "face.15" "face.24" \
  "face.25" "face.29" "face.30" "face.34" "face.35" "face.39" "face.40" \
  "face.44" "face.45" "face.64" "face.66"
physics create "catalyst-a" ctype "FLUID" volume "volume.5"
physics create "catalyst-c" ctype "FLUID" volume "volume.3"
physics create "channel-a" ctype "FLUID" volume "volume.7"
physics create "channel-c" ctype "FLUID" volume "volume.8"
physics create "gdl-a" ctype "FLUID" volume "volume.6"
physics create "gdl-c" ctype "FLUID" volume "volume.2"
physics create "membrane" ctype "FLUID" volume "volume.4"
physics create "current-a" ctype "SOLID" volume "volume.9"
physics create "current-c" ctype "SOLID" volume "volume.1"
window modify visible mesh
```

```
export fluent5 "pem-single-channel1011.msh"
save name "C:\\pem-single101110\\pem-single-channelnew.dbs"
save
export fluent5 "C:\\pem-single101110\\pem-single-channelnew.msh"
```

APPENDIX B:

THE DYNAMIC-Q OPTIMISATION ALGORITHM IN MATLAB

B-1 DYNQ.M

```
function [X,F]=dynq(x0,varargin);
tic
%
%       DYNAMIC-Q ALGORITHM FOR CONSTRAINED OPTIMISATION
%            GENERAL MATHEMATICAL PROGRAMMING CODE
%            ------------------------------------
%
% This code is based on the Dynamic-Q method of Snyman documented
% in the paper "THE DYNAMIC-Q OPTIMISATION METHOD: AN
ALTERNATIVE TO SQP?" by J.A. Snyman and A.M. Hay. Technical Report, Dept
Mech. Eng., UP.
%
%                       MATLAB implementation by A.M. HAY
%                 Multidisciplinary Design Optimisation Group (MDOG)
%            Department of Mechanical Engineering, University of Pretoria
%                                 August 2002
%
%                          UPDATED : 23 August 2002
%
%                            BRIEF DESCRIPTION
%                            -----------------

%  Dynamic-Q solves inequality and equality constrained optimisation
%  problems of the form:
```

```
%
%                 minimise F(X)  ,  X={X(1),X(2),...,X(N)}
%    such that
%                 Cp(X) <= 0       p=1,2,...,NP
%    and
%                 Hq(X) =  0       q=1,2,...,NQ
%    with lower bounds
%       CLi(X) = V_LOWER(i)-X(NLV(i)) <= 0   i=1,2,...,NL
%    and upper bounds
%       CUj(X) = X(NUV(j))-V_UPPER(j) <= 0   j=1,2,...,NU
%
% This is a completely general code - the objective function and the
% constraints may be linear or non-linear. The code therefore solves
% LP, QP and NLP problems.
%
%                   -----------------
%
% User specified functions:
%
% The objective function F and constraint functions C and H must be
% specified by the user in function FCH. Expressions for the respective
% gradient vectors must be specified in function GRADFCH.
%
% {The user may compute gradients by finite differences if necessary
%  - see example code in GradFCH}
%
% Side constraints should not be included as inequality constraints
% in the above subroutines, but passed to the dynq function as
% input arguments LO and UP. (Described below)
%
```

```
% In addition to FCH and GRADFCH the following functions are called
% by DYNQ and should not be altered:
%
DQLFOPC,DQFUN,DQCONIN,DQCONEQ,DQGRADF,DQGRADC,DQGRADH
%
% In addition the script HISTPLOT.m plots various optimisation
% histories. To suppress automatic plotting set PRNCONST=0 below.
%
%                 -----------------
%
%   synopsis:
%
%          [X,F] = dynq(x0,lo,up,dml,xtol,ftol,clim,np,nq,kloop);
%
%   outputs:
%       X  = optimal solution (1xN)
%       F  = optimal function value
%
%   inputs:
%       x0 = starting point (1xN)
%       lo = NLx2 matrix associated with lower limits on the variables
%             containing variable index NLV(i) in the first column and
%             associated value V_LOWER of that limit in the second column
%             (optional, otherwise assumed no lower side constraints)
%       up = NUx2 matrix associated with lower limits on the variables
%             containing variable index NUV(i) in the first column and
%             associated value V_UPPER of that limit in the second column
%             (optional, otherwise assumed no upper side constraints)
%      dml = the move limit which should be approximately the same order
%             of magnitude as the "radius of the region of interest"
```

```
%            = sqrt(n)*max-variable-range (optional, default =1)
%    xtol = convergence tolerance on the step size (optional, default =1e-5)
%    ftol = convergence tolerance on the function value (optional, default =1e-8)
%    clim = tolerance for determining whether constraints are violated
%            (optional, default =ftol*1e2)
%      np = number of inequality constraints (optional)
%      nq = number of equality constraints (optional)
%            Note: Both np and nq are optional and determined automatically
%            if not specified, but at the cost of an extra function evalution.
%   kloop = maximum number of iterations (optional, default = 100)
%
%    NOTE: use [] to activate default inputs, for example
%
%  [X,F]=dynq(x0,[],[],2); uses dml=2 but default values for all other inputs.
%
%      See FCH and GRADFCH for an example problem.
%
%   ---- This program is for educational purposes only ----


%*****PLOT OPTIMISATION HISTORIES AT END OF
PROGRAM?******************
%         YES: 1       OR          NO: 0
%
PRNCONST=1;
%*****************************************************************
***

clc;
```

```
N=length(x0);   % Determine number of variables
X=x0;

[dum,D]=size(varargin);
vars=cell(1,9);
vars(1:D)=varargin;

LO=vars{1};
UP=vars{2};
DML=vars{3};
XTOL=vars{4};
FTOL=vars{5};
CLIM=vars{6};
NP=vars{7};
NQ=vars{8};
KLOOPMAX=vars{9};

% default values
[NL,dum]=size(LO);
if NL>0
   NLV=LO(:,1)';
   V_LOWER=LO(:,2)';
else
   NLV=[];
   V_LOWER=[];
end
[NU,dum]=size(UP);
if NU>0
   NUV=UP(:,1)';
   V_UPPER=UP(:,2)';
```

```
else
   NUV=[];
   V_UPPER=[];
end
if isempty(DML)
   DML=1; end
if isempty(XTOL)
   XTOL=1e-5; end
if isempty(FTOL)
   FTOL=1e-8; end
if isempty(CLIM)
   CLIM=FTOL*1e2; end
if isempty(NP)|isempty(NQ)
   [F,C,H]=fch(X);
   NP=length(C);
   if isempty(C)
      NP=0;
   end
   NQ=length(H);
   if isempty(H)
      NQ=0;
   end
end
if isempty(KLOOPMAX)
   KLOOPMAX=100; end

%##########################################################################
###C
%*******************************************************************
***C
```

```
%     MAIN PROGRAM FOLLOWS: Do not alter!!!!
%*********************************************************************
***C
%#####################################################################
###C

%*****OPEN OUPUT
FILES*************************************************C
%
fidA=fopen('Approx.out','wt+');
fidD=fopen('DynamicQ.out','wt+');
fidH=fopen('History.out','wt+');
%
%*****SPECIFY INITIAL APPROXIMATION
CURVATURES*************************C
%
ACURV=0.D0;
BCURV=zeros(1,NP);
if NP==0
   BCURV=[];
end
CCURV=zeros(1,NQ);
if NQ==0
   CCURV=[];
end
%
%
%
%*****INITIALIZE
OUTPUT**************************************************C
```

```
FEASIBLE=0;

fprintf(fidA,' DYNAMICQ OUTPUT FILE \n');
fprintf(fidA,' -------------------- \n');
fprintf(fidA,' Number of variables [N]= %i \n',N);
fprintf(fidA,' Number of inequality constraints [NP]= %i \n',NP);
fprintf(fidA,' Number of equality constraints [NQ]= %i \n',NQ);
fprintf(fidA,' Move limit= %12.8e \n',DML);

fprintf(1,'\n DYNAMICQ OPTIMISATION ALGORITHM \n');
fprintf(1,' ------------------------------- \n');
% (MAXX=Maximum number of X-values to be displayed on screen)
MAXX=4;
if N<=MAXX
   fprintf(1,' Iter Function value  ? XNORM      RFD        ');
   fprintf(1,'X(%i)       ',1:N);
   fprintf(1,'\n ------------------------------------------');
   for I=1:N
      fprintf(1,'------------',1:N);
   end
   fprintf(1,'\n');
else
   fprintf(1,' Iter Function value  ? XNORM      RFD ');
   fprintf(1,'\n -------------------------------------------\n');
end

fprintf(fidD,' DYNAMICQ OPTIMISATION ALGORITHM\n');
fprintf(fidD,' -------------------------------\n');
fprintf(fidD,' Iter Function value       ? XNORM      RFD        ');
fprintf(fidD,'X(%i)           ',1:N);
```

```
fprintf(fidD,'\n');

fprintf(fidD,' ------------------------------------------------');
for i=1:N
   fprintf(fidD,'---------------');
end
fprintf(fidD,'\n');

% Initialize outer loop counter
KLOOP=0;

% Arbitrary large values to prevent premature termination
F_LOW=1.D6;
RFD=1.D6;
RELXNORM=1.D6;

C_A=zeros(1,NP+NL+NU+1);

%*****START OF OUTER OPTIMISATION
LOOP**********************************C

while KLOOP<=KLOOPMAX

%*****APPROXIMATE
FUNCTIONS*********************************************C

% Determine function values
   [F,C,H]=fch(X);

% Calculate relative step size
```

```
   if KLOOP>0
      DELXNORM=sqrt((X_H(KLOOP,:)-X)*(X_H(KLOOP,:)-X)');
      XNORM=sqrt(X*X');
      RELXNORM=DELXNORM/(1+XNORM);
   end

% Determine lowest feasible function value so far
   if KLOOP>0
   FEASIBLE=1;
      check=find(C<CLIM);
      if isempty(check)&NP>0;
         FEASIBLE=0;
      end
      check=find(abs(H)<CLIM);
      if isempty(check)&NQ>0;
         FEASIBLE=0;
      end
      for I=1:NL
         if C_A(I+NP)>CLIM
            FEASIBLE=0;
         end
      end
      for I=1:NU
         if C_A(I+NP+NL)>CLIM
            FEASIBLE=0;
         end
      end
   end

% Calculate relative function difference
```

```
    if F_LOW~=1.D6&FEASIBLE==1
       RFD=abs(F-F_LOW)/(1+abs(F));
    end

    if FEASIBLE==1&F<F_LOW
       F_LOW=F;
    end

% Store function values
    X_H(KLOOP+1,:)=X;   % Need to adjust from Fortran version since
    F_H(KLOOP+1)=F;     % Matlab does not accept 0 as a matrix index
    if NP>0
       C_H(KLOOP+1,1:NP)=C;
    end
    if NL>0
       C_H(KLOOP+1,NP+1:NP+NL)=C_A(NP+1:NP+NL);
    end
    if NU>0
       C_H(KLOOP+1,NP+NL+1:NP+NL+NU)=C_A(NP+NL+1:NP+NL+NU);
    end
    C_H(KLOOP+1,NP+NL+NU+1)=C_A(NP+NL+NU+1);
    if NQ>0
       H_H(KLOOP+1,:)=H;
    end

% Determine gradients
    [GF,GC,GH]=gradfch(X);

% Calculate curvatures
    if KLOOP>0
```

```
        DELX=X_H(KLOOP,:)-X_H(KLOOP+1,:);
        DELXNORM=DELX*DELX';

% Calculate curvature ACURV
        DP=GF*DELX';
        ACURV=2.*(F_H(KLOOP)-F_H(KLOOP+1)-GF*DELX')/DELXNORM;

        for J=1:NP
           DP=GC(J,:)*DELX';
% Calculate corresponding curvature BCURV(J)
           BCURV(J)=2.*(C_H(KLOOP,J)-C_H(KLOOP+1,J)-
GC(J,:)*DELX')/DELXNORM;
        end

        for J=1:NQ
           DP=GH(J,:)*DELX';
% Calculate corresponding curvature CCURV(J)
           CCURV(J)=2.*(H_H(KLOOP,J)-H_H(KLOOP+1,J)-
GH(J,:)*DELX')/DELXNORM;
        end
    end

%*****RECORD PARAMETERS FOR THE
ITERATION******************************C

% Write approximation constants to Approx.out
    fprintf(fidA,' Iteration %i \n',KLOOP);
    fprintf(fidA,' --------------\n');
    fprintf(fidA,' X=\n');
    for I=1:N
```

```
    fprintf(fidA,' %12.8f ',X(I));
  end
  fprintf(fidA,'\n F= %15.8e\n',F);
  for I=1:NP
    fprintf(fidA,' C(%i)=%15.8e',I,C(I));
  end
  for I=1:NQ
     fprintf(fidA,' H(%i)=%15.8e',I,H(I));
  end

  fprintf(fidA,' Acurv=%15.8e',ACURV);
  for I=1:NP
    fprintf(fidA,' Bcurv(%i)=%15.8e',I,BCURV(I));
  end
  for I=1:NQ
    fprintf(fidA,' Ccurv(%i)=%15.8e',I,CCURV(I));
  end

% Write solution to file
  if KLOOP==0
    fprintf(fidD,' %4i %+19.12e %i                    ',KLOOP,F,FEASIBLE);
  else
    if RFD~=1.D6
      fprintf(fidD,' %4i %+19.12e %i %9.3e
%9.3e',KLOOP,F,FEASIBLE,RELXNORM,RFD);
    else
      fprintf(fidD,' %4i %+19.12e %i %9.3e
',KLOOP,F,FEASIBLE,RELXNORM);
    end
  end
```

```
   fprintf(fidD,' %+13.6e',X);
   fprintf(fidD,'\n');

% Write solution to screen
   if KLOOP==0
      if N<=MAXX
         fprintf(1,' %4i %+14.7e %i                    ',KLOOP,F,FEASIBLE);
         fprintf(1,' %+9.2e',X);
         fprintf(1,'\n');
      else
         fprintf(1,' %4i %+14.7e %i\n',KLOOP,F,FEASIBLE);
      end
    else
      if N<=MAXX
         if RFD~=1.D6&FEASIBLE==1
            fprintf(1,' %4i %+14.7e %i %9.3e
%9.3e',KLOOP,F,FEASIBLE,RELXNORM,RFD);
         else
            fprintf(1,' %4i %+14.7e %i %9.3e
',KLOOP,F,FEASIBLE,RELXNORM);
         end
         fprintf(1,' %+9.2e',X);
         fprintf(1,'\n');
      else
         if RFD~=1.D6&FEASIBLE==1
            fprintf(1,' %4i %+14.7e %i %9.3e
%9.3e\n',KLOOP,F,FEASIBLE,RELXNORM,RFD);
         else
            fprintf(1,' %4i %+14.7e %i %9.3e\n',KLOOP,F,FEASIBLE,RELXNORM);
         end
```

```
    end
  end

% Exit do loop here on final iteration
  if KLOOP==KLOOPMAX|RFD<FTOL|RELXNORM<XTOL
    if KLOOP==KLOOPMAX
      fprintf(1,' Terminated on max number of steps\n');
      fprintf(fidD,' Terminated on max number of steps\n');
    end
    if RFD<FTOL
      fprintf(1,' Terminated on function value\n');
      fprintf(fidD,' Terminated on function value\n');
    end
    if RELXNORM<XTOL
      fprintf(1,' Terminated on step size\n');
      fprintf(fidD,' Terminated on step size\n');
    end
    fprintf(1,'\n');
    fprintf(fidD,'\n');
    break;
  end

%*****SOLVE THE APPROXIMATED
SUBPROBLEM********************************C

[X,F_A,C_A,H_A]=dqlfopc(X,NP,NQ,F,C,H,GF,GC,GH,ACURV,BCURV,CCURV,
DML...
  ,NL,NU,NLV,NUV,V_LOWER,V_UPPER,XTOL,KLOOP);

% Record solution to approximated problem
```

```
   fprintf(fidA,'Solution of approximated problem:\n');
   fprintf(fidA,'X=\n');
   for I=1:N
      fprintf(fidA,' %12.8f\n',X(I));
   end
   fprintf(fidA,' F_A=%15.8e\n',F_A);
   for I=1:NP+NL+NU+1
      fprintf(fidA,'C_A(%i)=%15.8e\n',I,C_A(I));
   end
   for I=1:NQ
      fprintf(fidA,'H_A(%i)=%15.8e\n',I,H_A(I));
   end

% Increment outer loop counter
   KLOOP=KLOOP+1;
end

% Write final constraint values to file

if NP>0
   fprintf(fidD,' Final inequality constraint function values:\n');
   for I=1:NP
      fprintf(fidD,'  C(%i)=%15.8e\n',I,C(I));
   end
end
if NQ>0
   fprintf(fidD,' Final equality constraint function values:\n');
   for I=1:NQ
      fprintf(fidD,'  H(%i)=%15.8e\n',I,H(I));
```

```
   end
end
if NL>0
   fprintf(fidD,' Final side (lower) constraint function values:\n');
   for I=1:NL
      fprintf(fidD,'  C(X(%i))=%15.8e\n',NLV(I),C_A(NP+I));
   end
end
if NU>0
   fprintf(fidD,' Final side (upper) constraint function values:\n');
   for I=1:NU
      fprintf(fidD,'  C(X(%i))=%15.8e\n',NUV(I),C_A(NP+NL+I));
   end
end

% Write final constraint values to screen
fprintf(1,' Constraint values follow:\n\n')
if NP>0
   fprintf(1,' Final inequality constraint function values:\n');
   for I=1:NP
      fprintf(1,'  C(%i)=%15.8e\n',I,C(I));
   end
end
if NQ>0
   fprintf(1,' Final equality constraint function values:\n');
   for I=1:NQ
      fprintf(1,'  H(%i)=%15.8e\n',I,H(I));
   end
end
if NL>0
```

```
    fprintf(1,' Final side (lower) constraint function values:\n');
    for I=1:NL
        fprintf(1,'  C(X(%i))=%15.8e\n',NLV(I),C_A(NP+I));
    end
end
if NU>0
    fprintf(1,' Final side (upper) constraint function values:\n');
    for I=1:NU
        fprintf(1,'  C(X(%i))=%15.8e\n',NUV(I),C_A(NP+NL+I));
    end
end

% Write history vectors

fprintf(fidH,' %3i%3i%3i%3i%3i%3i\n', KLOOP,N,NP,NL,NU,NQ);
for I=1:KLOOP+1
    fprintf(fidH,' %3i %15.8e',I-1,F_H(I));
    for J=1:N
        fprintf(fidH,' %15.8e',X_H(I,J));
    end
    fprintf(fidH,'\n');
end
if NP>0
    for I=1:KLOOP+1
        fprintf(fidH,' %3i',I-1);
        for J=1:NP
            fprintf(fidH,' %15.8e',C_H(I,J));
        end
        fprintf(fidH,'\n');
    end
```

```
end
if NL>0
   for I=1:KLOOP+1
      fprintf(fidH,' %3i',I-1);
      for J=NP+1:NP+NL
         fprintf(fidH,' %15.8e',C_H(I,J));
      end
      fprintf(fidH,'\n');
   end
end
if NU>0
   for I=1:KLOOP+1
      fprintf(fidH,' %3i',I-1);
      for J=NP+NL+1:NP+NL+NU
         fprintf(fidH,' %15.8e',C_H(I,J));
      end
      fprintf(fidH,'\n');
   end
end
if NQ>0
   for I=1:KLOOP+1
      fprintf(fidH,' %3i',I-1);
      for J=1:NQ
         fprintf(fidH,' %15.8e',H_H(I,J));
      end
      fprintf(fidH,'\n');
   end
end

fclose(fidD);
```

```
fclose(fidH);
fclose(fidA);

if PRNCONST
    histplot;
%    disp('Press a key to continue');
%    pause;
%    close all;
end
toc
```

B-2 FCH.M

```
function [F,C,H]=fch(X);
% Objective and constraint function evaluation for DYNAMIC-Q
%         (USER SPECIFIED)
%
%   synopsis:
%
%      [F,C,H]=fch(X);
%
%   outputs:
%      F  = objective function value
%      C  = vector of inequality constraint functions (1xNP)
%      H  = vector of equality constraint functions (1xNQ)
%
%   inputs:
%      X = design vector (1xN)
%
%                  -----------------
%
% The application of the code is illustrated here for the very simple
% but general example problem (Hock 71):
%
%     minimise  F(X) = X(1)*X(4)*(X(1)+X(2)+X(3))+X(3)
% such that
%             C(X) = 25-X(1)*X(2)*X(3)*X(4) <= 0
%     and
%             H(X) = X(1)^2+X(2)^2+X(3)^2+X(4)^2-40 = 0
%
%    and side constraints
```

```
%
%             1 <= X(I) <= 5 , I=1,2,3,4
%
%   Starting point is (1,5,5,1)
%
%   Solution of this problem is accomplished by:
%      (with FCH and GRADFCH unaltered)
%
%      x0=[1,5,5,1] % Specify starting point
%      lo=[1:4;1,1,1,1]' % Specify lower limits
%      up=[1:4;5,5,5,5]' % Specify upper limits
%      [X,F]=dynq(x0,lo,up); % Solve using Dynamic-Q
%
%   NOTE: This function should return C=[]; H=[]; if these are
%        not defined.
%
%   See also DYNQ and GRADFCH
%

%Objective Function
%Load Design Variables

%Get the Total Heat transfer

F = -LL4{2};

%Inequality Constraints
C(1)=(X(3)/(4*X(1)))-1;
C(2)=1-(2*X(3)/X(1));
C(3)=(X(4)/(4*X(2)))-1;
```

```
C(4)=1-(2*X(4)/X(2));

Volu = 0.05;
%Equality Constraints
H(1)=(X(1)^2*X(3))+(X(2)^2*X(4))-(4*Volu/pi);

% To eliminate error messages
% Do not delete

if ~exist('C')
   C=[];
end
if ~exist('H')
   H=[];
end
```

B-3 GRADFCH.M

```
function [GF,GC,GH]=gradfch(X);
% Objective and constraint function GRADIENT evaluation for DYNAMIC-Q
%        (USER SPECIFIED)
%
%   synopsis:
%
%     [GF,GC,GH]=gradfch(X);
%
%   outputs: Partial derivatives wrt variables X(I) of
%     GF  = objective function (1xN)
%     GC  = inequality constraint functions (NPxN)
%     GH  = equality constraint functions (NQxN)
%
%   inputs:
%      X = design vector (1xN)
%
%    COMPUTE THE GRADIENT VECTORS OF THE OBJECTIVE FUNCTION
F,
%    INEQUALITY CONSTRAINTS C, AND EQUALITY CONSTRAINTS H
%    W.R.T. THE VARIABLES X(I):
%          GF(I),I=1,N
%          GC(J,I), J=1,NP I=1,N
%          GH(J,I), J=1,NQ I=1,N
%
%   NOTE: This function should return GC=[]; GH=[]; if these are
%       not defined.
%
%   See also DYNQ, FCH
```

```
%

% Determine gradients by finite difference
FDFLAG=1;

if FDFLAG
   DELTX=1.D-4;    % Finite difference interval
   [F,C,H]=fch(X);
   N=length(X);
   for I=1:N
      DX=X;
      DX(I)=X(I)+DELTX;
      [F_D,C_D,H_D]=fch(DX);
      GF(I)=(F_D-F)/DELTX;
      if ~isempty(C)
         GC(1,1)=-X(3)/(4*X(1)^2);
         GC(1,2)=0;
         GC(1,3)=1/(4*X(1));
         GC(1,4)=0;
         GC(1,5)=0;
         GC(2,1)=2*X(3)/(X(1)^2);
         GC(2,2)=0;
         GC(2,3)=-2/X(1);
         GC(2,4)=0;
         GC(2,5)=0;
         GC(3,1)=0;
         GC(3,2)=-X(4)/(4*X(2)^2);
         GC(3,3)=0;
         GC(3,4)=1/(4*X(2));
         GC(3,5)=0;
```

```
      GC(4,1)=0;
      GC(4,2)=2*X(4)/(X(2)^2);
      GC(4,3)=0;
      GC(4,4)=-2/X(2);
      GC(4,5)=0;
    end
    if ~isempty(H)
    GH(1,1)=2*X(1)*X(3);
    GH(1,2)=2*X(2)*X(4);
    GH(1,3)=X(1)^2;
    GH(1,4)=X(2)^2;
    GH(1,5)=0;
    end
  end
end

% To eliminate error messages
% Do not erase
if ~exist('GC')
  GC=[];
end
if ~exist('GH')
  GH=[];
end
```

B-4 Execute_Finsim.m

```
%This program initiates DYNQ.M
clear all
clc
close all
x0=[+2.824638e-001 +1.513331e-001 +6.310029e-001 +5.814793e-001 +5.0000e-
002];
lo=[1 0.05
   2 0.05
   5 0.05];
up=[3 0.95
   4 0.95];
dml=0.0005;
xtol=[];
ftol=[];
clim=[];
np=4;
nq=1;
kloop=[];
[X,F] = dynq(x0,lo,up,dml,xtol,ftol,clim,np,nq,kloop);
```