

MATHEMATICAL MODEL DEVELOPMENT

In this chapter the complex mathematical requirements of the load reaction model and the average cylinder temperature model will be discussed in detail. At present the need exists in the market to implement these two models into the existing HWLC-systems, but it must be used to open the doors for future possibilities.

It has been found in section 1.7.3 that the mathematical relations on load reaction are still inconclusive. In this chapter the possibility to predict the effects of load-control actions would be viewed from device level, instead of the traditional system level.

Customer-comfort-levels for each user group are specified in the management rules in terms of a minimum allowable cylinder temperature. With open loop control as the predominant communication topology, it is impossible to measure each individual cylinder's temperature. The only remaining possibility is to predict the average cylinder temperature for each group before or during control actions. Existing prediction models will be adopted to ensure integration into existing HWLC-systems.

Case studies will also be done on actual municipal data to validate the integration of the two models when applied within existing HWLC-systems.

3.1 LOAD REACTION MODEL

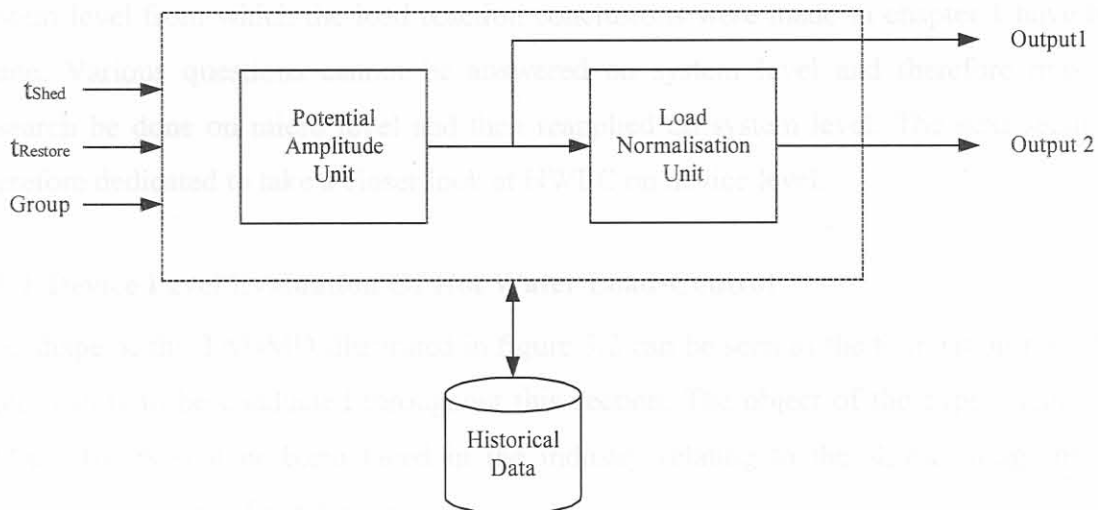


Figure 3.1: Composite layout of the Load Reaction Model.

The load reaction model was designed to predict the effect of a HWLC-action on the municipal and national load-profile. Two effects were noticed in chapter 1 when the hot water load was restored. The one is the increased load at the time of restoration (Potential Amplitude Curve) and afterwards the normalisation of the load (Load Normalisation Curve) occurred as secondary effect.

Figure 3.1 is a composite illustration of the load reaction model introduced in section 2.2.2 on page 50. The model itself comprises of two components namely:

- The Potential Amplitude Unit and
- The Load Normalisation Unit.

Both these units are required when predicting the effect of HWLC-actions, but it is possible to develop each unit separately according to the global system specifications and integrate it back into the model.

One HWLC-system is capable of controlling up to 16 million devices, but the average installation is in the order of 10,000 to 15,000 units. Data evaluation is usually done on system level from which the load reaction conclusions were made in chapter 1 have been made. Various questions cannot be answered on system level and therefore must the research be done on micro level and then reapplied on system level. The next section is therefore dedicated to take a closer look at HWLC on device level.

3.1.1 Device Level Evaluation Of Hot Water Load-Control

The shape of the TADMD illustrated in figure 3.2 can be seen as the foundation for all the experiments to be conducted throughout this section. The object of the experiments is to address the typical problem faced in the industry relating to the shortcomings of data extracted by means of notch testing.

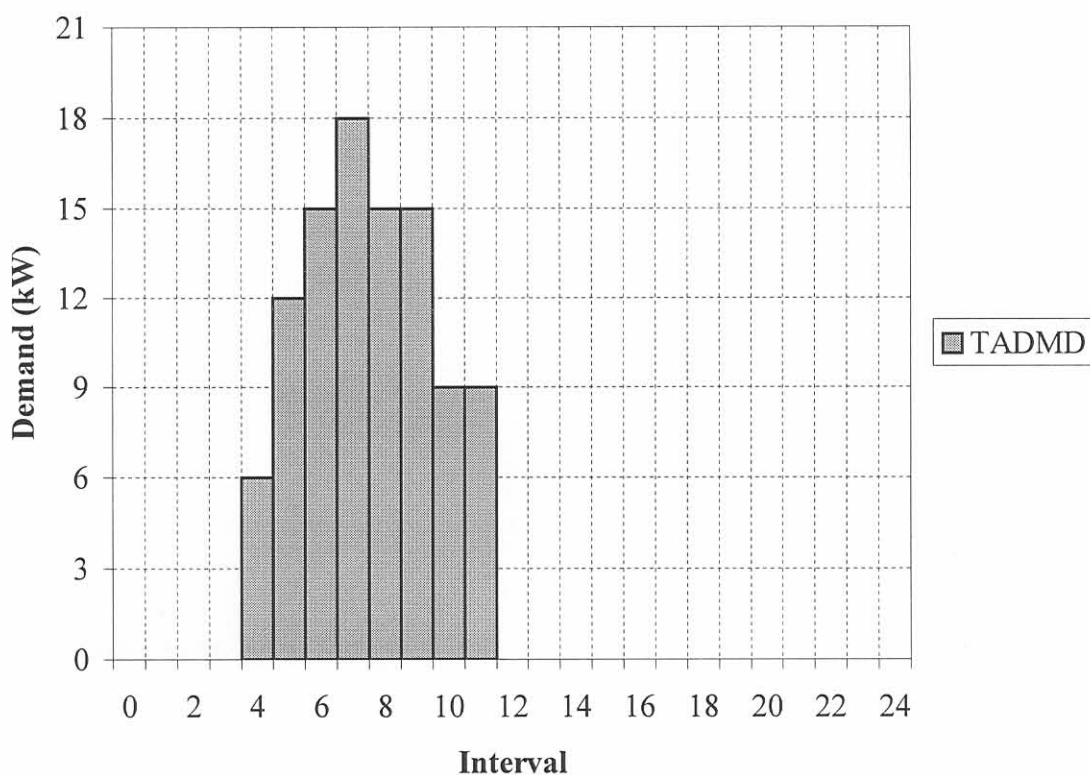


Figure 3.2: TADMD-profile of a group of eight hot water cylinders.

The following global assumptions have to be made to simplify the illustrations, but they will not necessarily influence the load reaction model as such. The assumptions are that:

- The group of cylinders consists of 8 members,
- Each with an element rating of 3kW,
- All at the same ambient temperature of 20°C,
- With a fixed thermostat level,
- No thermal losses and
- The cylinder is ON for an entire interval, with no switching within an interval.

From figure 3.2 one is able differentiate the exact amount of cylinders that consumed energy within a specific interval. The next experiment will prove that the most important information for HWLC-purposes is actually the composition of cylinders within a specific interval and not necessarily the shape of the TADMD itself.

3.1.2 The Need To Know The Exact Consumption Pattern

Let's assume for a moment that it is possible to extract the cylinder consumption pattern as illustrated in figure 3.3 and figure 3.5. Shading options are used in the TADMD to illustrate the contribution of various cylinders where applicable. The options are:

- The grey area that indicates a “don't care” status – these consumption patterns have no effect on the CLP or load normalisation periods and
- The solid colours signifies the allocated energy for load shifting purposes.

The experiment is started off by selecting two TADMD-profiles, equal in shape, with different consumption patterns embedded within intervals 11 and 12. In consumption pattern A, during intervals 10 and 11, there is a total of 6 different cylinders (Cyl A – Cyl F) that consumes energy. In pattern B only 3 cylinders (Cyl A- Cyl C) are in the ON state but their energy are consumed over of two intervals.

To illustrate the effect of the slight difference in consumption patterns, all the cylinders are switched off simultaneously at the end of the 9th interval and restored again at the start of the 12th. With the assumption that conversion of energy applies, the hot water cylinder simulator software can construct the load reaction curves as portrayed in figure 3.4 and figure 3.6. The simulator software routine is quite simple and it only has to conform to the following rules:

- The same amount of blocks of energy for each cylinder that was removed must be replaced after the load has been restored.
- Only one block per cylinder is allowed in an interval because a cylinder element consumes energy at a constant rate of 3kW per interval.

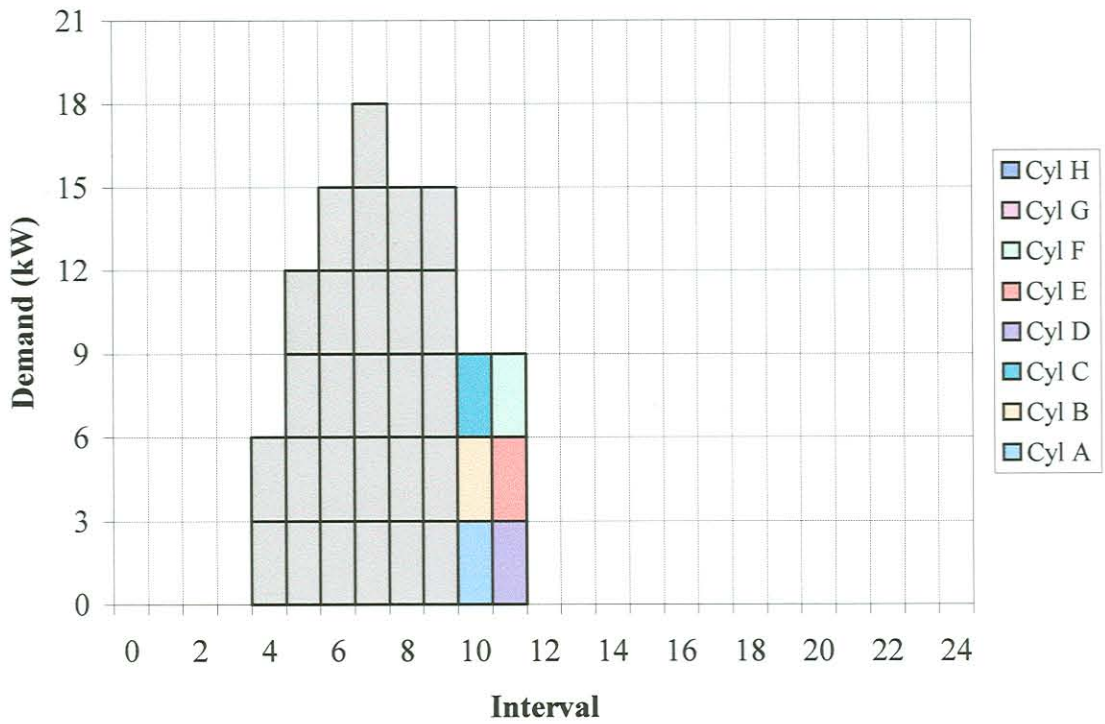


Figure 3.3: Hot water cylinder consumption pattern A.

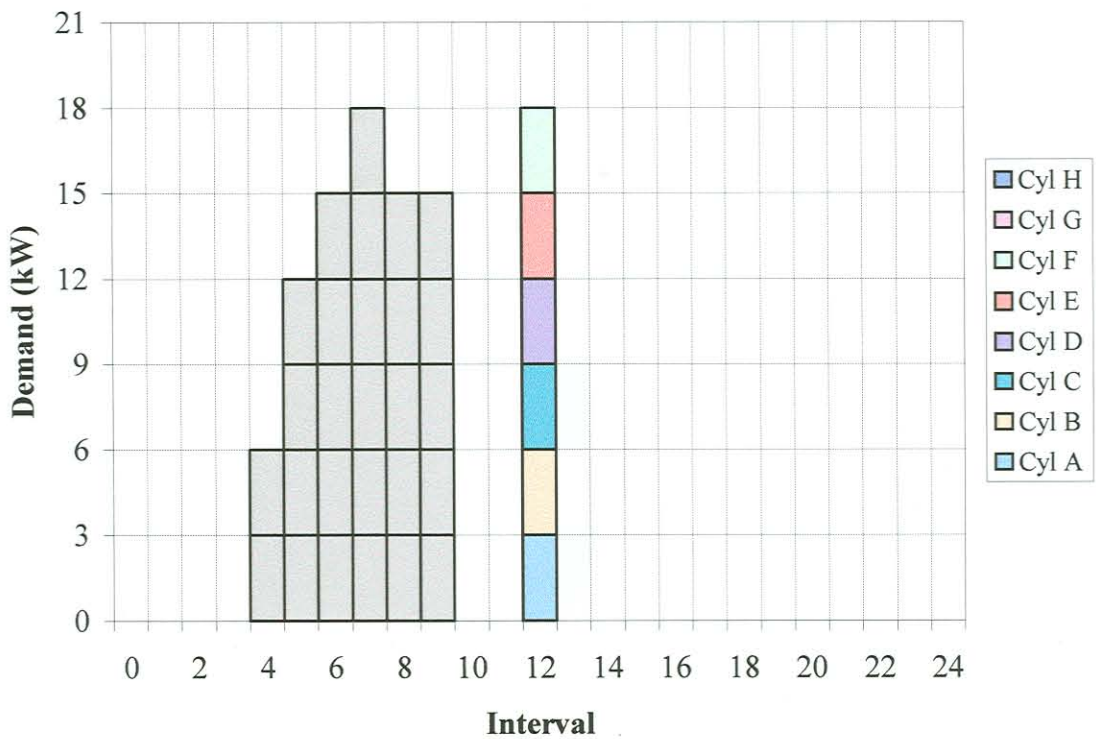


Figure 3.4: Load reaction of consumption pattern A to load curtailment.

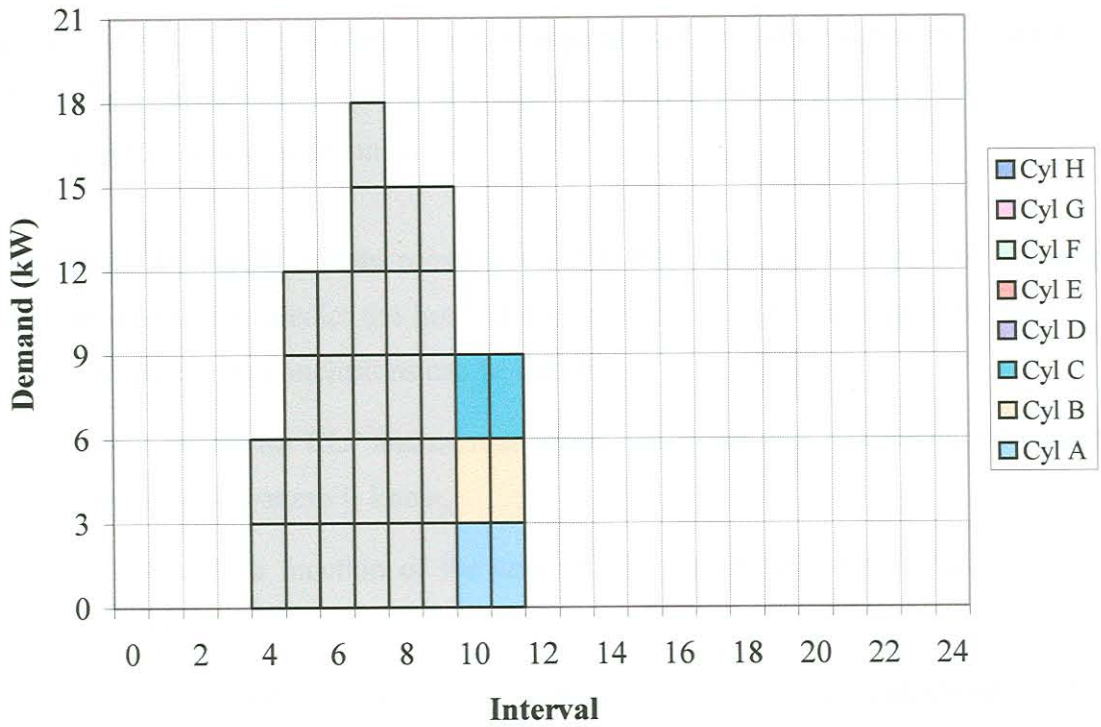


Figure 3.5: Hot water cylinder consumption pattern B.

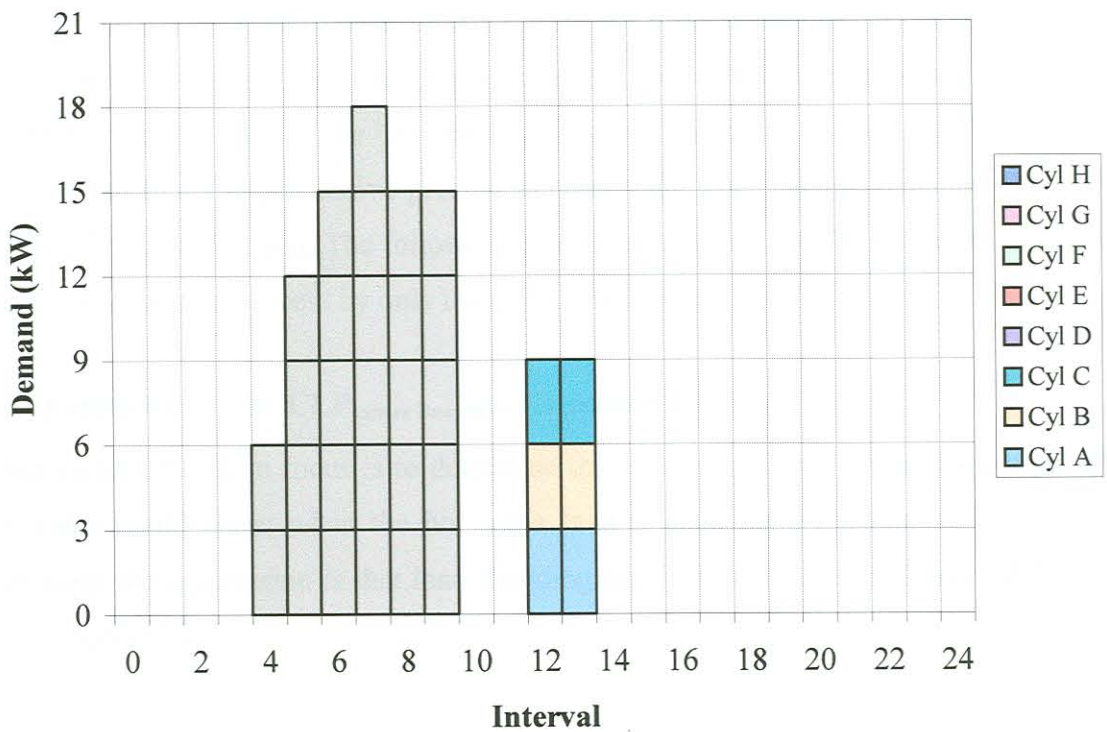


Figure 3.6 Load reaction of consumption pattern B to load curtailment.

The simulated results clearly illustrate that one TADMD can have multiple outcomes, depending on the actual cylinder consumption patterns. Consumption pattern A had a CLP of 18kW for a normalisation period of 1 interval whereas pattern B caused a CLP of 9kW within the same control environment.

Unfortunately the results clearly prove that a TADMD used on its own will not supply enough information to predict the hot water load reaction as result of applied control. At this stage the following conclusions can be drawn:

- An exact value for the CLP and normalisation curves can only be determined when the load consumption pattern is known,
- The CLP is only a function of the amount of cylinders that would have consumed energy during T_{Shed} .
- The load normalisation curve can be constructed by using the calculated CLP, the amount of energy extracted during T_{Shed} and the real-time consumption during normalisation.

At present no means exist to determine the exact hot water consumption patterns. The only other alternative is to define ways that enables one to extract the information within a TADMD-profile to full. At the University of Pretoria successful studies on TADMD data have resulted in a comprehensive process to predict the load reaction boundaries as result of a specific control action. The following experiments will guide the reader to understand the reasoning behind the process by only using TADMD data.

3.1.3 Formulating The CLP_{Lower Boundary} Extraction Process

In this section the main focus is to determine the conditions in which the lowest possible CLP value could occur when the hot water load is shed for a specific period. For this experiment the assumption is that load shedding occurs from the start of interval 7 till the end of interval 11.

The load reaction and reconstruction simulation was done on an iterative process until the boundary conditions occurred. The process to determine the lowest possible CLP for any situation can then be formulated by studying the TADMD conditions in which the lower boundary occurred.

From the TADMD of figure 3.1, it is possible to calculate the amount of cylinders that intended to consume energy during no-control conditions. The calculations showed that:

- 6 cylinders consumed energy in interval 7,
- 5 cylinders in interval 8,
- 5 cylinders in interval 9,
- 3 cylinders in interval 10 and
- 3 cylinders in interval 11.

The simulation results showed that the $CLP_{\text{Lower Boundary}}$ is in a sense visible with the naked eye. Results confirmed that the $CLP_{\text{Lower Boundary}}$ for the selected shedding period is fixed at 18 kW due to the consumption in 7th interval and is illustrated in figure 3.7 and figure 3.8.

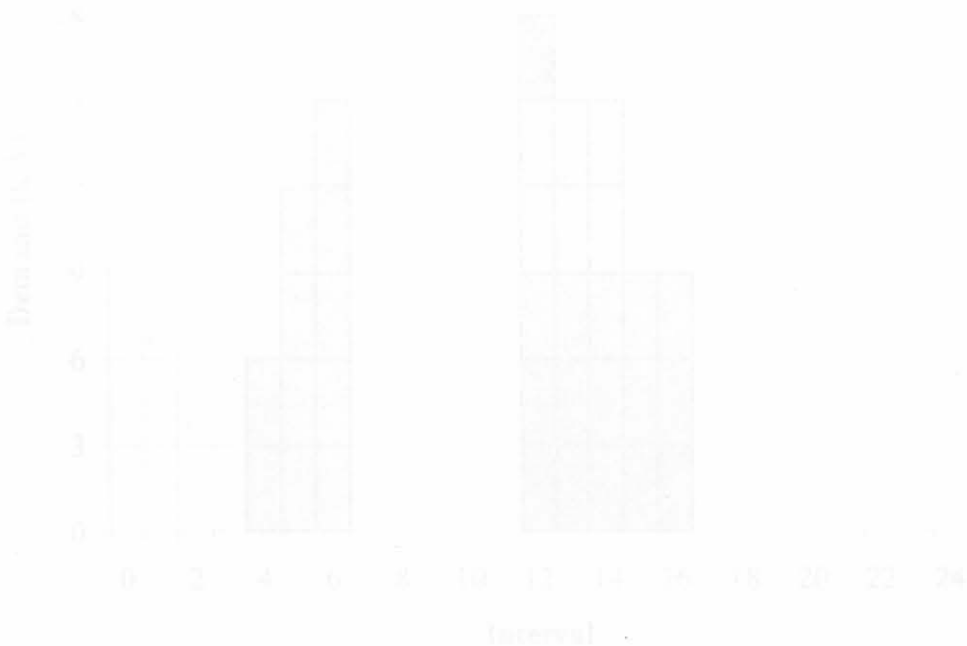


Figure 3.3: Lower boundary load reaction illustration.

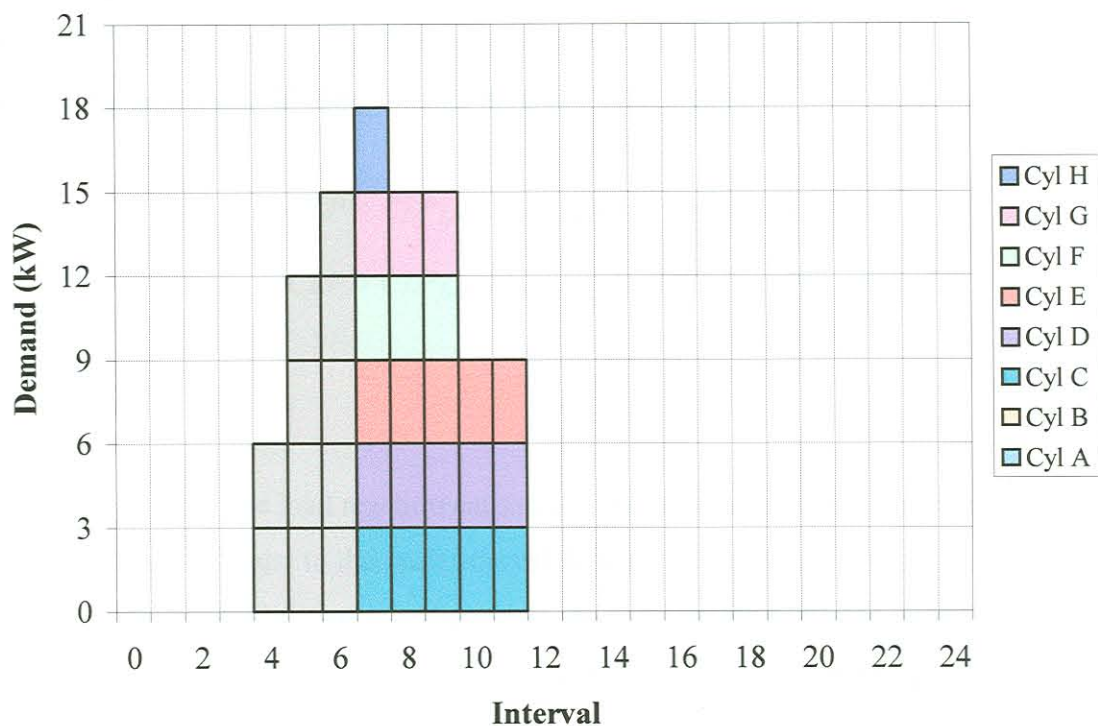


Figure 3.7: Consumption pattern for lower boundary illustration.

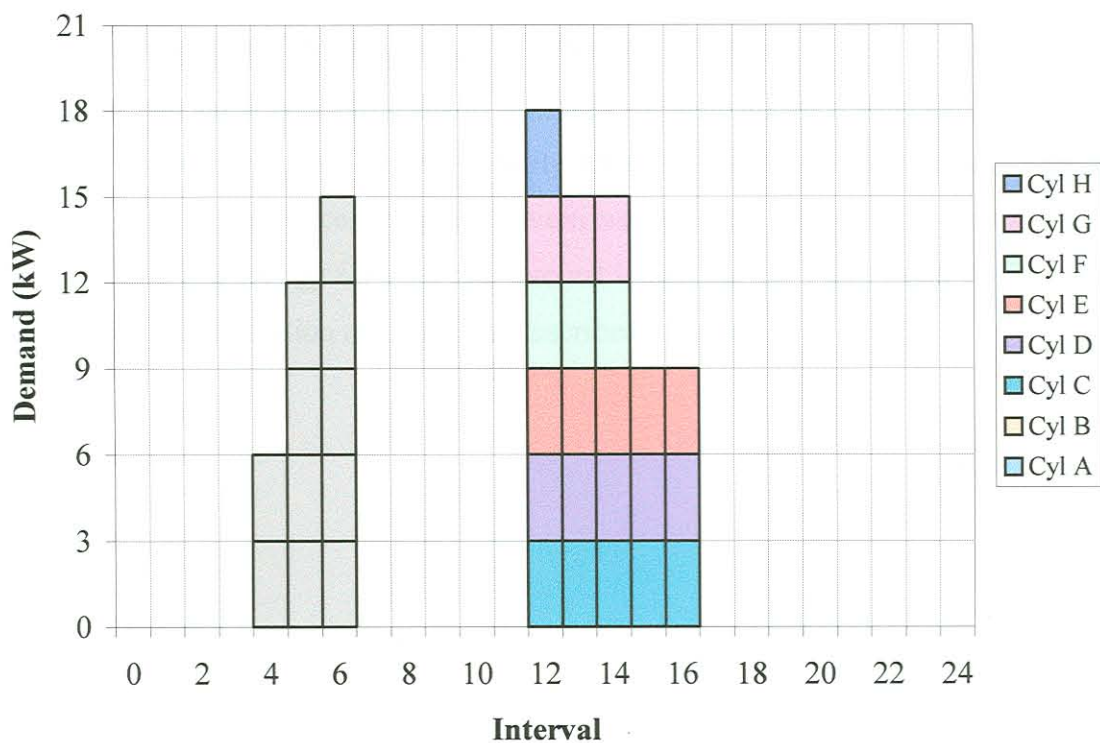


Figure 3.8: Lower boundary load reaction illustration.

The CLP can only be equal to the lower boundary when the cylinders in the peak demand interval coincides with the total population of cylinders that intended to consume energy during the entire T_{Shed} . Mathematically it can be described as:

$$CLP = CLP_{\text{Lower Boundary}} \text{ when } B = A \cap B \quad (3.1)$$

$$CLP > CLP_{\text{Lower Boundary}} \text{ when } B > A \cap B \quad (3.2)$$

where,

CLP	The load reaction caused by control actions for the period of T_{Shed} .
$CLP_{\text{Lower Boundary}}$	Equal to the peak interval demand during T_{Shed} .
A	Population of cylinders in the peak demand interval during T_{Shed} .
B	Population of cylinders that intended to consume energy during T_{Shed} .

Now that $CLP_{\text{Lower Boundary}}$ is fixed, the next step is to determine if a $CLP_{\text{Higher Boundary}}$ calculation process can be formulated.

3.1.4 Formulating The $CLP_{\text{Higher Boundary}}$ Extraction Process

Undoubtedly, the worst possible scenario in HWLC is when all the energy extracted during T_{Shed} is consumed during the interval directly after the load has been restored. This can only be possible when the consumption pattern was of such a nature that every cylinder intended to consume energy for only one interval length during T_{Shed} . Refer to figure 3.4, on page 63 for the simulation results of the described scenario.

Mathematically the higher boundary can be determined by:

$$CLP_{HigherBoundary} = \sum_{n=t_{shed}}^{t_{restore}} TADMD(n) \quad (3.3)$$

with,

$CLP_{HigherBoundary}$ Realistically the highest possible value of the CLP.

n Timeslot or interval number.

t_{shed} Timeslot or interval number in which shedding occurred.

$t_{restore}$ Timeslot or interval number in which restoring occurred.

$TADMD(n)$ The TADMD for the n^{th} timeslot or interval.

In theory this equation is true, but the calculated values seem outrageous when shedding occurs over long periods. Another possible boundary is the installed capacity and the limitation it puts on the calculated $CLP_{HigherBoundary}$. Equation 3.3 cannot be true when more energy is extracted during T_{Shed} than what all the controlled cylinders in the system is capable of consuming during one time interval. This phenomenon is clearly illustrated in figure 3.9 and figure 3.10.

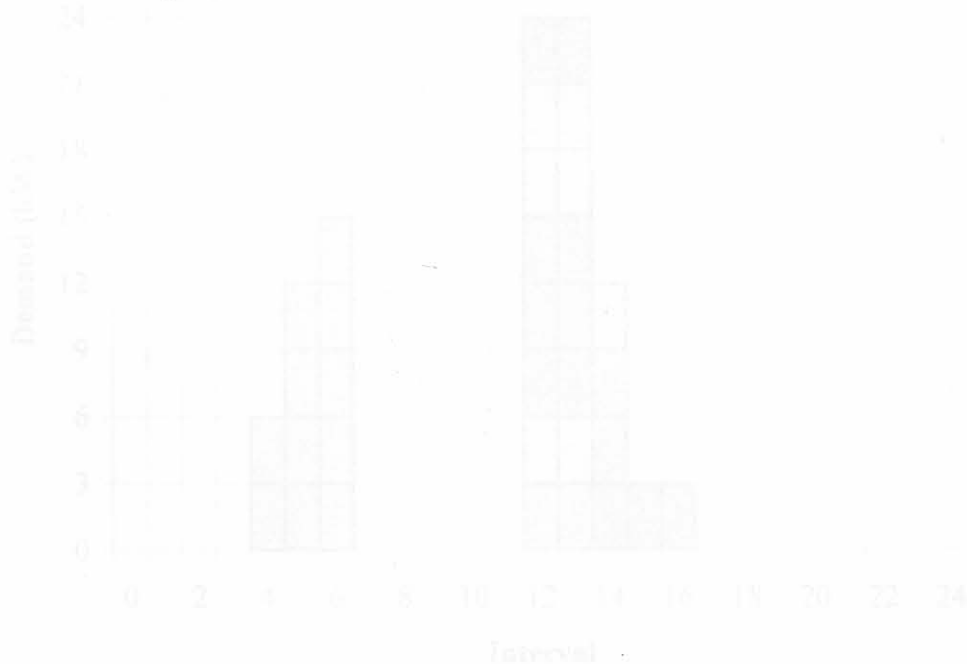


Figure 3.10: Load reaction pattern illustrating clipping at installed capacity.

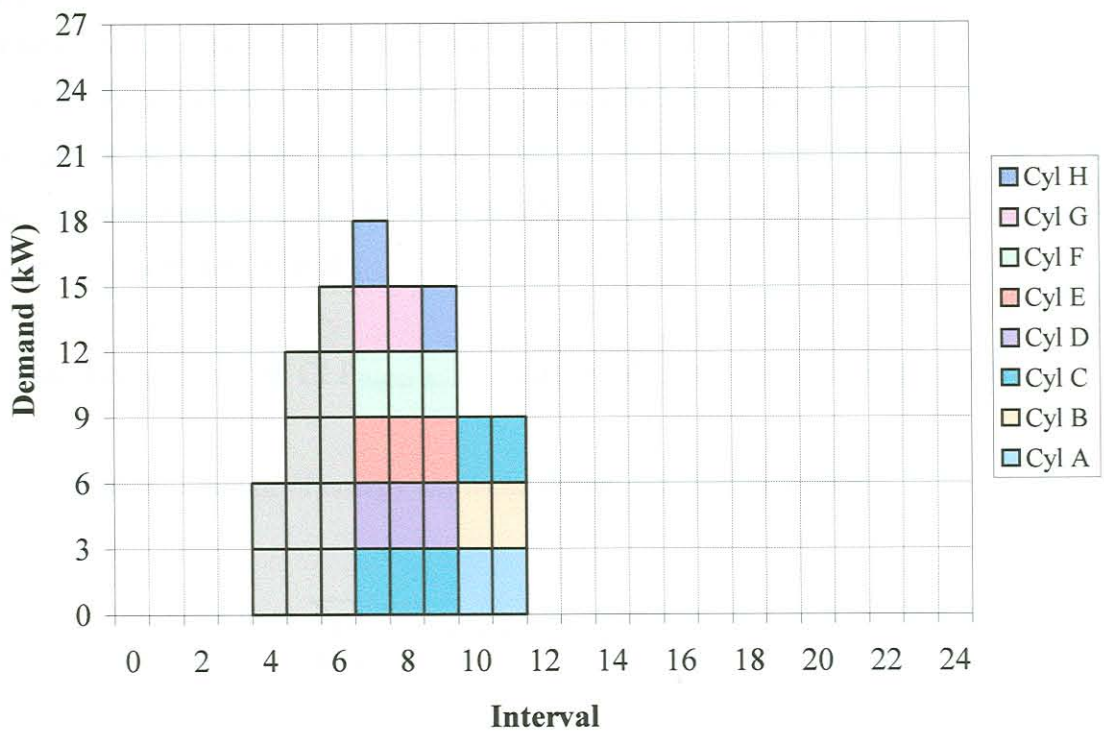


Figure 3.9: Consumption pattern for clipped higher boundary illustration.

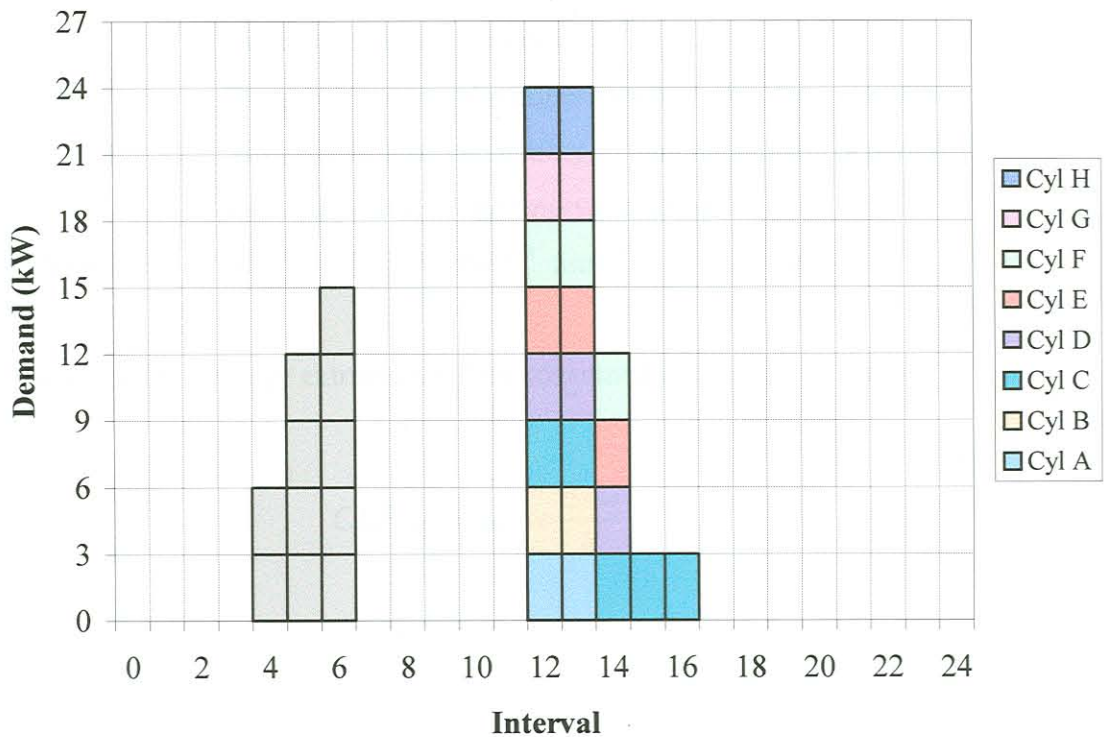


Figure 3.10: Load reaction pattern, illustrating clipping at installed capacity.

The predefined TADMD shape illustrated in figure 3.2 indicates that 22 blocks of energy will be extracted when the cylinders are switched off from the 7th till the 12th interval. In this case, equation 3.3 yields a $CLP_{\text{Higher Boundary}}$ of 66 kW (22 x 3kW) but the system is only capable of consuming 24 kW per interval. No matter what happens, the hot water system cannot consume more than its installed capacity per interval and in this example it is equal to 24 kW per interval.

With the data at hand, the $CLP_{\text{Higher Boundary}}$ can be calculated by following procedure:

(1) Calculate the amount of energy extracted during the shedding cycle.

$$E_{\text{Shed}} = \sum_{n=t_{\text{shed}}}^{t_{\text{restore}}} k \times TADMD(n) \quad (3.4)$$

where,

E_{Shed}	The amount of energy extracted as result of HWLC (J).
n	Timeslot or interval number.
t_{shed}	Timeslot or interval number in which shedding occurred.
t_{restore}	Timeslot or interval number in which restoring occurred.
k	Constant value of one interval or timeslot (s).
$TADMD(n)$	The TADMD for the n th timeslot or interval (W).

(2) Assume all the energy extracted will be consumed in one interval only.

$$CLP_{\text{HigherBoundary}} = \frac{E_{\text{Shed}}}{k} \quad (3.5)$$

- (3) Calculate the installed capacity for the amount of cylinders that has been switched off during the shedding cycle.

$$P_{Installed} = \sum_{m=1}^A P_{Cyl}(m) \quad (3.6)$$

- (4) The $CLP_{Higer\ Boundary}$ calculated in equation 3.5 cannot be greater than $P_{Installed}$. If it is the case then $CLP_{Higer\ Boundary}$ must be set equal to $P_{Installed}$.

Up till now in this section, the load normalisation materialised at a time when no additional consumption occurred. It could also be stated mathematically to say that the normalisation occurred during a period when the TADMD was equal to zero. Therefore the final phase would be to demonstrate the effect of real-time consumption during the load normalisation cycle.

3.1.5 The Effect Of Real-Time Consumption During Load Normalisation

Before this experiment can be conducted, the hot water cylinder simulator software has to be updated with a new rule to minimise confusion and to ease the description of the load reaction profile:

“During the load normalisation cycle the real-time consumption has priority over the shifted load.”

This addition only implies that real-time consumption is indicated on the load reaction curve at the time when it occurred. The shifted energy on the other hand can only be replaced when its allocated cylinder is not in use. From here on the consumption pattern shading also has to change a bit to accommodate the new simulation process.

The shading options used in the TADMD are:

- The grey area that indicates a “don’t care” status – these consumption patterns have no effect on the CLP or load normalisation periods,
- Solid colours signify the allocated energy for load shifting purposes and
- The diagonal shading interval that represents the real-time consumption during the load normalisation period.

Figure 3.11 depicts the changes made and figure 3.12 is the result of the simulated shedding of the 10th interval.

Figure 3.12: Load reaction profile when real-time consumption is taken into account.

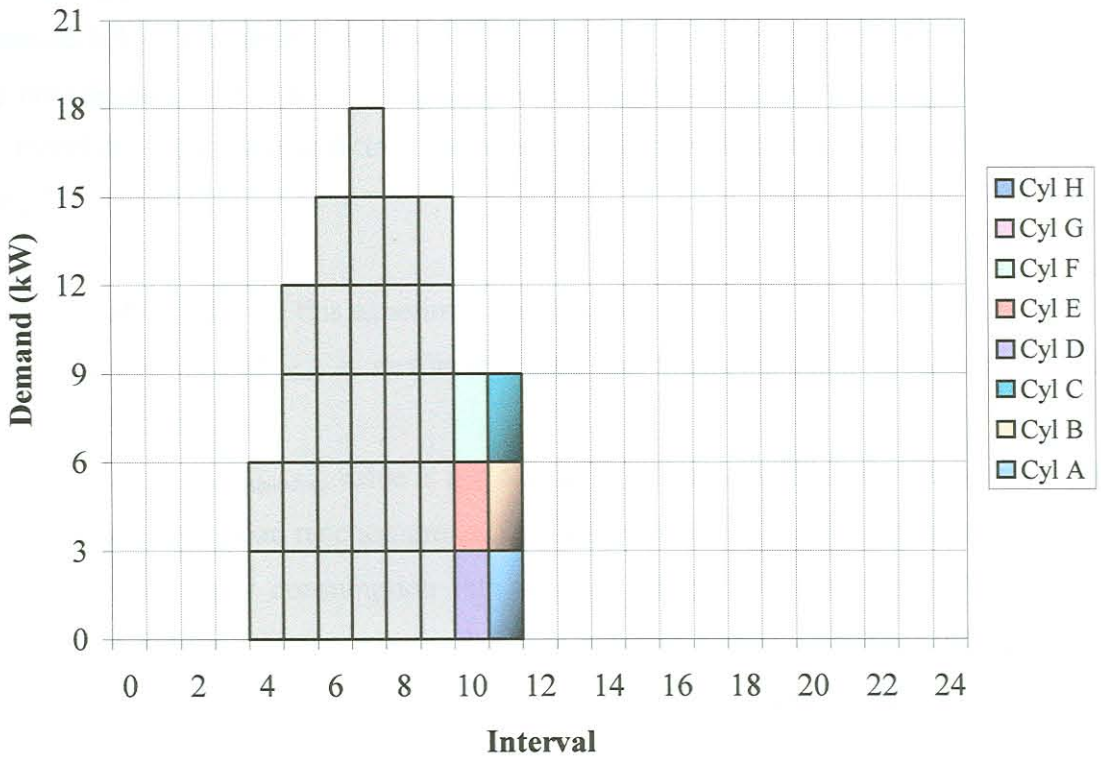


Figure 3.11: Consumption pattern illustrating controllable and real-time energy.

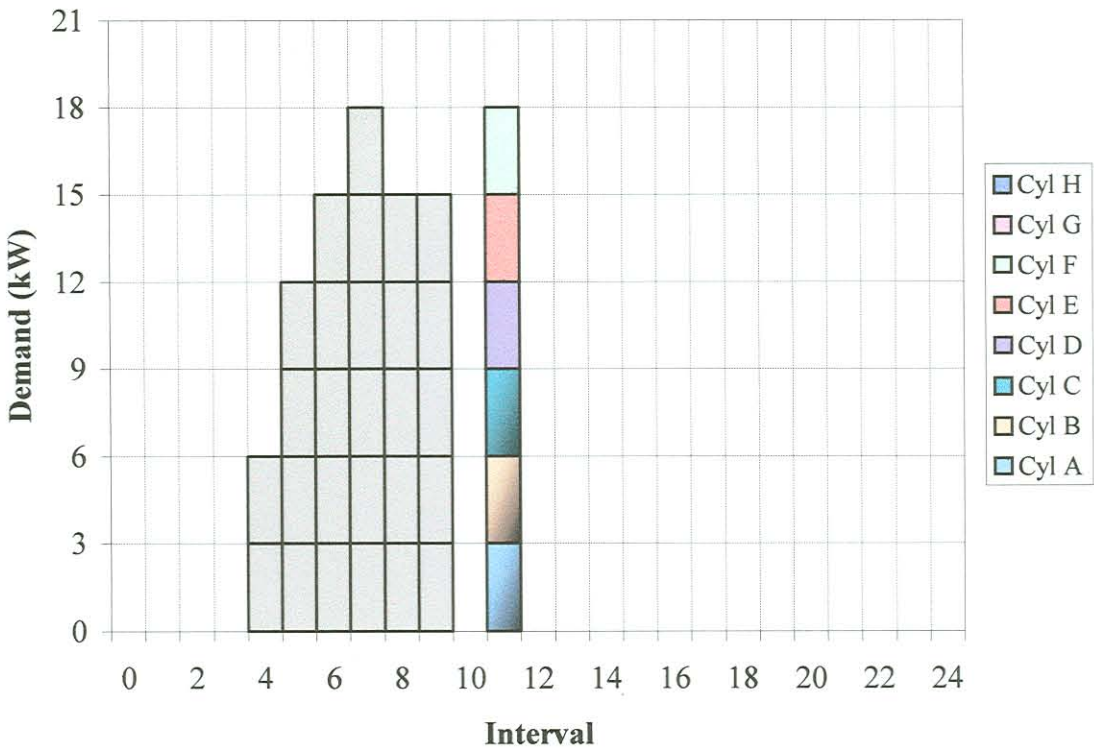


Figure 3.12: Load reaction profile when real-time consumption is taken into account.

The $CLP_{\text{Higher Boundary}}$ procedure is not yet complete because the procedure defined by equations 3.4 - 3.6 estimated that the highest possible CLP would be equal to 9 kW, which it is not the case. The real-time consumption in the 11th interval causes a CLP of 18 kW and therefore should the real-time consumption after load restoration also be taken into account.

The results obtained from this experiment are twofold:

- The $CLP_{\text{Lower Boundary}}$ is unaffected since real-time consumption can only increase the load reaction.
- The $CLP_{\text{Higher Boundary}}$ value is not correct and therefore does real-time consumption influence the load reaction profile. It is therefore better to take the effect of the real-time hot water consumption into account when predicting the higher boundary value during load restoration. The load reaction higher boundary ($LR_{\text{Higher Boundary}}$) can therefore be seen as the combined effect of CLP and real-time consumption.

The effect of the installed capacity on the $LR_{\text{Higher Boundary}}$ must be evaluated again before any final conclusions can be made. The next set of consumption patterns, figure 3.13 and figure 3.14, only illustrates that the clipping effect occurs again at the installed capacity level, even with real-time control taken into account.

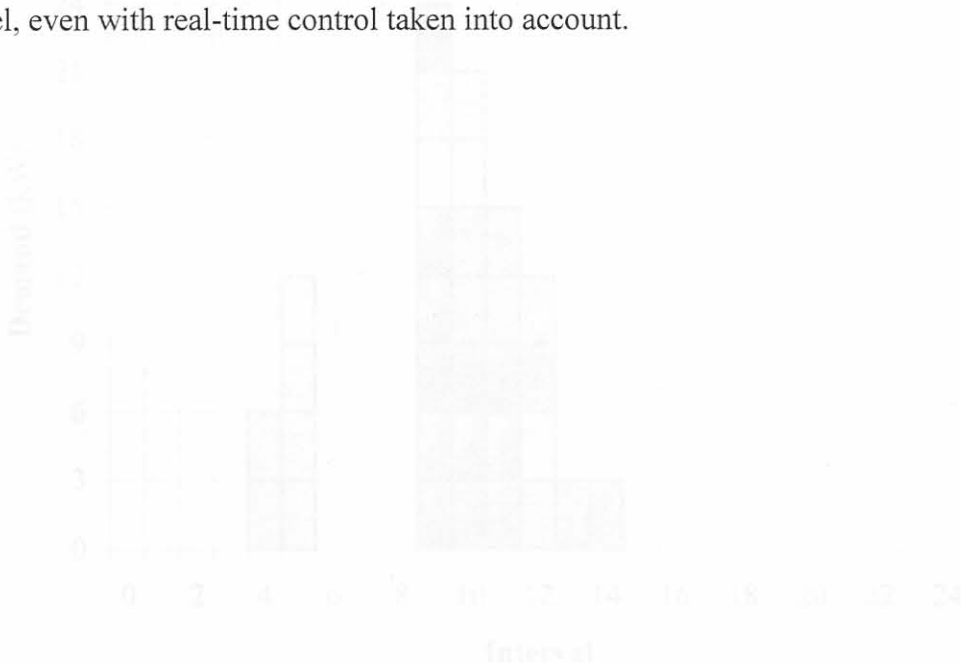


Figure 3.14: Load reaction profile illustrating clipping when real-time consumption is taken into account.

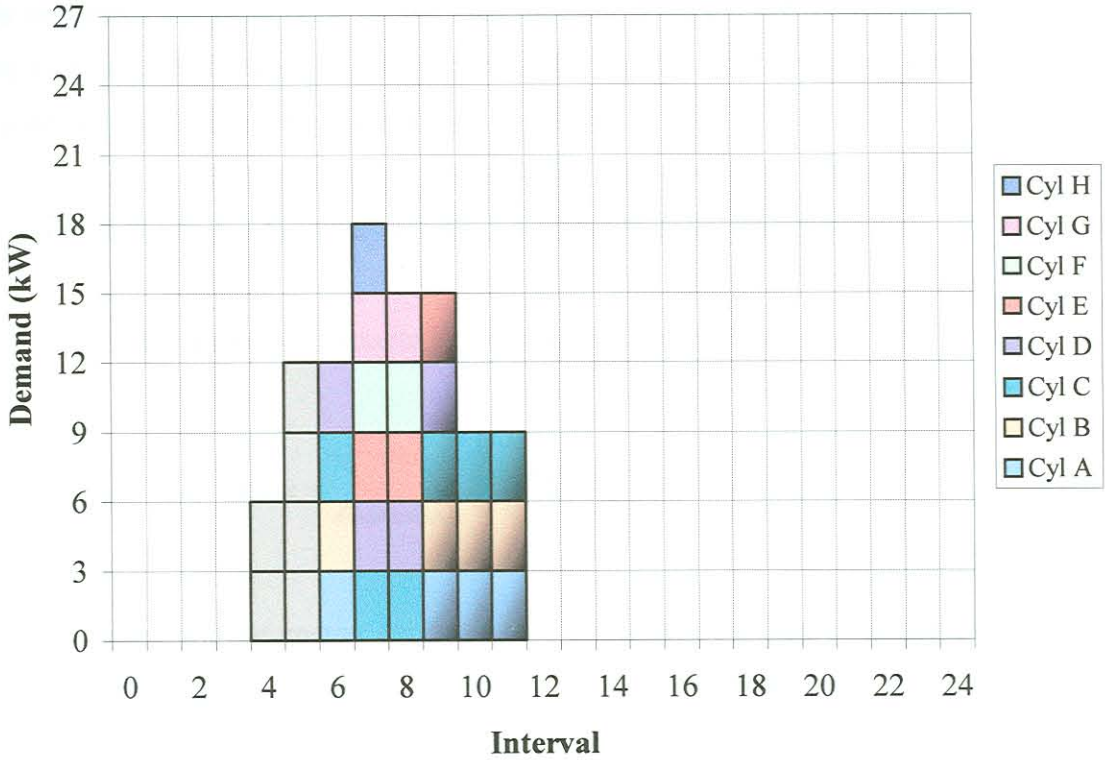


Figure 3.13: Cylinder consumption pattern for shedding intervals 6,7 and 8.

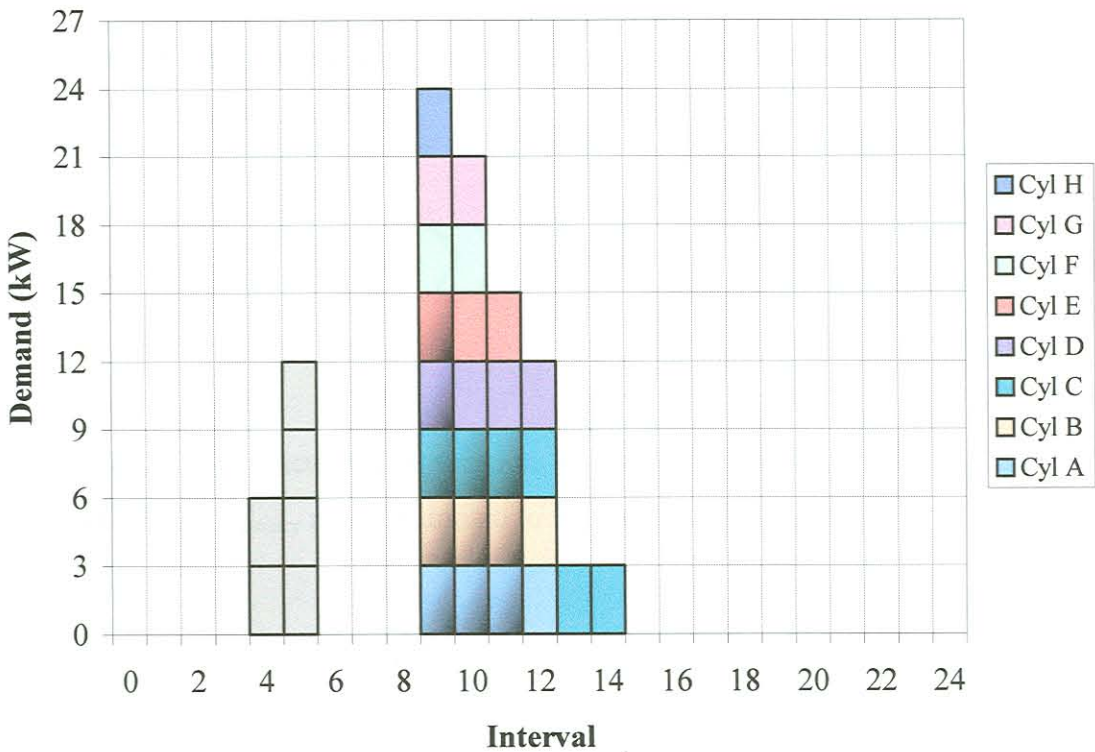


Figure 3.14: Load reaction profile illustrating clipping when real-time consumption is taken into account.

This concludes the work done on the load reaction boundary estimation process. The tests done to date proved that it is possible to predict the load normalisation boundaries while the load consumption patterns are not yet known.

In summary, the higher boundary can be calculated by following procedure:

- (1) Calculate the amount of extracted energy during the shedding cycle.

$$E_{Shed} = \sum_{n=t_{shed}}^{t_{restore}} k \times TADMD(n) \quad (3.7)$$

- (2) Assume that all the energy extracted as well as the real-time consumption within the restoration interval ($t_{restore}$) will be consumed in one interval only.

$$LR_{HigherBoundary} = \frac{E_{Shed}}{k} + TADMD(t_{restore}) \quad (3.8)$$

- (3) Calculate the installed capacity for the amount of cylinders that has been switched off during the shedding cycle.

$$P_{Installed} = \sum_{m=1}^A P_{Cyl}(m) \quad (3.9)$$

- (4) The $LR_{HigherBoundary}$ calculated in equation 3.8 cannot be greater than $P_{Installed}$. If it is the case then $LR_{HigherBoundary}$ must be limited to the value of $P_{Installed}$.

3.1.6 Model Implementation And Verification

The entire population of the hot water cylinders installed at the University of Pretoria is not enough to accurately represent a municipal environment. Therefore actual municipal data has been obtained from Eskom to ensure that the model verification is accurate and true.

During 1999, Eskom conducted a series of test from Monday, August 9 till Friday, August 13 on one of their newly installed HWLC-systems. The selected area is one of the fastest growing urban residential areas in South Africa with an exceptionally high population density factor. Personal comfort is in the order of the day since most of the consumers earn more than the average baseline, which explains why the cylinder ratio per household is equal to 1.27.

For the experiments conducted by Eskom, specified in table 3.1, 9,000 HWLC-units were used to determine the hot water dynamics of the area and effects of human behaviour on the HWLC-system.

Table 3.1: Schedule for tests conducted during August 1999.

Day	Description	Action
Monday	AMD for a public holiday	Notch on the ½ hour
Tuesday	ADMD for a weekday	Notch on the ½ hour
Wednesday	Emergency Interrupt Test	Shed all from 7:00 – 9:00
Thursday	Real Time Pricing Schedule	Shed all from 6:00 – 8:00
Friday	Wholesale Electricity Tariff Schedule	Shed all from 7:00 – 10:00

Aim Of The Study

The aim of this study is to use the data gathered during the tests conducted on Tuesday and Wednesday, and apply it to the CLP unit of the load reaction model. The procedure to calculate the higher and lower boundaries will be illustrated and then compared to the actual results obtained.

The notch test conducted on Tuesday, figure 3.15, can be used to construct a regular weekday TADMD-profile for the month of August, shown in figure 3.16. On Wednesday morning the entire controllable load of 9,000 cylinders were switched off at 7:00 and then restored again at 9:00. The load reaction curve was recorded and will be used to compare the actual values with the output predicted by the load reaction model.

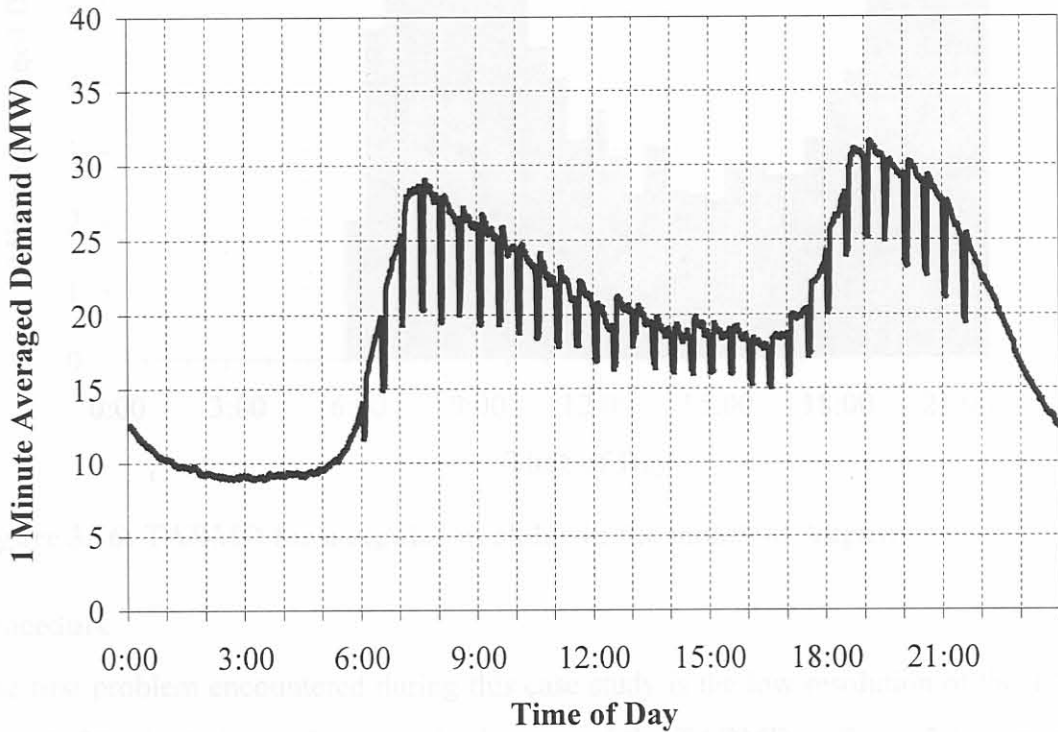


Figure 3.15: Notch test results for a regular weekday on Tuesday, 10 August 1999.

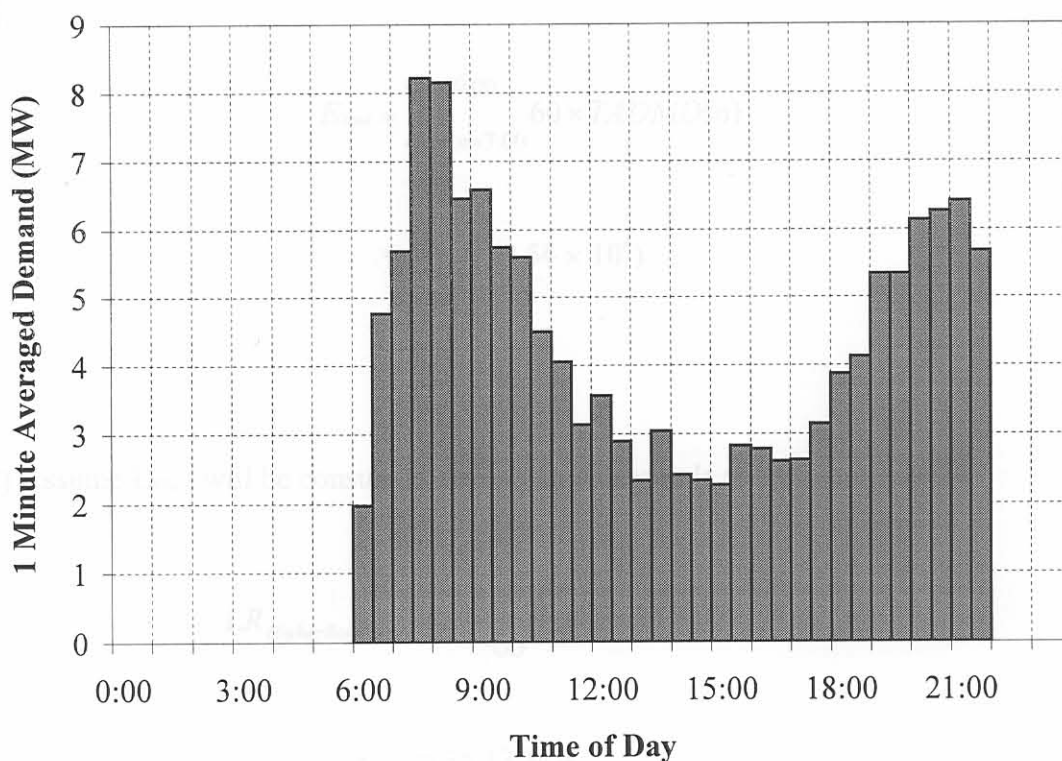


Figure 3.16: TADMD for a regular weekday in the month of August.

Procedure

The first problem encountered during this case study is the low resolution of the TADMD data. A 30-minute interval size, as in the case of the TADMD in figure 3.16, impairs the accuracy and versatility of the prediction process because prediction would preferably be done for every minute.

The one option would be to convert the uncontrollable load from a 1-minute interval size to a 30-minute average demand profile. In this case, it is a plausible option but definitely not versatile since shedding hardly occurs exactly on the half hour. Another option is to assume that the hot water consumption is constant between half hours and to divide the TADMD into minute intervals.

The $LR_{\text{Lower Boundary}}$ is always equal to the $CLP_{\text{Lower Boundary}}$, which in this case is 8.2 MW, found between 7:30 and 8:00. For the sake of the reader, the calculation of the $LR_{\text{Higher Boundary}}$ will be done step by step to illustrate the process defined in section 3.1.5.

(1) Calculate the amount of energy extracted between 7:00 and 9:00.

$$\begin{aligned}
 E_{Shed} &= \sum_{n=7:00}^{9:00} 60 \times TADMMD(n) \\
 &= 60 \times (28.54 \times 10^6) \\
 &= 1.71 \text{ GJ}
 \end{aligned}$$

(2) Assume E_{Shed} will be consumed entirely in 60 seconds at 9:00.

$$\begin{aligned}
 LR_{HigherBoundary} &= \frac{1.71 \times 10^9}{60} + 6.59 \times 10^6 \\
 &= 35.13 \text{ MW}
 \end{aligned}$$

(3) The assumption is that 9,000 units (3 kW, 150 litre) have been switched.

$$\begin{aligned}
 P_{Installed} &= 9000 \times 3000 \\
 &= 27 \text{ MW}
 \end{aligned}$$

(4) The $LR_{HigherBoundary}$ cannot be greater than $P_{Installed}$ and must therefore be equal to 27 MW.

(5) The base-load at 9:00 is equal to 19 MW, and therefore is the expected consumption for 9:00 between 27.2 MW and 46 MW.

Verification

A demand value of 35.2 MW was reached at 9:00 with a load reaction value of 16.2 MW. It means that only 4,050 of the entire population of cylinders wished to consume electricity between 7:00 and 9:00. The calculated boundaries are within specification, with the predicted lower boundary 51% less and the higher boundary 60% more than the measured load reaction value.

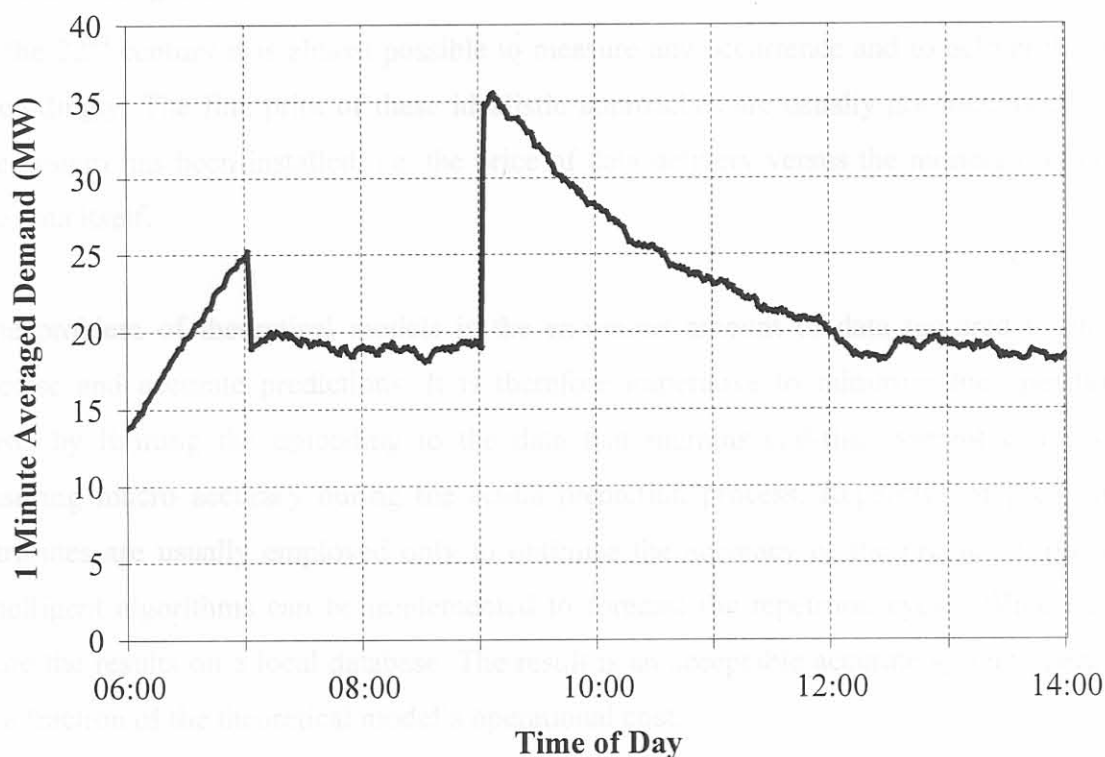


Figure 3.17: Measured load-profile for the Emergency Interrupt Test.

Various ways exist to refine the boundary calculation process and some of the ideas will be discussed in the conclusions section of chapter 5.

3.2 AVERAGE CYLINDER TEMPERATURE PREDICTION MODEL

Cylinder temperature modelling has been a favourite topic among authors [17, 21] at the University of Pretoria. Instead of re-inventing the wheel again, this section would rather focus on the process of implementing and integrating the average cylinder temperature predictions into the end-user model. Various evident pitfalls have been defined and will be outlined before the process definition phase can commence.

3.2.1 Refining The Data

In the 22nd century it is almost possible to measure any occurrence and to deliver the data accordingly. The fine print of these idealistic approaches are usually not mentioned until the system has been installed, i.e. the price of data delivery versus the monetary value of the data itself.

The problem of theoretical models is the enormous amount of data required to ensure precise and accurate predictions. It is therefore imperative to minimise the operational costs by limiting the uploading to the data that ingrain real-time variant constraints, ensuring macro accuracy during the actual prediction process. Repetitive or predictable attributes are usually employed only to optimise the accuracy of the prediction process. Intelligent algorithms can be implemented to forecast the repetition cycle offline and to store the results on a local database. The result is an acceptable accurate system operating at a fraction of the theoretical model's operational cost.

Another pitfall that is usually unattended is the effect of HWLC-actions on the consumer hot water consumption pattern.

3.2.2 Non Linear Consumption Patterns

Temperature prediction models developed by Delpont [17], indicates that hot water consumption is non linear when a cylinder is switched off during load-control actions.

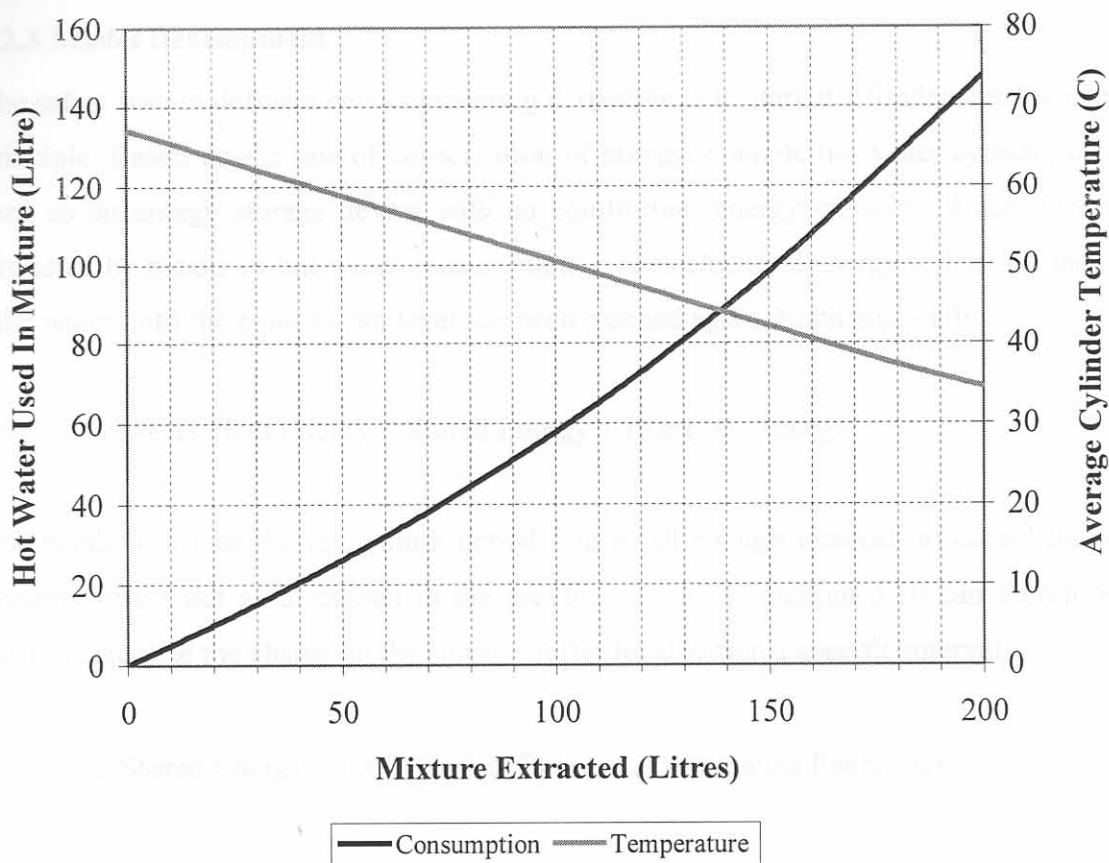


Figure 3.18: Non-linear residential hot water extraction during shedding periods.

Figure 3.18 was generated by the assumption that a residential consumer requested a constant water supply of 10 litres per minute at a mixture temperature of 40°C when taking a shower. The hot water is extracted at an initial temperature of 67 °C and then mixed with the cold water supply to obtain the mixture temperature.

As consumption continues the average cylinder temperature decreases because the HWLC-device has disabled the cylinder's heating element. This decrease in cylinder temperature initiates an increase in the hot water flow rate to maintain a mixture temperature 40°C. Therefore should the non-linear effect be taken into account when predicting the average cylinder temperature.

3.2.3 Model Development

The safest way to define a new engineering derivative is to start at a fundamental scientific principle. Based on the law of conservation of energy, a single hot water cylinder can be seen as an energy storage device with an equilibrium energy capacity. When energy is extracted by means of hot water consumption, it uses electrical energy to heat up the cold inlet water until the equilibrium level has been reached again. Mathematically:

$$\text{Electrical Energy} = \text{Stored Energy} + \text{Extracted Energy} \quad (3.10)$$

One needs to divide the integration period into small enough intervals to cancel the non-linearity effect out as discussed in the previous section. Equation 3.10 can therefore be used to calculate the change in the storage buffer level within a specific interval:

$$\Delta\text{Stored Energy}(t_x) = \text{Electrical Energy}(t_x) - \text{Extracted Energy}(t_x) \quad (3.11)$$

The amount of energy stored in the cylinder is directly proportional to its water temperature and the volume of the cylinder. Therefore, any change in stored energy would be directly responsible for a change in the cylinder temperature.

$$\Delta\text{Stored Energy}(t) = mc\Delta T(t) \quad (3.12)$$

The volume of the water inside the cylinder is also equal to the mass required for equation 3.12 because one litre of water weighs approximately one kilogram and the specific heat of water, c , is equal to $4,200 \text{ K/kg/}^\circ\text{C}$.

The next step is to evaluate the energy extracted out of the cylinder. According to the theoretical model developed by Delpont [17], it is mainly a function of:

- Water consumption patterns,
- The temperature of the water inside the cylinder,
- Inlet water temperature,
- Piping - and insulation losses and
- The cylinder orientation.

It is almost impossible, and definitely impractical to measure all the inputs individually. The data refinement process, discussed in section 3.2.1, has initiated the search for a cost effective solution.

Chapter 1 has shown that notch test data is a fair resemblance of the total amount of energy extracted out of the system. The TADMD-profiles have already been stored for the load reaction module and can just as well be used by the temperature prediction process. The extracted energy for a specific user group can therefore be calculated by using equation 2.2 on page 56:

$$\text{Extracted Energy}(\text{Group}, t_x) = \text{Interval Size} \times \text{TAMD}(\text{Group}, \text{Day}, \text{Month}, t_x)$$

The electrical energy consumed during a specific interval is a function of the:

- Element rating,
- The thermostat status (On or Off) and
- The HWLC-unit status (Shed or Restore)

3.2.4 Model Implementation

Now that the model has been defined and refined, it is possible to implement it and evaluate the scenario. The object of the model is not necessarily to predict the average cylinder temperature, but rather to predict the time of day that the hot water could be used. In other words, estimations are made to ensure that the average cylinder temperature is never below the level specified by the management rules.

Yet again is the value of measurement not viable and the requirement is to find an accurate representation of the real-time hot water electricity consumption. The only phrase that comes to mind is déjà vu. The output of the load reaction model embodies all of the above variants and therefore can the electrical energy consumed by a user group during any specific time interval be obtained by:

$$\begin{aligned} \text{Electrical Energy}(\text{Group}, t_x) &= \text{Interval Size} \times \text{Real-time Consumption Pattern}(\text{Group}, t_x) \\ &= \text{Interval Size} \times \text{Load Reaction Profile}(\text{Group}, t_x) \end{aligned} \quad (3.13)$$

The results obtained can now be substituted into equation 3.11 to attain the final equation relating the change in average cylinder for a specific user group and interval.

$$\Delta T(t_x) = \frac{\text{IntervalSize}}{mc} \times \text{RTConsumption}(\text{Group}, t_x) - \text{TADM}(\text{Group}, \text{Day}, \text{Month}, t_x)$$

The average cylinder temperature can be obtained by:

$$T(t_x) = T(t_0) + \sum_{n=1} \Delta T(t_n) \quad (3.14)$$

There is no need to verify the model since it is based on previous work done, along with basic scientific principles. An illustrative example follows, which will guide the reader through the process again to clarify any uncertainties.

3.2.4 Model Implementation

Now that the model has been defined and refined, it is possible to implement it into a real life scenario. The object of the model is not necessarily to predict the average cylinder temperature, but rather to predict the time of day that the risk of cold water could occur. In other words, estimations are made to ensure that the average cylinder temperature is never below the level specified by the management rules

Aim Of The Study

For this example, the model integration will again be illustrated on the data-set used in section 3.1.6 on page 82. This time the idea is to plot the change in average cylinder temperature due to the load-control actions taken at 7:00 and 9:00.

Procedure

The average cylinder temperature prediction model has been defined, but the integration process is not that simple. The data acquisition sequence, illustrated in figure 3.19, has been optimised to ensure that the requests to and from the database are the minimum.

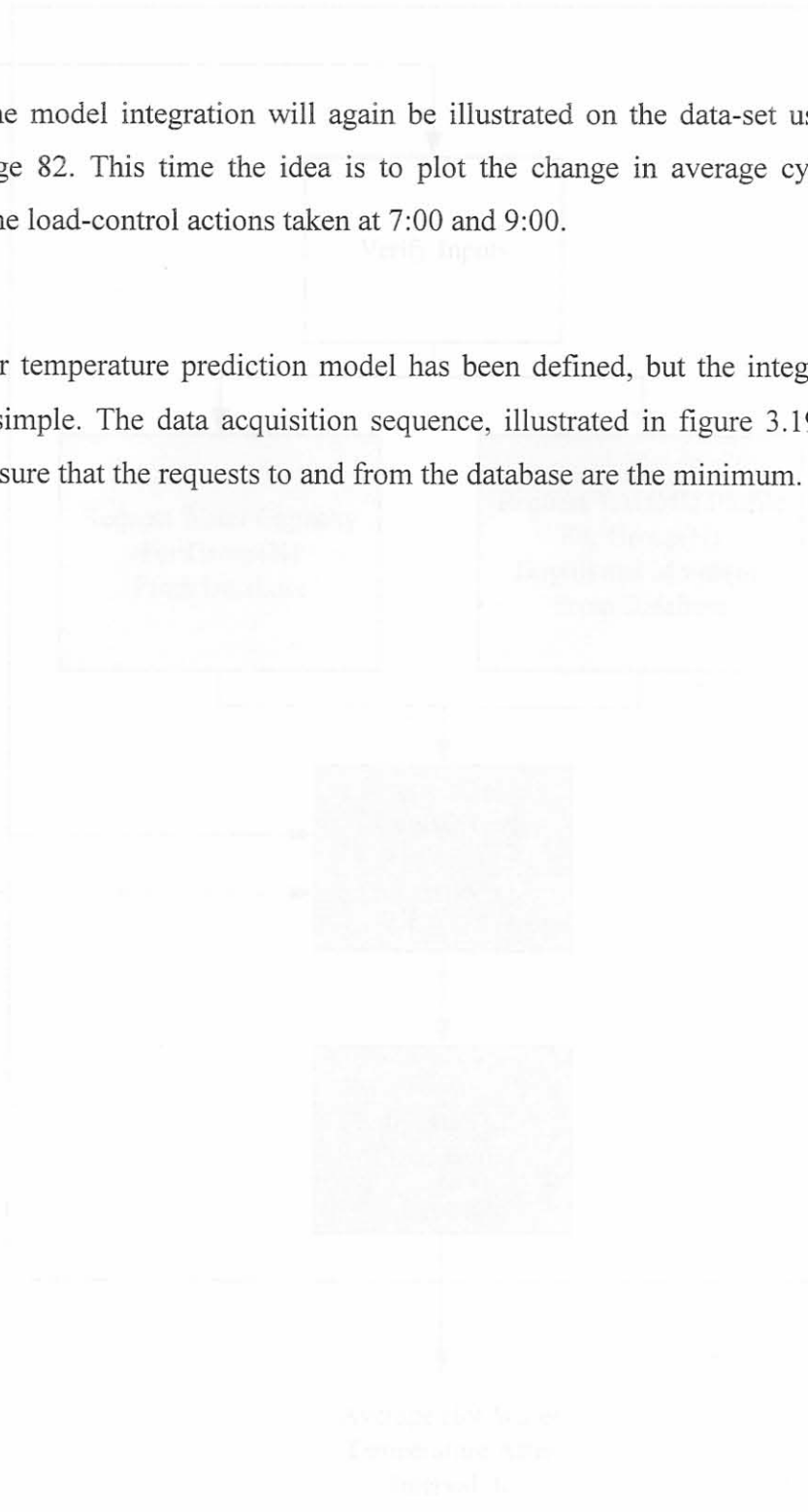


Figure 3.19: Implementation process of the average cylinder temperature prediction model.

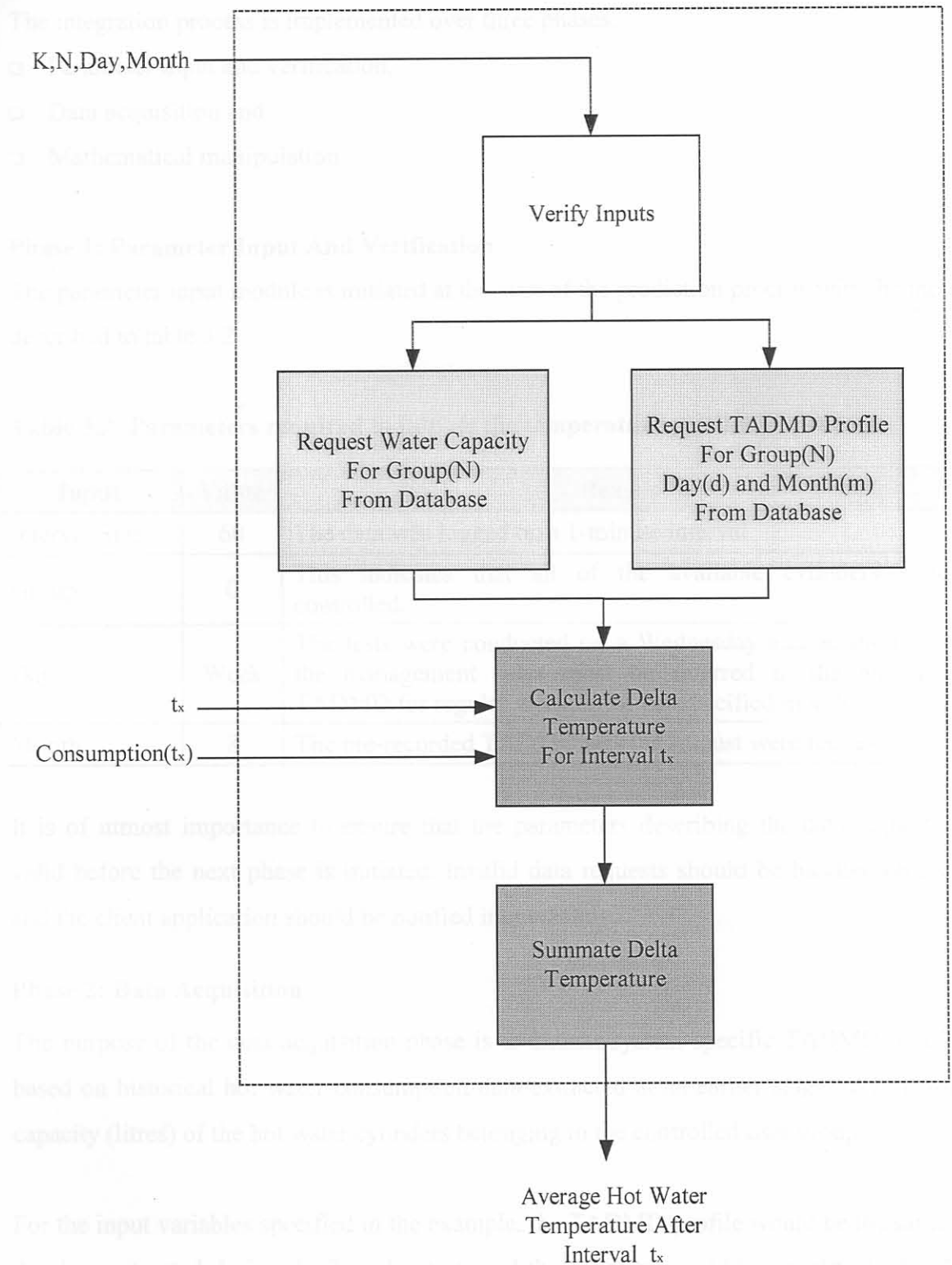


Figure 3.19: Implementation process of the average cylinder temperature prediction model.

The integration process is implemented over three phases:

- Parameter input and verification,
- Data acquisition and
- Mathematical manipulation.

Phase 1: Parameter Input And Verification

The parameter input module is initiated at the start of the prediction process with the inputs described in table 3.2.

Table 3.2: Parameters required to initiate the temperature prediction process.

Input	Value	Reason
Interval Size	60	The data was logged on a 1-minute interval.
Group	0	This indicates that all of the available cylinders were controlled.
Day	Week	The tests were conducted on a Wednesday and according to the management rules must be referred to the recorded TADMD for regular weekday of the specified month.
Month	8	The pre-recorded TADMD sets for August were requested.

It is of utmost importance to ensure that the parameters describing the data request are valid before the next phase is initiated. Invalid data requests should be handled on merit and the client application should be notified immediately.

Phase 2: Data Acquisition

The purpose of the data acquisition phase is to extract system specific TADMD-profiles, based on historical hot water consumption data extracted at an earlier stage, and the total capacity (litres) of the hot water cylinders belonging to the controlled user group.

For the input variables specified in the example, the TADMD-profile would be the same as the data extracted during the Tuesday tests and the capacity would be equal to 1.35×10^6 litre. These values are then stored in memory for future reference throughout the process for this specific day.

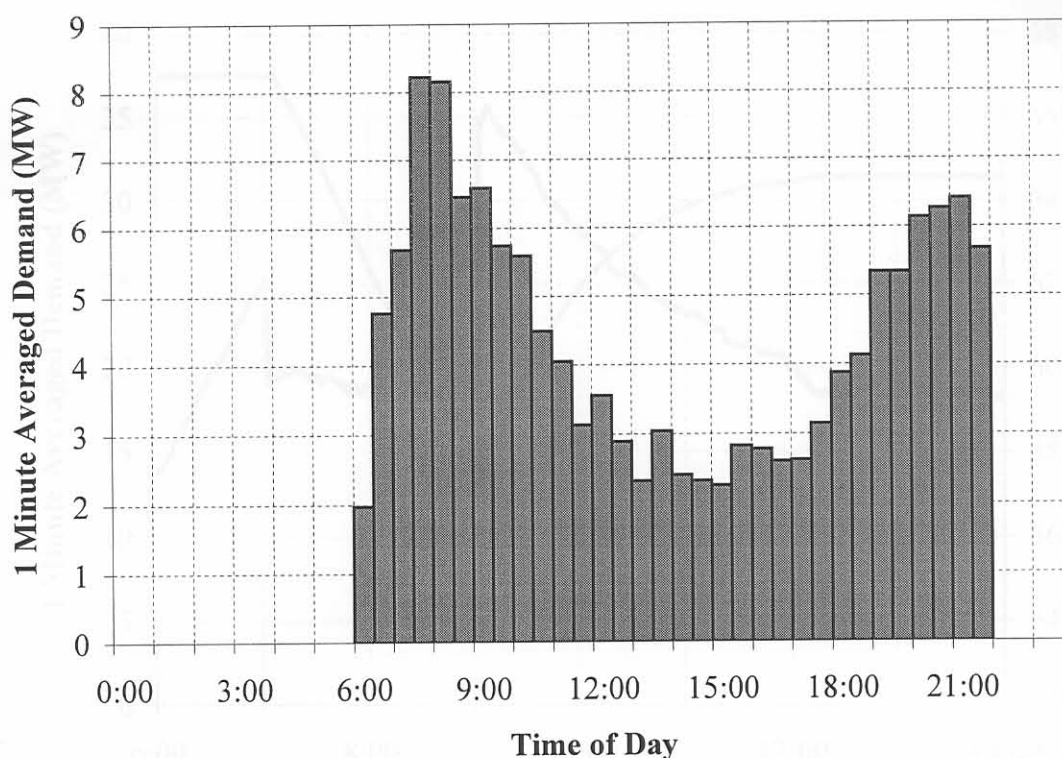


Figure 3.20: TADMD-profile representing the energy available for extraction purposes.

Phase 3: Mathematical Manipulation

The mathematical manipulation phase is an iterative process, running either in real-time as the consumption data becomes available or off-line on historical - and simulated data. The client application must pass the interval number and the user group's real-time consumption value for that specific interval to the calculation module in order to append the group's average cylinder temperature.

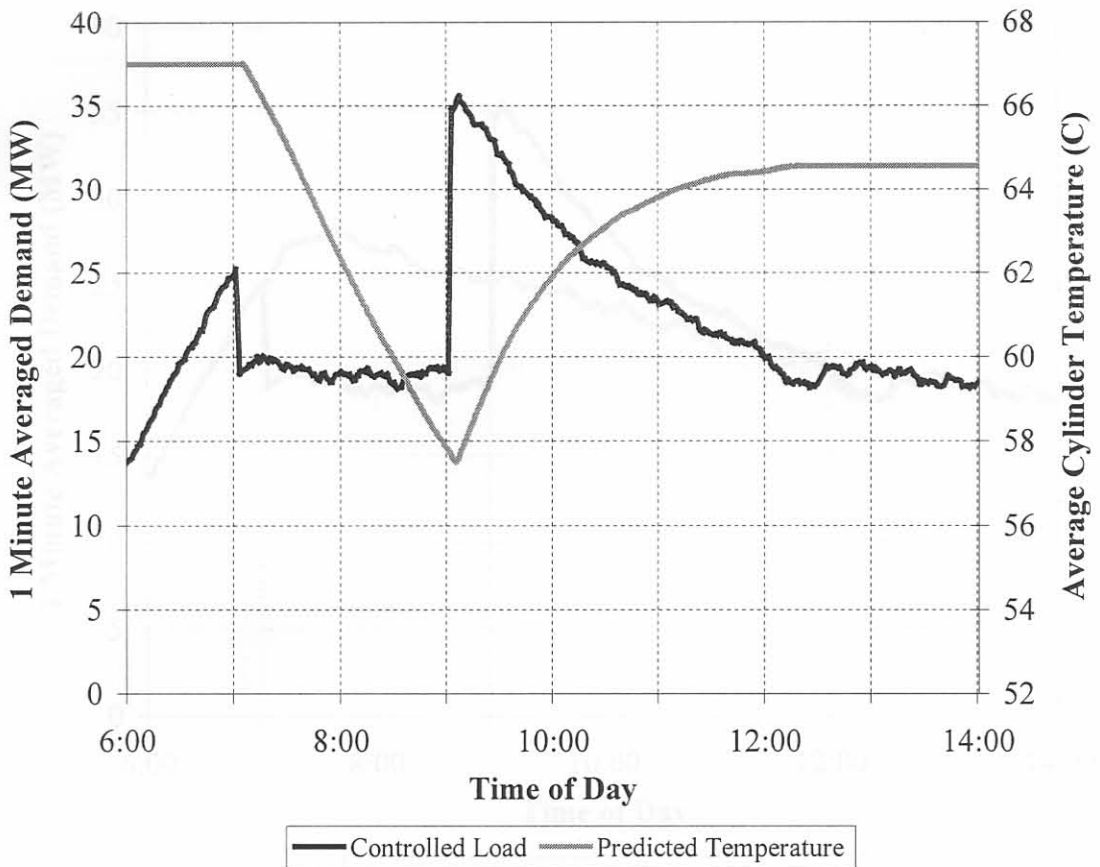


Figure 3.21: Average cylinder temperature prediction model output vs. load-control actions.

Results

The average cylinder temperature drops from 67°C to 57.5°C within the two-hour shedding period and takes more than 2 hours to heat the entire population again. Theoretically the load restoration time should be less than the shedding period, but consumption from various cylinders after restoration hindered the restoration cycle.

One also notices that the model stabilises the prediction temperature at 64.5°C . More energy was extracted during the 2-hour shedding period than what was electrically consumed again during restoration, which is verified by the overlaying illustration in figure 3.22.

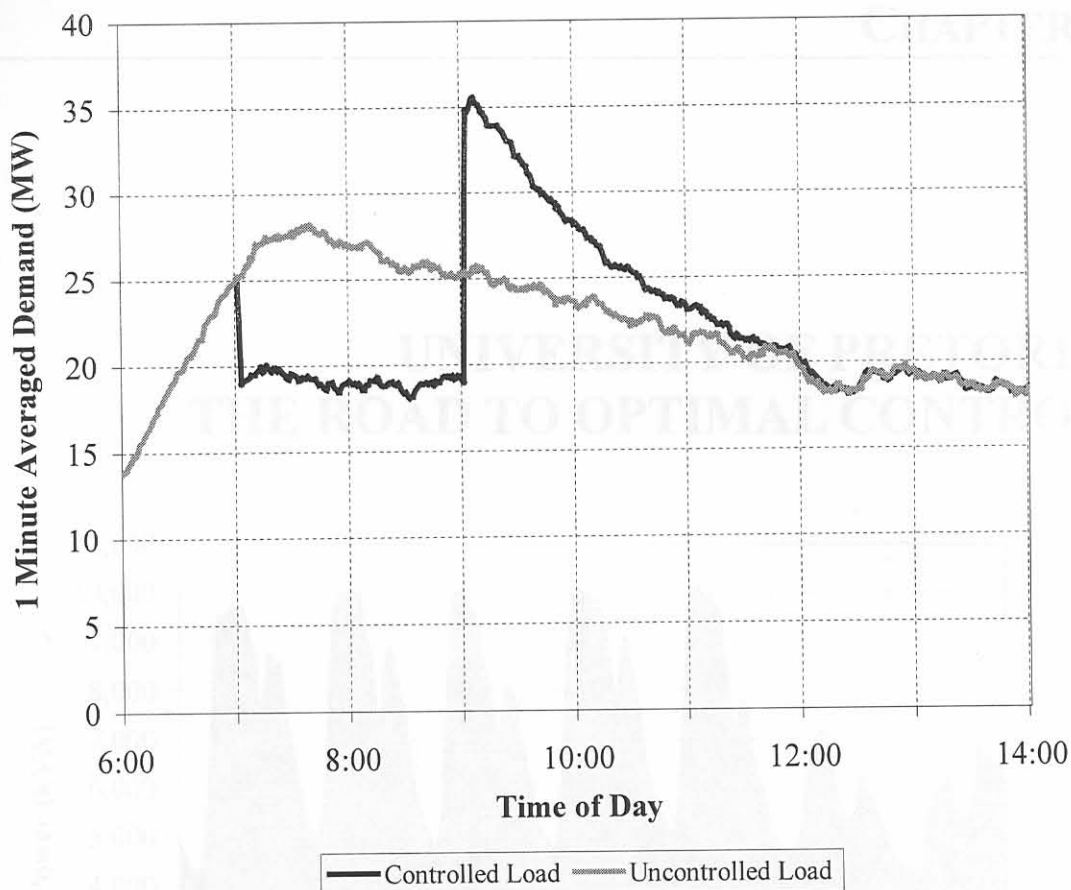


Figure 3.22: Tuesday uncontrolled load portrayed over the controlled load of Wednesday.

Mathematical calculations showed that 53.8 GJ were extracted but only 39.9 GJ were consumed to restore the load. This means that 13.9 GJ were supposed to be delivered but the users had already exhausted their 150-litre supply of hot water within the two-hour period.

The effectiveness of this model is suddenly questionable because certain users experienced cold water while the model predicted a minimum temperature of 57.8°C. The problem at hand is that the temperature prediction was done for the entire population of 9,000 cylinders and not the individual cases. The amount of cylinders used to calculate the mass directly influences the prediction accuracy. The conclusions and recommendations in chapter 5 will take a closer look at ways to optimise the accuracy of the model.