Analysis of the particle swarm optimization algorithm

by

Daniel N. Wilke

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Abstract

Title:	Analysis of the particle swarm optimization algorithm
Author:	Daniel Nicolas Wilke
Supervisors:	Prof. A.A. Groenwold Dr. S. Kok
Department:	Department of Mechanical and Aeronautical Engineering
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Increasing prominence is given to the role of optimization in engineering. The global optimization problem is in particular frequently studied, since this difficult optimization problem is in general intractable. As a result, many a solution technique have been proposed for the global optimization problem, e.g. random searches, evolutionary computation algorithms, taboo searches, fractional programming, etc. This study is concerned with the recently proposed zero-order evolutionary computation algorithm known as the particle swarm optimization algorithm (PSOA). The following issues are addressed:

1. It is remarked that implementation subtleties due to ambiguous notation have resulted in two distinctly different implementations of the PSOA. While the behavior of the respective implementations is markedly different, they only differ in the formulation of the velocity updating rule.

   In this thesis, these two implementations are denoted by PSOAF1 and PSOAF2 respectively.

2. It is shown that PSOAF1 is observer independent, but the particle search trajectories collapse to line searches in n-dimensional space.

   In turn, for PSOAF2 it is shown that the particle trajectories are space filling in n-dimensional space, but this implementation suffers from observer dependence.

It is also shown that some popular heuristics are possibly of less importance than originally thought; their greatest contribution is to prevent the collapse of particle trajectories to line searches.

3. A novel PSOA formulation, denoted PSOAF1* is then introduced, in which the particle trajectories do not collapse to line searches, while observer independence is preserved. However, the observer independence is only satisfied in a stochastic sense, i.e. the mean objective function value over a large number of runs is independent of the reference frame.

   Objectivity and effectiveness of the three different formulations are quantified using a popular unimodal and multimodal test set, of which some of the multimodal functions are decomposable. However, the objective functions are evaluated in both the unrotated, decomposable reference frame, and an arbitrary rotated reference frame.

4. Finally, a practical engineering optimization problem is studied. The PSOA is used to find the optimal shape of a cantilever beam. The objective is to find the minimum vertical displacement at the edge point of the cantilever beam. In order to calculate the objective function the finite element method is used. The meshes needed for the linear elastic finite element analysis are generated using an unstructured remeshing strategy. The remeshing strategy is based on a truss structure analogy.

Opsomming

Titel: Analise van die partikel swerm optimeringsalgoritme

Outeur: Daniel Nicolas Wilke

Leier: Prof. A.A. Groenwold
Dr. S. Kok

Departement: Departement Meganiese en Lugvaartkundige Ingenieurswese

Graad: Meester van Ingenieurswese

Sleutelwoorde: partikel swerm optimering, diversiteit, baan ineenstorting, lyn soektogte, waarnemer onafhanklikheid, invariansie, vorm optimering

Toenemende belangrikheid word aan die rol van optimering in ingenieurswese gegee. Veral die globale optimeringsprobleem word dikwels bestudeer, aangesien hierdie moelike optimeringsprobleem in die algemeen onoplosbaar is. Gevolglik is daar voorheen al verskeie oplossingstegnieke voorgestel vir die globale optimeringsprobleem, soos byvoorbeeld lukrake soektogte, evolusionêre berekeningsalgoritmes, taboe soektogte, fraksionele programmering, ens. Hierdie studie is vermoeid met die onlangs gepostuleerde nulde-orde evolusionêre berekeningsalgoritme wat bekend staan as die partikel swerm optimeringsalgoritme (PSOA). Die volgende kwessies word bespreek:

1. Daar word opgemerk dat twee verskillende formulerings van die PSOA bestaan, moontlik as gevolg van onduidelike notasie. Alhoewel die gedrag van die onderskeie implementerings dramaties verskil, verskil hulle slegs ten opsigte van die formulering van die snelheidsopdateringswet.

   In hierdie tesis word die onderskeie implementerings as PSOAF1 en PSOAF2 aangedui.

2. Verder word aangetoon dat PSOAF1 waarnemer onafhanklik is, maar dat die partikel bane in n-dimensionele ruimte na lyn soektogte ineenstort.

   Om die beurt, word daar vir PSOAF2 aangetoon dat die partikel bane ruimtevullend is in n-dimensionele ruimte, maar hierdie implementering is waarnemer afhanklik.

   Daar word ook gewys dat sommige gewilde heuristieke moontlik van minder belang is as

wat oorspronklik geag is. Daar word gewys dat hulle grootste bydrae waarskynlik is om partikel baan ineenstorting na lyn soektogte te voorkom.

3. ’n Nuwe PSOA formulering word dan voorgestel, naamlik PSOAF1*. Partikel trajekte stort nie na lyn soektogte ineen nie, terwyl waarnemer onafhanklikheid behou word. Waarnemer onafhanklikheid word egter slegs in ’n stogastiese sin bevredig, m.a.w. die gemiddelde doelwit funksie waarde is onafhanklik van ’n koördinaatstelsel, gesien oor ’n groot aantal verlope.

   Objektiwiteit en effektiwiteit van die drie formulerings word gekwantifiseer deur gebruik te maak van ’n gewilde unimodale en multimodale toets stel, waarvan die meerderheid multimodale funksies skeibaar is. Nietemin word die doelwit funksies geëvalueer in beide die ongeroteerde, skeibare, verwysingsraamwerk en ’n lukraak geroteerde verwysingsraamwerk.

4. Laastens word ’n praktiese ingenieurs optimeringsprobleem bestudeer. Die PSOA word aangewend om die optimale geometrie van ’n kantelbalk te vind. Die doelfunksie wat geminimeer word is die vertikale verplasing by die eindpunt van die kantelbalk. Die doelfunksies word bereken deur gebruik te maak van die eindige element metode. Die mase wat benodig word vir die linieêr elastiese eindige element analises word gegenereer deur van ’n ongestruktureerde hermasings-strategie gebruik te maak. Die hermasings-strategie is gebaseer op ’n vakwerk struktuur analoog.

Acknowledgments

Some have the pleasure of seeing distinguished men in their lifetime, some have the privilege of meeting them, but I had the honor to undertake a journey alongside them.
- D.N. Wilke
University of Pretoria, 2004.

I would like to dedicate this thesis to
my father and my mother

This section attempts to capture a mere snapshot of my thoughts and experiences over the last two years. This section is written in the form of an informal short story. To anyone who may feel affronted in any way by the writing style or content of this section, I offer my sincere apologies.

Conceit of the Absurd - A sailor's story

This story begins in the year 2002, as I reached my last year of formal enrollment as a sailor. My fellow sailor friends and I used to meet up at a local tap and talk about the adventures to come, and of possible treasures of gold.

As the year progressed I was adamant that I would set sail the following year for either Europe or the middle East, as I considered some lucrative offers of possible gold treasures and adventure in these distant and uncharted lands.

As the year came to an end, the sea tides turned.

I met Captain Groenwold and in the end I decided to trade adventures of Europe, and the middle East for adventures of another kind. The adventures that are about the journey and not the destination.

As the year 2003 dawned, Captain Groenwold and I embarked on an adventure. The adventure started calmly by sailing out of the harbor on a brig, affectionately referred to by the sailors as SORG. At the boat's command stood Captain Groenwold and on the deck stood I, a proud sailor.

We left the harbor and sailed into the open seas, that laid open for traveling and exploration. This was my first time out on the ocean. As the harbor disappeared on the distant horizon I did not quite know where I was, but I knew I was out there, somewhere. Months past as we encountered some light breezes and stormy clouds here and there.

Then one day Captain Groenwold got word that he was needed in a distant land. We anchored the vessel at the nearest harbor and on board came co-Captain Kok to take command of the brig. Our only contact with Captain Groenwold being the infamous message bottle system.

The vessel sailed further under co-Captain Kok's command, over the calm seas and oceans. Every now and then we would spot land here and there, for me the excitement grew as the frequency of sightings of land increased. Some days the breezes became stronger than others as we set forth towards the land. Not before long by the middle of 2004, the adventure turned into an epic of Gulliver's travels as we finally reached land. The epic started by us nearly shipwrecking the preceding day after I misread the map. Fortunately, co-Captain Kok instantly realized my mistake and recovered our situation. Nevertheless we anchored and set forth our exploration.

For months we set sail and anchored to explore various places, each an exotic place in it's own right. On one of the stops we picked up Captain Groenwold after his return from the distant lands. The adventure continued, I was constantly fascinated by each place, and not before long it dawned. What seemed to be vast and distant lands was actually one big island. We attempted to map what we could of this beautiful island but as our supplies where running low, we had to set forth the journey back home.

By the beginning of 2005, the adventure came to an end as our ship sailed into the harbor. As I disembarked our ship and touched home soil I realized "*I embarked on the right adventure*".

Formally I would like to express my sincere gratitude towards the following persons:

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- My father and my mother to whom I dedicate this thesis, I thank you for your continuous and steadfast support throughout my life.

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