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LINE OF SIGHT STABILIZATION OF AN OPTICAL INSTRUMENT USING GAINED MAGNETOSTRICTIVE ACTUATORS

by

CHRISTIAAN RUDOLF BESTER

Submitted in partial fulfillment of the requirements for the degree

PHILOSOPHIAE DOCTOR

in the Department of Mechanical and Aeronautical Engineering,

Faculty of Engineering,
Built Environment
and Information Technology

UNIVERSITY OF PRETORIA

FEBRUARY 2003

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Abstract

LINE OF SIGHT STABILIZATION OF AN OPTICAL INSTRUMENT USING GAINED MAGNETOSTRICTIVE ACTUATORS

Author: CR Bester
Supervisor: Prof MC van Schoor
Department: Mechanical and Aeronautical Engineering
Degree for which this thesis is submitted: Philosophiae Doctor

Line-of-sight stabilization of an optical instrument using magnetostrictive actuators is described in this study. Various stabilization methods, i.e. gyroscopic, hydraulic, piezoelectric, electrodynamic and magnetostrictive methods, are compared and magnetostrictive stabilization is selected for its relatively large stroke length, low input voltage and wide frequency bandwidth.

The system makes use of two magnetostrictive actuators, one at each end of the optical instrument, mounted between the moving base and instrument. Each actuator is equipped with cylindrical rods of Terfenol-D, a highly magnetostrictive material. Field coils are wound around the rods to produce a strain in the rods, thereby exciting angular motion of the instrument. Actuator stroke length is enhanced by means of a hingeless gain mechanism, rod prestressing and field biasing.

Dynamic characteristics of the system are modelled to facilitate actuator, coil and control system design. A linear, single-degree-of-freedom actuator model, in state-space and transfer function forms, is derived and coupled to a distributed model of the optical instrument, using the Rayleigh-Ritz method. Transfer functions between actuator coil voltages and instrument angular acceleration are derived. Normal mode shapes, natural frequencies and damping factors are predicted.

Design concepts for bias field, prestress, actuator gain and optical instrument support structure, are discussed and the most suitable concepts are selected. The required actuator gain, rod length and diameter, prestress spring stiffness, coil resistance and inductance are calculated. System components are designed in detail and safety of the design is checked.

The actuators are characterized quasi-statically to determine the saturation strain, linear range of operation and DC bias field. The system is dynamically characterized to obtain transfer functions between the coil voltage and instrument angular acceleration. The test setups are described and limitations of the setups are discussed. Test results are processed and discussed. A comparison with the modelled results shows that the model is highly inaccurate. Reasons for inaccuracies are given and updating of the model is motivated.

An updated model is obtained from the experimental results. The model is divided into electrical and mechanical subsystem models. The SDOF actuator models are replaced with 2DOF models (one for each actuator) and coupled to the instrument and base models, using substructure synthesis. The electrical and mechanical subsystem models are subsequently coupled. It is shown that the updated system model is considerably more accurate than the original model.

A linear, suboptimal, disturbance feedforward plus output feedback controller, with output integral feedback, is designed, implemented and tested. An H_2 optimal controller is designed and modified to improve robustness. The controller model is coupled to that of a suboptimal observer. An output integral feedback loop is added to further improve robustness. The controller is implemented in digital filter form. The test apparatus and procedure are described. Test results are processed and discussed. It is shown that the LOS stabilization system achieves 80% of the required isolation, over a frequency bandwidth of 0 Hz to 100 Hz.

A summary of the work done, conclusions that can be drawn from the results, problems encountered and recommendations for future work, are given.

Keywords: Line-of-sight, stabilization, active vibration isolation, magnetostriction, Terfenol-D, actuators, gain mechanisms

Uittreksel

SIGLYNSTABILISASIE VAN 'N OPTIESE INSTRUMENT MET BEHULP VAN MAGNETOSTRIKSIE-AKTUEERDERS MET WINSMEGANISMES

Outeur: CR Bester
Promotor: Prof MC van Schoor
Departement: Meganiese en Lugvaartkundige Ingenieurswese
Graad waarvoor hierdie proefskrif ingedien is: Philosophiae Doctor

Siglynstabilisasie van 'n optiese instrument, m.b.v. magnetostriksie-aktueerders, word in hierdie studie beskryf. Verskeie stabilisasiemetodes, nl. giroskopiese-, hidrouliese-, piezoëlektriese-, elektrodinamiese- en magnetostriksiemetodes, word vergelyk en magnetostriksiestabilisasie word verkies omdat dit 'n relatiewe lang slaglengte, lae insetspanning and wye frekwensieband het.

Die stelsel maak van twee magnetostriksie-aktueerders, een by elke punt van die optiese instrument, wat tussen die bewegende basis en instrument gemonteer is, gebruik. Elke aktueerder bevat silindriese stawe van 'n hoogs magnetostriktiewe materiaal, bekend as Terfenol-D. Klosse is rondom die stawe gewen om 'n veld in die stawe te bewerkstellig, wat 'n vervorming van die stawe teweegbring. Sodoende word hoekbeweging van die optiese instrument opgewek. Aktueerderslaglengte word vergroot deur 'n skarnierlose winsmeganisme, meganiese voorspanning van die stawe en veldvoorspanning.

Die dinamiese karakteristieke van die stelsel word gemodelleer om aktueerder-, klos- en beheerstelselontwerp te vergemaklik. 'n Lineêre, enkelvryheidgraadmodel van die aktueerder word in toestand- en oordragfunksievorm afgelei en m.b.v. die Rayleigh-Ritzmetode aan 'n verspreide massamodel van die optiese instrument gekoppel. Oordragsfunksies tussen die aktueerderklosspannings en hoekversnelling van die optiese instrument word afgelei. Normaalmodusvorme, natuurlike frekwensies en dempingsfaktore word voorspel.

Ontwerpkonsepte vir veldvoorspanning, meganiese voorspanning, aktueerderwinsmeganisme en ondersteuningstruktuur vir die optiese instrument, word bespreek en die mees geskikte konsepte word gekies. Die benodigde aktueerderwins, staaflengte en –diameter, voorspanningveerstyfheid, klosweerstand en –induktansie, word bereken. Detailontwerpe van die stelselkomponente word gedoen en ontwerpveiligheid word ondersoek.

Die aktueerders word kwasi-staties gekarakteriseer om die versadigingspanning, lineêre gebied en GS voorspanningsveld te bepaal. Die stelsel word dinamies gekarakteriseer om oordragsfunksies tussen die klosspanning en hoekversnelling van die instrument te verkry. Die toetsopstellings en beperkinge daarvan word bespreek. Toetsresultate word verwerk en bespreek. 'n Vergelyking tussen die gemodelleerde en gemete resultate toon dat die model hoogs onakkuraat is. Redes vir die onakkuraathede word gegee en modelopdatering word gemotiveer.

'n Opgedateerde model word uit die eksperimentele resultate verkry. Die model word in elektriese en meganiese substelselmodelle opgedeel. Die enkelvryheidsgraadmodelle van die aktueerders word met tweevryheidsgraadmodelle (een vir elke aktueerder) vervang en m.b.v.

substruktuursintese aan die instrument- en basismodelle gekoppel. Die elektriese en meganiese substelselmodelle word daarna gekoppel. Daar word aangetoon dat die opgedateerde model beduidend akkurater as die oorspronklike model is.

'n Lineêre, suboptimale, versteuringvorentoevoer-, plus uitsetterugvoerbeheerder, met uitsetintegraalterugvoer, word ontwerp, geïmplementeer en getoets. 'n H_2-optimale beheerder word ontwerp en gewysig om robuustheid te verbeter. Die beheerdermodel word aan 'n suboptimale waarnemermodel gekoppel. 'n Uitsetintegraalterugvoerlus word bygevoeg om robuustheid verder te verbeter. Die beheerder word as 'n digitale filter geïmplementeer. Die toetsapparaat en –prosedure word beskryf. Toetsresultate word verwerk en bespreek. Daar word aangetoon dat die siglynstabilisasiestelsel 80% van die benodigde isolasie oor 'n frekwensieband van 0 Hz tot 100 Hz lewer.

'n Opsomming van die werk wat gedoen is, gevolgtrekkings wat uit die resultate gemaak kan word, probleme ondervind en aanbevelings t.o.v. toekomstige werk, word gegee.

Sleutelwoorde: Siglyn, stabilisasie, aktiewe vibrasie-isolsie, magnetostriksie, Terfenol-D, aktueerders, winsmeganismes

Acknowledgements

To everybody who helped me through this study. I can compile a list that is much longer than the following, but no shorter.

Firstly, to my Lord and Saviour, the Holy, Almighty God, the Alpha and the Omega, Creator of everything that is, was and will be, for His support throughout my entire life, and especially during this extremely time-consuming and costly job.

My study leader, prof. Tienie van Schoor, for his profound knowledge of Structural Dynamics, and for keeping me on the right track when I was about to derail into the torture chambers of nonlinear, stochastic, multiple degree of freedom vibrations.

Dr Tupper Hyde, who spent his holiday time during his visit to South Africa, to give me useful tips on controller design.

Mr Wynand Avenant and prof. Jasper Steyn, for their financial support.

The friendly staff of Etrema in Ames, Iowa, for their hospitality during my visit there, as well as to the students in the Department of Mechanical Engineering at ISU, Dave, Jon and Toby.

Prof. Stephan Heyns, Gerrit Visser, Danie Smit and Herman Booysen, for lending me their precious laboratory equipment.

Jan Brandt, who made everything I asked him to, and Willie Vos, who drew everything I asked him to.

My friends Andre Ernst and Jacques Cilliers, who tried to train me as an electronics engineer. Thanks for helping me find the correct hardware and software that enabled me to complete the experimental work.

Prof. Jan Visser and prof. Albert Groenwold, who gave me computers to do my calculations and typing on.

My parents, for their relentless prayers, support and understanding, especially when I wanted to take a hammer and stop all vibrations in the laboratory permanently.

Elmi van der Dussen, for her support, even when I was extremely grumpy.

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List of symbols

a	coefficient
	polynomial coefficient
	length of minor axis of ellipse
A	amplitude
	area
	state-space coefficient matrix
b	coefficient
	length of major axis of ellipse
B	magnetic flux density
	state-space driving matrix
c	damping coefficient
C	coefficient vector
	coupling matrix
	damping matrix
	parameter vector
	programming language
	state-space output matrix
	spring index
C^*	normal mode damping matrix
d	diameter
	disturbance
	wire diameter of a coil spring
d^H	strain constant
d^σ	piezomagnetic cross-coupling constant
D	diameter
	differential (element of a controller)
	distance
	disturbance amplitude
	pitch circle diameter
	state-space transmission matrix
e	error
	error signal
E	error amplitude
	expected value
	global error
	Young's modulus
EI	flexural rigidity
f	frequency
	function
f_n	natural frequency
F	force
	force amplitude vector
g	function
	gravity acceleration (9,81 m/s^2)
G	feedforward transfer function
	gain
	transfer function

List of symbols (continued)

h	function
	height
H	constant height
	magnetic field strength
	output feedback gain
	discrete state-space output matrix
i	imaginary number $\left(\sqrt{-1}\right)$
I	coil current
	integral (element of a controller)
	moment of inertia, second moment of area
	unit matrix
j	imaginary number $\left(\sqrt{-1}\right)$
J	cost function
	discrete state-space transmission matrix
k	constant
	dimensionless constant
	number
	spring stiffness
	stiffness matrix
K	calibration factor
	constant
	correction factor
	coupled modal stiffness matrix
	gain
	linear state feedback gain matrix, linear state feedback gain vector
	stiffness matrix
$K*$	normal mode stiffness matrix
l	length
	number
	optical instrument length
L	inductance
	observer driving matrix
L_f	free inductance
L_0	clamped inductance
m	mass
	mass matrix
	number
M	dimensionless constant
	mass matrix
	modal mass matrix
	number
$M*$	normal mode mass matrix
n	noise
	number
N	cross-coupling weight
	nonlinearity (in describing function method)
	number

List of symbols (continued)

p	frequency ratio
	load vector
	numerator polynomial coefficient
	pitch
	pole (of a transfer function)
	polynomial coefficient
P	distributed load
	numerator polynomial
	perimeter
	power
	proportional (element of a controller)
	Riccati matrix
	spectral density
q	denominator polynomial coefficient
	normal mode displacement
	number
	polynomial coefficient
Q	denominator polynomial
	modal excitation force
	weight
r	reference input, reference signal
	root, zero (of a transfer function)
R	control weight, input weight
	reference amplitude
	resistance
s	Laplace-domain differential operator
	polynomial coefficient
S	constant
	Riccati matrix
	sensitivity function
t	thickness
	time
T	complimentary sensitivity function
	constant
	couple
	maximum time
	moment
	period
	torque
u	control signal
	input
U	control amplitude
	input amplitude
	matrix of eigenvectors
v	output noise signal

List of symbols (continued)

V	coil voltage
	Laplace-domain amplitude of a function
	voltage
	volume
V_i	input voltage
V_o	output voltage
w	distributed displacement
	process noise signal
	weight
W	distributed displacement amplitude
x	deflection
	displacement
	position
	state variable
	state vector
$\dot{x}$	state derivative vector
$\hat{x}$	estimated state vector
X	state amplitude vector
y	output
	displacement
	stroke length
$\hat{y}$	modelled output
	estimated output
$\ddot{y}$	output acceleration
Y	output amplitude
$\ddot{Y}$	acceleration amplitude
z	zero of a transfer function
	discrete domain differential operator
z^{-1}	discrete domain delay element
Z	impedance

Abbreviations

AC	alternating current
A/D	analogue-to-digital
AR	auto regressive
ARE	algebraic Riccati equation
ARIMA	auto regressive integrated moving average
ARMA	auto regressive moving average
ARMAX	auto regressive integrated moving average with exogenous inputs
ARX	auto regressive with exogenous inputs
AWG	American wire gauge
bei	imaginary part of Kelvin function of { }
ber	real part of Kelvin function of { }
BW	(frequency) bandwidth
cf	coupling factor

List of symbols (continued)

const	constant
CGS	classical Gram-Schmidt
CPU	central processing unit
CSD	cross-spectral density
dB	decibel
D/A	digital-to-analogue
DC	direct current
DE	differential equation
DF	describing function
DFT	Discrete Fourier Transform
DMA	direct memory access
DOF	degree of freedom
DSP	digital signal processing
e.g.	"exempli gratia", for example
foh	first order hold
FD	finite difference method
FE	finite element
FEM	finite element method
FFT	Fast Fourier Transform
FORSE	Frequency domain Observability Range Space Extraction
FRF	frequency response function
HP	high-pass
i.e.	"id est", that is
Im	imaginary part of { }
ITD	Ibrahim time domain
LOS	line of sight
LP	low-pass
LQ	linear quadratic
LQE	linear quadratic estimator
LQG	linear quadratic gaussian
LQR	linear quadratic regulator
LS	least squares
LTF	loop transfer function
LTI	linear time invariant
LTR	loop transfer recovery
LVDT	linear variable displacement transducer
MA	moving average
MDFT	multi-dimensional Fourier Transform
MDOF	multiple degree of freedom
MGS	modified Gram-Schmidt
MIMO	multiple input multiple output
MS	mean-square
NARMAX	nonlinear autoregressive moving average with exogenous inputs
NARX	nonlinear autoregressive with exogenous inputs
ORSE	Observability Range Space Extraction
pf	packing factor
p-p	peak-to-peak
P	proportional (controller)

List of symbols (continued)

PC	personal computer
PCD	pitch circle diameter
PD	proportional-differential (controller)
PDE	partial differential equation
PI	performance index
	proportional-integral (controller)
PID	proportional-integral-differential (controller)
PSD	power spectral density
RAM	random access memory
Re	real part of { }
R-K	Runge-Kutta
RLS	recursive least squares
RMS	root-mean-square
SDOF	single degree of freedom
SISO	single input single output
sup	supremum
SVD	singular value decomposition
TF	transfer function
TR	transmissibility
zoh	zero order hold

Subscripts

a	active (e.g. in active number of coils in a spring N_a)
	actuator (e.g. in actuator output y_a)
A	active (e.g. in active system transmissibility TR_A)
	amplitude (e.g. in field amplitude H_A)
asymm	asymmetric (e.g. in asymmetric assumed mode vector)
b	base (e.g. in base excitation force F_b)
B	bias (e.g. in bias field strength H_B, bias force F_B)
	bulk modulus (e.g. in fluid spring stiffness due to compressibilty K_B)
c	calibration (e.g. in disturbance accelerometer calibration factor K_{cd})
	coil
	controller (e.g. in controller transfer function G_c)
	corrected (e.g. in corrected spring wire shear stress τ_c)
ck	index (e.g. *k*-th coil transfer function numerator coefficient)
cl	index (e.g. *l*-th coil transfer function denominator coefficient)
	closed-loop (e.g. in closed-loop coefficient matrix A_{cl})
cr	critical (e.g. in critical buckling force F_{cr}, critical eddy current frequency f_{cr})
crisscross	indicating relative location between actuator and sensor
d	disturbance (e.g. in driving matrix B_d for disturbance *d*)
dehyst	dehysterized
distr	distributed (e.g. in distributed density ρ_{distr})
D	damping (e.g. in damping force F_D)
	differential (e.g. in differential control element)
DC	direct current (e.g. in TR_{DC}, i.e. transmissibility at 0 Hz)

List of symbols (continued)

e	ellipse (e.g. in perimeter of ellipse P_e)
	external (e.g. in external diameter D_e)
	inactive (e.g. in inactive number of coils in a spring N_e)
eff	effective (e.g. effective damping in a coupled electro-mechanical system)
eq	equivalent (e.g. in equivalent density ρ_{eq})
f	free (eg. in free inductance L_f)
	feedforward (e.g. in feedforward gain G_f)
filter	filter (e.g. in filter transfer function G_{filter})
fluid	fluid (e.g. in fluid stiffness k_{fluid})
g	gap (e.g. flux density in the air gap of a magnetic path)
G	gain (e.g. in gain mechanism damping ζ_G)
	gyration (e.g. in radius of gyration r_G)
h	feedback (e.g. in feedback gain G_h)
hyst	hysteresis
i	index (e.g. *i*-th element or mode)
	inner (e.g. in inner coil diameter d_i)
	input (e.g. in input voltage V_i)
I	current (e.g. in force per unit current F_I)
	integral (e.g. in integral feedback control gain H_I)
ii	index (e.g. *ii*-th diagonal element of matrix)
ij	index (e.g. *ij*-th element of matrix)
j	index (e.g. *j*-th element)
k	index (e.g. *k*-th element)
l	index (e.g. *l*-th element)
	number (e.g. in number of coil layers n_l)
loading	indicating driving direction (e.g. loading strain $\varepsilon_{loading}$)
m	magnetic (e.g. in flux density in magnet B_m)
	mean
	mechanical (e.g. in mechanical subsystem transfer function G_m)
M	maximum index, i.e. polynomial order (e.g. in b_M)
max	maximum (e.g. in maximum field $H_{\max}$)
min	minimum (e.g. in minimum field amplitude $H_{\min}$)
n	index (e.g. *n*-th element)
	natural (e.g. in natural frequency f_n)
	normal mode (e.g. in generalized normal mode displacement q_n)
	number
nc	natural, compressible (e.g. in natural frequency due to compressibility)
ni	natural, incompressible (e.g. in natural frequency due to incompressibility)
N	maximum index , i.e. polynomial order (e.g. in a_N)
o	outer (e.g. in outer coil diameter d_o)
	output (e.g. in output voltage V_o)
p	piston (e.g. in piston cross-sectional area (A_p)
	plant (e.g. in plant transfer function G_p)
	power (e.g. in time where maximum coil power occurs t_p)
P	passive (e.g. in passive system transmissibility TR_P)
parallel	indicating relative location between actuator and sensor

List of symbols (continued)

r	index (e.g. in *r*-th error coefficient e_r)
	reduced-order (e.g. in reduced-order stiffness matrix K_r)
	relative (e.g. in relative displacement y_r)
	rod (e.g. in rod force F_r)
ref	reference (e.g. in volume per unit volume V_{ref})
rigid	rigid body (e.g. in rigid body assumed mode vector)
rotation	indicating degree of freedom (e.g. in rotational transfer function $G_{rotation}$)
row, rows	index (e.g. of a row or rows in a column vector or matrix)
R	resonance (e.g. in resonance magnitude of a transfer function TF_R)
RMS	root-mean-square (e.g. in root-mean-square of output y_{RMS})
s	sample (e.g. in sample time T_s)
	shear (e.g. in spring material shear modulus G_s)
	structure (e.g. in structure assumed mode vector ψ_s)
	system (e.g. in system state vector x_s)
spr	spring (e.g. in spring stiffness k_{spr})
ss	steady-state (e.g. in steady-state Riccati matrix S_{ss})
symm	symmetric (e.g. in symmetric assumed mode vector)
S	static (e.g. in static transfer function magnitude TF_S)
t	total (e.g. in total number of coils of a spring N_t)
translation	indicating degree of freedom (e.g. in translational transfer function $G_{translation}$)
T	Terfenol-D
u	input (e.g. in driving matrix B_u for controllable input *u*)
unloading	indicating driving direction (e.g. unloading strain $\varepsilon_{unloading}$)
V	voltage (e.g. in input power spectral density P_V)
v	voltage (e.g. in time where maximum voltage occurs t_v)
w	wire (e.g. in coil wire thickness)
wc	wire coating (e.g. in coated wire thickness t_{wc})
W	Wahl (e.g. in Wahl shear stress correction factor K_W)
x	state (e.g. in state weight Q_x)
y	output (e.g. in output weight Q_y)
0 (zero)	clamped (e.g. in clamped inductance L_0, clamped permeability μ_0)
	reference value (e.g. in undeformed height of gain mechanism h_0)
	steady-state value
	zero-th value
1,2….	index (e.g. in gains K_1, K_2,….)

Superscripts

i	exponent
iv	fourth partial derivative with respect to spatial coordinate
j	exponent
k	exponent
l	exponent
*	normal (e.g. in normal, or uncoupled modal stiffness matrix K^*)
	optimal (e.g. in optimal output y^*)
T	transpose (of a matrix or vector)
-1	inverse (e.g. of matrix, parameter or variable)

List of symbols (continued)

Greek

α	angle dimensionless constant real part of eigenvalue
β	dimensionless constant
χ	integration variable eddy current loss factor
δ	Dirac delta function
Δ	change in parameter or variable
ε	strain
E	strain amplitude
ϕ	diameter magnetic flux normal mode shape phase angle
Φ	normal mode shape vector discrete state-space coefficient matrix
γ	attenuation factor dimensionless hysteresis parameter
Γ	discrete state-space driving matrix
η	integration variable loss factor
κ	constant statistical degrees of freedom
μ	micro permeability
μ^{σ}	free permeability
ν	Poissons ratio
φ	phase angle
π	constant (= 3,14159265358979......)
ρ	density resistivity
ρA	mass per unit length
σ	stress
$\bar{\sigma}[\ ]$	largest singular value of [ ]
Σ	sum
τ	period shear stress
θ	angle angular displacement dummy variable phase angle
$\ddot{\theta}$	angular acceleration
ω	angular frequency
ω_n	angular natural frequency

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List of symbols (continued)

Ω	constant frequency imaginary part of eigenvalue
ψ	angle assumed mode shape function
$\boldsymbol{\psi}$	assumed mode shape vector
ζ	dimensionless damping factor
Z	dimensionless modal damping matrix

Other

∞	infinity
H_2	type of optimal controller
H_∞	type of optimal controller
$\dot{}$	(dot) derivative with respect to time $\frac{d}{dt}(\ )$
$\ddot{}$	(double dot) 2nd order derivative with respect to time
ʹ	(prime) partial derivative with respect to spatial coordinate
ʹʹ	(double prime) 2nd order partial derivative with respect to spatial coordinate
¯	(overbar) mean or average value error (e.g. in estimated state vector $\bar{x}$)
^	(hat): approximate value estimated value (of state or output)
$\int$	integral
∂	partial derivative
$\lvert\ \rvert$	absolute value determinant magnitude
$\lvert[\ ]\rvert$	determinant of matrix
( )	function
{ }	vector
[ ]	matrix
2DOF	two degree of freedom