ANNEXURE FINITE ELEMENT METHOD AND ALGORITHM IN MSC.Marc

MSC.Marc is a general-purpose finite element (FE) program for advanced engineering analysis, which can be used to perform a wide variety of structural, fluid and coupled analyses using the finite element method (FEM) (MSC.Marc 2005).

The purpose of this annexure is to review the FEM for a better understanding of how MSC.Marc works.

A.1 Finite element method

The FE method basically has the following six steps. The success of any FE program depends in part on how the program implements these steps.

Step 1: Choose Shape Functions: The FEM expresses the displacement field, $\underline{u}(\underline{x})$, in terms of the nodal point displacement, $\underline{a}^e$, by using the shape functions, $\underline{N}(\underline{x})$, over the domain of the element Ω^e, as:

$$\underline{u}(\underline{x}) = \underline{N}(\underline{x})\ \underline{a}^e \tag{A.1}$$

Step 2: Establish the Material Relationship: The FEM expresses the dependent fields, such as the strain and stress, in terms of the nodal point displacement as:

$$\underline{\varepsilon}(\underline{x}) = \underline{L}\left[\underline{u}(\underline{x})\right] = \underline{B}\,\underline{a}^e \ ; \quad \underline{\sigma} = \underline{\sigma}(\underline{\varepsilon}) = \underline{D}\,\underline{\varepsilon}(\underline{x}) = \underline{D}\,\underline{B}\,\underline{a}^e \tag{A.2}$$

where

$\underline{L}$	Differential operator
$\underline{B} = \underline{L}\underline{N}(\underline{x})$	Strain – displacement operator
$\underline{D}$	Constitutive matrix

Step 3: Element Matrices: The FEM equilibrates each element with its environment, which can be expressed as:

$$\underline{K}^e \, \underline{a}^e + \underline{f}^e = 0 \tag{A.3}$$

where

$\underline{K}^e = \int_{\Omega^e} \underline{B}^T \, \underline{D} \, \underline{B} \, dV$ Represents physical properties such as stiffness

$-\underline{f}^e = \int_{\Omega^e} \underline{N}(\underline{x})^T \underline{b} \, dV + \int_{\Gamma^e} \underline{N}(\underline{x})^T \underline{t} \, dS + \underline{F}$ Represents loads experienced by the element.

These loads may be: body loads $\underline{b}$, such as weight or internal heat generation in volume Ω^e; surface loads $\underline{t}$, such as pressure on surface Γ^e; or concentrated loads $\underline{F}$.

Step 4: Assembly: The FEM assembles all the elements to form a complete structure in such a manner as to equilibrate the structure with its environment.

$$\underline{K} \, \underline{a} + \underline{f} = 0 \tag{A.4}$$

where

$\underline{K} = \sum_e \underline{K}^e$ Overall structural stiffness matrix

$\underline{f} = \sum_e \underline{f}^e$ Overall structural load vector

$\underline{a}$ Overall nodal unknowns (such as displacement) vector

Step 5: Solve the Equations: The FEM specifies the boundary conditions, namely the nodal point values on the boundary, and the system equations are partitioned as:

$$\begin{bmatrix} \underline{K}_{uu} & \underline{K}_{us} \\ \underline{K}_{su} & \underline{K}_{ss} \end{bmatrix} \begin{bmatrix} \underline{a}_u \\ \underline{a}_s \end{bmatrix} = - \begin{bmatrix} \underline{f}_a \\ \underline{f}_r \end{bmatrix} \tag{A.5}$$

where: $\underline{a}_u$ are the unknown nodal values; $\underline{a}_s$ are the specified nodal values; $\underline{f}_a$ are the applied nodal loads; and $\underline{f}_r$ are the nodal point reactions. Hence the solution becomes:

$$\underline{a}_u = - \underline{K}_{uu}^{-1} (\underline{f}_a + \underline{K}_{us} \underline{a}_s) \tag{A.6}$$

$$\underline{f}_r = - (\underline{K}_{su} \underline{a}_u + \underline{K}_{ss} \underline{a}_s) \tag{A.7}$$

Step 6: Recover: The FEM recovers the stresses by substituting the unknown nodal values found in Step 5 back into Step 2 to find the dependent fields, such as strain and stress.

A.2 Non-linear FE analysis and iteration solution

For the solution step, the following equation must be solved:

$$[K]\{a\} = \{F\} \quad \text{or} \quad \underline{I} - \underline{F} = 0 \tag{A.8}$$

where

$[K]$ Overall structural stiffness matrix

$\{a\}$ Overall nodal unknowns vector

$\{F\}$ Overall structural load vector.

$$\underline{I} = [K]\{a\}$$

$$\underline{F} = \{F\}$$

For non-linear equations, both the stiffness and external forces may be functions of the nodal displacements:

$$\underline{I}(\underline{a}) - \underline{F}(\underline{a}) = 0 \quad \text{(A.9)}$$

To solve a non-linear set of equations, MSC.Marc generally applies the following two solution methods:

a. Newton-Raphson (NR) method

This is an iterative method. The structural stiffness matrix is constantly updated at each iteration. Given a general non-linear equation $f(a) = 0$, and a known point a_i, a correction Δa_{i+1} can be calculated as follows:

$$\Delta a_{i+1} = \frac{f(a_i)}{f'(a_i)} \quad \text{(A.10)}$$

with

$$a_{i+1} = a_i + \Delta a_{i+1} \quad \text{(A.11)}$$

By defining the tangent stiffness:

$$f'(\underline{a}_i) \equiv \underline{K}_i^T(\underline{a}_i) = \frac{\partial}{\partial \underline{u}}(\underline{I}(\underline{a}_i) - \underline{F}(\underline{a}_i)) \quad \text{(A.12)}$$

and the residual:

$$f(\underline{a}_i) \equiv R(\underline{a}_i) = \underline{I}(\underline{a}_i) - \underline{F}(\underline{a}_i) \quad \text{(A.13)}$$

the Newton-Raphson method (equation A.10) can be rewritten in a more familiar form:

$$\underline{K}_i^T(\underline{a}_i)\Delta a_{i+1} = R(\underline{a}_i) \quad \text{(A.14)}$$

Gauss elimination techniques can be used to solve this set of equations for Δa_{i+1} .

With each iteration, the residual should decrease. If it does, the method converges to the correct solution.

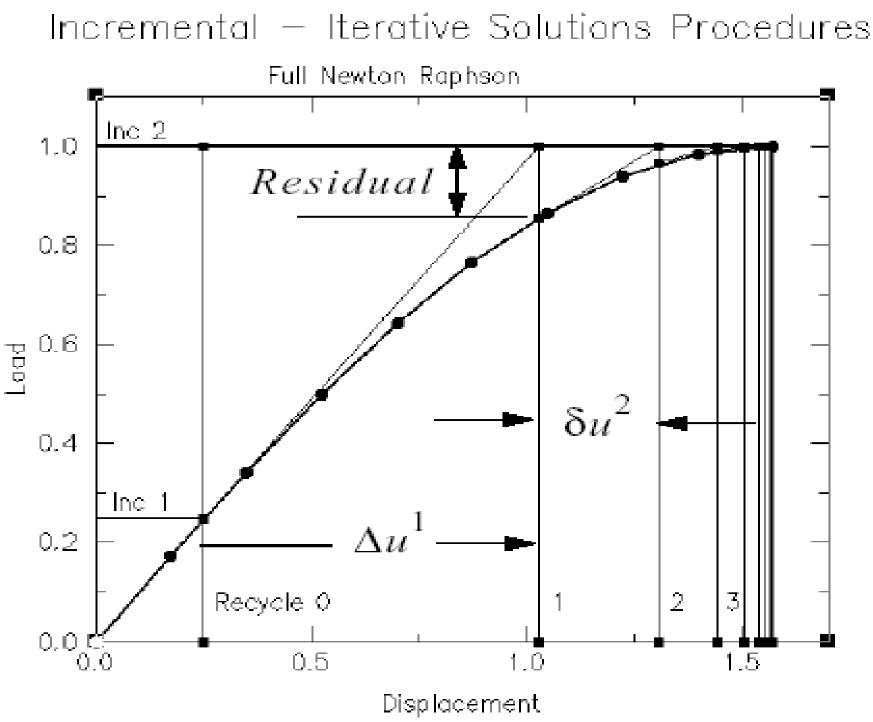

Figure A.1 - Full Newton-Raphson method (MSC.Marc 2005)

b. Modified Newton-Raphson (MNR) method

In this method, constant stiffness is applied within each load step and only updated at the beginning of the next load increment. There may be slow convergence behaviour.

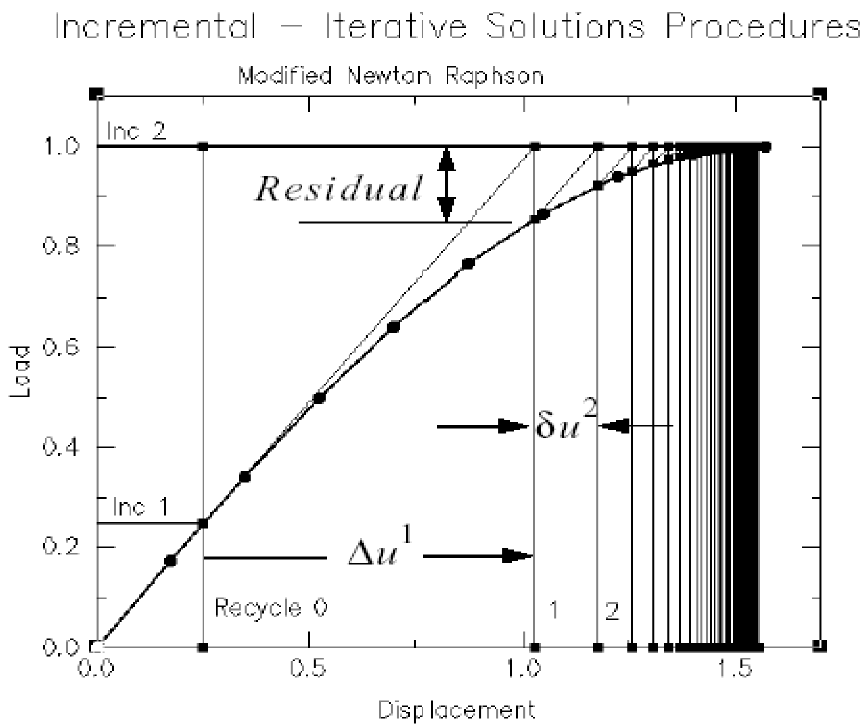

Figure A.2 - Modified Newton-Raphson method (MSC.Marc 2005)

A.3 Convergence checking

The iterative procedure is terminated when the convergence ratio is less than a criterion of tolerance.

a. Residual checking: Residuals and reactions

Relative: $$\frac{\|F_{residual}\|_{\max}}{\|F_{reaction}\|_{\max}} < Tol \qquad \text{(A.15)}$$

Absolute: $$\|F_{residual}\|_{\max} < Tol \qquad \text{(A.16)}$$

where

$\|F_{residual}\|_{\max}$ = maximum residual force

$\|F_{reaction}\|_{\max}$ = maximum reaction force

Tol = tolerance (default $Tol = 0.1$)

The residuals are the difference between the external forces and the internal forces at each node, namely:

$$\underline{F}_{residual} = \underline{F}_{external} - \int_{\Omega^e} \underline{B}^T \underline{D}\, \underline{B}\, dV \qquad \text{(A.17)}$$

The nodal reactions are from the system equations, namely equation (A.7):

$$\underline{F}_{reaction} = \underline{f}_r = -(\underline{K}_{su}\, \underline{a}_u + \underline{K}_{ss}\, \underline{a}_s) \qquad \text{(A.18)}$$

The maximum residuals and reactions occur at different degrees of freedom (dof) that have the largest magnitude, namely:

$$\|F_{residual}\|_{\max} = \text{Max}(F^i_{residual})\ ;\ \ \text{i} = 1, \text{maxdof} \qquad \text{(A.19)}$$

and

$$\|F_{reaction}\|_{\max} = \text{Max}(F^i_{reaction})\ ;\ \ \text{i} = 1, \text{maxdof} \qquad \text{(A.20)}$$

b. Displacement checking: Maximum displacement change and maximum displacement increment

Relative: $$\frac{\|\delta u\|_{\max}}{\|du\|_{\max}} = \frac{\|\Delta u^{i+1} - \Delta u^i\|_{\max}}{\|\Delta u^i\|_{\max}} < Tol \qquad \text{(A.21)}$$

Absolute: $$\|\delta u\|_{\max} < Tol \qquad \text{(A.22)}$$

where

$\|\delta u\|_{\max}$ = maximum displacement change

$\|du\|_{\max}$ = maximum displacement increment

Tol = tolerance (default $Tol = 0.1$)

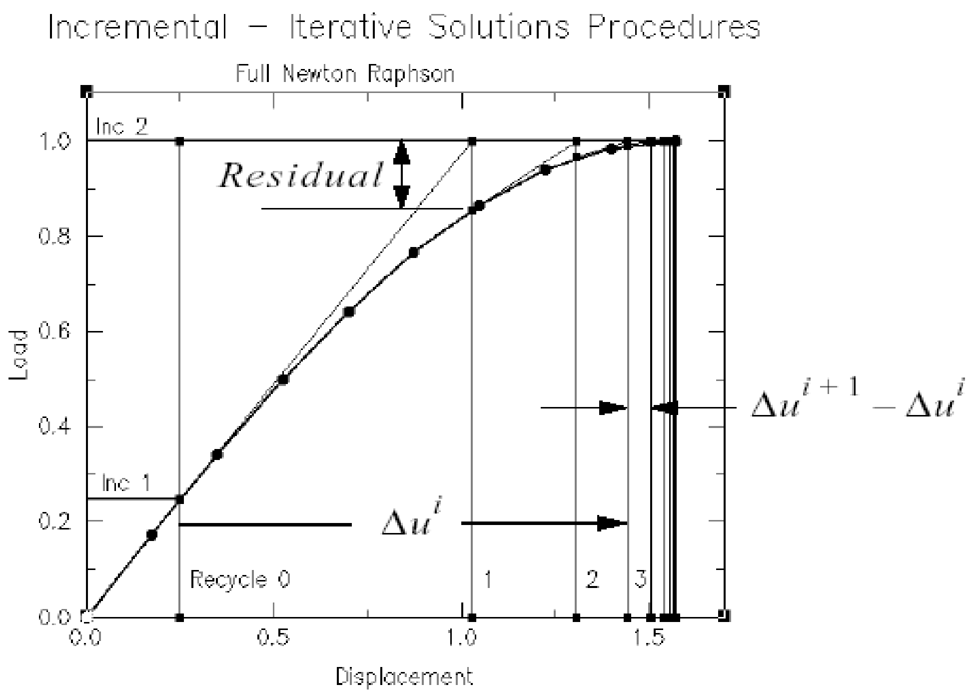

Figure A.3 - Convergence checking (MSC.Marc 2005)

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