CHAPTER I - INTRODUCTION

1.1 Background and overview

Concrete, as a material of low tensile strength, has been subject to cracking problems since it was first used in structural applications. Recognition of the importance of cracking in concrete structures has prompted great interest in research on the fracture modelling of concrete. Classical (strength-based) mechanics of materials have been proved to be inadequate to handle severe discontinuities, such as cracks in a material. With the advance of powerful finite element (FE) analysis techniques, intensified research efforts have been made over the past few decades in the application of fracture mechanics (FM) in the modelling of cracking phenomena in concrete and concrete structures. Plain and reinforced concrete structures have been extensively analyzed using this broad FE, FM approach.

For example, Valente (2003) used a crack band model to analyze statically and dynamically the collapsed baroque Noto Cathedral in Italy for the purpose of rebuilding the 60-m-high structure.

Shi, Ohtsu, Suzuki & Hibino (2001) extended the discrete crack approach to the numerical analysis of multiple cracks in a real-size tunnel specimen which had been experimentally tested.

The sudden collapse of the New York Schoharie Creek Bridge in 1987 due to the unstable cracking in the reinforced piers, caused by the rapid flow of a flood, led Swenson & Ingraffea (1991) to adopt discrete cracking models, including linear and non-linear FM, to evaluate the initiation, stability and propagation profile of the crack that caused the failure. The deadly (loss of ten lives) cracking problems of the bridge can be rationally explained by the use of FE-based models.

Other types of plain or reinforced concrete structures that experienced fracture-controlled problems, such as the pullout of anchor bolts, the thick-walled ring, beams, panels, frames, containment vessels and shells, have also been analyzed in the past using FE, FM models (ACI 1997).

In an important effort to apply the FM approaches that have been developed to problems of practical significance, concrete dams (which are normally huge, fracture-sensitive concrete structures) have received special attention from researchers and have formed an essential part of this broad area of research on concrete crack modelling.

Uncontrolled crack propagation in concrete caused the disastrous failures of Malpasset Dam in France in 1959 (SimScience website). The rapid crack propagation as evidenced in the failure process of the above dam, has emphasized the importance of developing an accurate crack modelling method to safeguard dams. The Kölnbrein arch dam in Austria and Koyna gravity dam in India are representative of the two main types of dam structures which have attracted most research efforts for FM crack modelling of concrete dams.

Gravity dams are structures that rely on their own weight for resistance against sliding and overturning to maintain stability. In ancient times dated back as early as 4000 years BC, gravity dams were built using masonry materials such as earth, rock and cut blocks, with both the upstream and downstream faces sloped and the base thickness being many times the height of the dam. Concrete was first used in building a 47-m-high gravity dam called the Lower Crystal Springs Dam in the USA which was completed in 1889. Because they are relatively simple to design and build, concrete gravity dams have become a major dam type throughout the world. With the development of design and analytical expertise, as well as of construction techniques and equipment, dams have become ever larger with regard to both height and volume, e.g. the world's largest gravity dam so far, Three Gorges Dam in China, has a height of 185 m and a water-storage volume of 39.2 billion m^3. If a dam on this scale were to fail and collapse, this could lead to probably the greatest disaster in human history. Therefore, the safety of huge structures such as concrete gravity dams is of the utmost concern to the engineers involved in the design, construction and post-built safety evaluation of dams.

A great deal of research on the numerical modelling of the cracking behaviour of concrete has been carried out during the past few decades. In the process, many concrete crack propagation models have been developed and applied in concrete cracking analyses. The early strength-based model, in which the crack was assumed to propagate when the calculated tensile stresses at the crack tip exceed a specified tensile strength of the concrete, has seldom been used in any recent concrete analyses due to its inherent lack of mesh objectivity (FE mesh discretization has a significant influence on the results). Linear elastic fracture mechanics (LEFM), in which crack growth occurs when the effective stress intensity factor exceeds the material's fracture toughness, has been widely used in the analysis of concrete in the past. Models based on non-linear fracture mechanics (NLFM) have now become popular for analysing concrete cracking due to the existence of a fracture process zone (FPZ) at the front of the crack tip.

Many concrete gravity dams, which are generally massive, plain concrete structures, have experienced cracking problems to various extents. Crack formation and propagation in concrete gravity dams could influence their structural stability and endanger the safety of the dams. Normally, the huge size of a concrete dam excludes direct experimental tests on the structural cracking behaviour under various loading conditions. Therefore, evaluation of the possible cracking trajectory in concrete dams by means of an accurate constitutive model, in order to simulate the cracking response of the concrete effectively, becomes vital and would be a useful tool for practising engineers to ensure the safety of dam structures. This requires developing a numerical model and techniques that can accurately analyse and appraise a dam structure, either for the purpose of designing a new dam or for evaluating the safety of an existing concrete dam.

The need for methods that can accurately predict the behaviour of cracking in concrete dams has led to intensified research in this field. In fact, many attempts have been made to develop a rigorous model to simulate the cracking mechanisms in and the behaviour of concrete dams, especially concrete gravity dams. To name a few, Ingraffea (1990) performed a case study on the Fontana Dam, a gravity dam in the United States, to elucidate the mechanisms for crack initiation and to predict the observed crack trajectory employing a 2-D discrete LEFM method. Bhattacharjee & Leger (1994) applied a 2-D smeared crack model based on NLFM crack propagation criteria to study the static fracture behaviour of the Koyna Dam, a gravity dam in India. A more detailed review of these attempts will be given in Chapter II.

Although many analytical methods based on fracture mechanics have been proposed for concrete dams in the last few decades, they have not yet become part of standard design procedures. In fact, few of the current researches from all over the world are being implemented by practising engineers when evaluating dam safety. Current practice for crack analysis in concrete dams is to implement either the traditional "no-tension" gravity design method, which is based on rigid body equilibrium and strength of materials to determine crack length, assuming horizontal planar crack extension, linear stress distribution and zero stress at crack tip, or a non-linear FE analysis including plasticity models and contact simulation.

There are several FE programs that can be used to analyse the cracking response of concrete structures, e.g. MSC.Marc, ABAQUS, ANSYS, DIANA, LUSAS, FRANC2D/3D, FRACDAM and MERLIN, etc. At the Department of Water Affairs and Forestry (DWAF) – the dam authority in South Africa, MSC.Marc is currently the main FE tool used for non-linear analysis, including crack prediction on concrete dams. The cracking analysis in MSC.Marc is limited to linear tensile strain softening and the constant shear retention factor. This means that MSC.Marc lacks the flexibility in analysing cracking behaviour that is offered by more advanced crack models (including multi-linear or non-linear softening, shear retention factor varied with the crack's normal strain, etc.). Although some commercial FE programs, such as DIANA include advanced crack models, the codes of these programs are not generally available to be modified/enhanced for research purposes. In addition, the existing programs yield significantly different results on the crack response of a concrete gravity dam, as demonstrated in the benchmark exercise carried out by the European Thematic Network Integrity Assessment of Large Dams between 2003 and 2005 (refer to http://nw-ialad.uibk.ac.at). The different results with regard to peak load, displacement and stress given by the different programs highlight the need to improve the cracking analysis capacity of the existing FE packages by developing, perhaps, a better method of crack analysis and its numerical implementation in an FE program. An improved crack model and method could give a more accurate crack response (crack profile, horizontal crest displacement, etc.) in concrete dams which could be used to evaluate the safety of dams.

1.2 Motivations and objectives of this study

This research aims to contribute to the continuing research efforts on mastering the mechanics of cracking in concrete dams.

In order to evaluate the stability and safety of concrete dams more accurately, it is necessary to develop a better model and method for analysing cracking problems in concrete dams.

The objectives of the research are as follows:

- To evaluate the existing constitutive crack models critically and to adopt a suitable constitutive crack model using non-linear smeared fracture mechanics for simulating and investigating the cracking process in concrete dam structures.
- To develop a more accurate strain softening relation and to calibrate the parameters.
- To develop a numerical program specially for implementing the constitutive model in order to carry out fracture analysis of concrete dams under static loading conditions.
- To validate the constitutive model and numerical techniques by investigating the cracking behaviour of concrete structures that have been researched experimentally and/or numerically in the past.
- To investigate dam concrete softening parameters.
- To investigate the cracking behaviours of concrete gravity dams for better evaluation of dam safety.

1.3 Scope of this study

This research is focused on the development of a suitable crack modelling and analysis method for the prediction and study of fracturing in concrete gravity dams, and consequently, for the evaluation of dam safety against cracking. The research is limited to the two-dimensional (2-D) static cracking analysis of concrete gravity dams. The following areas are not covered in this research:

- Three-dimensional (3-D) cracking, although the research could be extended from 2-D to 3-D with some additional effort.
- Dynamic cracking.
- The water pressure that develops inside the crack as the crack grows.
- The coupling between different crack modes, and different cracks.
- Time dependent behaviour such as creep and shrinkage.

1.4 Methodology of this study

The research begins with a thorough literature review of previous investigations into the subject area and similar research. The theory and development of constitutive crack models are followed to establish the material crack model used in this study. The implementation of the proposal crack model is undertaken through the development of a sub-program specially coded for this research. The constitutive crack model proposed and the implementation procedure of the proposed crack model in an FE program are validated by analyzing and comparing the results obtained with the previously investigated concrete beams, gravity dams and model dam. After the verification process, the crack model and the sub-program are applied to analyze and predict the fracture response and to evaluate the related dam safety against the cracking of an existing, full-size concrete gravity dam. Finally, conclusions are drawn and recommendations are made based on this study.

1.5 Organization of this study

Chapter I gives the background, motivation, objectives, scope and methodology of this study. Chapter II is a comprehensive literature review of the development of crack models for application with concrete, especially concrete dams. The review focuses mainly on the evolution of the crack models proposed by other researchers in the world during the past few decades and gives a critical appraisal of the pros and cons of the crack models. A brief description of the analytical and design methods adopted for concrete gravity dams is given for readers who are not familiar with the design and safety evaluation procedure for concrete gravity dams. Past investigations into the fracture analysis of concrete dams are also discussed in Chapter II.

Chapter III presents the constitutive crack model adopted for smeared crack analysis of concrete structures in this research. It describes the crack onset criterion and direction, strain softening during the fracture process and post-crack features (such as non-orthogonal crack criteria for a new crack to occur, and whether the crack is fixed or rotating, definition of crack closing and reopening, and crack mechanisms for unloading and reloading), etc. A bilinear strain softening diagram for mode I and a shear softening relationship for mode II are proposed.

In Chapter IV, a numerical program capable of constitutive modelling crack initiation and the crack propagation of mixed modes is developed specially for smeared crack analysis of fracture behaviours in concrete dams. The program is incorporated into a commercial general-purpose code called MSC.Marc in order to carry out a complete FE analysis from the pre-processing involved in setting up the mesh and the loading/boundary conditions etc., and the solving of equilibrium equations, to post-processing of the results obtained. Preliminary verification of the program that has been developed is carried out on some elementary specimens. Three cracking verification cases in DIANA are selected to further benchmark the program that has been developed in this chapter.

In Chapter V, the proposed crack model and the numerical techniques that have been developed are thoroughly evaluated and benchmarked in static fracture analyses of different plain concrete beam structures under either mode I or mixed mode loading conditions. The fracture response of the beams and the parameter study on the proposed bilinear softening relationship are discussed in this chapter.

The proposed crack modelling techniques are needed to be evaluated and benchmarked in static fracture analyses of concrete dams. For this purpose, in Chapter VI, a scaled-down model of a gravity dam, a full-scale “benchmark” gravity dam and an existing gravity dam (the Koyna Dam in India) under static loading conditions are selected to do the benchmark because these dams were previously investigated for fracture behaviours by other researchers.

With confidence in the crack modelling techniques developed in this research having been built up by means of the benchmark exercises detailed in the previous chapters, Chapter VII is devoted to using the crack model to predict the static fracture response of a concrete gravity dam in South Africa and to evaluate the safety of the dam – the stability of the dam has been a matter of concern for the dam safety authority.

The last chapter, Chapter VIII, gives the conclusions of this research and makes recommendations for further study and application of concrete crack models.

Figure 1.1 gives a schematic outline of the organization of this study.

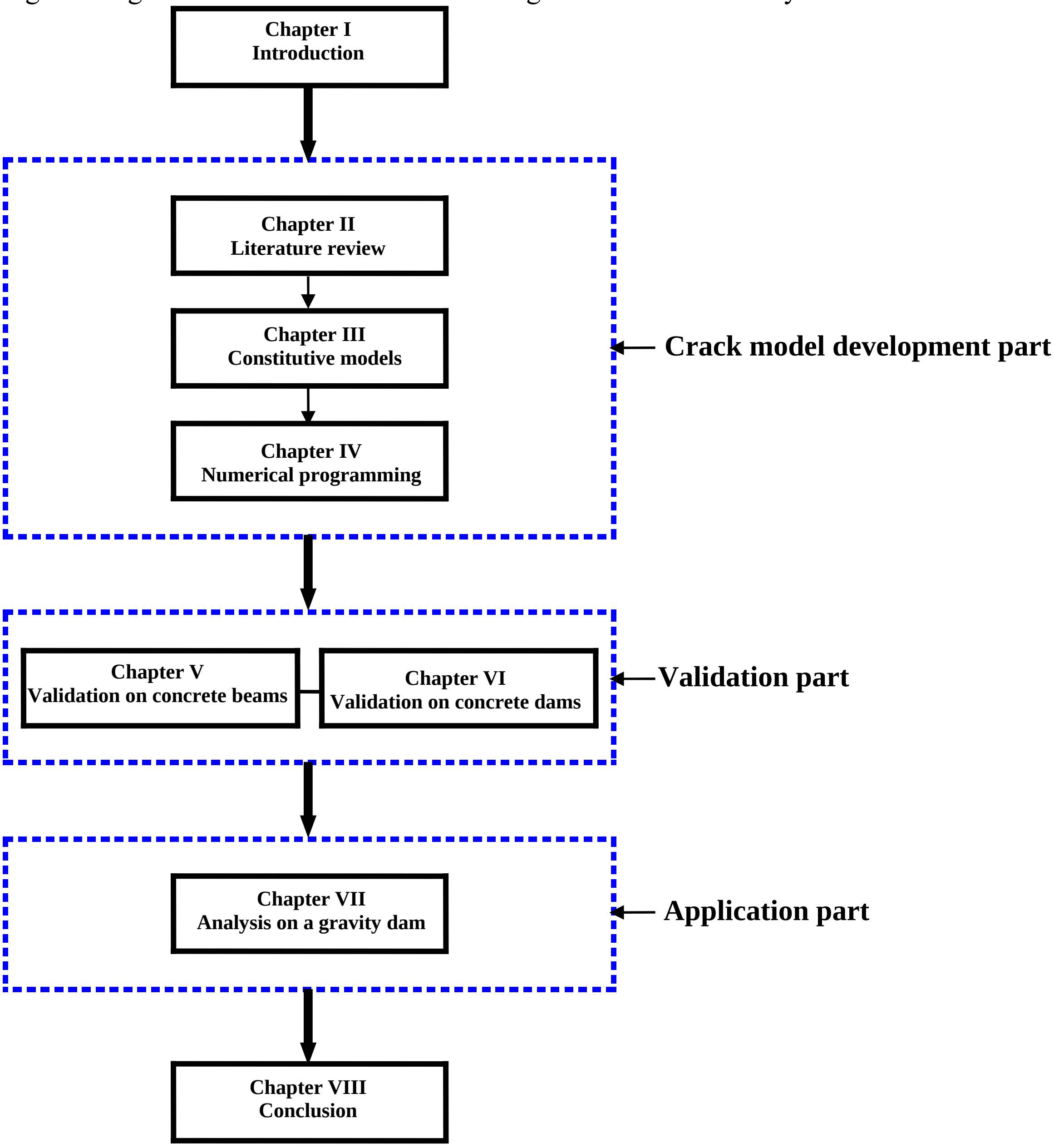

Figure 1.1 - Outline of the research

CHAPTER II - LITERATURE REVIEW ON GRAVITY DAM DESIGN AND ON THE DEVELOPMENT IN FRACTURE ANALYSIS OF CONCRETE DAMS

The constitutive modelling of cracking behaviours and crack representations in numerical implementation are the two main issues in the study of cracking in concrete structures. Crack modelling in concrete structures has undergone great development in the past, especially with regard to smeared constitutive models. The general development of research into the crack modelling of concrete, especially smeared constitutive modelling of concrete cracking and its application to concrete dams, is presented. This review of past and current research into the modelling of concrete cracking should provide a clear background to and platform for any further research in this field.

The methodology adopted to date for analyzing the cracking problems of concrete dams is reviewed to show the historical trends in the development of analysis methods for correctly predicting fracturing behaviours in concrete dams. Current research efforts to improve the modelling of cracking in concrete dams are also pointed out.

The use of finite element (FE) analysis for modelling cracks is an important step in developing crack models suitable for the accurate simulation of the cracking process in concrete structures. Much effort has been put into finding appropriate ways to represent cracking in FE formulation. The main methods proposed so far are presented.

Accurate determination of the fracture energy (a material parameter of fracture mechanics) of massive dam concrete, which uses large aggregates, has a significant influence on the fracture analysis of concrete dams. For this reason, research on this key material parameter for fracture in concrete dams is described.

Past investigations into cracking analyses for concrete gravity dams under static loadings are discussed to highlight the human pursuit of complete safety for dam structures, even under cracking situations.

2.1 Causes of cracking in concrete gravity dams

The low tensile resistance of concrete is the main reason why cracking is a common phenomenon in concrete dams. There are many causes that could contribute to cracking in concrete dams, either individually or collectively. A broad classification of these causes is given below (Linsbauer 1990).

Inadequate design and construction methods: Geometrical "flaws or defects" such as notches, corners, jagged interface between dam and foundation, and inadequate preparation of the construction joints and concrete block joints. All these defects can lead to local stress concentration and deformation restraint.

Material problems: Volume changes due to shrinkage, creep, heat of hydration or chemical changes such as alkali-aggregate reaction.

Structural behaviour: Tensile stresses induced by varied static loadings, earthquake loadings, temperature changes and differential settlement of the foundation. Uplift pressure and overflow can also cause severe cracking in dams and endanger their safety.

Normally, surface cracks caused by, e.g., concrete shrinkage or creep cannot really threaten the structural safety of concrete dams and are not the type of cracks that dam engineers regard as a concern for structural safety. Cracks penetrating deep inside dams, caused by excessive stresses or strains (which develop as a result of load application) or by material volume changes (such as alkali-aggregate reaction), are the main concern for engineers because these cracks can lead to considerable changes to the structural behaviour and failure resistance of the dam structure. In general, the state of stress and strain in the concrete mass will determine and control the fracture mechanism in concrete.

2.2 Brief description of methods of analysis and design criteria for concrete gravity dams

For the design of concrete dams, it is necessary to determine the forces that can be expected to affect the stability of the structure. The forces (shown in Figure 2.1) that must be considered for the design and analysis of concrete gravity dams under static loadings are those due to:

- Hydrostatic pressure (including tailwater loading) H, H', V, V'
- Silt pressure (sediment loading) S
- Uplift forces U
- Weight of structure (self weight) Gr

Other loads, including wind and waves, ice and temperature loading, are sometimes considered in design.

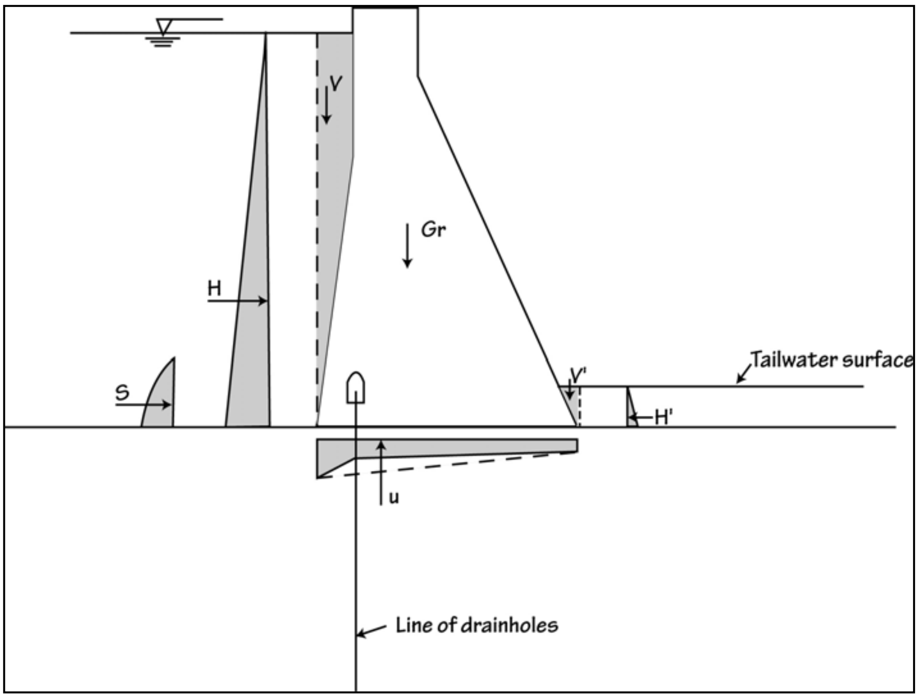

Figure 2.1 - Forces acting on a gravity dam

In addition to the normal static loading conditions, it may be necessary to apply earthquake loads. It is not likely, however, that all of these loads will occur at the same time. Table 2.1 below lists the load combinations for concrete dam design in South Africa (Chemaly 1995).

TABLE 2.1 - Definition of load combinations in South Africa

Load category	**Load combinations**
Normal loading	A. RDD + Gr + PU + S + TW
Abnormal loading	B. FSL + Gr + PU + S + OBE
	C. RDD + Gr + FU + S + TW
	D. RMF + Gr + PU + S + TW
Extreme loading	E. FSL + Gr + PU + S + TW + MCE

FSL: Water level at full supply level

FU: Full uplift (relief holes blocked or no drainage system)

Gr: Self weight of dam

MCE: Maximum credible earthquake

OBE: Operationally based earthquake

PU: Partial uplift (with pressure-relief holes functioning)

RDD: Water level at recommended design discharge (1 in 200-year flood)

RMF: Water level at regional maximum flood

S: Silt loading (after 100 years' deposition)

TW: Tailwater level

Over the years, methods of analyzing concrete gravity dams have been developed and improved – from the classical method based on linear elastic calculations to the FE method which can carry out far more accurate non-linear analysis under more complex loading conditions.

The classical method of calculating stresses is based on the assumption of a linear stress distribution on a horizontal plane. The gravity dam is idealized as a cantilever beam. The stresses are computed by applying the following classical formula:

$$\sigma = \frac{P}{A} \pm \frac{My}{I} \qquad (2.1)$$

Where

P Normal force acting on the selected cross-section

A Area of the cross-section

M Bending moment acting on the cross-section

y Distance to the centre of the cross-section

I Moment of inertia of the cross-section.

The calculated stresses in a horizontal plane are limited to meet the design criteria for permissible stresses in a concrete dam. The design criteria for stress distribution in a concrete gravity dam in South Africa are shown in Table 2.2 (Kroon 2002):

TABLE 2.2 - Design criteria for normal stresses in concrete gravity dams (South Africa)

	Normal loading (A)	**Unusual loadings (B, C, D and E)**
Tensile stresses	None	0,2 MPa
Compressive stresses	0,25 f_c	0,25 f_c

Note: f_c is the compressive strength of the concrete in a standard cube after one year. The maximum tensile stress of 0,2 MPa can only be allowed on a dam site where the foundation rock is sound and not excessively horizontally jointed.

A gravity dam is also designed to be safe against sliding and overturning. The stability of a gravity dam against overturning is guaranteed by dimensioning the dam so that the resultant of all forces acting on any horizontal plane within the dam and foundation, intersects the corresponding base plane within its middle third of the length. This will effectively prevent tensile stresses in a dam.

The stability of a dam against sliding is of major concern to dam engineers. The factor of safety against sliding is defined using the following formula:

$$F.O.S = \frac{CA + P\tan\varphi}{H} \tag{2.2}$$

Where

C Ultimate cohesion of concrete or rock

φ Angle of internal friction

A Area of the basis of contact

P Sum of the vertical forces, including the uplift forces

H Sum of the horizontal forces.

The design criteria adopted in South Africa for safety against sliding of concrete dams are listed in Table 2.3.

TABLE 2.3 - Design criteria for safety against sliding in concrete gravity dams (South Africa)

	Normal loading (A)	**Abnormal loadings (B, C, D)**	**Extreme loading (E)**
F.O.S. for peak	2,0 – 4,0	1,5 – 2,0	> 1,0 – 1,2
F.O.S. for residual	1,5 – 2,0	> 1,0 – 1,2	> 1,0

Note: Peak is the maximum shear properties (such as C and φ) in the interface between the wall concrete and foundation rock. Residual means the remained shear properties in the interface for long term.

The development of computing power and the FE method allowed engineers to analyse non-linearity in concrete dam behaviour, including dam-foundation interactions, material plasticity, thermal stresses, etc. The FE method and numerical techniques have been improved and gradually introduced into the codes of practice, leading to better and safer designs (Galvez *et al.* 1996). Currently, two principal analytical methods, namely the classical method and the FE method, are being used in the design of concrete dams.

2.3 Analysis of cracking in concrete dams

With the conventional design methodology described in Section 2.2, concrete dams are usually designed to have "no tension" in any part of the dam under normal service loads and to withstand minimum tensile stresses only under extreme loading cases. However this "no tension" design has never been justified theoretically. The work of Bažant (1990) reveals that even apparently conservative "no tension" design cannot always be regarded as safe if a certain size (e.g. base width) of dam is exceeded. In his paper, Bažant used a simple example of a dam model with a predefined horizontal crack at the base to demonstrate that the "no tension" solution could yield a higher maximum load than linear elastic fracture mechanics (LEFM). The size effect plot also shows that there is a critical size of dam after which the "no tension" design gives a higher maximum load than LEFM. In reality, most, if not all, of the existing concrete dams in the world cannot really be said to be in a perfect crack-free condition, even if many of these were designed to have "no tension".

The rigid body equilibrium strength-based criterion was initially adopted where it was assumed that a crack would propagate whenever the principal tensile stress at the crack tip exceeded a specified tensile strength of the concrete. This was the only criterion for determining crack growth in concrete dams before the late 1970s (Saouma, Bruhwiler & Boggs 1990).

The strength criterion for crack analysis of concrete dams is based on the assumption that a crack will propagate horizontally in a plane and extend to a point where the stress becomes zero, and that the stress distribution is linear along the uncracked length of the dam wall in that plane. This kind of cracking analysis method suffers from the following shortcomings:

- The shear stress cannot be considered.
- Strictly speaking, this shallow beam theory cannot be applied to concrete dams which usually have low aspect (height/base width) ratios.
- The stress singularity at the tip of a crack cannot be taken into account.

The analysis of cracking based on this rather arguable assumption is not compatible with continuum mechanics (Linsbauer 1990). The diagram in Figure 2.2 is an illustrative example of the forces and stress distribution in a cracked concrete gravity dam analyzed by means of the conventional method.

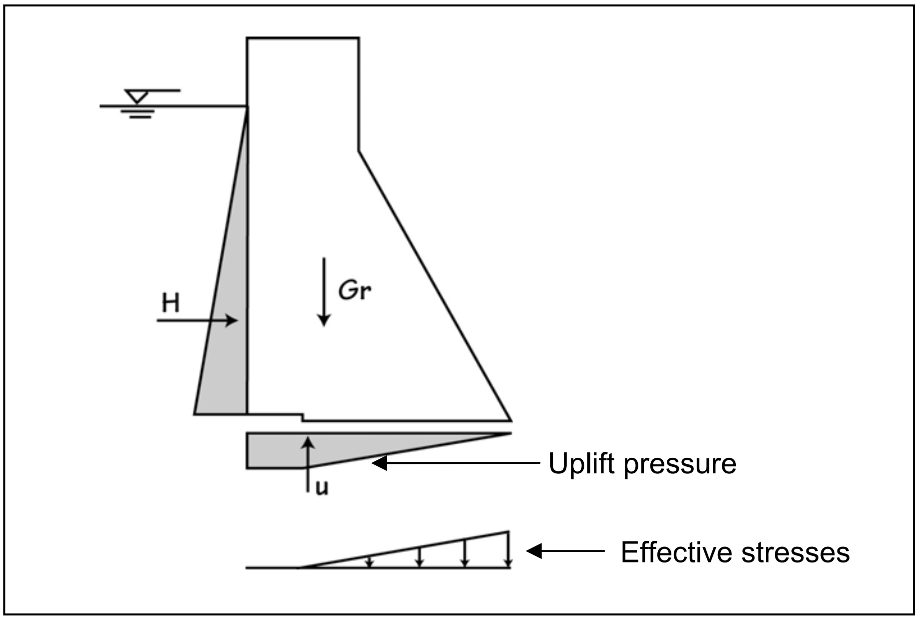

Figure 2.2 - Diagram of the forces and stresses used in the classical analysis method for a concrete gravity dam

Significant advances in the FE method make it a very useful tool for investigating cracking in concrete dams. However, when the strength-based approach is applied in the FE method, it is often found that if the mesh around the crack tip is refined, the stresses become progressively larger and the results are said to be "mesh-unobjective". This leads to the conclusion that strength-based models are unsuitable for modelling the stress singularities at a crack tip and that they are inadequate for analyzing cracking in concrete structures.

It is well known that cracking in concrete is a dominant source of the non-linearity experienced in concrete dams. To study cracking behaviour in a dam and to gain an understanding of how the cracking that is normally caused by high stress concentrations can redistribute the stress in a dam, non-linear FE analysis using material plasticity models (such as Drucker-Prager and Mohr-Coulomb) and contact simulation of the cracks has been adopted. This approach allows prediction of the scope of the cracking and the potential effect on leaking in concrete dams.

Fracture mechanics, based on fracture energy principles, deals with cracking in materials and is ideally suited for studying crack development and propagation in concrete structures. The application of fracture mechanics to modelling the cracking process in concrete dams for the purpose of safety evaluation has drawn a great deal of interest and attention world-wide. At the 15th International Congress on Large Dams (ICOLD) conference held in Lausanne in 1985, researches on the analysis of cracking responses in concrete dams using fracture mechanics were accepted and presented (Linsbauer 1990). Although many analytical methods based on fracture mechanics have been proposed for concrete dams in the last decades, they have not yet been introduced into standard design procedures.

During the past decades, LEFM has been widely used in the analysis of concrete dams, especially gravity dams. Due to the existence of a fracture process zone (FPZ) (refer to Figure 2.3) at the front of the crack tip, although sometimes, small compared with the size of the dam, strictly speaking, a model based on non-linear fracture mechanics (NLFM) should be adopted in all cracking analyses of concrete dams. NLFM has gained recognition among the researchers and become the main trend for the fracture analysis of concrete dams.

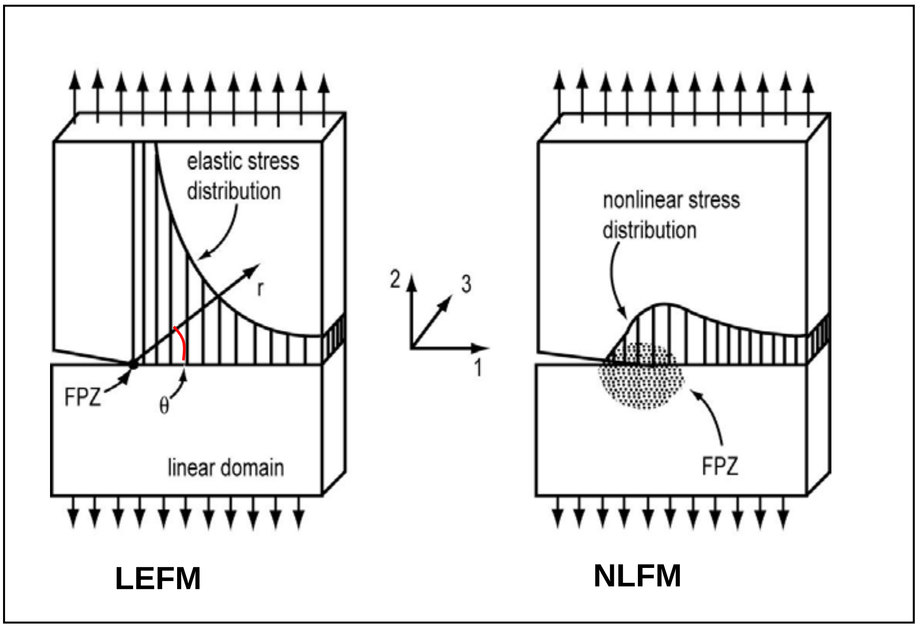

Figure 2.3 - Fracture process zone in LEFM and NLFM (Bhattacharjee & Leger 1992)

Detailed descriptions of LEFM and NLFM, the crack models based on them and their past application in the analysis of concrete dams are given in later sections (2.5.3, 2.5.4 and 2.7) of this chapter.

2.4 Finite element approaches for modelling cracking in concrete

Constitutive modelling of the crack behaviour of concrete relies on the FE program's ability to install the constitutive model and simulate the cracking profile. At present, the methods most frequently adopted in FE analysis to model cracking are as discussed below.

Discrete crack approach: This approach represents a crack as a discrete gap along the inter-element boundary. Inter-element boundaries are separated to simulate cracking. This involves the addition of nodes which influence element connectivity and the stiffness matrix. The analysis is complicated by a continuous change of the FE topology during the analysis (Bhattacharjee & Leger 1992). A pre-defined crack path is sometimes needed beforehand to define the orientation of the cracks.

Smeared crack approach: In this approach, the stiffness of the material in an element (or at an integration point) is modified to simulate an infinite number of closely spaced cracks 'smeared' over the region under consideration. The advantage of the method lies in its simplicity and cost-effectiveness since the topology of the FE mesh remains unchanged and no restrictions are imposed on the orientation of the crack (Bhattacharjee & Leger 1992). This approach still has several deficiencies, namely its tendency to cause diffused crack patterns, the directional bias and stress locking.

There are other FE approaches that could be used to model cracking in concrete. For example, Kuo (1982) proposed an interfaced smeared crack model (ISCM) which combines the advantages of the discrete and smeared approaches. Graves & Derucher (1987) proposed an improved interfaced smeared crack model on the basis of Kuo's work to find a satisfactory 'pushing-back' procedure (by altering the local element displacements until the stresses at the cracking interface are brought close to zero through an iteration process) at the local level. Other authors have mentioned the lattice approach as another numerical method with possibilities (Galvez, Cervenka, Cendon & Saouma 2002; Cai, Robberts & van Rensburg 2004).

2.5 Crack modelling of concrete

Concrete is made up of different constituents (or phases) and is by nature a heterogeneous material. The cracking process in concrete is very complicated and the crack surface is tortuous (see Figure 2.7). To model this complex process adequately demands continued research efforts to find methods capable of accurately predicting and simulating the cracking response in concrete.

A non-linear static analysis of cracked concrete requires a constitutive model that is able to represent the locations phenomenon (i.e. to identify locations where cracks will initiate, predict crack growth and model crack coalescence) and to model this process up to collapse of the structure. In general, five main phases can be identified and these are discussed in the following sections:

- pre-fracture material stress-strain behaviour
- crack (fracture) initiation
- crack propagation criteria
- crack modelling, and
- post-crack behaviour.

2.5.1 Pre-fracture material stress-strain behaviour

Before cracking, concrete in tension can be sufficiently modelled as an isotropic, linear elastic material. The behaviour of concrete under high compressive loading is normally modelled as non-linear. However, in structures such as concrete gravity dams, the compression stresses are low enough that it is adequate to assume a linear elastic constitutive law. It is true that some non-elastic softening close to the peak tensile stress, before a crack is initiated, will be ignored in the above assumption. Nevertheless, a non-linear, plastic stress-strain law can always be adopted due to the rapid advance of FE analysis capacity. Most of the previous investigations into concrete cracking, especially in concrete gravity dams, have assumed a pre-cracking linear, elastic behaviour under both tensile and compressive loadings.

For concrete cracking analysis, the plane stress state is probably the analysis most often adopted for verification. In plane stress analysis, the linear, elastic incremental stress–incremental strain relationship is expressed as follows:

$$\begin{Bmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\sigma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\gamma_{xy} \end{Bmatrix} \tag{2.3}$$

Where

E Young's modulus

ν Poisson's ratio

x the global horizontal direction

y the global vertical direction.

2.5.2 Crack initiation

To study the non-linearity caused by cracking in concrete, it is necessary to know where the cracking starts. Thus it is important to set up crack initiation criteria in the model. Researchers have proposed various criteria to indicate crack initiation:

- The conventional criterion for a homogeneous structure is to assume that a new crack will initiate when the principal tensile stress reaches the uniaxial tensile strength of the concrete. Bhattacharjee & Leger (1993) have pointed out that this criterion is not entirely satisfactory in a 2-D or 3-D FE analysis, because: (i) the material stress-strain response is non-linear prior to reaching the peak strength, and (ii) the principal stress and strain, used as the response indicators, are not directly proportional due to Poisson's effect.

- In a 3-D analysis, cracking can be assumed to occur when the stress reaches a failure surface (or more specifically, the crack detection surface) on a meridian plane (Chen 1982).

- Difficulty in finding the uniaxial tensile strength experimentally has led to the modulus of rupture, as obtained from a beam test, being used as a crack initiation criterion (Linsbauer, Ingraffea, Rossmanith & Wawrzynek 1989a).

- Another crack initiation criterion states that a linear elastic relationship is assumed until the tensile strain energy density in the analysis equals the pre-peak area under a uniaxial stress-strain diagram as obtained from a laboratory specimen, as shown in Figure 2.4 (Bhattacharjee & Leger 1993; Bhattacharjee & Leger 1994).

$$\tfrac{1}{2}\sigma_1\varepsilon_1 \geq \int_0^{\varepsilon} \sigma \, d\varepsilon = \tfrac{1}{2}\sigma_i\varepsilon_i = \frac{\sigma_i^2}{2E} \tag{2.4}$$

To obtain the tensile strain energy, $\frac{1}{2}\sigma_1\varepsilon_1$, we need to know the maximum principal stress, σ_1, and strain, ε_1, at a material point. σ_i is the apparent tensile strength, defined as 25 ~ 30% higher than the tensile strength of concrete, σ_t, while ε_i is the corresponding strain.

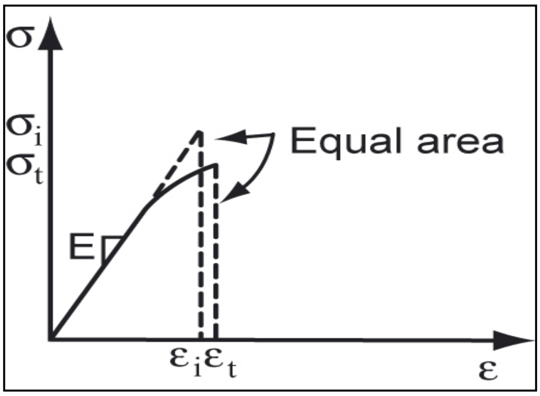

Figure 2.4 – Crack initiation criterion (Bhattacharjee & Leger 1994)

- Onate, Oller, Oliver & Lubliner (1988) used a fully elasto-plastic model for the concrete and assumed that cracking occurred where the effective plastic strain was greater than zero. It was further assumed that the crack developed in a direction orthogonal to the direction of the maximum principal strain at the point.

The conventional crack initiation criterion remains the most accepted due to its simplicity and conceptual straightforwardness (Araujo & Awruch 1998; Leclerc, Leger & Tinawi 2003; Planas, Elices, Guinea, Gomez, Cendon & Arbilla 2003, etc). Most researchers also agree that the crack direction should be orthogonal to the maximum principal stress or strain.

2.5.3 Crack propagation criteria

➢ <u>Strength-based criterion</u>

As discussed in Section 2.3, the strength-based criterion assumes that a crack will propagate when the predicted stress at the tip of the crack exceeds the tensile strength of the material. In this way, the criterion is identical to the conventional crack initiation criterion.

➢ <u>Fracture mechanics criteria</u>

Fracture mechanics predicts the propagation of cracks on the basis of the energy dissipated by the structure during fracturing. It is now well established that fracture in concrete is not concentrated in a point at the crack tip, but rather occurs within a finite zone ahead of the crack, defined as the FPZ. Micro-cracking of the material in the FPZ helps to explain the observed softening behaviour of material in this region.

The non-homogeneous nature of concrete causes further complications:

(i) Cracks do not propagate along a straight line, but rather follow a tortuous path.

(ii) The exact position of the crack tip is difficult to determine because of aggregate bridging in the crack and variations in the size of the FPZ.

Fracture mechanics can be broadly classified into two categories: LEFM and NLFM. Models based on LEFM assume a linear elastic material and that crack extension is accompanied by a sudden release of surface stresses. At the tip of a crack, the stress becomes singular. Crack growth occurs when the effective stress intensity factor (including the appropriate modes of I – opening, II – sliding and III – tearing – see Figure 2.5) equals the material fracture toughness.

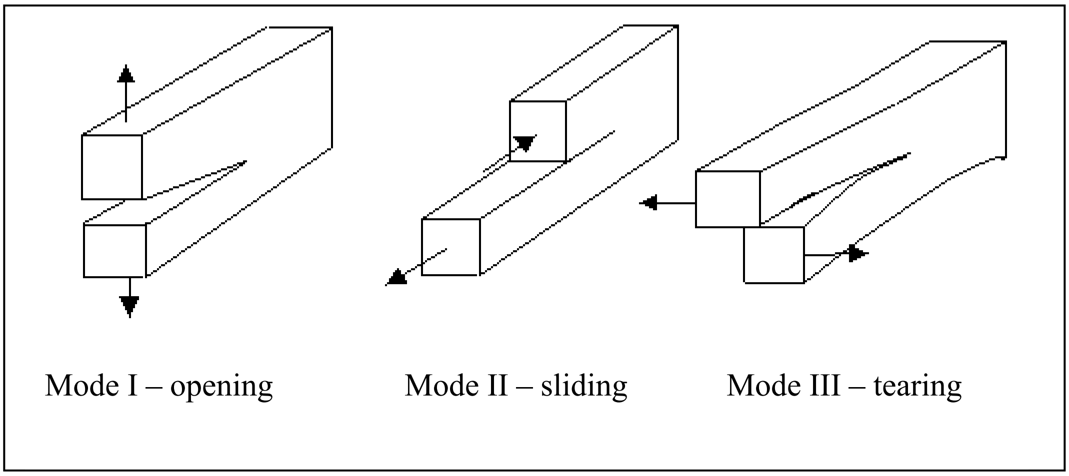

Figure 2.5 - Modes of fracture

Since the presence of FPZ is ignored, LEFM should be applied only to concrete structures in which the FPZ is much smaller than the dimensions of the structure under consideration. LEFM can, therefore, successfully be applied to most parts of a large concrete structure, such as a gravity dam. However, a concrete gravity dam, normally a very stiff structure, may have long and narrow (small opening displacement) cracks. In this case, the FPZ cannot be treated as small and be ignored, which means that sometimes LEFM cannot be applied, even to large gravity dams.

NLFM recognizes the non-linear material behaviour by including the strain-softening behaviour of the concrete in the FPZ. In 1990, Saouma *et al.* adopted that crack propagation occurs when the stress at the tip of the FPZ reaches the tensile strength. Since the 1980s, research into crack analysis models based on NLFM has been intensified. The following section, Section 2.5.4, focuses mainly on the development of smeared cracking models based on NLFM, although the other crack models based on, for example, discrete fracture, LEFM and strength-based criteria, are also addressed.

2.5.4 Crack models

Strain softening has been modelled by various types of constitutive laws. Apart from NLFM, endochronic theory, plastic-fracturing theory, plasticity with decreasing yield limit and, recently, continuum damage theory are also used (Pijaudier-Cabot & Bažant 1987; Bažant & Kim 1979; Ghrib & Tinawi 1995).
Various crack propagation criteria and fracture models based on these criteria have been proposed in the literature, but only the major developments are presented here.

Two major categories of crack models – discrete and smeared – are described in this section, which shows the development of the major fracture models. The overall development of cracking models is illustrated in Figure 2.10.

- <u>Discrete fracture models</u>

- **Discrete model 1: Linear elastic fracture mechanics (LEFM)**

The criterion for crack growth in LEFM, which is applicable only to cracked structures, is as follows:

$$K \geq K_{IC} \tag{2.5}$$

Where K is the stress intensity factor, which is a measure of the strength of the singularity around the tip of a crack. K can be expressed and computed by:

$$K = f_{ij}(\theta)\sigma_{ij}\sqrt{2\pi r} \tag{2.6}$$

As shown in Figure 2.6, σ_{ij} are the stresses at a distance, r, from the crack tip and at an angle, θ, from the crack plane. $f_{ij}(\theta)$ are known trigonometric functions of θ (depending on the specimen, crack geometry and loading, etc.). At the crack tip, the stress is theoretically singular. Thus, as the crack tip is approached, the stress asymptotically approaches infinity. Hence:

$$K = \sqrt{\pi a}\,\sigma \qquad (\theta = 0;\ r \to 0)$$

Where

a the crack size

σ the applied stress.

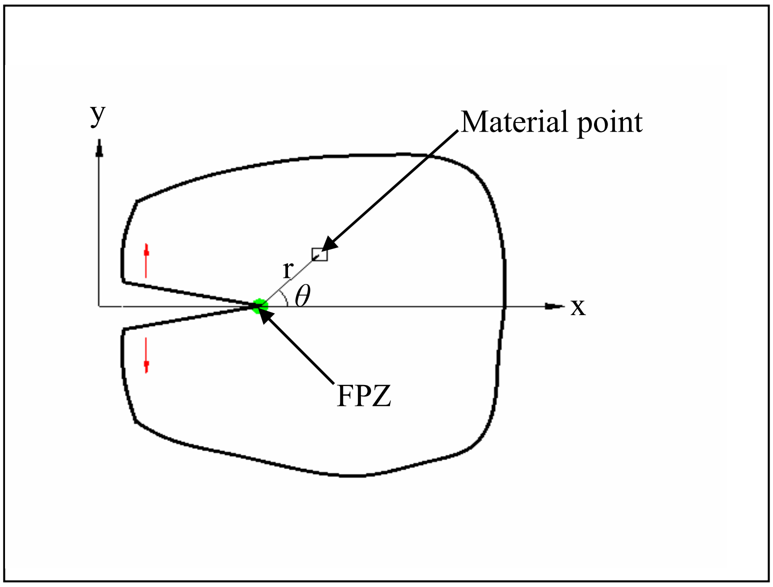

Figure 2.6 - Crack in an arbitrary body and coordinate system (LEFM)

K_{IC} is the fracture toughness, which is a measure of the material's resistance to cracking and can be determined experimentally.

The stress intensity factor, K, and the fracture toughness, K_{IC}, should be determined in accordance with the three different fracture modes (I – opening, II –sliding and III – tearing). For 2-D analysis, modes I and II are normally considered, although mode I – opening is usually the dominant mode in concrete fracturing.

- **Discrete model 2: Fictitious crack model** (Hillerborg, Modeer & Petersson 1976)

As can be seen in Figure 2.7, the FPZ is characterized as a fictitious crack lying ahead of the actual crack tip. Three material parameters are required in this model: tensile strength f_t, specific fracture energy G_f, and the shape of the softening curve $\sigma(\delta)$. G_f is

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regarded as a material property and represents the energy absorbed per unit area of crack:

$$G_f = \int_0^{\delta_f} \sigma(\delta)\, d\delta \tag{2.7}$$

where δ_f is the critical crack separation displacement when the softening stress is equal to zero.

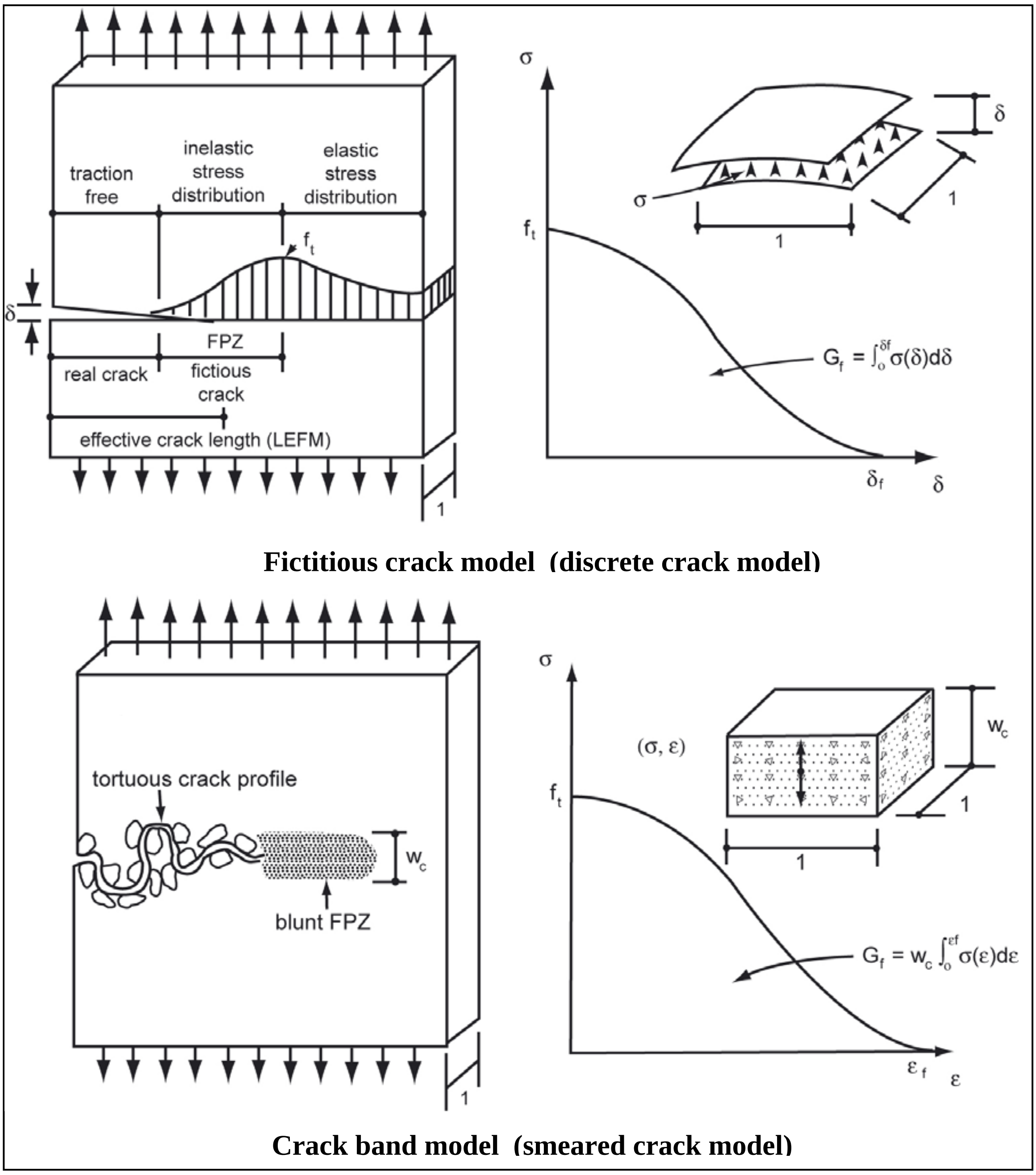

Figure 2.7 - Representative NLFM discrete and smeared crack models (Bhattacharjee & Leger 1992)

- **Discrete model 3: Effective elastic crack approach**

  The FPZ in concrete can also be modelled by a single Griffith-Irwin energy dissipation mechanism. By setting $\sigma(\delta) = 0$, it is implied that no energy is required to overcome the cohesive pressure in separating the crack surfaces. A two-parameter fracture model by Jenq & Shah (1985) and a size effect model by Bažant & Kazemi (1990) are typical examples.

The following three discrete models have also been proposed (Rots 2002):

- Decomposed crack model (Rots 1988)
- Plasticity-based interface model (Lourenco 1996)
- Model based on total relative displacement (Rots 1988)

➢ <u>Smeared fracture and constitutive models</u>

- **Smeared model 1: Orthotropic stress-strain relations** (Rashid 1968)

  The classical smeared crack model was based on the conventional strength-based crack initiation/propagation criterion, with zero post-cracking strength perpendicular to the crack. Although good results were obtained for many practical applications, the method proved to be mesh-unobjective and converged to an incorrect failure mode, with zero energy dissipation. The model's results did not reflect the size effect seen in test results. The constitutive law of this model in 2-D application is as follows (Rots 1989).

$$\begin{Bmatrix} \Delta\sigma_{nn} \\ \Delta\sigma_{ss} \\ \Delta\sigma_{ns} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mathrm{E} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_{nn} \\ \Delta\varepsilon_{ss} \\ \Delta\varepsilon_{ns} \end{Bmatrix} \tag{2.8}$$

Where

n the direction normal to the crack (mode I – opening) *s* the direction tangential to the crack (mode II – shearing).

In this model, both the normal and shear stiffness of the crack become zero immediately after the crack is formed. The orientation of the crack is fixed upon crack formation.

- **Smeared model 2: Mode II shear retention improvement** (based on Rashid's orthotropic model)

Numerical difficulties and distorted crack patterns were sometimes experienced with the application of the above orthotropic model (Rots 1989). Retaining a reduced shear stiffness across the crack can improve the performance of the model which has the following stress-strain relation:

$$\begin{Bmatrix} \Delta\sigma_{nn} \\ \Delta\sigma_{ss} \\ \Delta\sigma_{ns} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mathrm{E} & 0 \\ 0 & 0 & \beta G \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_{nn} \\ \Delta\varepsilon_{ss} \\ \Delta\varepsilon_{ns} \end{Bmatrix} \tag{2.9}$$

A constant shear retention factor is usually adopted in the application of the model. A more realistic, crack-opening-dependent shear retention factor was also applied previously in order to reflect the fact that shear stress transferred in a crack would decrease as the crack propagated further, with an increase in the crack's normal strain.

The shear retention factor β ($0 \le \beta \le 1$) is used to account for aggregate interlock in the concrete cracking process, which can reduce the numerical difficulties.

- **Smeared model 3: Mode I tension softening improvement** (based on the above smeared model 2)

$$\begin{Bmatrix} \Delta\sigma_{nn} \\ \Delta\sigma_{ss} \\ \Delta\sigma_{ns} \end{Bmatrix} = \begin{bmatrix} E_S & 0 & 0 \\ 0 & \mathrm{E} & 0 \\ 0 & 0 & \beta G \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_{nn} \\ \Delta\varepsilon_{ss} \\ \Delta\varepsilon_{ns} \end{Bmatrix} \tag{2.10}$$

$E_S = \mu E$

A sudden drop in the tensile strength to zero in the above smeared crack model 2 may also cause numerical difficulties similar to those caused by shear drop to zero (Rots 1989). Many displacement-controlled tensile tests reveal a gradual softening in the stress-strain relation after crack initiation. For this reason, further improvements were made to introduce a normal strain-softening concept into the fixed crack model. By doing this, 'over-stiff' results can be reduced; such results are often seen despite the effort of adopting mode II shear retention β. Linear strain softening is often used by inserting a negative normal retention factor μ ($-1 \leq \mu \leq 0$).

- **Smeared model 4: Extended crack band model (CBM)** (proposed by Bažant & Oh 1983)

Bažant & Oh (1983) improved the above fixed crack models by taking Poisson coupling after crack formation into consideration. In their original crack band model, they ignored the shear retention effect by not including the shear modulus term ($\beta G = 0$). The following extended crack band model was proposed to reinsert the shear modulus on a retention basis (βG).

$$\begin{Bmatrix} \Delta\sigma_{nn} \\ \Delta\sigma_{ss} \\ \Delta\sigma_{ns} \end{Bmatrix} = \begin{bmatrix} \frac{\mu E}{1-\nu^2\mu} & \frac{\nu\mu E}{1-\nu^2\mu} & 0 \\ \frac{\nu\mu E}{1-\nu^2\mu} & \frac{E}{1-\nu^2\mu} & 0 \\ 0 & 0 & \beta G \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_{nn} \\ \Delta\varepsilon_{ss} \\ \Delta\varepsilon_{ns} \end{Bmatrix} \tag{2.11}$$

Where

$\mu = \frac{E_s}{E}$

ν Poisson's ratio

E_s the strain-softening modulus (negative value).

It is assumed that the FPZ develops within a fixed bandwidth, propagating as a blunt front. A typical value for the bandwidth, w_c, would be three times the size of the aggregate. Although the CBM (see Figure 2.7) has shown good agreement with all the basic experimental data for concrete specimens, it has the following disadvantages (Bažant & Lin 1988):

(i) The bandwidth determines the element size and subdivision of the bandwidth is not allowed.
(ii) If the crack follows a zigzag path, the rugged opposite sides of the crack could incorrectly transfer stresses that would normally not be present in an open crack – a phenomenon referred to as 'stress locking'.
(iii) The choice of mesh could influence the direction of fracture propagation.
(iv) The bandwidth of the cracking zone cannot be altered.

For the linear strain-softening relationship shown in Figure 2.8, which is often assumed, the E_s can be obtained using the following formula:

$$G_f = h_c(1 - \frac{E}{E_s})\frac{f_t^2}{2E} \quad \Rightarrow \quad E_s = \frac{f_t^2 E}{f_t^2 - \dfrac{2EG_f}{h_c}} \tag{2.12}$$

Where

f_t the tensile strength

E the elastic modulus

G_f the fracture energy

h_c the crack characteristic length (in CBM, $h_c = w_c$).

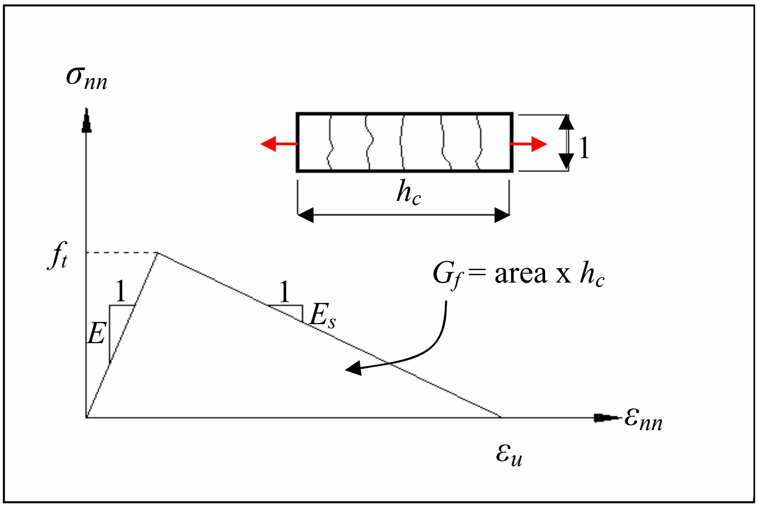

Figure 2.8 - Stress-strain diagram for the crack band model

- **Smeared model 5: Non-orthogonal model** (de Borst & Nauta 1985)

In this non-orthogonal smeared crack model, the total strain increment is considered to be composed of an intact concrete strain increment $\Delta\varepsilon^{co}$ and a cracked strain increment $\Delta\varepsilon^{cr}$:

$$\Delta\varepsilon = \Delta\varepsilon^{co} + \Delta\varepsilon^{cr} \tag{2.13}$$

The constitutive relationship for the cracked concrete is given by:

$$\Delta\sigma = \left\{D^{co} - D^{co}N\left(D^{cr} + N^T D^{co} N\right)^{-1} N^T D^{co}\right\}\Delta\varepsilon \tag{2.14}$$

Where

D^{co} the constitutive matrix for the intact concrete between cracks

D^{cr} the constitutive matrix for the cracks (in the local coordinate direction)

N a transformation matrix.

The model has the following advantages:

- A mixed-mode crack matrix can be formed.
- Non-orthogonal multi-crack formation can be modelled.
- Crack formation can be combined with other non-linear phenomena, such as plasticity, creep and thermal effects.

Rots (2002) also pointed out two disadvantages of this model:

- Difficulty of implementation due to the complicated algorithms involving internal iterations.
- Difficulty of choosing parameters for the shear retention functions and the threshold angles. There is no theoretical or experimental guideline to determine these values for different applications.

- **Smeared model 6: Non-local crack constitutive model** (Bažant & Lin 1988)

  The principal idea of this non-local crack model is to use the non-local concept only for those variables that control 'damage' and not for the strains or stresses in the constitutive relation. The disadvantages of the above smeared model 4 (CBM) can be eliminated by using a non-local constitutive model. Variables causing strain softening are treated as non-local, while all other variables are treated as local. The most important feature of this model is that the effect of structure size on the ultimate capacity and on the post-peak slope of the load–deflection diagram can be correctly presented (ACI 1997). The application of a non-local model in the analysis of a dam may be limited since at least three elements are required on the crack band, resulting in a very fine mesh. The analysis is complicated by the spatial averaging of local response quantities (Bhattacharjee & Leger 1994).

  When the model was used for practical applications, the following inconveniences became apparent (Pijaudier-Cabot & Bažant (1987):

  - The non-local concept is applied for all responses, including the elastic or plastic hardening behaviour.
  - An overlay with a local continuum is necessary for avoiding certain zero-energy periodic modes of instability.

  Pijaudier-Cabot & Bažant (1987) developed a modified non-local formation which avoids these two inconveniences.

- **Smeared model 7: Localized smeared fracture models** (Bhattacharjee & Leger 1994)

  The models of plane stress use a simplified definition of the constitutive material behaviour and have been shown to be computationally economical. A local axis system *ns* is selected for the fractured material, where the direction *n* is normal to the smeared cracks (refer to Figure 2.9). If E_n is the secant modulus of the fractured material, then the 2-D constitutive matrix relating local stresses and strains is defined as:

$$[D]_{ns} = \frac{E}{1-\eta\nu^2}\begin{bmatrix} \eta & \eta\nu & 0 \\ \eta\nu & 1 & 0 \\ 0 & 0 & \mu\dfrac{1-\eta\nu^2}{2(1+\nu)} \end{bmatrix} \tag{2.15}$$

Where

$$\eta = \frac{E_n}{E}; \qquad \mu = \frac{1+\nu}{1-\eta\nu^2}\left(\frac{\eta\varepsilon_n - \varepsilon_s}{\varepsilon_n - \varepsilon_s} - \eta\nu\right)(0 \le \mu \le 1)$$

ε_n and ε_s normal strain components in the local axis normal to and parallel with the fractured plane respectively.

The local constitutive relationship matrix $[D]_{ns}$ can be transformed to the global coordinate directions as follows:

$$[D]_{xy} = [T]^T\,[D]_{ns}\,[T] \tag{2.16}$$

Where

[T] strain transformation matrix defined as follows in terms of the inclination of the normal to a crack plane, θ:

$$[T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & \cos\theta\sin\theta \\ \sin^2\theta & \cos^2\theta & -\cos\theta\sin\theta \\ -2\cos\theta\sin\theta & 2\cos\theta\sin\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \tag{2.17}$$

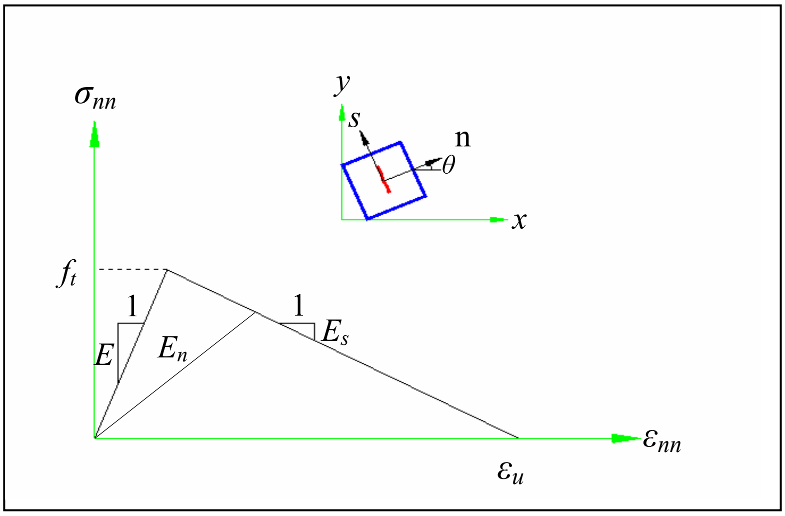

Figure 2.9 - Stress-strain diagram in local coordinates for smeared crack model 7

- **Other smeared models**

Rots (2002) also listed and elaborated on three other smeared crack models with their merits and demerits.

- Total-strain based model (Feenstra *et al*. 1998): In this model, material is modelled by stress-total strain relations.
- Plasticity based model (Feenstra 1993): The tension and compression of the model are approached in a unified way.
- Micro-plane crack model (Bažant & Oh 1985): The model is similar to the non-orthogonal crack model.

Two other smeared crack models were also proposed (Planas, Elices, Guinea, Gomez, Cendon & Arbilla 2003):

- Strong singularity crack model (Oliver *et al*. 2002): This model is similar to the classical local models (such as the crack band model) with an improvement in the strong singularity kinematics, which is able to make a jump in displacements appear naturally in a solution of the continuum approach.

- Gradient crack model (Peerlings *et al*. 2001): This model is similar to the non-local model. It assumes that the stress at a material point is derived from both the strain at that point and its spatial derivatives.

To summarize the available crack models, the flow chart in Figure 2.10 categorizes and lists them into a systematic way.

2.5.5 Summary of crack models discussed

Since the late 1960s, many concrete crack propagation criteria have been developed and applied to analyze cracking in concrete structures. The early strength-based criterion has seldom been used in recent analyses due to its inherent lack of mesh objectivity. LEFM has been widely used in the analysis of concrete dams, in particular gravity dams, as shown in Section 2.7 below. NLFM manages to account for the FPZ in front of the crack tip, providing improved modelling of cracking in concrete. Most of the recent models proposed in the literature are based on NLFM.

Concrete dams are huge structures and models requiring a fine FE mesh, such as the CBM (Bažant & Oh 1983) and non-local models (e.g. Bažant & Lin 1988), are not recommended. The use of a cohesive (fictitious) discrete crack model seems to be gaining popularity, although the computational costs are very high. The non-orthogonal smeared crack model proposed by de Borst & Nauta (1985) appears to be very promising due to its ability to handle simultaneously non-linear concrete behaviour and cracking, non-orthogonal multi-crack formation and crack rotation, and due to it having no stringent mesh size requirement.

Some features that should be considered in concrete cracking models are briefly discussed below.

- **Mixed mode:** In the papers by Planas *et al*. (2003) and Rots & de Borst (1987), although it is pointed out that fractures predominantly form and propagate in mode I, both sets of researchers agree that pure mode I fractures do not occur, which means that mode II cannot be totally ignored.

Galvez *et al.* (2002) further investigated mixed-mode fracturing and their numerical results agreed quite well with those from two experimental sets of mixed-mode fracture of concrete beams. The benchmark results showed that a mode II parameter change has little influence on the numerical predictions. They suggested further research on the influence of the parameters of mode II in the mixed-mode (I/II) fracture of concrete.

- **Crack direction:** The direction of crack propagation has been determined predominantly in the literature to be orthogonal to the direction of maximum principal stress or strain. Martha, Llorca, Ingraffea & Elices (1991) described and compared three crack-direction criteria, namely (i) maximum circumferential stress theory, (ii) minimum strain energy density theory and (iii) maximum energy release rate theory, and concluded that a suitable criterion had still not been found and that further research was necessary. Feng, Pekau & Zhang (1996) adopted the strain energy density factor criterion, which assumes that the direction of crack propagation is towards the minimum region of strain energy density factor. Two assumptions (hypotheses) had to be made in order to obtain a simplified model of 3-D crack propagation for arch dams.

- **Coupling effect:** In most of the cracking models, the coupling effect between the shear stiffness and normal stiffness is generally ignored due to the fact that most applications are restricted to small crack strains. 2-D modelling of the crack shear transferred in rough cracks and the influence of the crack width and normal stresses, etc. on crack shear has been done by various researchers – Bažant & Gambarova (1984); Riggs & Powell (1986); Yoshikawa, Wu & Tanabe (1989) and Divakar & Fafitis (1992), etc. To the authors' knowledge, 3-D crack shear modelling is still an untouched area, at least in the field of concrete dams.

- **Crack closing and reopening:** Most crack models adopt the secant modulus approach for unloading (crack closing). For reloading, the constitutive models follow the same route of unloading until the normal strain in the crack exceeds the previously reached strain. Bhattacharjee & Leger (1992) reviewed a few available studies on this matter and suggested further rational investigation.

2.5.6 Shear resistance of fractured concrete

Fracture could lead to a significant change in the direction of the principal stresses. Aggregate interlocking on the rough crack surfaces results in the development of shear stresses in the cracked concrete. Shear transfer along rough cracks is known to be pressure-sensitive and to cause dilation. Three boundary conditions for the normal pressure imposed on a rough crack are usually considered in the modelling of shear resistance in a crack. They are:

- Constant normal stress condition
- Constant crack width condition
- Variable normal stress and crack width conditions.

Divakar & Fafitis (1992) developed a micro-mechanical interface shear model to predict the shear transfer under the above three boundary conditions. Four mechanisms (sliding; interlocking, overriding and fracturing) of shear transfer were considered. This micro-mechanical model, which took into account the internal structure of the material and the nature of the rough surface, was satisfactorily verified by the experimental results. Bažant & Gambarova (1984) introduced a crack band micro-plane model to describe crack shear in concrete.

A simple shear retention factor, β, is often used to reduce the shear modulus in the constitutive matrix. However, this ignores shear dilation and the dependence of shear on the crack opening displacement. A constant shear modulus fails to account for the variation in shear strain, caused by the strain normal to the crack, which has been observed experimentally. To overcome this problem, Rots & de Borst (1987) adopted a bilinear shear modulus approach and Balakrishnan & Murray (1988) proposed a method of decreasing the shear modulus linearly with increasing normal strain.

2.5.7 Post-crack behaviour

The post-fracture behaviour forms an important part of the crack constitutive model. A crack is normally assumed to propagate in a direction perpendicular to the maximum principal stress that initiated the first cracking. Fixed orthogonal crack models assume that an additional crack plane will only form orthogonally to the first crack plane. Later developments (Rots & Blaauwendraad 1989) resulted in the rotating crack model and the fixed, multi-directional (non-orthogonal) crack model.

Cervera, Oliver & Herrero (1990) presented an elastic-fracturing constitutive model, of which the post-fracture concrete behaviour is the most important part, for progressive cracking in large dams due to the swelling of concrete. The model allows for one or two independent sets of cracks appearing at the same point in two scenarios, as follows:

- Two cracks will be orthogonal if they are formed simultaneously and no further cracking will be allowed at that point.
- If one crack occurs first, further loading may cause a sequential, second crack to occur, which can be allowed to form in a non-orthogonal direction to the first (primary) crack.

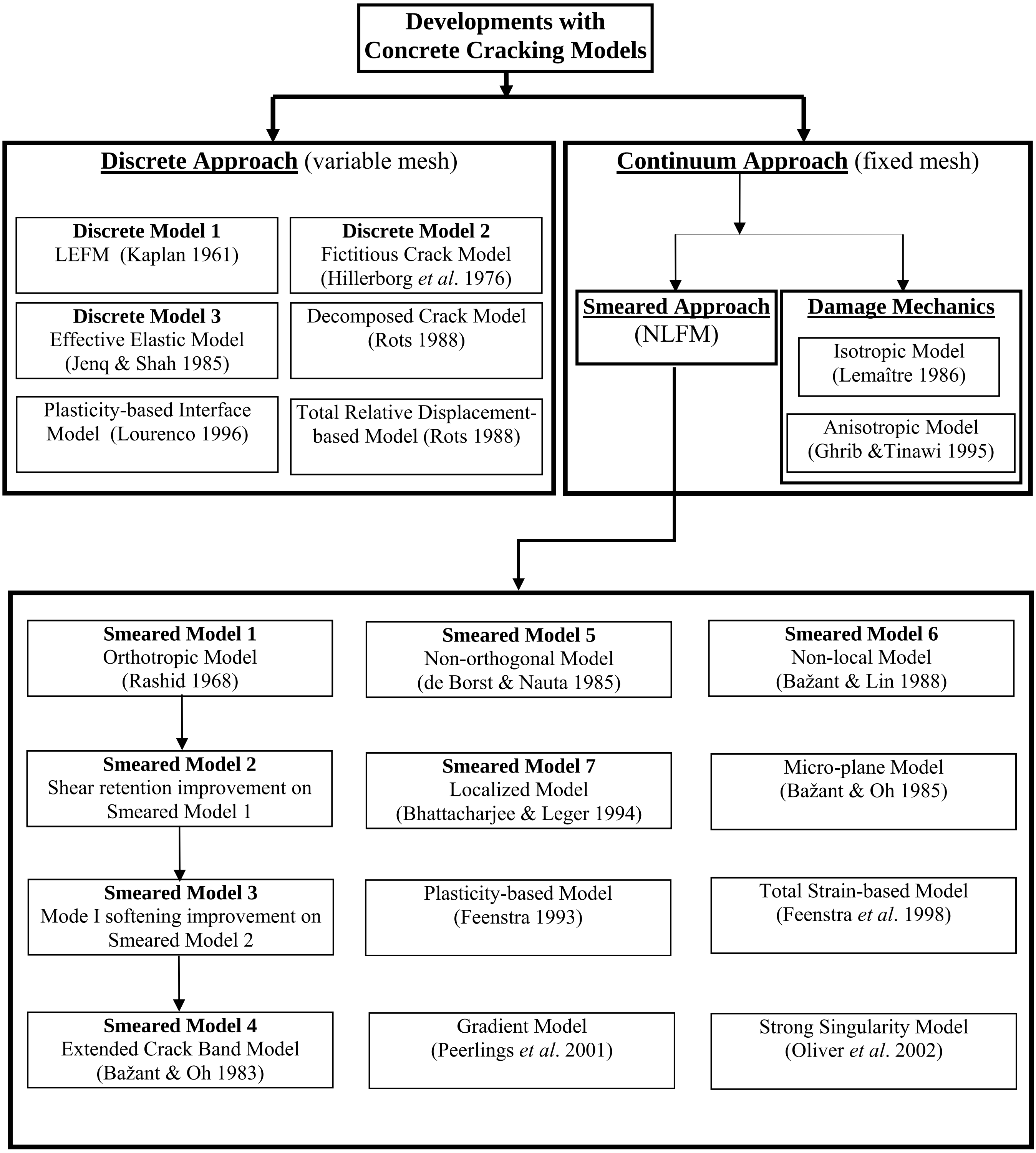

Figure 2.10 - Flowchart of overall cracking models proposed for concrete fracture

2.6 Fracture energy G_f of dam concrete

Accurate determination of the fracture energy of concrete, especially dam concrete, is not an easy task because the amount of the fracture energy G_f will vary with many factors, namely type and size of specimen, type of aggregates, maximum grain size, concrete strength and moisture, type of cement and additives, loading rate, etc. Dam concrete is usually different from normal concrete in the following ways:

- Large aggregate size: maximum aggregate size is usually 100 ~ 150 mm
- Low water-cement ratio: to improve strength
- Low cement content: to reduce thermal cracking and shrinkage during curing.

Very high discrepancies of the fracture energy G_f of dam concrete have been reported, as discussed below.

Trunk & Wittmann (1998) conducted a series of tests on normal and dam concrete with different specimen sizes up to 3 200 mm. For normal concrete, a fracture energy G_f of 121 ~ 322 N/m was obtained. For dam concrete, a fracture energy G_f of 219 ~ 482 N/m was determined.

Brühwiler (1988) carried out wedge-splitting tests on different specimens – cylindrical, drilled cores from three existing concrete dams. Fracture energies G_f of 175, 235 and 257 N/m were obtained for these three dam concrete specimens which had diameters of 200 ~ 300 mm.

Espandar & Lotfi (2000) adopted a fracture energy G_f of 600 N/m in the analysis of the Shahid Rajaee arch dam in Iran.

Espandar & Lotfi (2003) stated that RILEM gave a fracture energy G_f in the range of 70 ~ 200 N/m for normal concrete and suggested that the fracture energy of dam concrete should be three times higher than that of normal concrete. A higher fracture energy has often been used in the practical analysis of dam concrete. Fracture energies G_f of as high as 1 375 and 2 200 N/m have been seen to be used before.

Bhattacharjee & Leger (1994) pointed out that the fracture energy generally observed for dam concrete is typically in the range of 100 ~ 200 N/m.

The ICOLD report (2001) stated that fracture energy increases with the maximum aggregate size. For normal concrete, if the maximum grain size is in the range of 2 ~ 38 mm, the fracture energy was found to be in the range of 50 ~ 200 N/m. Fracture energies of up to 280 N/m were obtained for the maximum grain size of 76 mm. The maximum aggregate size normally used in concrete dams is in the range of 100 ~ 150 mm, which would result in an even higher fracture energy.

Saouma, Broz, Bruhwiler & Boggs (1991) obtained fracture energy G_f of 80 ~ 140 N/m for dam concrete.

The fracture energy of concrete increases with the size of the specimens and becomes a constant value after a critical large specimen size is reached. Therefore, sufficiently large specimens have to be used for any experiments to accurately determine the fracture energy. The fracture energy obtained on small specimens needs to be corrected for the size effect.

Bažant and his co-workers (Bažant, Kim & Pfeiffer 1986; Bažant & Pfeiffer 1987) proposed a 'size effect law' for the correct determination of the fracture energy G_f as follows:

$$G_f = \frac{g(\alpha)}{AE} \qquad (\alpha = \frac{a_0}{d}) \qquad (2.18)$$

Where

$g(\alpha)$ the non-dimensionalized energy-release rate, known for the chosen specimen shape from LEFM

A the slope of the size-effect regression line of σ_N^{-2} versus d

σ_N nominal stress at maximum load

d the characteristic dimension (depth) of the specimen

a_0 the traction-free crack length (notch length).

Linsbauer (1991) carried out the wedge-splitting tests on drilling core samples of dam concrete with two diameters, 150 mm and 190 mm. The diameter of each sample allowed ten drilling cores to be tested. A large spread of fracture energies were reported to have been obtained from the experiment. For the samples of 150 mm in diameter, the fracture energy ranged from 59.9 to 177.3 N/m, with an average value of 109.1 N/m. The larger samples of 190 mm in diameter samples gave fracture energies in the range of 109.1 to 230.8 N/m, with an average value of 155.4 N/m.

He, Plesha, Rowlands & Bažant (1992) also carried out large-scale wedge-splitting compact tension tests on dam concrete in order to study the fracture mechanics properties of dam concrete for different loading rates and specimen sizes. Dam concrete specimens were cast with a maximum aggregate size of 76 mm. The size-effect law presented by Bažant *et al*. (1986) discussed above was adopted to compute the fracture energy of the dam concrete from the test data. The experimental results showed that the fracture energy falls within the range of 200 to 300 N/m for dam concrete.

It is also reported from experiments that the fracture energy G_f increases with the compressive strength of the concrete and the loading rate.

Three test methods are usually employed for the determination of the concrete fracture energy G_f, namely the uniaxial tensile test, the bending test and the wedge-splitting test. The double cantilever-beam test, the compact tension specimen test and the double torsion specimen test have also been used in the past.

In conclusion, the fracture energy of dam concrete with large maximum aggregates is much higher than that of normal concrete. The past investigations and experiments yielded huge discrepancies in the magnitudes of the fracture energy of the dam concrete (mostly between 80 and 600 N/m), which would make the choice of fracture energy for the crack modelling of concrete dams a rather uncertain matter. Therefore, a sensitivity study on this fracture parameter should be considered in the fracture analysis of a concrete dam. The fracture energy G_f = 100 ~ 300 N/m is probably the most possible value for concrete dams.

2.7 Past investigations of the static cracking problems of concrete gravity dams

Over the past decades, many attempts have been made to investigate the cracking problems in concrete gravity dams by using various cracking analysis methods. Discrete LEFM seems to be the most popular approach for dam fracture analysis and it is used extensively in modelling the cracking of gravity dams. Ayari (1988), in his PhD study, adopted the discrete LEFM approach in analyzing the static fracture response of concrete gravity dams.

Chappell & Ingraffea (1981) used LEFM to model fracture in the Fontana gravity dam (USA) which had experienced the first traces of cracking in 1949. The cracking problem in this dam was formally acknowledged in 1972. Reasonably accurate predictions of crack trajectory and stability were obtained (SIMSCIENCE website). Again, Ingraffea (1990) used a 2-D mixed-mode, discrete LEFM model as a forensic tool to analyze crack propagation in the Fontana Dam. The observed crack profile, which started from the middle of the downstream face (caused by a combination of cyclic, reversible thermal expansion and concrete growth due to the alkali-silica reaction) and then dipped down through the gallery, was reproduced by this mixed-mode LEFM crack analysis. He further used the method in a generic gravity dam for the purpose of evaluating the dam's design and analyzing its stability. The factor of safety against sliding predicted by LEFM is, in general, less conservative (i.e. has a higher value) than that of the classical method.

Linsbauer (1990) developed a diagram (critical crack vs. depth of crack level and fracture toughness) based on LEFM for assessing cracking in gravity dams which can be applied in determining "the stability of horizontal cracks in the top three quarters of any gravity dam" with a triangular dam profile of width-to-height ratio of 0.8. The value of LEFM in analyzing cracking in concrete dams has therefore been demonstrated.

Gioia, Bažant & Pohl (1992) carried out a 2-D mixed-mode discrete LEFM analysis on an identical model of the Koyna Dam – a concrete gravity dam in India. FRANC – a discrete

LEFM program, was adopted for the prediction of crack growth. Curves of overflow-displacement at the top were plotted to compare the results obtained from analyses of no-tension, plasticity and fracture mechanics. The conclusions drawn from the analysis are that the classical no-tension design criterion is not always safe and that the safety of dams should be evaluated on the basis of fracture mechanics.

Kumar & Nayak (1994) carried out a 2-D mixed-mode, discrete LEFM analysis in a case study of the Lakhwar gravity dam. Seven load cases with six different cracks at different locations were used to study the effect of parameters such as dam height, B/H (base-height ratio), type of singularity at the heel, etc. The results show that the most significant parameters affecting the tensile stress and the stress intensity factor are E_R/E_C (ratio of Young's modulus of foundation rock and concrete), B/H and the type of singularity at the heel. LEFM can "successfully" determine the location of cracks.

Plizzari, Waggoner & Saouma (1995) experimentally tested and numerically analyzed (using LEFM) cracking in gravity dam models in order to establish a centrifugal testing technique for modelling fracturing in concrete gravity dams.

Plizzari (1997) further used LEFM to predict crack propagation in concrete gravity dams. A parametric study of triangular-shaped dams was performed, assuming a horizontal crack at the dam/foundation interface. He proposed a scale law to determine the maximum hydrostatic pressure that a cracked gravity dam can carry.

Compared with the wide applications of LEFM in the analysis of the cracking of concrete dam structures, the NLFM criterion was applied much less in the past to concrete structures, apparently due to the complexity of applying it. Nevertheless, Bhattacharjee & Leger (1994) used the NLFM criterion in a 2-D smeared crack model for analyzing a model gravity dam and the Koyna gravity dam in India. A rotating crack model and a fixed crack model with variable shear resistance factors (FCM-VSRF) were used to analyze the Koyna Dam. The relation of overflow and displacement at the top was plotted to allow comparison with the results of LEFM and plasticity analyses. The crack profile predicted was good. Sensitivity studies of the response to fracture parameters, such as fracture energy and initial crack depth, and to different crack models (rotating or fixed) were carried out. The FCM-VSRF model normally provides a stiffer response due to significant stress-locking. The fracture energy G_f and the initial crack depth a_0 do not have much influence on the ultimate structural response.

Bhattacharjee & Leger (1995) again employed a rotating smeared crack model to predict the static fracture behaviour of the Koyna Dam. They proposed the concept of 'effective porosity' to model the water intrusion and the consequent uplift pressure inside the cracks which was found to greatly reduce the ultimate resistance of the dam structure. Comparison with the conventional 'no-tension' gravity method revealed that this method of analysis for concrete dams may not always be as conservative as is usually thought.

Ghrib & Tinawi (1995) presented damage mechanics models based on anisotropic formulation to predict the static response of concrete dams. A 1:40 reduced model of a gravity dam tested by Carpinteri, Valente, Ferrara & Imperato (1992) was used to verify the accuracy of the proposed model. The damage models provide an accurate prediction of the ultimate load. They also provide crack profiles "similar" to the experimental results. The Koyna Dam was also used to compare the proposed damage mechanics models with the other numerical investigations under overflow hydrostatic loading. It is stated that the proposed models are mesh-objective and accurate, and can be used for assessing the ultimate capacity of a concrete dam and the dam's safety margin.

Saouma & Morris (1998) used a 2-D LEFM and NLFM interface crack model (ICM) to analyze the Greyrock gravity dam in the USA. The program MERLIN was coded so that the criterion for crack propagation was a strength-based one in which the tensile stress could not exceed the tensile strength. Fracture mechanics analysis was employed to evaluate the dam safety and to highlight the need for practice engineers to accept this method for evaluating dam safety. Two crack orientations – straight and kinking – were considered. The analysis based on fracture mechanics revealed that the classical rigid body equilibrium is more conservative and that it would be more economical to use the method based on fracture mechanics.

Araújo & Awruch (1998) adopted both discrete NLFM crack model and continuum damage theory in an analysis of the cracking of the Tucuruí gravity dam in Brazil, which was due to thermal effects during the construction phase, and verified the dam's safety against cracking.

Cervera *et al*. (1990) used 2-D and 3-D elastic-fracturing models in the analysis of a 79-m-high gravity dam in Spain, called the Mequinenza Dam, which had experienced cracking on the upstream wall and the interior corridors. Fracture due to swelling of the concrete was modelled, including the mechanism of water intrusion, concrete extension, tensile fracturing and seasonal thermal straining.

Cervera, Oliver & Galindo (1992) again developed an FE constitutive model to study cracking due to concrete hydration in the large Mequinenza gravity dam. For short-term behaviour, a continuum damage constitutive model was used. For long-term creep behaviour, visco-elastic effects were modelled and considered. Good agreement between the numerical simulation and the measurements was obtained.

Barpi & Valente (2001) used a 1:40-scale gravity dam model previously tested by Carpinteri *et al*. (1992) to verify the capability of the cohesive crack model to correctly predict the size effect, using fuzzy parameters. They found that the cohesive crack model could indeed predict the size effect, thus explaining the behaviour of a dam model based on experimental results from much smaller specimens.

Shi, Suzuki & Nakano (2003) used an extended fictitious crack model to model multiple cracks in concrete dams. Only mode I cracks were considered for simplicity. Bi-linear strain softening was adopted. Three FE models of generic gravity dams with initial notches were used to verify the analytical procedure developed for the cracking analysis of concrete dams. They demonstrated that the model is able to investigate the ultimate response of concrete dams, identify the potentially damaging cracks and predict the crack profile, without any restrictions on the numbers and locations of cracks.

Saouma *et al*. (1990) gave a detailed review of the application of fracture mechanics to concrete dams. The applicability of various models to concrete dams, and the limitations of the models, were discussed and practical examples of fracture mechanics models were presented. Compared with the classical method, the fracture mechanics approach is less conservative and the results are less sensitive to material properties.

2.8 Concluding remarks and recommendations

It is a demanding and challenging task to develop a constitutive model that effectively includes all the characteristics of cracking in concrete, yet remains sufficiently simple for practical implementation. An accurate model should include the tortuous crack path and the non-linear inelastic material behaviour in the FPZ.

Constitutive modelling of the tensile resistance of concrete using the FE method has progressed from conventional strength-based models to models based on fracture mechanics and energy-conserving principles. Current trends in research show a movement from 2-D to 3-D modelling, from LEFM to NLFM, and from a single crack with a predefined location to multiple cracks with unbiased location and taking into consideration the effect of water pressure inside the cracks. The complexity of the problem seems to be the reason why a generally accepted 3-D fracture model for concrete has not yet been presented. However, the constitutive model proposed by de Borst & Nauta (1985) appears to be very promising.

From the literature review, it can be concluded that further research is needed into developing a constitutive model of concrete dams, to be implemented into FE analysis. The following aspects should be addressed:

- The fracture strain softening has been well studied, but further attention should be given to the multi-linear and non-linear mode I softening diagram for the concrete normally used in concrete dams.
- The post-cracking treatment of shear in the constitutive model requires more attention. The influence of normal displacements (or strains) on the shear modulus should be investigated both experimentally and numerically.
- Mixed-mode criteria should be used to address the complex stresses often encountered in dams.

- The smeared crack model is recommended for incorporation into the existing FE programs.
- More research effort into crack propagation direction criteria in concrete dams is needed.