UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

FINITE ELEMENT MODELLING OF CRACKING IN CONCRETE GRAVITY DAMS

Q. CAI

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Finite element modelling of cracking in concrete gravity dams

QINGBO CAI

A thesis submitted in partial fulfilment of the requirements for the degree of

PHILOSOPHIAE DOCTOR (ENGINEERING)

In the

FACULTY OF ENGINEERING, BUILT ENVIRONMENT AND INFORMATION TECHNOLOGY

UNIVERSITY OF PRETORIA

June 2007

THESIS SUMMARY

Finite element modelling of cracking in concrete gravity dams

by

Q. CAI

Supervisor:	Professor B.W.J. van Rensburg
Co-Supervisor:	Dr. J.M. Robberts
Department:	Civil Engineering
University:	University of Pretoria
Degree:	Philosophiae Doctor (Engineering)

Evaluating the safety of unreinforced concrete structures, such as concrete dams, requires an accurate prediction of cracking. Developing a suitable constitutive material model and a reliable computational procedure for analysing cracking processes in concrete has been a challenging and demanding task.

Although many analytical methods based on fracture mechanics have been proposed for concrete dams in the last few decades, they have not yet become part of standard design procedures. Few of the current research findings are being implemented by practising engineers when evaluating dam safety.

This research is focused on the development of a suitable crack modelling and analysis method for the prediction and study of fracturing in concrete gravity dams, and consequently, for the evaluation of dam safety against cracking. The research aims to contribute to the continuing research efforts into mastering the mechanics of cracking in concrete dams.

An analytical method for the purpose of establishing a crack constitutive model and implementing the model for the fracture analysis of concrete structures, in particular massive concrete gravity dams under static loading conditions, has been developed, verified and applied in the safety evaluation of a concrete gravity dam.

The constitutive material model is based on non-linear fracture mechanics and assumes a bilinear softening response. The crack model has various improved features: (1) an enhanced mode I bilinear strain-softening approach has been put forward; (2) a new formula for bilinear softening parameters has been developed and their relation with linear softening has been outlined; (3) the influence of bilinear softening parameters on the cracking response has been studied; and (4) an enhanced modification to the shear retention factor which depends on the crack normal strain is included.

The material model has been incorporated into a finite element analysis using a smeared crack approach. A sub-program was specially coded for this research.

The validity of the proposed cracking model and the computational procedure developed for the purpose of analyzing the tensile fracture behaviour of concrete structures has been confirmed by verification on various concrete structures, including beams, a dam model and actual gravity dams.

The crack modelling technique developed was successfully used in evaluating the safety of an existing concrete gravity dam in South Africa and adequately predicted the cracking response of the dam structure under static loadings.

The main conclusions drawn are as follows:

- Both mode I and mode II fracture have been modelled successfully.
- The proposed bilinear softening model remains relatively simple to implement but significantly improves on predicting the softening response of “small-scale” concrete structures.
- Both plane stress and plane strain crack analyses have been considered and can be confidently adopted in two-dimensional applications.

- The proposed method is mesh objective.
- The crack modelling method developed can correctly predict the crack propagation trajectory and the structural behaviour with regard to fracturing in concrete structures.
- If not considering shear stress concentration near the tip of a crack, constitutive crack analysis normally indicates a higher safety factor and a higher Imminent Failure Flood (IFF) than the classical methods in the analysis of concrete gravity dams for safety evaluation.

Keyterms: Concrete gravity dams, constitutive crack model, non-linear fracture mechanics, crack modeling, dam safety, computational procedure, crack propagation, bilinear softening, smeared crack approach.

ACKNOWLEDGEMENTS

I wish to express my appreciation to the following organization and people who made this thesis possible:

(a) Professor B.W.J. van Rensburg, my supervisor, and Dr. J.M. Robberts, my co-supervisor, for their constant guidance, profound interest in and valuable advice with this difficult research topic.

(b) Dr. C. Oosthuizen for his support and encouragement during the course of the study.

(c) Mr. P. Nightingale for his assistance on finding the information on the Van Ryneveld's Pass Dam.

(d) My family for their support, sacrifices and patience during the study.

(e) The permission of the Director-General of the Department of Water Affairs and Forestry (DWAF) to publish this thesis is gratefully acknowledged. The views expressed are those of the author, and not necessarily those of the Department.

TABLE OF CONTENTS

THESIS SUMMARY **1**

ACKNOWLEDGEMENTS **4**

TABLE OF CONTENTS **5**

LIST OF TABLES **8**

LIST OF FIGURES **9**

NOTATION **15**

CHAPTER I INTRODUCTION **21**

1.1 Background and overview 21
1.2 Motivations and objectives of this study 25
1.3 Scope of this study 25
1.4 Methodology of this study 26
1.5 Organization of this study 26

CHAPTER II LITERATURE REVIEW ON GRAVITY DAM DESIGN AND ON THE DEVELOPMENT IN FRACTURE ANALYSIS OF CONCRETE DAMS **29**

2.1 Causes of cracking in concrete gravity dams 30
2.2 Brief description of methods of analysis and design criteria for concrete gravity dams 30
2.3 Analysis of cracking in concrete dams 34
2.4 Finite element approaches for modelling cracking in concrete 38
2.5 Crack modelling of concrete 39

    2.5.1 Pre-fracture material stress-strain behaviour 39
    2.5.2 Crack initiation 40
    2.5.3 Crack propagation criteria 42
    2.5.4 Crack models 44
    2.5.5 Summary of crack models discussed. 55
    2.5.6 Shear resistance of fractured concrete 57
    2.5.7 Post-crack behaviour. 57

2.6 Fracture energy G_f of dam concrete 60
2.7 Past investigations of the static cracking problems of concrete gravity dams 63
2.8 Concluding remarks and recommendations 67

CHAPTER III CONSTITUTIVE MODELS AND PARAMETERS STUDY **69**

3.1 Pre-softening constitutive relationship 69
3.2 Crack onset criterion and crack direction 71
3.3 Constitutive relationship during concrete cracking 72

    3.3.1 Plane stress application used in this research. 78

3.3.2 Plane strain application used in this research. 79

3.4 Mode I tensile softening 80
3.5 Mode II shear softening 84
3.6 Fixed/rotating, unloading/reloading and closing/reopening of cracks 85
3.7 Width of crack blunt front and mesh objectivity 89
3.8 Element selection for crack analysis 91
3.9 Concluding remarks 91

CHAPTER IV NUMERICAL TECHNIQUE AND PROGRAM FOR FINITE ELEMENT CONSTITUTIVE CRACKING ANALYSIS 93

4.1 Program framework for the cracking analysis of concrete 94

4.1.1 Framework for the implementation of the constitutive model in the FE analysis of concrete structures 94
4.1.2 Sub-pragram coded in MSC.Marc to implement the crack constitutive model. 95
4.1.3 Possible numerical implementation problems 99

4.2 Verification study with MSC.Marc and other specimens investigated in the past 103

4.2.1 Built-in crack model in MSC.Marc for specimens 1 and 2 103
4.2.2 The smeared model adopted for specimens 1 and 2 104
4.2.3 The smeared model adopted for specimens 3 and 4 105
4.2.4 FE models benchmarked 105
4.2.5 Discussion of results of the verification 113

4.3 Verification study with DIANA 119

4.3.1 Cracking with linear tensile softening – plane strain 121
4.3.2 Cracking with bilinear tensile softening – plane strain 121
4.3.3 Cracking with alternating loading – plane strain 122

4.4 Concluding remarks 123

CHAPTER V STATIC FRACTURE ANALYSIS OF CONCRETE STRUCTURES 125

5.1 Introduction 125
5.2 Case 1: three point, centre-loaded, single-notched beam 126
5.3 Case 2: single-notched shear beam 132
5.4 Case 3: mesh objectivity and second-order elements validation 138
5.5 Conclusion 146

CHAPTER VI STATIC FRACTURE ANALYSIS OF CONCRETE GRAVITY DAMS 148

6.1 Introduction 148
6.2 Model concrete dam 149
6.3 A concrete gravity dam adopted by NW-IALAD 153
6.4 Koyna Dam 158

CHAPTER VII SAFETY EVALUATION OF A CONCRETE GRAVITY DAM IN SOUTH AFRICA BASED ON FRACTURE ANALYSIS 172

7.1 Introduction 172

7.2 Description of the gravity dam and finite element model 172
7.3 Material properties and constitutive fracture parameters 175
7.4 Bilinear strain-softening shape parameters 176
7.5 Fracture analysis and evaluation of the dam safety 180

    7.5.1 Parametric study on the fracture energy of concrete and rock 181
    7.5.2 Parametric study on the bilinear shape parameters α_1 and α_2 184
    7.5.3 Parametric study on the tensile strength of concrete and rock 190
    7.5.4 Parametric study on the crack onset threshold angle ϕ 192
    7.5.5 Parametric study on the maximum shear retention factor 194
    7.5.6 Comparison with linear elastic and plasticity analyses 196

7.6 Evaluation of dam safety against sliding (shear) 200
7.7 Conclusions 200

CHAPTER VIII CONCLUSIONS AND RECOMMDATIONS 203

8.1 Conclusions 204
8.2 Recommendations 207
8.3 Closure 208

ANNEXURE 209

REFERENCES/BIBLIOGRAPHY 217

LIST OF TABLES

Table 2.1 Definition of load combinations in South Africa 32
Table 2.2 Design criteria for normal stresses in concrete gravity dams (South Africa) 33
Table 2.3 Design criteria for safety against sliding in concrete gravity dams (South Africa) 34
Table 3.1 Direction cosines of local axes in global axis 74
Table 3.2 Direction cosines of local axes in global axis (2-D) 75
Table 5.1 Loads from elastic bending theory and FE analyses for different mesh finenesses – first-order elements 139
Table 5.2 Loads from elastic bending theory and FE analyses for different mesh finenesses – second-order elements 140
Table 6.1 Model parameters (model dam) 149
Table 6.2 Model parameters (NW-IALAD) 154
Table 6.3 Model parameters (Koyna Dam) 159
Table 7.1 Material properties of concrete and rock 176

LIST OF FIGURES

Figure 1.1 Outline of the research 28
Figure 2.1 Forces acting on a gravity dam 31
Figure 2.2 Diagram of the forces and stresses used in the classical analysis method for a concrete gravity dam 36
Figure 2.3 Fracture process zone in LEFM and NLFM (Bhattacharjee & Leger 1992) 37
Figure 2.4 Crack initiation criterion (Bhattacharjee & Leger 1994) 41
Figure 2.5 Modes of fracture 43
Figure 2.6 Crack in an arbitrary body and coordinate system (LEFM) 45
Figure 2.7 Representative NLFM discrete and smeared crack models (Bhattacharjee & Leger 1992) 46
Figure 2.8 Stress-strain diagram for the crack band model 50
Figure 2.9 Stress-strain diagram in local coordinates for smeared crack model 7 54
Figure 2.10 Flowchart of overall cracking models proposed for concrete fracture 59
Figure 3.1 Crack direction and local axis system for 2-D and 3-D applications 71
Figure 3.2 Crack initiation criteria for a 2-D application 72
Figure 3.3 Coordinate system and traction vectors across a crack for 3-D application 73
Figure 3.4 Linear, bilinear and curved mode I strain-softening diagram of “crack” 82
Figure 3.5 Linear elastic – mode I strain-softening diagram of cracked concrete 83
Figure 3.6 Definition of bilinear mode I strain-softening diagram of “crack” 83
Figure 3.7 Bilinear mode I strain-softening diagrams for $\alpha_1 = 1/3$; $\alpha_2 = 0.1$, 0.2 and 0.3 (local coordinate) 84
Figure 3.8 Relationship between shear retention factor and “crack” strain (local coordinate) 85
Figure 3.9 Diagram of unloading/reloading and closing/reopening (in crack strain) 88
Figure 3.10 Diagram of unloading/reloading and closing/reopening (in total strain) 88
Figure 3.11 Crack characteristic length h_c of a quadrilateral element (first order with full integration) 90
Figure 3.12 Quadrilateral element of first order with full integration used in the research 91
Figure 4.1 General FE crack analysis procedure for concrete structures 94
Figure 4.2 Flow chart of the overall organization for coding the sub-program HYPELA 101
Figure 4.3 Flow diagram for finite element analysis process in MSC.Marc 102
Figure 4.4 Uniaxial stress-strain diagram 104
Figure 4.5 First-order plane stress element with full integration 106
Figure 4.6 FE model and model input (specimen 1) 108
Figure 4.7 Applied displacement load vs. time (specimen 1) 108
Figure 4.8 FE model – beam of four elements (specimen 2) 109
Figure 4.9 Only one element softening (specimen 2) 109

Figure 4.10 Applied load vs. time (specimen 2) 109
Figure 4.11 Strain-softening diagram (specimen 3) 110
Figure 4.12 Applied load vs. time (specimen 3) 111
Figure 4.13 Scenario 1: One element 111
Figure 4.14 Scenario 2: Two elements 111
Figure 4.15 Scenario 3: Three elements 111
Figure 4.16 Scenario 4: Four elements 111
Figure 4.17 Scenario 5: Five elements 111
Figure 4.18 FE model – beam of 16 elements (specimen 4) 112
Figure 4.19 Strain-softening diagram (specimen 4) 112
Figure 4.20 Applied load vs. time (specimen 4) 112
Figure 4.21 Only the elements adjacent to rigid boundary softening (specimen 4) 113
Figure 4.22 Stress-strain plots for softening modulus E_s = -2 000 MPa (specimen 1) 114
Figure 4.23 Stress-strain plots for softening modulus E_s = -20 000 MPa (specimen 1) 114
Figure 4.24 Stress-strain plots for softening modulus E_s = -50 000 MPa (specimen 1) 115
Figure 4.25 Stress-strain plots (softening modulus E_s = -2 000 MPa) (specimen 2) 116
Figure 4.26 Stress-strain plots (softening modulus E_s = -5 000 MPa) (specimen 2) 116
Figure 4.27 Stress-strain plots (softening modulus E_s = -20 000 MPa) (specimen 2) 117
Figure 4.28 Averaged strain for different numbers of elements in the model (specimen 3) 118
Figure 4.29 Force-displacement response (specimen 4) 119
Figure 4.30 Second-order plane strain element 120
Figure 4.31 Boundary and loading 120
Figure 4.32 Crack stress and crack strain response (PET1CR) 121
Figure 4.33 Crack stress and crack strain response (PET2CR) 122
Figure 4.34 Loading factor f at steps (PECLOP) 122
Figure 4.35 Crack stress and crack strain response (PECLOP) 123
Figure 5.1 Finite element model (Case 1) 129
Figure 5.2 Linear, bilinear and non-linear strain softening 129
Figure 5.3 Load-load point deflection for strain-softening branches in Figure 5.2 130
Figure 5.4 Bilinear strain softening with α_1 = 0.25 and α_2 = 0.1, 0.2 and 0.3 respectively 130
Figure 5.5 Load-load point deflection for strain-softening branches in Figure 5.4 131
Figure 5.6 Bilinear strain softening with α_1 =1/3 and α_2 = 0.1, 0.2 and 0.3 respectively 131
Figure 5.7 Load-load point deflection for strain-softening branches in Figure 5.6 132
Figure 5.8 Finite element model (Case 2) 135
Figure 5.9 Load – CMSD 135
Figure 5.10 Snap-back in load – deflection at point C 136

Figure 5.11 Load – CMOD 136
Figure 5.12 Crack profiles 137
Figure 5.13 Predicted deformation 137
Figure 5.14 Geometric configurations and boundary conditions 140
Figure 5.15 Coarse model 1 – 6 elements in depth 141
Figure 5.16 Medium model 1 – 12 elements in depth 141
Figure 5.17 Fine model 1 – 24 elements in depth 142
Figure 5.18 Comparison of mesh objectivity (models 1) 142
Figure 5.19 Comparison of element objectivity (models 1) 143
Figure 5.20 Coarse model 2 – 6 elements in depth 144
Figure 5.21 Medium model 2 – 12 elements in depth 144
Figure 5.22 Fine model 2 – 24 elements in depth 145
Figure 5.23 Comparison of mesh objectivity (models 2) 146
Figure 6.1 Finite element model of concrete dam model and applied loads 151
Figure 6.2 Strains and crack profiles in the model dam 152
Figure 6.3 Total force vs. CMOD response in the model dam 152
Figure 6.4 Geometric configurations of concrete dam (NW-IALAD) 155
Figure 6.5 Finite element model of concrete dam with rock foundation (NW-IALAD) 156
Figure 6.6 Strain and crack plots for NW-IALAD dam 157
Figure 6.7 Relationship of water level (overflow) vs. crest displacement (NW-IALAD) 158
Figure 6.8 Finite element model of Koyna Dam and applied loads 159
Figure 6.9 Comparison of predicted responses to overflow load for different crack models (G_f = 100 N/m) (Koyna Dam) 163
Figure 6.10 Comparison of predicted responses to overflow load for different crack models (G_f = 200 N/m) (Koyna Dam) 163
Figure 6.11 Influence of fracture energy G_f on predicted structural response for linear softening models (Koyna Dam) 164
Figure 6.12 Influence of fracture energy G_f on predicted structural response for bilinear softening models (Koyna Dam) 164
Figure 6.13 Influence of bilinear softening parameters $\alpha_1 = 0.3$ and $\alpha_2 = 0.1$, 0.2 and 0.3 respectively on predicted structural response (Koyna Dam) 165
Figure 6.14 Influence of bilinear softening parameters $\alpha_1 = 0.4$ and $\alpha_2 = 0.1$, 0.2 and 0.3 respectively on predicted structural response (Koyna Dam) 165
Figure 6.15 Influence of bilinear softening parameters $\alpha_1 = 0.44$ and $\alpha_2 = 0.1$, 0.2 and 0.3 respectively on predicted structural response (Koyna Dam) 166

Figure 6.16 Influence of bilinear softening parameters α_1 = 0.3, 0.4 and 0.44, and α_2 = 0.1 respectively on predicted structural response (Koyna Dam) 166
Figure 6.17 Influence of bilinear softening parameters α_1 = 0.3, 0.4 and 0.44, and α_2 = 0.2 respectively on predicted structural response (Koyna Dam) 167
Figure 6.18 Influence of bilinear softening parameters α_1 = 0.3, 0.4 and 0.44, and α_2 = 0.3 respectively on predicted structural response (Koyna Dam) 167
Figure 6.19 Influence of maximum shear retention factor β_{max} on predicted structural response (Koyna Dam) 168
Figure 6.20 Influence of threshold angle on predicted structural response (Koyna Dam)....................... 168
Figure 6.21 Crack profile (bilinear softening, fracture energy G_f = 200 N/m) (Koyna Dam) 169
Figure 6.22 Crack profile (bilinear softening α_1 = 0.3 and α_2 = 0.2, fracture energy G_f = 100 N/m) (Koyna Dam) 169
Figure 6.23 Crack profile (bilinear softening α_1 = 0.4 and α_2 = 0.1) (Koyna Dam)............................. 170
Figure 6.24 Crack profile (bilinear softening α_1 = 0.4 and α_2 = 0.2) (Koyna Dam)............................. 170
Figure 6.25 Crack profile (bilinear softening α_1 = 0.44 and α_2 = 0.2) (Koyna Dam)........................... 171
Figure 6.26 Crack profile (bilinear softening α_1 = 0.44 and α_2 = 0.3) (Koyna Dam)........................... 171
Figure 7.1 Van Ryneveld’s Pass Dam (view from downstream) 173
Figure 7.2 Finite element model of Van Ryneveld’s Pass Dam 174
Figure 7.3 Finite element model of Van Ryneveld’s Pass Dam (close-up for dam wall) and hydrostatic and sediment loadings applied 175
Figure 7.4 Bilinear strain softening (tensile stress vs. crack opening displacement)............................ 178
Figure 7.5 Bilinear strain softening (tensile stress vs. local crack strain) 178
Figure 7.6 Crest horizontal displacement vs. overflow for various values of fracture energy.............. 183
Figure 7.7 Crack profile for G_f^c = 100 N/m and G_f^r = 400 N/m.. 183
Figure 7.8 Crack profile for G_f^c = 200 N/m and G_f^r = 400 N/m.. 183
Figure 7.9 Crack profile for G_f^c =300 N/m and G_f^r = 400 N/m.. 184
Figure 7.10 Crack profile for G_f^c = 300 N/m and G_f^r = 400 N/m (deformed shape)............................ 184
Figure 7.11 Bilinear softening shapes with α_1 = 0.25 and α_2 = 0.05, 0.1, 0.2 and 0.3.......................... 185
Figure 7.12 Bilinear softening shapes with α_1 = 1/3 and α_2 = 0.05, 0.1, 0.2 and 0.3 185
Figure 7.13 Bilinear softening shapes with α_1 = 0.4 and α_2 = 0.05, 0.1, 0.2 and 0.3........................... 186
Figure 7.14a Crest horizontal displacement vs. overflow level for strain-softening relationships with α_1 = 0.25 and α_2 = 0.05, 0.1, 0.2 and 0.3 187

Figure 7.14b Crest horizontal displacement vs. overflow level for strain-softening relationships with α_1 = 1/3 and α_2 = 0.05, 0.1, 0.2 and 0.3 ... 187
Figure 7.14c Crest horizontal displacement vs. overflow level for strain-softening relationships with α_1 = 0.4 and α_2 = 0.05, 0.1, 0.2 and 0.3 ... 188
Figure 7.15 Crack profile for α_1 = 0.25 and α_2 = 0.05 ... 188
Figure 7.16 Crack profile for α_1 = 0.25 and α_2 = 0.1 ... 188
Figure 7.17 Crack profile for α_1 = 0.25 and α_2 = 0.2 ... 189
Figure 7.18 Crack profile for α_1 = 0.25 and α_2 = 0.3 ... 189
Figure 7.19 Crack profile for α_1 = 1/3 and α_2 = 0.05 ... 189
Figure 7.20 Crack profile for α_1 = 1/3 and α_2 = 0.1 ... 189
Figure 7.21 Crack profile for α_1 = 1/3 and α_2 = 0.2 ... 189
Figure 7.22 Crack profile for α_1 = 1/3 and α_2 = 0.3 ... 189
Figure 7.23 Crack profile for α_1 = 0.4 and α_2 = 0.05 ... 190
Figure 7.24 Crack profile for α_1 = 0.4 and α_2 = 0.1 ... 190
Figure 7.25 Crack profile for α_1 = 0.4 and α_2 = 0.2 ... 190
Figure 7.26 Crack profile for α_1 = 0.4 and α_2 = 0.3 ... 190
Figure 7.27 Crest horizontal displacement vs. overflow level for various values of concrete strength ... 191
Figure 7.28 Crack profile for f_t^c = 0.002 MPa and f_t^r = 2.5 MPa ... 192
Figure 7.29 Crack profile for f_t^c = 0.2 MPa and f_t^r = 2.5 MPa ... 192
Figure 7.30 Crack profile for f_t^c = 1.0 MPa and f_t^r = 2.5 MPa ... 192
Figure 7.31 Crack profile for f_t^c = 1.5 MPa and f_t^r = 2.5 MPa ... 192
Figure 7.32 Crest horizontal displacement vs. overflow level for various threshold angles ... 193
Figure 7.33 Crack profile for threshold angle of 0.1° ... 193
Figure 7.34 Crack profile for threshold angle of 15° ... 193
Figure 7.35 Crack profile for threshold angle of 30° ... 194
Figure 7.36 Crack profile for threshold angle of 45° ... 194
Figure 7.37 Crack profile for threshold angle of 60° ... 194
Figure 7.38 Crest horizontal displacement vs. overflow level for various maximum shear retention factors ... 195
Figure 7.39 Crack profile for β_{max} = 0.05 ... 195
Figure 7.40 Crack profile for β_{max} = 0.1 ... 195
Figure 7.41 Crack profile for β_{max} = 0.2 ... 196

Figure 7.42 Crack profile for $\beta_{max} = 0.3$ 196

Figure 7.43a Crest horizontal displacement vs. overflow level for various analysis methods 197

Figure 7.43b Crest horizontal displacement vs. overflow level for various analysis methods 197

Figure 7.43c Crest horizontal displacement vs. overflow level for various analysis methods 198

Figure 7.44 Crest horizontal displacement vs. overflow 199

Figure 7.45 Crack profile for overflow level at 17 m 199

Figure 7.46 Crack profile at the end of unloading 199

NOTATION

Given below is a list of the principal symbols and notations used in the thesis. All symbols and notations are defined in the text when they appear.

Stresses and Strains

σ_{ij}	Stress tensor
S_{ij}	Stress deviator tensor
σ_m	Mean normal (hydrostatic) stress
σ	Stress
$\sigma_1, \sigma_2, \sigma_3$	Principal stresses
σ_x	Normal stress in x direction
σ_y	Normal stress in y direction
σ_z	Normal stress in z direction
σ_{xy}	Shear stress in xy plane
σ_{yz}	Shear stress in yz plane
σ_{zx}	Shear stress in zx plane
σ_{nn}	Stress normal to crack
σ_{ss}	Stress parallel to crack
σ_{ns}	Shear stress in crack
$\{\sigma\}$	Stress vector in global coordinate
$\{\sigma'\}$	Stress vector in local coordinate
S^{cr}	Crack stresses in local coordinate
S_n^{cr}, S_{nn}^{cr}	Mode I normal stress in local coordinate
S_{ns}^{cr}	Mode II shear stress in local coordinate
S_{nt}^{cr}	Mode III shear stress in local coordinate
ε_{ij}	Strain tensor
ε	Strain

$\varepsilon_1, \varepsilon_2$	Principal strains
ε_x	Normal strain in x direction
ε_y	Normal strain in y direction
ε_z	Normal strain in z direction
ε_{xy}	Shear strain in xy plane
ε_{yz}	Shear strain in yz plane
ε_{zx}	Shear strain in zx plane
ε_u	Ultimate normal tensile strain of no-tension resistance
$\varepsilon_n, \varepsilon_{nn}$	Strain normal to crack
$\varepsilon_s, \varepsilon_{ss}$	Strain parallel to crack
ε_{ns}	Shear strain in crack
ε^{co}	Intact concrete strain in global coordinate
$\varepsilon^{cr}, \varepsilon_i^{cr}$	Crack strain in global coordinate
$\{\varepsilon\}$	Strain vector in global coordinate
$\{\varepsilon'\}$	Strain vector in local coordinate
e_n	Normal strain of cracked concrete in local coordinate
e_n^e	Elastic normal strain of concrete at the tensile strength
e_n^u	Ultimate normal strain of crack concrete
e_n^f	Ultimate normal crack strain in local coordinate
e_i^{cr}	Crack strain in local coordinate
e_{nn}^{cr}	Mode I normal crack strain in local coordinate
γ_{ns}^{cr}	Mode II shear crack strain in local coordinate
γ_{nt}^{cr}	Mode III shear crack strain in local coordinate
I_1	First invariant of stress tensor
J_2	Second invariant of stress deviator tensor
J_3	Third invariant of stress deviator tensor

Material Parameters

D^{co}	Constitutive matrix of the intact concrete
D^{cr}	Constitutive matrix of cracks
D_i^I	Mode I stiffness of a crack(i)
D^{II}, D_i^{II}	Mode II stiffness
D^{III}	Mode III stiffness
$D_{i,l}^I$	Mode I stiffness of a crack(i) for linear strain softening
$D_{i,bl}^I$	Mode I stiffness of a crack(i) for bilinear strain softening
$\underline{D}$	Constitutive matrix
E	Young's modulus
E_s	Strain softening modulus
E_n	Secant modulus
f_c	Compressive strength of concrete
f_t	Tensile strength of concrete
f_t^c, f_t^r	Tensile strength of concrete or rock
G	Shear modulus
G_f	Specific fracture energy
G_f^c, G_f^r	Fracture energy of concrete or rock
h_c	Crack characteristic length
$\underline{K}^e$	Stiffness matrix of an element
$\underline{K}$	Overall structural stiffness matrix
$[K]$	Constitutive matrix in global coordinate
$[K']$	Constitutive matrix in local coordinate
K	Stress intensity factor
K_{IC}	Fracture toughness
p	Constant defining shear softening shape
α_1, α_2	Bilinear softening shape parameters

β	Shear retention factor
$\beta_{\max}$	Maximum shear retention factor
μ	Normal retention factor
ν	Poisson's ratio
w_c	Crack band width

Miscellaneous Symbols

a	Depth of crack
$\underline{a}^e$	Nodal point displacement of an element
$\underline{a}$	Overall nodal displacement vector
$\underline{B}$	Stress-displacement operator
d	Depth of beam
Gr	Self weight
$\underline{f}^e$	Loads on an element
$\underline{f}$	Overall structural load vector
h	Width of dam at the level of initial notch
$\underline{L}$	Differential operator
l_1, l_2, l_3	Direction cosines of local axes (n, s, t) to global x axis
n_1, n_2, n_3	Direction cosines of local axes (n, s, t) to global y axis
m_1, m_2, m_3	Direction cosines of local axes (n, s, t) to global z axis
$\underline{N}(\underline{x})$	Shape functions
N, N_i	Transformation matrix of crack quantities between the global and local coordinate
MPa	Megapascals stress or pressure
n	Direction normal to crack
s	Direction parallel to crack
t	Direction parallel to crack
P_0	Load to cause crack-tip tensile stress equal to the tensile strength
P_u	Peak load

$[R]$	Transformation matrix of stress, strain and stiffness between the global and local coordinate systems
$\underline{u}(\underline{x})$	Displacement field
ΔT	Temperature drop in degree Celsius
Tol	Convergence tolerance
W, W_1, W_2	Crack opening
x, y, z	Cartesian coordinates
Δ	Increment of quantities
φ	Frictional angle
ϕ	Threshold angle of a crack
θ	Angle of the local axis system with the global coordinate system
U_x	Displacement in x-direction
U_y	Displacement in y-direction

Abbreviations and Acronyms

BLS	Bilinear softening
B&L(1993)	Bhattacharjee & Leger (1993)
B&L(1994)	Bhattacharjee & Leger (1994)
CBM	Crack band model
CMOD	Crack mouth opening displacement
CMSD	Crack mouth sliding displacement
CS	Cornelissen et al's softening
DWAF	Department of Water Affairs & Forestry
FE	Finite element
FM	Fracture mechanics
F.O.S	Factor of safety
FPZ	Fracture process zone
FSL	Full supply level
FU	Full uplift
H:V	Slope ratio of horizontal to vertical
ICM	Interface crack model
ICOLD	International Congress on Large Dams

IFF	Imminent failure flood
LEFM	Linear elastic fracture mechanics
LS	Linear softening
ISCM	Interfaced smeared crack model
NLFM	Non-linear fracture mechanics
NOC	Non-overspill crest
NW-IALAD	Network Integrity Assessment of Large Concrete Dams
PU	Partial uplift
R&B(1989)	Rots & Blaauwendraal (1989)
R&D(1987)	Rots & de Borst (1987)
RDD	Recommended design discharge
RDF	Recommendation design flood
RL	Reduced level
RMF	Regional maximum flood
SEF	Safety evaluation flood
TW	Tailwater level
OBE	Operationally based earthquake
MCE	Maximum credible earthquake