CHAPTER 7

REVIEW OF THE STRUCTURE OF THE ORANI MODEL

7.1 INTRODUCTION

The computable general equilibrium (CGE) model that will be utilised in this analysis is based on the ORANI model of the Australian economy. The ORANI model of Australia is a highly disaggregated applied general equilibrium model of the Australian economy, which was first developed in the 1970s. It uses the Johansen approach of linearisation to find a solution to the set of general equilibrium prices and quantities. Since its development, this model has been used as the launching pad for developing new models in various parts of the world. The group of models is known as the ORANI-G family of CGE models, and has served as the basis for models of countries such as Vietnam, South Korea, Thailand, the Philippines, South Africa and Pakistan (http://www.monash.edu.au/policy/oranig.htm).

Because the CGE model that will be used in this study is based on the ORANI-G methodology, the theoretical structure of the ORANI model will be discussed. The theoretical discussion will be complimented in the next chapter by a discussion of the South African database that will be employed within this study and the derivation of the elasticities of the model.

As already mentioned in Chapter 6, the development of the theoretical framework for a CGE model consists of the following components:

i. Equations for intermediate and primary factor inputs.
ii. Equations representing household and other final demands for commodities.
iii. Pricing equations relating commodity prices to costs.
iv. Market clearing equations for primary factors and commodities.

The ORANI specification as described for each of these components will subsequently be discussed (as described by Dixon *et al* (1980) and Horridge *et al* (2000)). Dixon *et al* (1980) describes the theory and background of the original model in detail, while the layout of Horridge *et al* (2002) encompasses the latest version of the ORANI-G model, which includes changes and adaptations to the original model. The notation that is used in this chapter is derived from the Dixon *et al* (1980) literature and is adapted if applicable.

7.2 THE ORANI-G MODEL AND EQUATIONS FOR INTERMEDIATE AND PRIMARY FACTOR INPUTS

ORANI-G addresses the modelling of the production side of the economy by assuming that the production of commodities in each industry takes place by combining domestic and imported commodities, capital, different types of labour, land and "other costs" as inputs. These inputs are combined by a certain production technology to produce a specified level of output. In the ORANI-G model it is assumed that all factors of production are variable. This means that, with respect to capital and agricultural land, producers act as if they rent both these inputs. It is, however, assumed that both capital and land are not transferable between industries.

As is the case in most CGE models, the production specification is nested. At the top nest of the ORANI-G model, the demand for inputs in each industry (j) is determined by minimising the cost function of the firm subject to a Leontief production function. The inputs entering the production structure at this top nest are composite commodities (i), a primary input (g+1,s) and "other costs" (g+2). The production function in the top nest is therefore given by:

$$Leontief\left\{\frac{X_{ij}^{1}}{A_{ij}^{1}}\right\} = A_{j}^{1} Z_{j}$$

where

X_{ij}^{1} is the effective input of good or factor i into current production in industry j.

Z_{j} is industry j's activity level.

A_{ij}^{1} and A_{j}^{1} is technological coefficients that allows for technological change.

Because the Leontief specification is assumed, the production function exhibits constant returns to scale and there are no substitution possibilities between the inputs in the production function. This assumption, which has been inherited from input-output analysis, has been justified by numerous studies that have failed in attempts to establish whether changes in relative prices result in changes in relative input quantities (Sevaldson, 1976). It must be noted, however, that more recent work by Jorgenson and others has shown that there could be feasibility in estimating material/material and material/primary factor substitution elasticites at a detailed level for the USA. This could be important for general equilibrium analysis of issues that involve sharp changes in the relative prices of competitive inputs (Dixon *et al*, 1992). In the South African context, the assumption of a Leontief technology at the top nest is retained and it is assumed that no substitution possibilities exist between the primary factor composite, the intermediate inputs or the "other cost tickets".

At the second level of the production nest, the composite commodities $X^1_{(is)j}$ that are used by each industry consist of a combination of domestically produced (s=1) and imported (s=2) goods, which are combined according to a CES technology. Also on the second level of the production nest, the primary input ($X^1_{(g+1,s)j}$) consists of a combination of labour (s=1), capital (s=2) and land (s=3), which is also combined with a CES technology. The second level of the production technology can be expressed as:

$$X^1_{ij} = \underset{s=1,2}{CES}\left\{\frac{X^1_{(is)j}}{A^1_{(is)j}}\right\}$$

and

$$X^1_{(g+1,s)j} = \underset{s=1,2,3}{CES}\left\{\frac{X^1_{(g+1,s)j}}{A^1_{(g+1,s)j}}\right\}$$

where

$i = 1,....,g$ (that is g, different products)

$j = 1,.....h$ (that is h, different industries)

The use of a CES production technology to combine domestically produced and imported commodities indicates that these two sources may not be perfectly substitutable for each other and that the demand for these inputs will change according to relative price changes. This treatment of imported and domestically produced goods follows Armington's (1969 and 1970) treatment of imported and domestically produced goods and is usually adopted in order to accommodate the phenomenon of countries both importing and exporting the same good (cross hauling).

Substitution possibilities are also allowed between the three different primary inputs through the use of the CES technology to combine them. This allows for substitution possibilities between the different primary factor inputs, based on changes in relative prices. If the price of one primary production factor increases relative to the average price of all primary inputs, the demand for the input will decrease relative to the total primary factor input demand.

At the third level of the production nest, the m different types of labour ($X^1_{(g+1,1,m)j}$) are also combined by means of a CES production technology to obtain the aggregated labour input, which enters into the primary factor composite. That is:

$$X^{1}_{(g+1,1,m)j} = CES\left\{\frac{X^{1}_{(g+1,1,m)j}}{A^{1}_{(g+1,1,m,j)}}\right\}$$

As is the case for the primary factor composite and intermediate inputs, the CES specification allows for substitution possibilities between different types of labour, based on relative price changes between these different skill types.

In order to solve the above production nest for the input demand functions, the level of output in each industry, as well as the prices of the inputs (except that for the composite labour demand), are treated as exogenous. Therefore, given the above production specifications, the following cost minimisation problem needs to be solved in order to obtain the input demand equations:

Choose:

$X^{1}_{ij}, X^{1}_{(is)j}, X^{1}_{(g+1,s)j}, X^{1}_{(g+1,1,m)j}$

to minimise

$$\sum_{i=1}^{g}\sum_{s=1}^{2} P^{1}_{(is)j} X^{1}_{(is)j} + \sum_{m=1}^{M} P^{1}_{(g+1,1,m)j} X^{1}_{(g+1,1,m)j} + \sum_{s=2}^{3} P^{1}_{(g+1,s)j} X^{1}_{(g+1,1,s)j} + P^{1}_{g+2} X^{1}_{g+2,j}$$

where:

X^{1}_{ij} is the demand for effective intermediate and primary inputs by industry j.
$X^{1}_{(is)j}$ is the demand for imported and domestic intermediate inputs, i, by industry j.
$X^{1}_{(g+1,s)j}$ is the demand for the "composite" primary factor input of industry j, that consists of capital, labour and land.
$X^{1}_{(g+1,1,m)j}$ is the demand for labour of different skill groups by industry j.
$P^{1}_{(is)j}$ is the price of the domestic and imported intermediate input.
$P^{1}_{(g+1,1,m)j}$ is the price of labour of skill group m in industry j.
$P^{1}_{(g+1,s)j}$ is the price primary factor s in industry j.
P^{1}_{g+2} is the price of other cost tickets.

Because the solution of the system of non-linear equations in the ORANI model is based on Johansen's linearisation technique, the solution to the above minimisation problem should therefore enter the ORANI model in the form of percentage change.

The equations below represent the solutions to the above cost minimisation problems in form of percentage change.

i. Intermediate input demand equation:

$$x^1_{(is)j} = z_j - \sigma^1_{ij}(p^1_{(is)j} - \sum_s S^1_{(is)} p^1_{(is)j}) + a^1_j + a^1_{ij} + a^1_{(is)j} - \sum_s S^1_{(is)} a^1_{(is)j}$$

$$i = 1,.....g$$
$$s = 1,2$$
$$j = 1,....h$$

The lower case denotes the percentage change forms of the variables that have been defined above. $S^1_{(is)j}$ is the share of the total cost of good i, from source s, into the inputs of industry j.

$$S^1_{(is)j} = \frac{P^1_{(is)j} X^1_{(is)j}}{\sum_s P^1_{(is)j} X^1_{(is)j}}$$

and σ_{ij} is the elasticity of substitution between imported and domestic goods as inputs into production of industry j. This specification indicates that if the price of one of the sources increases relative to the other, this will result in a decrease in the demand for the input from this source.

ii. Demand functions for primary factors

$$x^1_{(g+1,v)j} = z_j - \sigma^1_{(g+1,v)j}(p^1_{(g+1,v)j} - \sum_s S^1_{(g+1,v)j} p^1_{(g+1,v)j}) + a^1_j + a^1_{g+1,j} + a^1_{(g+1,v)j} - \sum_s S^1_{(g+1,v)} a^1_{(g+1,v)j})$$

$$v = 1,2,3$$
$$j = 1,...g$$

$S^1_{(g+1,v)}$ is the share of labour, capital and land in industry j's payments for primary factor inputs and is expressed as:

$$S^{1}_{(g+1,v)j} = \frac{P^{1}_{(g+1,v)j} X^{1}_{(g+1,v)j}}{\sum_{v=1}^{3} P^{1}_{(g+1,v),j} X^{1}_{(g+1,v)j}}$$

Once again, increases in the cost of industry j of any particular factor relative to a weighted average of the costs of all three factors leads to substitution away from that factor in favour of the other two factors.

Although the price of capital and land is exogenous to the cost minimisation problem, the price of labour is not exogenous and is a function of the different types of labour included in the model. The cost of labour is therefore defined (in percentage change form) as:

$$p^{1}_{(g+1,1)j} = \sum_{q=1}^{M} p^{1}_{(g+1,1,q)j} S^{1}_{(g+1,1,q)j} + \sum_{q=1}^{M} a^{1}_{(g+1,1,q)j} S^{1}_{(g+1,1,q)j}$$
$$j = 1,.....h$$

If the technology coefficient ($a^{1}_{(g+1,1,q)j}$) is set to zero, then the percentage change in the cost of labour ($p^{1}_{(g+1,1)j}$) is a weighted average of the percentage changes in the cost to industry j of units of labour from the different skill groups. The weights are the shares of each skill group in j's total labour cost ($S^{1}_{(g+1,1,q)j}$). Given this cost, the demand for different types of labour can be determined. The percentage change form of this demand equation is shown below.

iii. Demand function for labour of different skill types

$$x^{1}_{(g+1,1,q)j} = x_{(g+1,1)j} - \sigma^{1}_{(g+1,1)j}(p^{1}_{(g+1,1,q)j} - \sum_{q} S^{1}_{(g+1,1,q)j} p^{1}_{(g+1,1,q)j}) + a^{1}_{(g+1,1,q)j} -$$
$$\sigma^{1}_{(g+1,1)j}(a^{1}_{(g+1,1,q)j} - \sum_{q} S^{1}_{(g+1,1,q)j} p^{1}_{(g+1,1,q)j})$$
$$v = 1,2,3$$
$$j = 1,...g$$

$S^{1}_{(g+1,v)}$ is the labour specific cost share defined by:

$$S^1_{(g+1,1,q)j} = \frac{P^1_{(g+1,1,q)j} X^1_{(g+1,1,q)j}}{\sum_{q=1}^{M} P^1_{(g+1,1,q),j} X^1_{(g+1,1,q)j}}$$

In summary, the demand function for labour of certain skill types relates the demand for labour of a certain skill type to the industry's demand for labour in general, to the costs of the different types of labour and also to various technical change variables. If there is no technical change, an increase in the price of labour of a certain type (for example skilled labour) relative to the price of other types of labour, will cause the use of this type of labour to increase more slowly than the use of other types of labour.

The above four equations give the solution to the cost minimising problem of the firm in the ORANI model and represents the linearised demand equations for intermediate inputs, capital, land, labour and labour of different skill types.

7.3 A REVIEW OF EQUATIONS USED FOR HOUSEHOLDS AND OTHER FINAL DEMAND EQUATIONS IN ORANI

i. Household demands for consumer goods

Household demands for different goods are represented by a nested structure. It is assumed that households derive utility from the consumption of goods and services and that each household maximises its utility subject to a budget constraint. At the top nest, each household determines the allocation of household expenditure between different commodity composites by maximising a Klein-Rubin utility function subject to a CES composite of each product and an aggregate consumer budget.

$$U = \frac{1}{Q} \prod (X_i^3 - \delta_i)^{\theta_i}$$

$$\textit{subject to}:$$

$$\frac{X_i^3}{Q} = \underset{s=1,2}{CES} \left\{ \frac{X_{is}^3}{Q} \right\}$$

$$i = 1, \ldots . g$$

$$\textit{and}$$

$$\sum_{s=1}^{2} \sum_{i=1}^{g} P_{is}^3 \frac{X_i^3}{Q} = C$$

where Q is the number of households and X_{is}^3 and P_{is}^3 are the quantities consumed and prices paid by households for units of good *i*. δ_i and θ_i are behavioural parameters with $\delta > 0$ and $\sum_i \theta_i = 1$. C is the aggregate consumer budget constraint and remains exogenous. The solution to this utility maximising problem of the consumer results in the linear expenditure demand equations for each good:

$$X_i^3 = \delta_i + \theta_i \frac{\varsigma}{P_i^3}$$
$$where:$$
$$\varsigma = C - \sum_i \theta_i P_i$$

As is evident from the expression above, the expenditure on each good is a linear function of prices and expenditure. δ_i is the subsistence requirements of each good that a household needs and purchases regardless of the price of the commodity. The ς is the residual of the consumer's budget after subsistence expenditures are deducted and can be viewed as the luxury expenditure. The θ_i represents the shares that each of the luxury goods receives from the "luxury budget" (the marginal budget shares).

X_{is}^3 and P_{is}^3 are the quantities consumed and prices paid by households for units of good *i* from source s, with s=1 referring to domestic sources and s=2 referring to imports. The A^3's are positive coefficients, introduced to allow for changes in tastes. C remains exogenous and Q is the number of households in the economy.

The linearised solution to the above maximisation problem is given by the following demand equations which represent the households' decision between imported and domestically produced goods, as well as the households' demand for each of the *g* commodities in the economy.

$$x_i^3 - q = \varepsilon_i (c - q) + \sum_{K=1}^{g} \eta_{ik} p_k^3 + a_i^3 + \sum_{k=1}^{g} \eta_{ik} (a_k^3 + \sum_{s=1}^{2} S_{ks}^3 a_{ks}^3)$$
$$i = 1,\dots,g$$
$$and$$
$$x_{is}^3 = x_i^3 - \sigma_i^3 (p_{is}^3 - \sum_{s=1}^{2} S_{is}^3 p_{is}^3) + a_{is}^3 - \sigma_i^3 (a_{is}^3 - \sum_{s=1}^{2} S_{is}^3 a_{is}^3)$$

where S_{is}^{3} is again the share of good *i* from source s in the household's consumption budget. The parameters ε_i and η_{ik} are the expenditure and own- and cross-price elasticities that satisfy the usual restrictions of homogeneity, symmetry and Engel's aggregation. These parameters are the effects on (per) household consumption of effective units of good *i* arising from a one percent increase in the general price of good *k*.

ORANI assigns values for the elasticities by making use of the Klein-Rubin utility function:

$$U = (\overline{X}_1^3, \ldots . \overline{X}_g^3) = \sum_i^1 \delta_i \ln(X_i^3 - \theta_i)$$

where δ_i and θ_i are parameters and the elasticities can then be derived:

$$\varepsilon_i = \frac{\delta_i}{S_i^3}$$

$$\eta_{ik} = \frac{-\delta_i S_k^{*3}}{S_i^3}$$

and

$$\eta_{ii} = -\varepsilon_i - \sum_{k \neq i} \eta_{ik}$$

where

$$S_i^3 = \frac{\overline{P}_i^3 \overline{X}_i^3}{\sum_k \overline{P}_k^3 \overline{X}_k^3}$$

$$S_i^{*3} = \frac{\overline{P}_i^3 \theta_i^3}{\sum_k \overline{P}_k^3 \overline{X}_k^3}$$

In summary, the ORANI household demand specification can be described as follows:

- Consumers maximise a single utility function subject to a budget constraint.
- The utility function that is used to derive the elasticities is the Klein Rubin utility function results in a linear expenditure demand system.
- The consumption of good *i*, in general is defined by a CES aggregate of the consumption of good *i* from domestic and foreign sources. Consumers are assumed to substitute

between the two sources of supply of good *i* in response to changes in the relative prices of good *i* from the two sources.

- Consumers are assumed to substitute between good i and good k in response to changes in the relative general prices of k and *i*.

Changes in household preferences can be simulated using quantity augmenting variables that closely parallel those of changes in technology.

ii. Demands for inputs for the production of fixed capital

The demand for inputs to construct a unit of fixed capital is determined by a nested structure. At the top level of this nest, the ORANI model assumes that a unit of fixed capital in industry j can be created according to a Leontief technology that combines the different inputs *i* that are available in the economy. It is assumed that primary factors of production are not used in the production of fixed capital. The production of investment goods can therefore be expressed as:

$$A_j^2 Y_j = Leontief\left\{\frac{X_{ij}^2}{A_{ij}^2}\right\}$$

where Y_j is the number of fixed capital units that is created for industry *j*, X_{ij}^2 is the direct effective input of good *i* that is used to create capital for industry j, and A_j^2 and A_{ij}^2 are positive coefficients that represent technological change in the production of fixed capital. The level of fixed investment that industry *j* will conduct is assumed to be exogenous when the demand for inputs are derived. The treatment of the firm's decision on the level of investment is discussed towards the end of this chapter.

At the second level of this nest, the inputs that are used in the first nest are created by a combination of both imported and domestically produced goods. As in the case of the production of intermediate goods, imported and domestically produced products are combined by a CES technology in the production of fixed capital. This technology can be expressed as:

$$X_{ij}^{2} = CES\left\{\frac{X_{(is)j}^{2}}{A_{(is)j}^{2}}\right\}$$

where $X_{(i1)j}^{2}$ and $X_{(i2)j}^{2}$ are inputs of good *i*, from domestic (s=1) and imported (s=2) sources and the $A_{(is)j}^{2}$ is another set of technological coefficients.

Because it is assumed that markets are perfectly competitive, producers of capital for the different industries do not have any price setting power, and have to take input prices as if they are determined by the market. Therefore, to determine the amount of inputs that the capital producer will demand from the domestic or foreign market, the producer will choose a combination that minimises his/her cost subject to the CES production technology. The producer therefore choose $X_{(is)j}^{2}$ to minimise:

$$\sum_{i=1}^{g}\sum_{s=1}^{2} P_{(is)j}^{2} X_{(is)j}^{2}$$

subject to

$$X_{ij}^{2} = CES\left\{\frac{X_{(is)j}^{2}}{A_{(is)j}^{2}}\right\}$$

where $P_{(is)j}^{2}$ is the price of good *i* from source *s* when it is used as an input for creating capital for industry *j*.

The linearised solution to the above minimisation problem is given by:

$$x_{is}^{2} = y_{j} - \sigma_{ij}^{2}\left(p_{(is)j}^{2} - \sum_{s=1}^{2} S_{(is)j}^{2} p_{(is)j}^{2}\right) + a_{j}^{2} + a_{ij}^{2} + a_{(is)j}^{2} - \sigma_{ij}^{2}\left(a_{(is)j}^{2} - \sum_{s=1}^{2} S_{(is)j}^{2} a_{(is)j}^{2}\right)$$

$$i = 1,\ldots,g$$

$$j = 1,\ldots,h$$

where $S_{(is)j}^{2}$ is the share of good *i* from source *s* in the total cost of good *i*, used in the creation of capital in industry *j*, and is expressed as:

$$S^{2}_{(is)j} = \frac{P^{2}_{(is)j} X^{2}_{(is)j}}{\sum_{s} P^{2}_{(is)j} X^{2}_{(is)j}}$$

It is clear that a change in the price of one source of commodity *i*, relative to the other source, will allow for substitution possibilities between the different sources.

In summary, the assumptions concerning the construction of fixed capital are:

- Fixed capital for industry *j* is created by combining effective units of produced inputs according to a Leontief production function.
- The effective input of good *i* to construct a unit of capital for industry *j* is a CES combination of inputs from domestic and foreign sources.
- The composition of units of capital varies across industries.

iii. Demand for export products from the rest of the world

The ORANI-G model distinguishes between two different groups of exports. Namely the individual export commodities and the collective export commodities. The individual export commodities include all the main export commodities of the exporting country, and it is assumed that the demand equations for these commodities are downward sloping and a function of the foreign price elasticity of demand for these goods. These demand equations can be expressed as follows:

$$X_{i}^{4ind} = A_{i}^{4indQ} \left(\frac{P_{i}^{ind4}}{\phi * A_{i}^{4indP}}\right)^{\gamma_i}$$

where X_{i}^{4ind} is the individual export demand for good *i*, P_{i}^{4ind} is the domestic price for individual export good *i* (which includes transport costs), and ϕ is the exchange rate that converts the domestic price of the export good to a foreign price. The foreign price elasticity of export demand of good *i* is given by γ_i while A_{i}^{4indQ} and A_{i}^{4indP} are technological coefficients that allow for horizontal and vertical shifts in the individual export demand schedules of the export demand equations.

The linearised demand equation for individual export commodities is:

$$x_i^{4ind} - a_i^{4indq} = \gamma_i (p_i^{4ind} - \phi - a_i^{4indp})$$

This indicates that an increase in the foreign currency price of collective export commodity, *i*, will result in a decrease in the foreign demand for this commodity, as is implied by the price elasticity of this export product.

In contrast to the individual export demand equations, the foreign demand for collective export commodities is inversely related to the prices of all the collective export commodities and usually includes all the commodities for which individual export demand equations seem inappropriate. Typical inclusions are the service commodities where export volumes do not necessarily depend on the corresponding price of the commodity. A "collective exports" commodity is therefore created through a Leontief technology, expressed as:

$$Leontief \left\{ \frac{X_i^{4col}}{A_i^{4col}} \right\} = X^{4col}$$

where X^{4col} is the Leontief aggregate of the individual collective export commodities (X_i^{4col}). The demand for the aggregate (X^{4col}) is a function of the collective foreign price of these commodities and can be expressed as:

$$X^{4col} = A^{4colQ} \left(\frac{P^{4col}}{\phi * A^{4colP}} \right)^{\gamma col}$$

where X^{4col} is the collective export demand, P^{4col} is the domestic price for the collective export price (which includes transport costs), and ϕ is the exchange rate that converts the domestic price of the collective exports to a foreign price. The foreign price elasticity of the collective export good is given by γ^{col} while A^{4colQ} and A^{4colP} are technological coefficients that allow for horizontal and vertical shifts in the collective export demand schedule. The linearised collective demand equation is expressed as:

$$x^{4col} - a^{4colq} = \gamma col \ (p^{4col} - \phi - a^{4colp})$$

As in the case with the individual export demand equations, the demand for the collective export product will decrease if the foreign price increases, as indicated by the collective export price elasticity.

iv. "Other" final demands and demands for margins

"Other" final demands consist mainly of government demands for both domestically and imported goods. The ORANI-G methodology includes no theoretical explanation for this category of final demands and expresses the linearised "other" demand equations simply as:

$$x_{is}^{5} = c_r h_{is}^{5} + f_{is}^{5}$$

where x_{is}^{5} is the percentage change in "other" demands for good *i* from source s. c_r is the percentage change in real aggregate household expenditure and can be expressed as:

$$c_r = c - \xi^{3}$$

where ξ^{3} is the consumer price index. f_{is}^{5} are shift variables and the h_{is}^{5} are parameters that allow other final demands to change relative to household demands. If h_{is}^{5} are set equal to 1, and the f_{is}^{5} are set equal to zero, "other demands" will move in line with real household expenditure. However, if h_{is}^{5} are set equal to zero, government consumption is exogenously determined by f_{is}^{5}.

In the ORANI framework it is assumed that the demand for margins associated with the delivery of inputs for current production and capital construction is proportional to the demand for the specific inputs that it is used for. It can be expressed as:

$$X_{r1}^{(is)jk} = A_{r1}^{(is)jk} X_{(is)j}^{k}$$
$$i, r = 1, \ldots, g$$
$$j = 1, \ldots., h$$
$$k, s = 1,2$$

where $X_{r1}^{(is)jk}$ is the quantity of good *r1* used as a margin to facilitate the flow of good *i* from source *s* to industry *j* for purpose k. $A_{r1}^{(is)jk}$ is a technical coefficient that indicates the proportionality of the margin demand to the total demand for the input. In the absence of any technical change, this demand specification allows the demand for margins to be proportional to commodity flows with which the margins are associated. The linearised demand for margins associated with the delivery of inputs for current production and capital construction is given by:

$$x_{r1}^{(is)jk} = x_{(is)j}^{k} + a_{r1}^{(is)jk}$$

Similarly, margin flows that are associated with the delivery of commodities to households and "other" users are handled by the following equations:

$$X_{r1}^{(is)k} = A_{r1}^{(is)k} X_{is}^{k}$$
$$k = 3,5$$
$$r, i = 1,\ldots,g$$
$$s = 1,2$$

These equations represent the demand for margins associated with the delivery of commodities to households and "other users", while

$$X_{r1}^{(is)4} = A_{r1}^{(is)4} X_{is}^{4}$$
$$i, r = 1,\ldots.,g$$

represent the margin demand associated with the delivery of commodities from export producers to the port of delivery.

In percentage change form, the margin demand for capital, household, and exports demand are given by:

$$x_{r1}^{(is)k} = x_{is}^{k} + a_{r1}^{(is)k}$$
and
$$x_{r1}^{(is)4} = x_{is}^{4} + a_{r1}^{(is)4}$$

7.4 A REVIEW OF PRICING EQUATIONS THAT RELATE COMMODITY PRICES TO COSTS IN THE ORANI MODEL

The ORANI-G methodology distinguishes between 5 sets of commodity prices. These are purchaser prices, basic values, prices of capital units, foreign currency export prices and foreign currency import prices. In determining the relationship between these prices it is assumed that there are no pure profits in any economic activity and that basic prices are uniform across all users and producing industries for both domestic and imported goods.

i. Basic values of products

Basic values for domestic goods are the prices received by producers. Basic values therefore exclude sales taxes and margin costs. With regards to the basic value of imports, they represent the prices received by importers. Given the assumptions of no pure profits and uniformity of basic prices across the economy, the basic value of each product *i* can be expressed as the sum of the value of intermediate inputs, the value of labour, the value of capital used, the value of land used, and the value of other cost tickets. This can be expressed as (with no change in the notation as already defined):

$$\sum_{i=1}^{g} P_{i1}^{0} X_{(i1)j}^{0} = \sum_{i=1}^{g}\sum_{s=1}^{2} X_{(is)j}^{1} P_{(is)j}^{1} + \sum_{m=1}^{M} P_{(g+1,1,m)j}^{1} X_{(g+1,1,m)j}^{1} + \sum_{s=2}^{3} P_{(g+1,s)j}^{1} X_{(g+1,s)j}^{1} + P_{(g+2)j}^{1} X_{g+2,j}^{1}$$

In percentage change form this relationship is:

$$\sum_{i=1}^{g} p_{i1}^{0} H_{(i1)j}^{0} = \sum_{i=1}^{g}\sum_{s=1}^{2} p_{is}^{1} H_{(is)j}^{1} + \sum_{m=1}^{M} p_{(g+1,1,m)}^{1} H_{(g+1,1,m)j}^{1} + \sum_{s=2}^{3} p_{(g+1,s)j}^{1} H_{(g+1,s)j}^{1} + p_{g+2,j}^{1} H_{g+2,j}^{1} + a(j)$$

where

$$a(j) = a_j^0 + \sum_{r=1}^{N(j)} a_{(r^*)j}^{0} H_{(r^*)j}^{1} + \sum_{i=1} a_{(i1)j}^{0} H_{(i1)j}^{0} + a_j^1 + \sum_{i=1}^{g+2} a_{ij}^{1} H_{ij}^{1} + \sum_{i=1}^{g}\sum_{s=1}^{2} a_{(is)j}^{1} H_{(is)j}^{1} + \sum_{s=1}^{3} a_{(g+1,s)j}^{1} H_{(g+1,s)j}^{1} + \sum_{m=1}^{M} a_{(g+1,1,m)j}^{1} H_{(g+1,1,m)j}^{1}$$

The H's in the linearised equation represent the revenue and cost shares of each input in the production process. $H_{(i1)j}^{0}$ and $H_{(r^*)j}^{0}$ are the shares of industry *j*'s revenue accounted for by its sales of commodity *i1* and composite commodity *r*. $H^1_{(is)j}, H^1_{ij}$ and $H^1_{(g+1,1,m)j}$ are the shares of *j*'s costs accounted for by inputs of *is*, by inputs of *i* from all sources and by inputs of labour of skill *m*.

Intuitively the linearised equation is easily explained if no technical change were to take place. For each industry *j*, the expression implies that a weighted average of the percentage changes in the basic prices of outputs equals a weighted average of the percentage changes in the relevant purchasers prices of inputs. The technological change terms are included to insure that the zero pure profits condition applies in the case of a technical change event that reduces/increases the efficiency of production.

ii. The price of a unit of capital

As in the case of basic prices, the value of a unit of capital is equal to the cost of its production. Because no primary inputs are used in the production of capital, the value of a unit of capital can be expressed as the sum of the value of the intermediate inputs used:

$$\pi_j Y_j = \sum_{i=1}^{g}\sum_{s=1}^{2} P^2_{is} X^2_{(is)j}$$

In linearised form this relationship is expressed as:

$$\pi_j = \sum_{i=1}^{g}\sum_{s=1}^{2} p^2_{(is)j} H^2_{(is)j} + a^2_j + \sum_{i=1}^{g} a^2_{ij} H^2_{ij} + \sum_{i=1}^{g}\sum_{s=1}^{2} a^2_{(is)j} H^2_{(is)j}$$

where π_j is the percentage change in the price of a unit of capital for industry j and the $H^2_{(is)j}$ and H^2_{ij} are cost shares. They are, respectively, the share of good *i* from source s and the share of good *i* from all sources in the cost of constructing a unit of capital for industry *j*. Once again, if there is no technical change in the production of capital, a percentage change in the cost of a unit of capital for industry *j* is a weighted average of the percentage changes in the prices of inputs, the weights being the cost shares.

There is a difference between the cost of using (or renting) a unit of capital ($P^1_{(g+1,2)j}$) and the cost of buying or producing a unit of capital π_j. The ratio $\frac{P^1_{(g+1,2)j}}{\pi_j}$ represents the gross rate of return on units of capital for industry *j*. In the ORANI-G methodology, this ratio could enter the investment decision of industry *j*. This is discussed at the end of this chapter.

iii. Determining the basic price of imported good i

The basic price of imports is the sum of the foreign price of the imported goods (expressed in domestic currency units), and tariffs or quotas on the specific good. This relationship can be expressed as:

$$P_{i2}^{0} = P_{i2}^{m}\phi + G(i2,0)$$

where P_{i2}^{0} is the basic price of imported good *i*, P_{i2}^{m} is the foreign currency price of imported good *i*, ϕ is the exchange rate and G(i2,0) is the tariff in domestic currency raised on a unit import of *i*.

The linearised expression for the basic price of imports is then given by:

$$p_{i2}^{0} = (p_{i2}^{m} + \phi)\xi_1(i2,0) + g(i2,0)\xi_2(i2,0)$$

where $\xi_1(i2,0)$ and $\xi_2(i2,0)$ are, respectively, the shares in the basic price of imported good *i* accounted for by the foreign currency price in domestic currency, and the tariff, G(i2,0).

iv. Determining the equation that relates prices of domestic goods to export prices

The prices of the export goods are the basic price of the good and the taxes and margins that are involved in delivering the good at domestic ports. This price is then converted into domestic currency by making use of the exchange rate. It can be expressed as:

$$P_{i1}^{e}\phi = P_{i1}^{0} + G(i1,4) + \sum_{r=1}^{g} A_{r1}^{(i1)} P_{r1}^{0}$$

where P_{i1}^{e} is the export price of good *i*, G(i1,4) is the export tax per unit of export of *i1* and $A_{r1}^{(i1)} P_{r1}^{0}$ is the value of each margin that is used to deliver the export product to the domestic port of delivery. In the case of an export subsidy, G(i1,4) will be negative. The above relationship can be expressed in linear form as:

$$(p_{i1}^{e}+\phi)=p_{i1}^{0}\zeta_{1}(i1,4)+g(i1,4)\zeta_{2}(i1,4)+(\sum_{r=1}^{g}M_{r1}^{(i1)4}p_{(r1)}^{0})\zeta_{3}(i1,4))+\sum_{r=1}^{g}M_{r1}^{i1}a_{r1}^{(i1)4}\zeta 4(i1,4)$$

where $\zeta_1(i1,4),\zeta_2(i1,4),\zeta_3(i1,4)$ and $\zeta_4(i1,4)$ are, respectively, the shares accounted for by the basic value, the export tax and the margins in the domestic currency price paid by foreigners for units of good *i1* at domestic ports. $M_{r1}^{(i1)4}$ is the share in the total cost of margin services involved in transferring good *i1* from domestic producers to the ports of exit represented by the use of good *r1*.

v. Determining the equation that relates purchaser's prices paid by domestic users of good *i* from domestic and imported source to its basic values.

Purchaser's prices are the sums of basic values, sales taxes and margins. Sales taxes are treated as *ad valorem* on basic values. ORANI-G relates purchaser's prices to basic values by using the following equation:

$$P_{(is)j}^{k}=P_{is}^{0}+G(is,jk)+\sum_{r=1}^{g}A_{r1}^{(is)jk}P_{r1}^{0}$$

and

$$P_{is}^{3}=P_{is}^{0}+G(is,3)+\sum_{r=1}^{g}A_{r1}^{(is)3}P_{r1}^{0}$$

where the G's are tax terms (for example G(is,jk) is the tax associated with the sale of good *i* from source s to industry *j*). The first equation therefore equates the price paid in industry *j* for good *is* to the sum of the basic value of good *is* and the cost of the relevant taxes and margins. The second equation describes the purchaser's price of good, *is*, when households use it.

It is important to note that the G's are taxes on sales and not on production. In the ORANI-G methodology, producer taxes can be included via other cost tickets. Taxes are allowed to vary across users (which is not a feature of producer taxes).

The above two equations can be expressed in percentage change form as:

$$p^{k}_{(is)j} = p^{0}_{is}\zeta_1(is,jk) + g(is,jk)\zeta_1(is,jk) + (\sum_{r=1}^{g} M^{(is)jk}_{r1} p^{0}_{r1})\zeta_3(is,jk) + (\sum_{r=1}^{g} M^{(is)jk}_{r1} a^{0}_{r1})\zeta_3(is,jk)$$

and

$$p^{3}_{(is)j} = p^{0}_{is}\zeta_1(is,3) + g(is,3)\zeta_1(is,3) + (\sum_{r=1}^{g} M^{(is)3}_{r1} p^{0}_{r1})\zeta_3(is,3) + (\sum_{r=1}^{g} M^{(is)3}_{r1} a^{0}_{r1})\zeta_3(is,3)$$

where the coefficients ζ and M, are once again the share coefficients of basic values, taxes and margins. For example, $\zeta_1(is,3)$ is the share in the purchasers' price to households of good *is* accounted for by the basic price, while $M^{(i1)3}_{r1}$ is the share in the total cost of margin services involved in transferring good *i1* from producers to households, accounted for by the use of good *r1*.

The five equations described above determine the prices in the ORANI-G general equilibrium model. They can be summarised as follows:

- The first system of equations relates changes in the basic prices of outputs to changes in the purchaser prices of inputs and to changes in technology for current production.
- The second system of price equations relates changes in the cost of capital to changes in the purchaser prices of inputs to capital creation and to changes in the technologies for capital creation.
- The third system of price equations defines the changes in the basic prices of imports in terms of changes in their foreign currency prices, the exchange rate and the tariff rates.
- The fourth system of price equations relates basic prices of domestic commodities to export prices and tariffs.
- The fifth system of price equations relates changes in the prices paid by domestic users to changes in basic prices, the relevant taxes and changes in the cost of margin services.

7.5 A REVIEW OF THE MARKET CLEARING EQUATIONS IN ORANI

The ORANI model distinguishes between market clearing equations that ensure that demand equals supply for both domestically produced commodities and the primary factors of production.

i. An equation that equates the supply and demand of each of the domestically produced goods.

The first market clearing relationship ensures that the total supply of goods is equal to the total demand for goods. As seen above, total demand in the ORANI model consists of:

- Demand for intermediate inputs into current production.
- Demand for inputs to the production of capital equipment.
- Demand for consumption goods.
- Export demand.
- "Other demands".
- Demand for margins on the delivery of goods to households and to "other users".
- Demand for margins on the delivery of goods to industrial users for current production and capital creation.
- Demand for margins on the delivery of exports from domestic producers to ports of exit.

This market clearing relationship for goods and services can therefore be expressed as:

$$X_{r1}^{0} = \sum_{j=1}^{h} X_{(r1)j}^{1} + \sum_{j=1}^{h} X_{(r1)j}^{2} + X_{r1}^{3} + X_{r1}^{4} + X_{r1}^{5} + \sum_{i=1}^{g}\sum_{s=1}^{2}\sum_{j=1}^{h}\sum_{k=1}^{2} X_{r1}^{(is)jk} + \sum_{i=1}^{g}\sum_{s=1}^{2}\sum_{k=3,5} X_{r1}^{(is)k} + \sum_{i=1}^{g} X_{r1}^{(i1)4}$$

where

$X_{r1}^{0} = \sum_{i=1}^{h} X_{(r1)j}^{0}$ is the total supply of commodities in the economy.

In percentage change form, this market clearing relationship can be written as:

$$x_{r1}^{0} = \sum_{j=1}^{h} x_{(r1)}^{1} B_{(r1)j}^{1} + \sum_{j=1}^{h} x_{(r1)j}^{2} B_{(r1)j}^{2} + x_{r1}^{3} B_{r1}^{3} + x_{r1}^{4} B_{r1}^{4} + x_{r1}^{5} B_{r1}^{5} + \sum_{i=1}^{g}\sum_{s=1}^{2}\sum_{j=1}^{h}\sum_{k=1}^{2} x_{r1}^{(is)jk} B_{r1}^{(is)jk}$$
$$+ \sum_{i=1}^{g}\sum_{s=1}^{2}\sum_{k=3,5} x_{r1}^{(is)k} B_{r1}^{(is)k} + \sum_{i=1}^{g} x_{r1}^{(i1)4} B_{r1}^{(i1)4}$$

The B's represent the shares of the sales of domestically produced goods that are absorbed by the various types of demands.

ii. An equation that equates the supply of labour of skill m (L_m) to demand.

The second market clearing equation ensures that the supply and demand for labour is equal within each skill group. This relationship can be expressed as:

$$L_m = \sum_{j=1}^{h} X^1_{(g+1,1,m)j}$$

This equation implies that labour is homogenous within each skill group and is shiftable between industries. It does not imply that ORANI is a full employment model. Setting the L_m variables exogenously at their full employment levels will impose full employment in the model. However, as an alternative, wages might be set exogenously and the L_m variables would become endogenous. The model will then generate the employment levels, L_m corresponding to the given wage rates.

The equation, in percentage change form, is:

$$l_m = \sum x^1_{(g+1,1,m)j} B^1_{(g+1,1,m)j}$$

The B's in this equation are employed shares. That is, $B^1_{(g+1,1,m)}$ is the share of total employment of labour of type *m* that is accounted for by industry *j*.

iii. An equation that equates the supply and demand for capital in each industry

The third market clearing equation ensures that the supply and demand for capital is equal within each industry and can be expressed as:

$$K_j(0) = X^1_{(g+1,2)j}$$

This expression indicates that it is assumed that capital is industry specific, and therefore cannot be shifted between industries. This market clearing equation can be expressed in percentage change terms as:

$$k_j(0) = x^1_{(g+1,2)j}$$

iv. An equation that equates the demand and supply for land in each industry

The final market clearing equation within the ORANI model equates the demand and supply of land in each industry and is expressed as:

$$N_j = X^1_{(g+1,3)j}$$

As in the case of capital, it is assumed that land is industry specific and cannot be shifted between industries. The percentage change form of this market clearing equation is given by:

$$n_j = x^1_{(g+1,3)j}$$

These four market-clearing equations complete the description of the structural equations of the ORANI model that will be employed within this study. There are, however, additional equations within this model which either complete the description of the economy or allow for easier interpretation of the results.

7.6 ADDITIONAL EQUATIONS FOR EXPLAINING GROSS DOMESTIC PRODUCT AND ALLOCATION OF INVESTMENT ACROSS INDUSTRIES

Apart from the above equations, the ORANI model also includes equations that explain gross domestic product and allocation decisions across industries. The first set of these equations allows for the calculation of gross domestic product from both the income and the expenditure side of the economy. GDP from the income side is calculated by simply adding the total payments to labour, capital, land and other cost tickets, as well as production and indirect tax revenue, while GDP from the expenditure side is calculated by subtracting total imports from the sum of total investment use, government demands, inventories, household demands and exports.

The ORANI model contains three alternative investment rules, which determine the amount of investment that each industry undertakes. The model does not, however, attempt to explain aggregate investment in fixed capital. It only attempts to explain how aggregate investment is allocated across the relevant industries. The first rule relates the creation of new capital stock in each industry to the profitability of each industry. If one assumes that the rate of return on fixed capital is given by $R_j(0) = \frac{P^1_{(g+1,2)j}}{\pi_j}$, and that investors expect that industry j's rate-of-return schedule in one period's time will have the form

$$R_j(1) = R_j(0)(\frac{K_j^1(1)}{K,(0)_j})^{-\beta_j}$$

where β_j is a positive parameter, $K_j(0)$ is the current level of capital stock in industry j and $K_j(1)$ is the level at the end of one period then investment will increase if the price of capital increases. It is further assumed that aggregate total investment is allocated across industries in a manner which equates the expected rates of return which means that there exists some rate of return ω, such that

$$(\frac{K_j(1)}{K_j(0)})^{-\beta_j} R_j(0) = \varpi$$

which allows new capital for industry j to be acquired according to:

$$K_j(1) = K_j(0) + Y_j$$

The second rule allows investment to be determined exogenously according to the aggregate investment of the economy and is an appropriate assumption for those industries which have a close relationship with government spending. The third rule allows investment to move along fixed capital growth rates where investment follows the industry capital stock.

Because the labour market will be analysed within this study, it is important to note that the ORANI model does not allow any theory for the labour supply of an economy. The model allows an option of setting employment exogenously, with market clearing wage rates determined endogenously, or setting the wage rates exogenously and allowing employment to be demand determined, as it is assumed that supply of each skill type is elastic.

The labour market decisions are usually made at an economy-wide level, but could be applied individually and differentially to different industries or types of labour.

7.7 CONCLUSION

This chapter briefly summarised the theoretical equations that underlie the ORANI-G modelling methodology. It has shown that the model consists of equations that represent both the production and demand side of an economy, as well as pricing and market clearing equations. This methodology will be applied to a South African database to test the effect of a revenue neutral tax on coal.