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Some mathematical problems in the dynamics of stochastic second-grade fluids

by

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DECLARATION

I, the undersigned, hereby declare that the thesis submitted herewith for the degree Philosophiae Doctor to the University of Pretoria contains my own, independent work and has not been submitted for any degree at any other university.

Name: Paul André Razafimandimby

Date: May 2010

In Loving Memory of My Father.

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Abstract

In the present work we initiate the investigation of a stochastic system of evolution partial differential equations modelling the turbulent flows of a bidimensional second grade fluid.

Global existence and uniqueness of strong probabilistic solution (but weak in the sense of partial differential equations) are expounded. We also give two results on the long time behavior of the strong probabilistic solution of this stochastic model. Mainly we prove that the strong probabilistic solution of our stochastic model converges exponentially in mean square to the stationary solution of the time-independent second grade fluids equations if the deterministic part of the external force does not depend on time. In the time-dependent case the strong probabilistic solution decays exponentially in mean square. These results are obtained under Lipschitz conditions on the forces entering in the model considered.

We also establish the global existence of weak probabilistic solution when the Lipschitz condition on the forces no longer holds.

Finally, we show that under suitable conditions on the data we can construct a sequence of strong probabilistic solutions of the stochastic second grade fluid that converges to the strong probabilistic solution of the stochastic Navier-Stokes equations when the stress modulus α tends to zero.

All these results are new for the stochastic second-grade fluid and generalize the corresponding results obtained for the deterministic second-grade fluids.

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