APPENDIX A

$$TOC = c_0 + c_1\mu + c_2 \frac{(k+1)\lambda}{2k\mu(\mu-\lambda)} + c_3 \frac{\mu k(k+1)\lambda^2}{2(\mu-\lambda)(k\mu+\lambda)^2}$$

$$TOC = c_0 + c_1\mu + c_2 \frac{(k+1)\lambda(k\mu-\lambda)^2}{2k\mu(\mu-\lambda)(k\mu-\lambda)^2} + c_3 \frac{\mu k(k+1)\lambda^2 k\mu}{2(\mu-\lambda)(k\mu+\lambda)^2 k\mu}$$

$$= \frac{(c_0+c_1\mu)2k\mu(\mu-\lambda)(k\mu-\lambda)^2 + c_2\lambda(k+1)(k\mu-\lambda)^2 + c_3\lambda^2 k^2\mu^2(k+1)}{2k\mu(\mu-\lambda)(k\mu-\lambda)^2}$$

$$= \frac{(c_0+c_1\mu)2k\mu\{-2(\mu-\lambda)(k\mu-\lambda)-(k\mu-\lambda)^2\} + c_2(k+1)\{(k\mu-\lambda)^2 - 2\lambda(k\mu-\lambda) + c_3(k+1)\{2\lambda k^2\mu^2\}}{2k\mu\{-2(\mu-\lambda)(k\mu-\lambda)-(k\mu-\lambda)^2\}}$$

The final analytic expression for TOC is given by

$$TOC = \frac{4k\mu^3(k-1)^2(c_0+c_1\mu) - 2c_2(k+1)(k-1)(k-3)\mu^2 + 2c_3 k^2\mu^3(k+1)}{4k\mu^3(k-1)^2}$$

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